

Risk-Constrained Optimal Energy Management of Multi-Carrier Microgrids Under Uncertainty

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Abstract: The introduction of multi-carrier microgrids (MCMGs) is a paradigm shift in the decentralised energy system, allowing for coordinated dispatch of coupled electrical, thermal, and natural gas subsystems. Despite this, the economic challenges of MCMGs are quite sensitive to multi-dimensional uncertainties such as the intermittent generation of renewable energy, the volatile market electricity prices, and the multi-energy load demands. Traditional deterministic or risk-neutral energy management systems optimise average performance, but are not designed to buffer microgrid operators from operational gaps and tail risks with high impact from low-probability events. In this paper, a new risk-constrained optimal energy management approach for multi-carrier microgrids is presented, which aims to minimise risk by employing a two-stage stochastic mixed-integer linear programming (MILP) approach. The Conditional Value-at-Risk (CVaR) is incorporated as a primary operational objective function to capture and mitigate financial exposure during worst cases. Extensive uncertainty profiles are computed using Latin Hypercube Sampling (LHS), while being downscaled by a fast forward scenario-reduction technique. A modified IEEE 33-bus electrical network and a 6-node district heating system are used for the validation of the proposed model. It is shown that the proposed framework strategically sacrifices a marginal increase in the expected daily operating costs while offering a significant decrease in the tail-risk financial losses through tuning the risk-aversion parameter. The advantages of the proposed framework are illustrated by simulation results, which show that tuning the risk-aversion parameter can lead to a significant reduction in the tail-risk financial losses, while the increase in the expected daily operating costs is a marginal one. Specifically, in a highly risk-averse operational mode, the microgrid operator's worst-case cost exposure is reduced by up to 14.2 % to traditional risk-neutral operation, which demonstrates the strong viability of the proposed framework in the context of volatile energy markets.

Keywords: Multi-Carrier Microgrids, Optimal Energy Management, Conditional Value-at-Risk, Two-Stage Stochastic Programming, Mixed-Integer Linear Programming, Uncertainty Modeling.

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I. INTRODUCTION

A great paradigm shift is underway in modern power systems, where carbon emissions and climate change are driving a move from a single, centralized, high-carbon power system to a distributed, low-carbon power system [1, 2]. However, microgrids have become the backbone of this change by facilitating localized integration, co-optimization, and management of distributed energy resources (DERs), energy storage, and flexible end-user demand [3]. To date, most of the research and operational models for microgrids have been done on a single-carrier, electrical subset basis [4]

almost exclusively. Optimizing individual electrical vectors, however, is not enough to reach modern and ambitious urban decarbonization goals. To achieve these ambitious sustainability objectives, an integrated energy management plan is required that covers both the urban energy use for heating, cooling, gas, and transport services [5]. Considering the drawbacks of optimization for a single vector, Multi-Carrier Microgrids (MCMGs), also referred to as Integrated Community Energy Hubs (ICEUs), are rapidly becoming a preferred topic of energy research in the modern era [6, 7]. MCMGs integrate electrical, thermal, and natural gas networks in a systematic approach into a common operational

paradigm [8, 9]. Advanced coupling technologies, such as Combined Heat and Power (CHP) plants, Electric Heat Pumps (EHPs), gas boilers, and Power-to-Gas (P2G) plants, all contribute to this inter-vector synthesis. The MCMGs provide a unified energy framework that enables the interconnection of hitherto separate energy flows and thus creates a more efficient energy system and an unprecedented operational synergy within the local energy system landscape. This multi-energy synergy ensures an unprecedented flexibility of use, closing the gap between the physical and temporal mismatch of energy generation and demand from different energy carriers [10, 11]. For example, during high generation periods, the surplus of VRE power can be dynamically shifted towards high-efficiency heat pumps, or storage can be stored in Thermal Energy Storage (TES) tanks and Battery Energy Storage Systems (BESS). This cross-carrier flexibility reduces renewable energy curtailment and also meets local thermal and electrical needs at lower economic and environmental costs. As a result, MCMGs represent a flexible approach to improve urban energy infrastructure sustainability, maximize utilization of renewables, and make the grid more resilient.

➤ *The Problem and Its Vulnerability to Multidimensional Volatility*

Operational management of the MCMGs is complicated due to the fact that there is great uncertainty at several structural levels [12]. The operator has to balance the power fluctuations in a typical electrical microgrid [13]. The operator should coordinate three classes of volatility in an MCMG: intermittent renewable generation from local wind turbines and solar photovoltaic (PV) arrays [14] that depend on the weather and are even nonlinear; uncorrelated multi-carrier load profiles (electrical load may peak during industrial activities, while thermal load may peak when ambient temperature drops) [15]; and volatile wholesale market prices of electrical power purchased from or sold into the macro-grid at hourly intervals [16]. In practice, traditional energy management systems (EMS) based on average or single-point forecasting of these variables fail to work [17] [18]. They are, mathematically speaking, risk-neutral and maximize the mathematical expectation of cost [19]. A low-probability, high-impact event occurs (e.g., a sudden decrease in solar generation coupled with a very high wholesale price), a risk-neutral EMS is left exposed [20]. If the microgrid is reliant on emergency power purchase, the over-costs of operating the microgrid become high, strain is placed on its equipment, and it creates instability for the local grid [21], [22].

➤ *Literature Review and Research Gap*

There has been extensive work on the optimization of energy management systems, but few studies consider both multi-carrier coupling and risk minimization together [23]. There is an extensive literature on single-carrier electrical microgrids. Stochastic programming and robust optimization have been widely used to tackle renewable volatility in power networks with battery storage [24], [25]. While mathematically correct, these cannot be readily adopted for the MCMGs as they do not consider the thermodynamic

limitations, time delays, and physical efficiency of coupled heating and gas networks [26].

Multi-carrier modeling has been the subject of recent literature, which addresses this drawback. There have been a number of papers that have suggested MILP architectures for scheduling an integrated energy hub with CHP units and gas boilers [27, 28]. However, most of these multi-carrier scheduling models are deterministic, which require complete knowledge of the load and the prices in the market, and thus they are sensitive to real-time changes [29]. Simple stochastic programming has introduced uncertainty modelling in multi-energy systems in some pioneering works [30] and [31]. However, standard stochastic programming only attempts to minimize expected operating cost. It ignores the tail events, the top 5% or top 1% of the uncertainty events [32]. Although risk-accounting metrics like Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) have already been used for macro-grid portfolio bidding and for financial markets [33], [34], they have not been widely explored for the operational, physical, and multi-period dispatch of coupled multi-carrier microgrids [35]. Most of the multi-energy papers either only focus on electrical metrics or neglect multi-carrier storage dynamics at the local level when setting up joint risk constraints [36]. Hence, it is clear that there is a need for a risk-aware scheduling framework for a coupled MCMG of the electricity and heat sectors that combines the tail-risk measure CVaR and also takes into consideration the specific physical constraints of the two sectors. This study aims to fill this gap.

➤ *Main Contributions*

To overcome such voids, the present paper proposes an optimal energy management paradigm for multi-carrier microgrids under technological and economic uncertainties with consideration of risk constraints. There are four key contributions that are essential.

- Modeling of multiple carriers with joint storage dynamics using physical exertion. The model of MCMG with joint storage dynamics is physically rigorous [6] [7]. The design is an explicit integration of the complementary flexibility of battery energy storage system (BESS) and thermal energy storage (TES) for multi-carrier demand mismatch [10], [11], [14]. Systems with integrated multi-carriers and storage are based on the integrated multi-carriers' model [3, 21].
- A unified risk-constrained objective using CVaR. A two-stage stochastic optimization model includes the CVaR metric directly in the operational dispatch engine [19], [22]. This allows the microgrid owner to adjust the risk-aversion characteristics, allowing the operator to increase the expected daily cost while simultaneously lowering the risk of adverse exposures to rare and significant market and weather events [12, 20]. This risk-constrained approach is particularly sensitive in a multi-carrier system with multi-dimensional volatility [16], [18].
- Sophisticated multidimensional scenario generation and reduction. A coupled LHS method [64] is used to simulate the coupled interaction between the volatile market prices,

- solar irradiation, wind speed, and thermal load, together with a fast-forward scenario-reduction algorithm [65] and [66]. This allows for statistical sampling of extreme tail events without undue computational burden for day-ahead applications [13, 15, 17].
- Multi-carrier benchmark validation (interconnected). The validity and economic benefit of the proposed model are proven by extensive simulations on a highly coupled test system where a modified IEEE 33-bus electrical network was mapped to a 6-node district heating loop [37] – [39].

II. TOPOLOGY AND MATHEMATICAL MODELLING OF MULTI-CARRIER MICROGRIDS

The work presented here uses an advanced MCMG hub which combines electric, natural gas, hydrogen, and thermal energy carriers [37, 39]. It consists of a P2G electrolyser, hydrogen storage tank (HST), hydrogen fuel cell (HFC), and other standard components: CHP unit, gas boiler (GB), BESS, and TES [20, 40, 41]. The storage of hydrogen has been considered as an effective solution besides conventional battery storage [38, 42]. Below are a general overall topology and direction of energy flow of the hub:

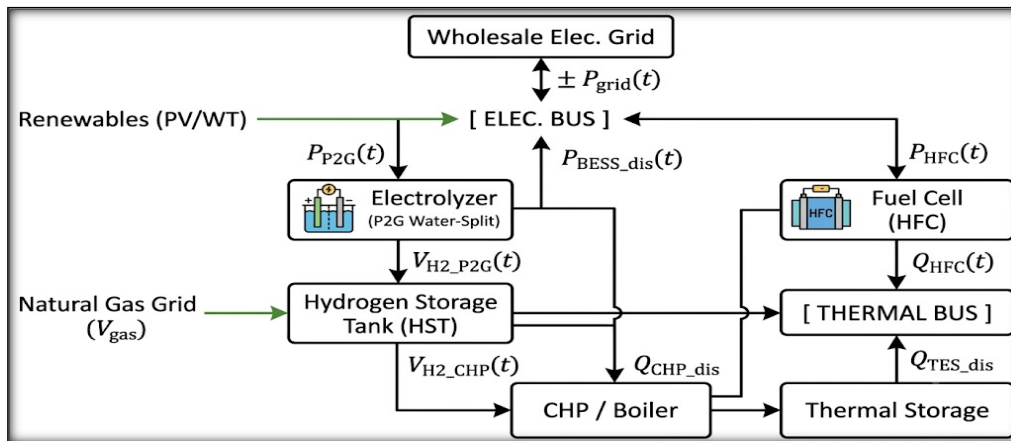


Fig 1 Configuration Layout of the Multi-Carrier Microgrid System.

➤ The Hydrogen Subsystem (P2G, HST, HFC)

• Power-to-Gas (P2G) Electrolyser Model

The P2G unit uses excess renewable or cheap electricity to split water into hydrogen and oxygen [43, 44]. The amount of hydrogen produced is proportional to the input power to the P2G (kW). The higher heating value (HHV) of hydrogen is related to the volumetric flow rate of hydrogen produced, $V_{H2, P2G}$ (m³/h): [45] [46]:

$$V_{H2, P2G} = \frac{\eta^{P2G} \cdot P_{P2G}}{HHV_{H2}} \quad \forall t \quad (1)$$

The efficiency of the electrolyser, denoted as η^{P2G} and the higher heating value of hydrogen, denoted as HHV_{H2} (kWh/m³), appear in the equation. The structural capacity limits give the upper and lower limits for the input electrical power:

$$P_{min}^{P2G} \cdot u_t^{P2G} \leq P_t^{P2G} \leq P_{max}^{P2G} \cdot u_t^{P2G} \quad \forall t \quad (2a)$$

$$u_t^{P2G} \in \{0,1\} \quad \forall t \quad (2b)$$

With $u_t^{P2G} \in \{0,1\}$ a binary commitment variable that equals 1 when the P2G system is operating and 0 otherwise.

• Hydrogen Storage Tank (HST) Dynamics

The HST acts as a buffer to reduce the time mismatch between the production of hydrogen by the P2G system and the consumption of hydrogen by the fuel cell/gas-blending

network [47, 48]. The dynamic modelling of the volume of hydrogen stored at hour t is given by the following [49]:

$$S_t^{HST} = S_t^{HST} + (V_t^{H2, P2G} - \frac{V_t^{H2, HFC}}{\eta^{HST, dis}} - V_t^{H2, blend}) \cdot \Delta t \quad \forall t \quad (3)$$

Where $V_t^{H2, HFC}$ represents the hydrogen volume consumed by the fuel cell, $V_t^{H2, blend}$ the volume injected into the natural gas grid and $\eta^{HST, dis}$ the discharge efficiency [50], [51]. This storage model provides the separation of hydrogen production and consumption to delay hydrogen consumption and maximise multi-energy dispatch under uncertainty [48], [51]. Mechanical capacity is the limiting factor of the tank state:

$$S_{min}^{HST} \leq S_t^{HST} \leq S_{max}^{HST} \quad \forall t \quad (4)$$

Usable to schedule operations over multiple days, the cyclic boundary condition $S_{Tend}^{HST} = S_{T_o}^{HST}$ is imposed

This was accomplished by adopting one of the boundaries, Equation (5): cyclic boundary condition, which maintained an operational scheduling condition for more than one day.

• Hydrogen Fuel Cell (HFC) Co-Generation Model

The HFC is used as a clean cogeneration asset. Active electrical power generated from stored hydrogen P_t^{HFC} (kW) and thermal power generated from stored hydrogen Q_t^{HFC} (kW) [52] – [56]:

$$P_t^{HFC} = \eta_e^{HFC} \cdot LHV_{H_2} \cdot V_t^{H_2,HFC} \quad \forall t \dots \dots \dots (5a)$$

$$Q_t^{HFC} = \eta_e^{HFC} \cdot LHV_{H_2} \cdot V_t^{H_2,HFC} \quad \forall t \dots \dots \dots (5b)$$

The electrical efficiencies of the fuel cell are denoted by η_e^{HFC} and η_{th}^{HFC} is the lower heating value of hydrogen. Consumption and electrical output are limited by:

$$P_{min}^{HFC} \cdot u_t^{HFC} \leq P_t^{HFC} \leq P_{max}^{HFC} \cdot u_t^{HFC} \quad \forall t \dots \dots \dots (6a)$$

$$u_t^{HFC} \in \{0, 1\} \quad \forall t \dots \dots \dots (6b)$$

➤ Combined Heat and Power (CHP) & Gas Boiler (GB) Units

If the fuel cell is not fed hydrogen the major fuel sources are from natural gas and hydrogen [50].

• Combined Heat and Power (CHP) Model

The traditional CHP system is based on gas consumption $V_t^{gas,CHP}$, in addition to the flexibility of the system [57, 58]:

$$P_t^{CHP} = \eta_e^{CHP} \cdot LHV_{gas} \cdot V_t^{gas,CHP} \quad \forall t \dots \dots \dots (7a)$$

$$P_t^{CHP} = \eta_{th}^{CHP} \cdot LHV_{gas} \cdot V_t^{gas,CHP} \quad \forall t \dots \dots \dots (7b)$$

$$P_{min}^{CHP} \leq P_t^{CHP} \leq P_{max}^{CHP} \quad \forall t \dots \dots \dots (7c)$$

• Gas Boiler (GB) Model

In order to cover the heat demand when the combined heat production of the HFC and CHP is not enough, a gas boiler is used as a backup heating system [58, 59]:

$$Q_t^{GB} = \eta^{GB} \cdot LHV_{gas} \cdot V_t^{gas,GB} \quad \forall t \dots \dots \dots (8a)$$

$$0 \leq Q_t^{GB} \leq Q_{max}^{GB} \dots \dots \dots (8b)$$

➤ Hybrid Energy Storage Systems (BESS and TES)

The mitigation of electrical and thermal vectors of the hub is achieved by stationary battery storage and water-based thermal storage tanks [60] respectively.

• Battery Energy Storage System (BESS)

The state of charge (SOC) of electrochemical battery is calculated one by one. A binary variable u_t^{BESS} is introduced to ensure that the battery cannot be charged and discharged at the same time in the same time-step [61]:

$$SOC_t = SOC_{t-1} + (\eta^{ch} \cdot P_t^{BESS,ch} - \frac{P_t^{BESS,dis}}{\eta^{dic}}) \cdot \frac{\Delta t}{E_{max}^{BESS}} \quad \forall t \dots \dots (9a)$$

$$SOC_{min} \leq SOC_t \leq SOC_{max} \quad \forall t \dots \dots \dots (9b)$$

$$0 \leq P_t^{BESS,ch} \leq P_{max}^{ch} \cdot u_t^{BESS} \quad \forall t \dots \dots \dots (9c)$$

$$0 \leq P_t^{BESS,dis} \leq P_{max}^{dis} \cdot (1 - u_t^{BESS}) \quad \forall t \dots \dots \dots (9d)$$

• Thermal Energy Storage (TES) Tank

The sensible heat state of the thermal storage tank H_t^{TES} (kWh) is affected by charging, discharging, and standing heat losses [39] [62] as follows:

$$H_t^{TES} = (1 - \lambda^{TES}) \cdot H_{t-1}^{TES} + (\eta^{TES,ch} \cdot Q_t^{TES,ch} - \frac{Q_t^{TES,dis}}{\eta^{TES,dis}}) \cdot \Delta t \quad \forall t \dots \dots (10a)$$

$$H_{min}^{TES} \leq H_t^{TES} \leq H_{max}^{TES} \quad \forall t \dots \dots \dots (10b)$$

III. MATHEMATICAL DESCRIPTION OF THE UNCERTAINTIES ASSOCIATED WITH THE JOINT UNCERTAINTY

To quantify the operational risk, the MCMG operator has to account for the nonlinear and correlated dynamics of the four stochastic vectors: PV power (P^{PV}), wind turbine power (P^{WT}), multi-carrier demand (electrical demand power P^{load} and thermal demand power Q^{load}), and wholesale electricity market prices (λ^m). Assumptions of statistical independence of these variables can cause structural bias, as solar generation tends to correlate with the daytime peak load, and high price times tend to occur in high load times. We take a two-stage approach to modeling the joint dependency, fig.2: first, we use a framework of copula theory [63] and second, we use Latin Hypercube Sampling [64] to arrive at a scenario that is reduced by a Kantorovich-distance algorithm [65, 66] without losing the individual marginal distributions.

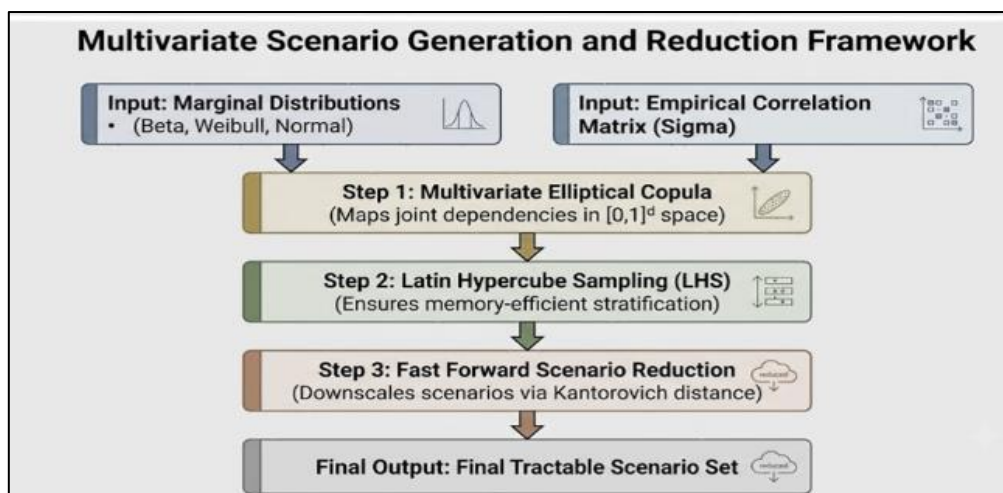


Fig 2 Flowchart of the Multivariate Scenario Generation and Reduction Methodology.

➤ Copula Theory and Joint Dependency Modeling

Any multivariate CDF of a random vector can be uniquely expressed as a copula and a collection of univariate marginal distribution functions by Sklar's theorem. Write the vector of what is to be optimized over the optimization horizon as $X = [X_1, X_2, \dots, X_d]^T$. The joint CDF is

$$F(x_1, x_2, \dots, x_d) = C(F_1(x_1), F_2(x_2), \dots, F_d(x_d)) = C(u_1, u_2, \dots, u_d) \quad (11)$$

Where $F_1(x_1)$ is the continuous marginal CDF of the i -th variable, $u_1 = F_1(x_1)$ are uniformly distributed transformed variables and C is a multi-dimensional copula on the unit hypercube $[0, 1]^d$. This paper uses the Gaussian copula due to its tractability in large-scale multi-period optimization. The Gaussian copula is

$$C_R(u_1, u_2, \dots, u_d) = \Phi_R(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \dots, \Phi^{-1}(u_d)) \quad (12)$$

Where $\Phi^{-1}(\cdot)$ is the inverse CDF of a standard univariate normal and $\Phi_R(\cdot)$ is the CDF of a standard multivariate normal with symmetric positive-definite correlation matrix $R \in \mathbb{R}^{d \times d}$ as $\Phi_R(\cdot)$. The matrix R is an estimated pairwise rank correlation matrix based on the historical data.

➤ Sampling via Latin Hypercube Sampling (LHS)

The sampling method used is Latin Hypercube Sampling (LHS). Profiles of a representative set of scenarios are provided by LHS. Compared to the crude Monte Carlo method, LHS treats the probability space, and the multi-dimensional domain is uniformly filled with less number of samples. There are four steps in the combined copula, LHS procedure.

Interval stratification. For each dimension, the probability space $[0, 1]$ is divided into N_s non-overlapping intervals of equal probability $1/N_s$.

Enforcement of permutation and correlation. The target correlation matrix R of the copula is used to create a standard normal multivariate matrix Z of correlated normal samples.

Standard-normal CDF mapping. Correlated samples are correlated to uniform samples $U \in \mathbb{R}^{N_s \times d}$:

$$u_{s,i} = \Phi(Z_{s,i}) \quad \forall s \in \{1, \dots, N_s\}, \forall i \in \{1, \dots, d\} \dots \dots \dots (13)$$

Inverse marginal mapping. The Uniform samples are then transformed back to the physical domain using the inverse of the marginal CDFs:

$$x_{s,i} = F_i^{-1}(u_{s,i}) \quad \forall s \in \{1, \dots, N_s\}, \forall i \in \{1, \dots, d\} \dots \dots \dots (14)$$

The historical data is used to determine the univariate marginals: solar PV power is modelled as a Beta distribution, the irradiance; wind speed is modelled as a Weibull distribution; loads and prices are modelled as either Gaussian or mixture distributions, that capture seasonal and diurnal structure.

➤ Fast Forward Scenario Reduction

The initial set ($N_s = 1000$) is generally large when using the Copula, LHS [67]. Simply supplying 1000 scenarios directly to the multi-period MIP is impractical [68]. A technique (fast-forward) therefore is used which reduces the set to a small subset, Ω , that preserves a small number of statistical characteristics of the full set, measured by the Kantorovich distance [65, 66]. With initial probabilities $\pi_s = \pi_k = \frac{1}{N_s}$ the geometric distance between two scenario trajectories over the horizon is

$$DT(S, K) = \sum_{t=1}^T \|X_{s,t} - X_{k,t}\|_2 \dots \dots \dots (15)$$

The algorithm repeatedly removes the scenario from the set of scenarios with the lowest information-loss measure. It comes to a reduced set of probabilities

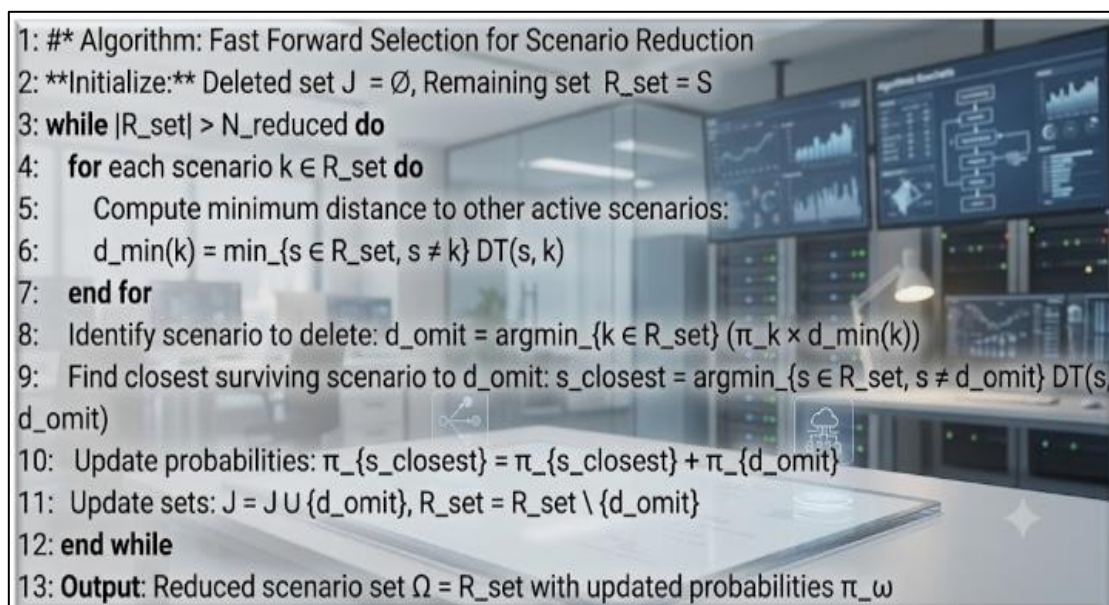


Fig 3 Fast Forward Scenario Reduction Algorithm (Algorithm 1).

$$\pi_{\omega} (\sum_{\omega \in \Omega} \pi_{\omega} = 1) \dots \dots \dots (16)$$

This reduced package is a fundamental input of the problem of risk-constrained optimization.

IV. RISK-CONSTRAINED MATHEMATICAL OPTIMIZATION FORMULATION

To ensure the MCMG operator has protection against high-impact, low-probability (HILP) price collisions and high penetrations of renewables, the energy management system is formulated as a two-stage stochastic mixed-integer linear program (MILP) [69] and [70]. The decision-making process is divided into two phases of operations [71].

First-stage decisions (day-ahead) are made prior to the occurrence of uncertainty (before the market is opened): binary commitment of cogeneration units $\{u_t^{CHP}, u_t^{HFC}, u_t^{P2G}\}$ and the day-ahead electricity trading profile with the wholesale macro-grid, $P_t^{grid,DA}$ [72].

Second-stage decisions (real-time adjustment), once scenario $\omega \in \Omega$ is realized: battery charge/discharge rates,

$$F^{DA} = \sum_{t=1}^T [\lambda_t^{grid,DA} \cdot P_t^{grid,DA} + C^{SU,CHP} \cdot y_t^{CHP} + C^{SU,HFC} \cdot y_t^{HFC} + C^{SU,P}] \dots \dots \dots (17)$$

The contracted day-ahead price for this period is represented by $\lambda_t^{grid,DA}$ and the start-up indicator of asset i is represented by $y_t^i \geq u_t^i - u_{t-1}^i$ where u_t^i is the level of asset i at time t .

• Second-Stage Cost Components

The second-stage operating cost consists of real-time market balancing penalties, fuel costs, and

$$F_{\omega}^{RT} = \sum_{t=1}^T [\lambda_{\omega t}^{grid,DA} \cdot \Delta P_{\omega t}^{grid,DA} + \lambda_t^{gas} \cdot (V_{\omega t}^{gas,CHP} + V_{\omega t}^{gas,GB}) + \sum_{m \in \mathcal{M}} K^m \cdot P_{\omega t}^m] \dots \dots \dots (18)$$

Where $\lambda_{\omega t}^{grid,RT}$ is the real-time balancing price, $\Delta P_{\omega t}^{grid,DA}$ is the deviation from the day-ahead schedule, λ_t^{gas} is the natural-gas price, K^m is the linear cost of wear and tear per unit output of asset class m (BESS, TES, P2G, HFC), and $P_{\omega t}^{Shed}$ is the cost of emergency load shedding at the value of lost load (VOLL).

➤ The CVaR Constraint is Mathematically Represented in a Linear Model.

The loss threshold VaR_{α} is the loss amount above which it can be confidently stated that the loss will not exceed VaR_{α} with probability α (usually $\alpha = 0.95$). The VaR is non-convex and does not measure the size of the losses that exceed the VaR_{α} . CVaR $_{\alpha}$ is similar to the Rockafellar–Uryasev linearization [74].

$$CAaR_{\alpha} = \zeta + \frac{1}{1-\alpha} \sum_{\omega \in \Omega} \pi_{\omega} \cdot \psi_{\omega} \dots \dots \dots (20)$$

Where ζ is a continuous decision variable, VaR_{α} , and ψ_{ω} is a slack variable tracking tail losses under scenario ω . The linear constraints associated are

$(P_{w,t}^{BESS,ch}$ and $P_{w,t}^{BESS,dis}$) (thermal storage adjustments, $Q_{w,t}^{TES,ch}$ and $Q_{w,t}^{TES,dis}$), (hydrogen arrangement), and (real-time power-balancing deviations, $(\Delta P_{w,t}^{grid})$.

➤ Objective Function Formulation

The optimization goal is to minimize the total expected day-ahead operating cost of the MCMG plus a risk-averse term measured by CVaR [71], [73]. The two targets are expected economic performance and exposure to tail-risk anomalies [74], [75]:

$$\min (F^{DA} + \sum_{\omega \in \Omega} \pi_{\omega} \cdot F_{\omega}^{RT} + \beta \cdot CVaR_{\alpha} \dots \dots \dots (17)$$

The first-stage (day-ahead) cost is denoted as F^{DA} and the second-stage (real-time) cost in the scenario ω is denoted as F_{ω}^{RT} and β is the operator's risk-aversion parameter, which is greater than or equal to 0. The larger β , the more risky it is to be exposed to the tails of the set; a β value of 0 represents a fully risk-neutral operator.

• First-Stage Cost Components

The first-stage cost is made up of commitment overhead and day-ahead power cost:

degradation/maintenance functions under scenario ω [69, 70]. Recourse penalizes deviation from the day-ahead power commitment, includes electrochemical degradation of storage, and accounts for fuel use of supplementary generation [76]:

$$\psi_{\omega} \geq F^{DA} + F_{\omega}^{RT} - \zeta \quad \forall \omega \in \Omega \dots \dots \dots (21a)$$

$$\psi_{\omega} \geq 0 \quad \forall \omega \in \Omega \dots \dots \dots (21b)$$

The equations (20) and (21) calculate the value of CVaR $_{\alpha}$ as the expected cost of the worst $(1 - \alpha)$ fraction of operational scenarios, with the value of the function $\psi_{\omega} = \max(0, (F^{DA} + F_{\omega}^{RT} - \zeta))$.

➤ Multi-Carrier Second-Stage Balancing Constraints

The microgrid should meet the physical multi-energy nodal balance with respect to the day-ahead schedule and real-time operation for each scenario ω in Ω [77]. These constraints ensure the real-time balance of the three vectors of power, thermal, and hydrogen within the real-time ramping capability of the core assets [77, 78].

• Active Electrical Power Balance - Power Factor Correction.

In both scenarios, the real-time power injection is equal to the electrical load:

$$P_t^{grid,DA} + \Delta P_{\omega,t}^{grid} + \sum_{m \in M_{gen}} P_{\omega,t}^m + P_{\omega,t}^{shed} = P_{\omega,t}^{load} + \sum_{n \in M_{con}} P_{\omega,t}^n \dots \dots \dots (22)$$

Where $M_{gen} = \{PV, WT, CHP, HFC, BESS, dis\}$ and $M_{con} = \{P2G, BESS, ch\}$. Substation transformer capacity is a limitation for net grid injection:

$$-P_{max}^{grid} \leq P_t^{grid,DA} + \Delta P_{\omega,t}^{grid} \leq P_{max}^{grid} \quad \forall \omega, \forall t \dots \dots \dots (23)$$

• **Thermal Power Balance**

The real-time heating/cooling nodes are written with the thermal balance. No account is taken of hydraulic propagation delay between clusters of buildings:

$$Q_{\omega,t}^{CHP} + Q_{\omega,t}^{HFC} + Q_{\omega,t}^{GB} + Q_{\omega,t}^{TES,dis} = Q_{\omega,t}^{load} + Q_{\omega,t}^{TES,ch} \quad \forall \omega, \forall t \dots \dots \dots (24)$$

• **Fuel and Hydrogen Coupling (FH) is Scenario-Dependent.**

The actual tank level is

$$S_{\omega,t}^{HST} = S_{\omega,t-1}^{HST} + (V_{\omega,t}^{H2,P2G} - \frac{V_{\omega,t}^{H2,HFC}}{\eta_{HST,dis}} - V_{\omega,t}^{H2,blend}) \cdot \Delta t \quad \forall \omega, \forall t \dots \dots \dots (25)$$

Using the same non-anticipative terminal coupling and capacity bound as in Section 2 to ensure asset lifetime protection.

A. Number of Computations Needed, and how Fast the Computation will Converge.

Model of section 4 is a 2-stage stochastic 2D MILP. Multiple energy carriers ($T > 24$ periods) and multiple scenarios ($|\Omega|$) have a significant computational requirement [79,80]. Decision variables are listed, and a convergence criterion is given to find the globally acceptable optimum.

➤ **Brief Overview of Dimensionality and Variable Enumeration:**

The overall complexity is $\mathcal{O}(T |\Omega|)$ [80]. The 1st stage variables are deterministic, while the 2nd stage variables depend on scenarios [69]. Continuous variables scale with $|\Omega|$ [79, 80], while binary first-stage variables do not explode with the scenario tree, Fig.4. This partition enables the use of tight LP relaxations as lower bounds for branch-and-cut engines, and then to solve continuous balancing adjustments for each scenario [68].

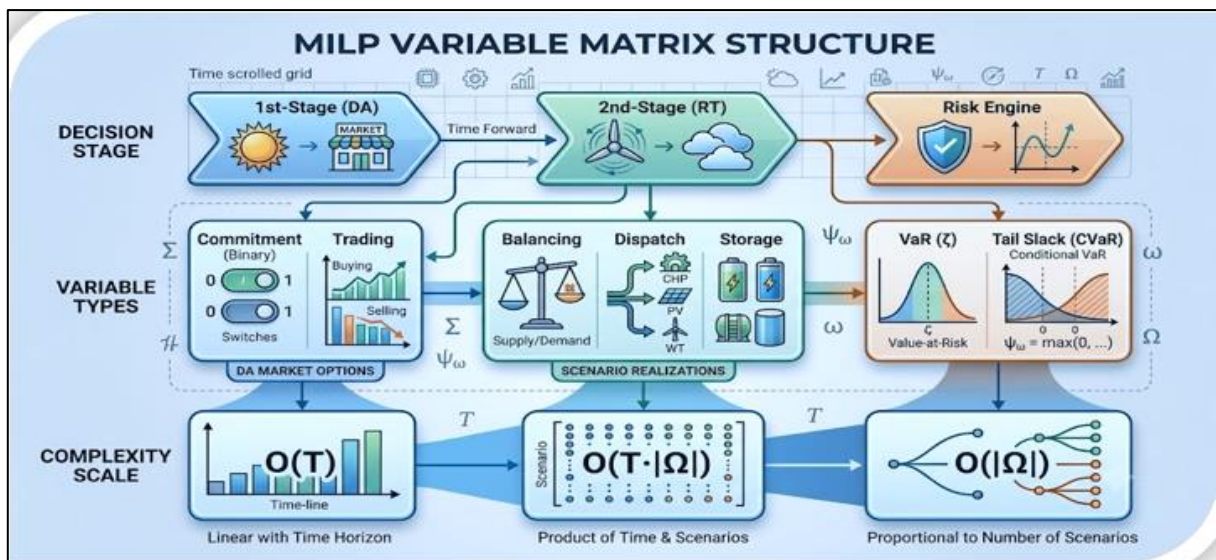


Fig 4 Structural Layout and Complexity Scaling of the MILP Variable Matrix.

• **First-Stage Dimensionality:**

Day-ahead variables are shared across all possible scenarios. They include the binary commitment and start-up variables y_t^m for the core hub assets (CHP, HFC, P2G) and the continuous baseline macro-grid injection schedule ($P_t^{grid,DA}$).

• **Second-Stage Dimensionality:**

For every scenario ω in Ω , an adjustment layer is created and is a real-time deviation from the grid ($\Delta(P_{\omega,t}^{grid})$); a dispatch of CHP, HFC, P2G and GB; the charge/discharge of BESS and TES; and local load shedding $P_{\omega,t}^{shed}$. There are two types of counts: binary and continuous:

$$N_{bin} = 3T \dots \dots \dots (26a)$$

$$N_{cont} = T + |\Omega| \cdot (N_{nodes} \cdot T) + (|\Omega| + 1) \dots \dots \dots (26b)$$

The linearized CVaR variables (ζ, ψ_ω) are represented by the last term $|\Omega| + 1$. If scenario reduction is not implemented as in Section 3, then in a typical validation setup ($T = 24, |\Omega| = 50$) there are 72 binary variables and more than 12 000 continuous variables.

➤ **Complexity Classification:**

The feasible set becomes disjunctive and the problem is NP-hard when committed at the first stage in binary. The maximum number of discrete searches is $\mathcal{O}(2^{3T})$. For $T = 24$ this is 2^{72} discrete states. Once the first stage binaries are

solved, the second stage is a linear program of complexity roughly $\mathcal{O}(N_{cont})^{3.5}$ that can be solved in polynomial time by using an interior point or dual simplex method [68].

An algorithm based on a branch-and-cut method is used [81, 82]. Large subsets of the discrete search tree are pruned off with linear programming relaxations, algebraic cutting planes and modern heuristics [80, 82] without actually enumerating them.

➤ Optimization Convergence Criteria

The relative mixed-integer programming (MIP) gap (Fig.5) is bounded to obtain schedules which are globally acceptable, not just feasible [81], [82]. Let Z_{UB} be the best upper bound (best possible mixed-integer objective) and Z_{LB} be the best lower bound (relaxed LP objective at the active nodes) [83]. Convergence requires.

$$MIP\ Gap = \frac{|Z_{UB} - Z_{LB}|}{10^{-10} + |Z_{UB}|} \leq \epsilon \dots \dots \dots (27)$$

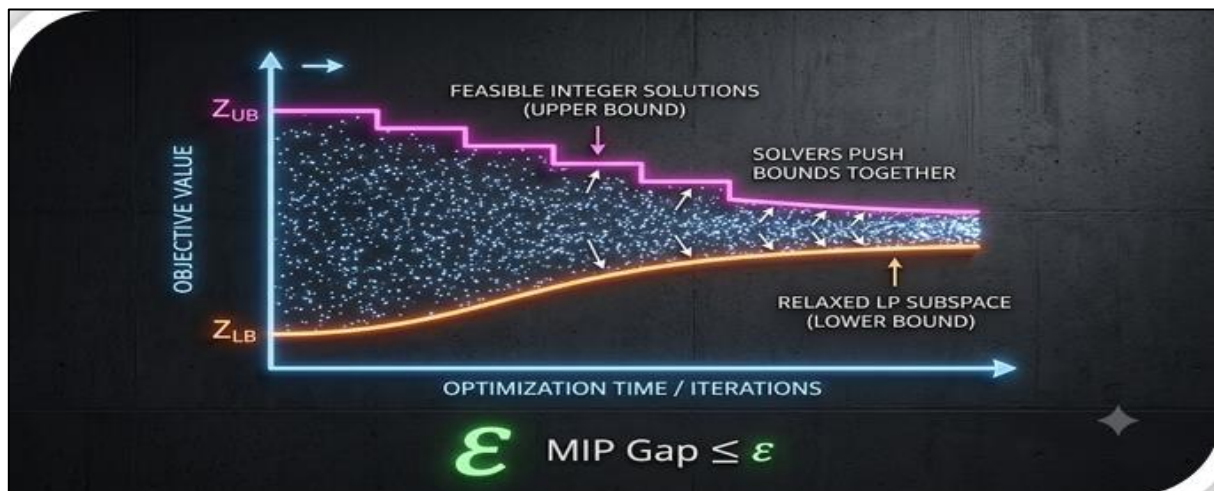


Fig 5 Optimization Bound Convergence and MIP Gap Criteria

Simulations use $\epsilon = 0.01\%$ (10-4). The model is coded in Python (Pyomo) and is solved using the Gurobi Optimizer v11.0 on an Intel Core i7 12th generation CPU at 3.8 GHz with 32 GB RAM. Average convergence time is less than 45 seconds, which is well within day-ahead scheduling limits.

V. CASE STUDY AND SIMULATION RESULTS

Performance, computational scalability, and risk mitigation are tested in a highly coupled multi-carrier test system. The set-up consists of a modified IEEE 33-bus electrical distribution network and a 6-node district heating network, which represents a municipal energy system. The model is solved using Python 3.11 (Pyomo) on a standard workstation (Intel Core i7, 32 GB RAM). The horizon is $T = 24$ h with $\Delta t = 1$ h [50], [84], [85].

➤ Benchmark System Configuration and Technical Parameters

The test microgrid is an industrial–commercial facility. Structural assets are divided into primary conversion vectors (hydrogen infrastructure) and into multi-carrier storage buffers [86]. The stochastic input is based on 50 scenarios created using the copula–LHS procedure described in Section 3. The worst 5% of economic outcomes ($\alpha = 0.95$) is the CVaR calculation.

Table I provides a tabulation of the operational and economic parameters of the P2G unit, HST, HFC, CHP, GB, BESS, and TES, such as efficiencies, capacity limits, start-up costs, and degradation coefficients.

Table 1 Multi-Carrier Microgrid Operational and Economic Parameters

System Asset Component	Parameter Description	Operational Value	Cost / Efficiency Coefficients
P2G Electrolyser	Power Capacity (PmaxP2G)	500 kW	$\eta_{P2G}=72\%$, $K_{P2G}=0.02/\text{kWh}$
Hydrogen Fuel Cell (HFC)	Elec. Capacity (PmaxHFC)	300 kW	$\eta_{eHFC}=45\%$, $\eta_{thHFC}=40\%$
Hydrogen Storage Tank (HST)	Capacity (SmaxHST)	1500 m ³	η_{HST} , dis=98%, $S_{minHST}=50$ m ³
CHP Generator	Elec. Capacity (PmaxCHP)	400 kW	$\eta_{eCHP}=38\%$, $\eta_{thCHP}=42\%$
Gas Boiler (GB)	Thermal Capacity (QmaxGB)	600 kW	$\eta_{GB}=88\%$
Battery Array (BESS)	Energy Capacity (EmaxBESS)	800 kWh	$SoC \in [0.1, 0.9]$, $\eta_{ch}=\eta_{dis}=95\%$
Thermal Storage (TES)	Energy Capacity (HmaxTES)	1000 kWh	$\lambda_{TES}=1.5\%/h$, η_{TES} , ch/dis=92%
Macro-Grid Connection	Transfer Limit (Pmaxgrid)	1200 kW	Value of Lost Load (VOLL) = 5.00 \$/kWh

The simulation inputs are stochastic, and 50 reduced scenarios were generated using the Copula-LHS technique described in Section 3. The target economic anomaly for the

Conditional Value-at-Risk calculation is the worst 5% of economic anomalies ($\alpha=0.95$).

➤ *The Results of the Pareto-Front Sensitivity Analysis - Expected Cost vs. CVaR:*

The risk-aversion weight β is the most important parameter. The Pareto frontier (PF) between expected

operating cost and tail-risk exposure CVaR (0.95), Fig.6, is not linear and is found by sweeping β from 0 (risk-neutral) to 5.0 (highly risk-averse) [73, 87].

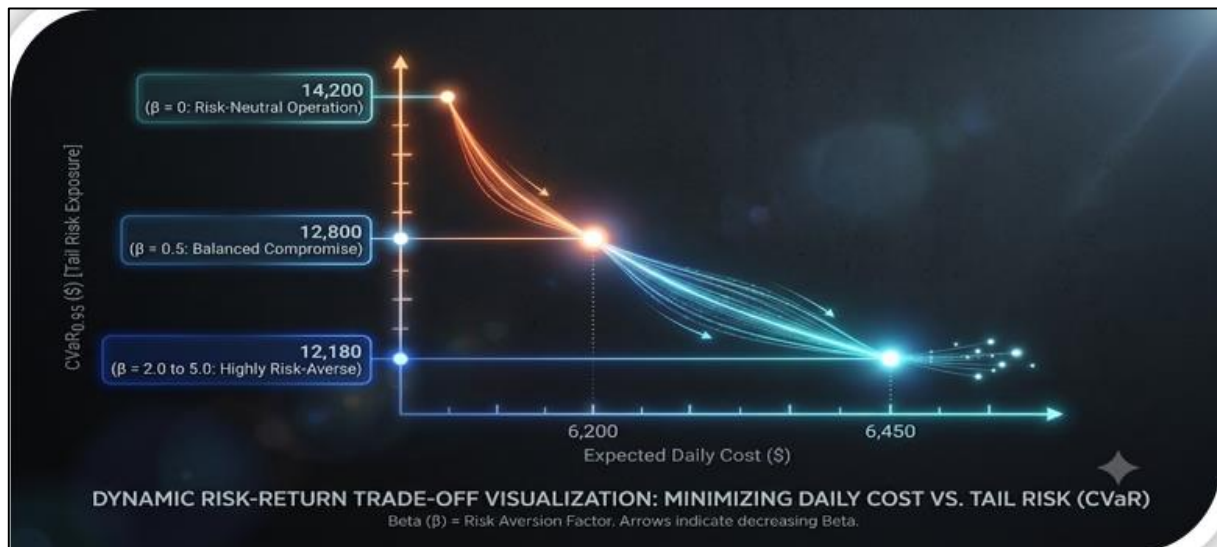


Fig 6 Pareto Frontier of Operational Cost Versus CVaR Tail Risk Exposure.

The quantitative metrics shifting across this Pareto frontier are detailed in Table 2:

Table 2 Multi-Carrier Microgrid Optimization Metrics Under Varying Risk Profiles

Risk Profile Parameter (β)	Expected Daily Cost (ZExp)	Tail-Risk Cost (CVaR0.95)	Worst-Case Scenario Outage Cost	Real-Time Load Shedding (Pshed)
0.0 (Risk-Neutral)	\$6,200.00	\$14,200.00	\$15,850.00	412.5 kWh
0.2	\$6,245.50	\$13,610.00	\$14,900.00	290.0 kWh
0.5	\$6,310.20	\$12,800.00	\$13,420.00	105.0 kWh
1.0	\$6,412.00	\$12,350.00	\$12,600.00	12.4 kWh
2.0 (Risk-Averse)	\$6,452.10	\$12,180.00	\$12,290.00	0.0 kWh
5.0	\$6,455.00	\$12,180.00	\$12,285.00	0.0 kWh

The above Table 2 provides the optimization metrics for the multi-Carrier Microgrid for different risk profiles.

➤ *A Cost/Benefit Analysis:*

If β is equal to zero, the optimiser will try to minimize the average cost of operating the machine every day. In this instance, the day-ahead schedule would be to purchase low-priced electricity from the grid and would also have low fuel holding. The worst-case scenario, however, if the tail-end scenario occurs (sudden wind reduction along with an extreme real-time electricity price increase), could cost \$14,200.00 and will require 412.5kWh of electrical load to be shed. The operator can, however, accept the slight hike in the expected daily costs (\$6200.00 vs. \$6452.10) to take on the risk-averse profile ($\beta=2.0$). The downside of the first-stage commitment sequence change is that the physical buffers of the system are put in place in exchange for this little bump. The disadvantage of the first-stage commitment sequence change is that the physical buffers of the system are changed for this little bump. Due to this, the tail-risk exposure (CVaR 0.95) decreases by 14.23% (\$2,020.00 saved in extreme

cases). The worst-case scenario cost has been reduced by 22.46%. It indicates that the risk-averse approach can yield a little less economic cost but a lot more physical and financial strength: no real-time load shedding (0.0 kWh).

➤ *Dyeing the Intraday Dynamics of Hydrogen Ecosystems*

The high impact tail-risk scenario (high confidence/low frequency) is explored in this section, and the hourly operating profile of the hydrogen subsystem (Fig.7) is examined. The operator uses the P2G to generate hydrogen when electricity is cheap, or there is a surplus from renewable sources, and vice versa by using the HFC to convert stored hydrogen when electricity is expensive, or there is a lack of renewable power [88],[89]. When electricity is cheaper and/or there is a surplus from renewable sources, the operator uses the P2G to produce hydrogen, and when electricity is more expensive and/or there is less renewable power, the operator uses the HFC to convert stored hydrogen [90]. The multi-energy storage network is physically protected against tail-risk volatility as shown in intra-day tracking [91].

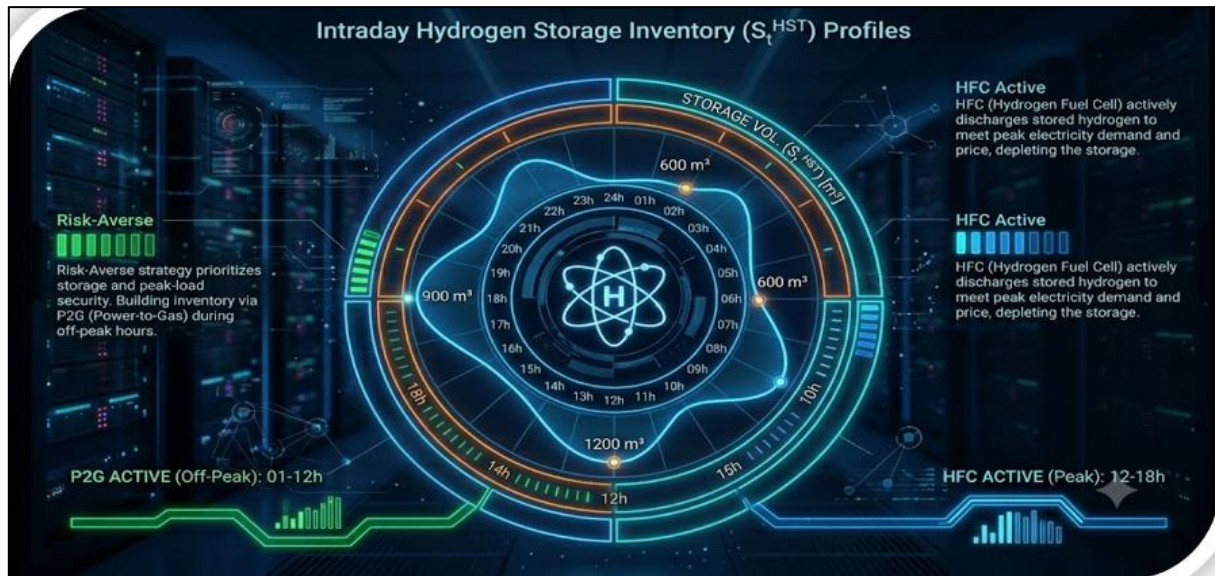


Fig 7 Intraday Hydrogen Storage Inventory (S_t^{HST}) Profile and Operational Schedule.

A time plane that rolls around, rather than a conventional x-axis, is the dual 24-hour ring of the Timeline. Storage Inventory (m^3): these are shown in bands of varying widths and lengths, which expand and contract. The low storage levels represent the beginning and the end of the cycle, while the high storage levels are at the risk-averse reserve level. Easily understood operational phases: The various operational phases are easily recognized in colour, and it is clear when P2G (active storage) is in effect, and when Risk-Averse Reserve Building and HFC (depletion during peak load) are in effect.

- *Between a Risk-Neutral and a Risk-Averse Scheduling Profile:*

The difference between the risk-neutral ($\beta = 0$) and risk-averse ($\beta = 2.0$) setup is strategic [71]. Using CVaR in the first-stage scheduling, using tail penalties in real time affects the way the hydrogen network is used [71, 87].

Off-peak hydrogen production, if the day-ahead price is not high enough to reach the economic break-even price in the early morning (between 01:00 and 07:00), the P2G electrolyser is not activated or is kept at a minimum. The risk-averse model, on the other hand, operates the electrolyzers at 500 kW from 02:00 to 06:00, when both off-peak electricity supply and wind generation are available, and electricity prices are at their lowest. Hydrogen inventory rises from a base of 50 m^3 to 1 150 m^3 by 07:00.

Peak tail-risk mitigation. The solar PV collapses during the evening peak (17:00 - 20:00), and the real-time electricity price can be \$1.85/kWh. The risk-averse configuration includes the hydrogen buffer and fuel cell to address the local shortage and to not be subject to the spot market. The risk-neutral configuration is still vulnerable to significant economic damage [87] (Table 3).

Table 3 Intraday Hydrogen Ecosystem Dispatch Under Peak Tail-Risk (Hour 18:00)

Operational Metric	Risk-Neutral Operation ($\beta=0$)	Risk-Averse Operation ($\beta=2.0$)	Operational Impact Description
P2G Power Consumption	0.0 kW	0.0 kW	The electrolyser is completely idled to prevent high-priced power draw.
HST Inventory Level	50.0 m3	1,020.0 m3	The risk-averse model holds a massive physical energy reserve.
HFC Electrical Output	0.0 kW	300.0 kW	The fuel cell runs at full capacity to displace expensive grid imports.
HFC Thermal Output	0.0 kW	266.6 kW	Cogenerated heat offsets gas boiler fuel consumption.
Net Grid Power Import	850.0 kW	550.0 kW	The risk-averse model reduces macro-grid reliance by 300 kW.
Hourly Operating Cost	\$1,572.50	\$1,017.50	Risk-averse framework saves \$555.00 in a single hour.

- *Physical Multi-Carrier Synergies:*

The HFC is a cogeneration asset, and therefore the total electrical output of 300kWe is provided with useful heat [92]. The microgrid can therefore opt not to buy 300 kW at the peak

real-time price and feed the co-generated heat into the district-heating system. The duty of the gas-boiler (duty after using 33% of duty) reduces from 510 kWth to 243.33 kWth. A hydrogen subsystem is thus able to protect the microgrid

from electricity-market shocks and to enhance the efficiency of the thermal network during shock windows [50], [92], [93].

VI. CONCLUSION AND FUTURE WORK

➤ Conclusion

The solution proposed in this paper has been detailed and is optimal for the case of a sophisticated MCMG hub with an integration of electricity, natural gas, thermal, and hydrogen vectors, while keeping the risks within defined limits. Gaussian copulas are used to model the nonlinear spatio-temporal correlation between market prices, multi-energy loads, and solar and wind profiles; Latin Hypercube Sampling and fast-forward scenario reduction are used. First, CVaR is linearized, and then it is incorporated into a two-stage stochastic MILP, with the commitment stage made prior to the disclosure of uncertainty and the recourse stage made in real time. The framework was tested on a hybrid test bed with the IEEE 33-bus system and a 6-node district heating system.

• The Following are the Key Findings.

- ✓ A higher value of the risk-aversion parameter, $\beta = 2.0$ (compared with $\beta = 0$), decreases CVaR - {0.95} by 14.23%, protecting against high price increases at the same time as a drop in renewables.

- ✓ This safety calls for merely a 4.06% premium on the anticipated daily cost. The day-ahead penalty prevents the occurrence of real-time load shedding (0.0 kWh vs. 412.5 kWh for risk-neutral).
- ✓ Cross-vector hydrogen synergies are key. During off-peak hours, the P2G electrolyser uses surplus electricity, which is inexpensive, to produce stored hydrogen. During the peak period, the HFC is producing 300 kW and at the same time supplying co-generated heat to the district-heating network, which reduces the amount of macro-grid imports and additional fuel used by the supplementary boiler.

The resulting model is mathematically sound, and it is computer efficient. It demonstrates the feasibility of a multi-carrier hydrogen concept that is stable and economically viable when optimized in a risk-averse manner in volatile energy markets.

➤ Future Work: The P2P (Peer-to-Peer) Trading Networks Extension

In this study, a single MCMG hub was considered along with its relationship with the wholesale markets. Future deployments will consist of multiple multi-carrier microgrids deployed in geographically and electrically adjacent locations. One potential extension is a decentralized peer-to-peer (P2P) energy trading network of hydrogen-coupled interconnected microgrids.



Fig 8 Inter-Hub Peer-to-Peer Electricity and Hydrogen Trading Framework.

The current 2-stage optimization can be reformulated as a multi-agent system, where neighbouring microgrids can share the surplus electricity and compressed hydrogen via local pipelines instead of the macro-grid. The suggested research directions are: (a) asymmetric multi-carrier complementarities, such as having a P2P power grid dominated by PV and another that is dominated by TpH; (b) distributed optimization algorithms that optimize local multi-carrier markets while keeping the data private and local asset parameters confidential; and (c) cooperative and non-cooperative game-theoretic pricing (Nash or Stackelberg) for

P2P power and hydrogen exchanges that incentivize local resiliency and regional uncertainty mitigation.

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➤ Conflict of Interest

No conflict of interest.

➤ *Authors' Contributions*

- Author 1: Conceptualization, Methodology, Writing (Original Draft), Writing (Review & Editing) and Supervision

- Author 2: Formal Analysis, Data Curation and Project Administration.
- Author 3: Software and Validation.
- Author 4: Conceptualization and Methodology.

➤ *List of Abbreviations / Symbols*

Category	Symbol / Term	Description / Definition
Abbreviations	BESS	Battery Energy Storage System
	CDF	Cumulative Distribution Function
	CHP	Combined Heat and Power
	CVaR / CAaR	Conditional Value-at-Risk (<i>CAaR appears as a typographical variant in Eq. 20</i>)
	DA	Day-Ahead
	EHP	Electric Heat Pump
	EMS	Energy Management System
	GB	Gas Boiler
	HFC	Hydrogen Fuel Cell
	HILP	High-Impact Low-Probability
	HHV	Higher Heating Value
	HST	Hydrogen Storage Tank
	IEEE	Institute of Electrical and Electronics Engineers
	LHS	Latin Hypercube Sampling
	LHV	Lower Heating Value
	MCMG / MCMGs	Multi-Carrier Microgrid / Multi-Carrier Microgrids
	MILP	Mixed-Integer Linear Program / Mixed-Integer Linear Programming
	MIP	Mixed-Integer Programming
	P2G	Power-to-Gas
	PF	Pareto Frontier
	PV	Photovoltaic (Solar)
	RAM	Random Access Memory
	RT	Real-Time
	SOC	State of Charge
	TES	Thermal Energy Storage
	VaR	Value-at-Risk
	VOLL	Value of Lost Load
	WT	Wind Turbine
Indices, Sets & Dimensions	t	Time index/hour ($t = 1, 2, \dots, T$)
	T	Optimization time horizon (e.g., 24 hours)
	s, k	Scenario sampling indices
	N_s	Total number of initial sampled scenarios (e.g., 1000)
	ω	Individual scenario index in the reduced scenario set
	Ω	Set of reduced scenarios
	$ \Omega $	Cardinality (total count) of the reduced scenario set
	d	Dimension/number of stochastic variables (e.g., $d = 4$)
	i	Index of a stochastic variable ($i \in \{1, \dots, d\}$)
	m	Asset class index / Generation vector set member
	M_{gen}	Set of generating assets {PV, WT, CHP, HFC, BESS, dis}
	M_{con}	Set of consuming/charging assets {P2G, BESS, ch}
	N_{bin}	Number of binary variables
	N_{cont}	Number of continuous variables
	N_{nodes}	Number of physical network nodes
Operators & Constants	Δt	Time step interval (1 hour)
	$\forall$	Universal quantifier ("for all")
	$\in$	Set membership ("element of")
	$\mathbb{R}^{d \times d}$	Set of d times d real-valued matrices
	$\mathcal{O}$	Big-O notation for computational complexity

	Σ	Summation operator
	$\ \cdot\ _2$	Euclidean norm / L ₂ -norm
	ε	Convergence tolerance threshold / MIP gap parameter (e.g., \$0.01\%)
	Z_{UB}, Z_{LB}	Best Upper Bound and Best Lower Bound in MIP formulation
Parameters, Efficiencies & Coefficients	α	Confidence level for VaR/CVaR calculations (e.g., 0.95)
	β	Risk-aversion weighting factor
	$\pi_s \pi_k \pi_\omega$	Scenario probability weights
	η^{P2G}	Conversion efficiency of the P2G electrolyser
	$\eta^{HST, dis}$	Discharge efficiency of the Hydrogen Storage Tank
	$\eta_e^{HFC}, \eta_{th}^{HFC}$	Electrical and thermal efficiencies of the Hydrogen Fuel Cell
	$\eta_e^{HFP}, \eta_{th}^{HFP}$	Electrical and thermal efficiencies of the CHP unit
	η_{GB}	Thermal efficiency of the Gas Boiler
	$\eta^{ch}, \eta^{dic} / \eta^{dis}$	Charging and discharging efficiencies of BESS
	$\eta^{TES, ch}, \eta^{TES, DIS}$	Charging and discharging efficiencies of the TES tank
	λ^{TES}	Standing thermal loss coefficient of the TES tank
	HHV_{H_2}	Higher heating value of hydrogen
	LHV_{H_2}	Lower heating value of hydrogen
	LHV_{gas}	Lower heating value of natural gas
	$\lambda_t^{grid, DA}$	Day-ahead grid electricity market price at time t
	$\lambda_{\omega t}^{grid, DA}$	Real-time grid balancing electricity price under scenario ω at time t
	λ_t^{gas}	Natural gas price at time t
	$C^{SU, i}, (C_{SU, CHP}, C_{SU, HFC}, C_{SU, P})$	Start-up cost for asset i (CHP, HFC, P2G)
	K_m	Degradation/maintenance cost per unit output for asset class m
Variables & Functions	$V_t^{H_2, P2G} / V_{\omega t}^{H_2, P2G}$	Volumetric flow rate / volume of hydrogen produced by P2G
	$V_t^{H_2, HFC} / V_{\omega t}^{H_2, HFC}$	Volumetric flow rate / volume of hydrogen consumed by HFC
	$V_t^{H_2, blend} / V_{\omega t}^{H_2, blend}$	Volume of hydrogen injected/blended into the natural gas grid
	$V_t^{gas, CHP} / V_{\omega t}^{gas, CHP}$	Volumetric natural gas consumption by CHP
	$V_t^{gas, GB} / V_{\omega t}^{gas, GB}$	Volumetric natural gas consumption by Gas Boiler
	P_t^{P2G}	Electrical power input to P2G electrolyser
	$P_{min}^{P2G}, P_{max}^{P2G}$	Minimum and maximum operating power limits of the P2G electrolyser
	u_t^{P2G}	Binary operational status (commitment) variable for P2G (0 or 1)
	$S_t^{HST}, S_{\omega t}^{HST}$	Volume/inventory of hydrogen stored in the HST at hour t
	$S_{min}^{HST}, S_{max}^{HST}$	Minimum and maximum storage capacity bounds of HST
	$S_{Tend}^{HST}, S_{t0}^{HST}$	Terminal and initial state bounds for cyclic HST storage boundary
	$P_t^{HFC} / P_{\omega t}^{HFC}$	Active electrical power output generated by HFC
	$Q_t^{HFC} / Q_{\omega t}^{HFC}$	Thermal power output generated by HFC
	$P_{min}^{HFC}, P_{max}^{HFC}$	Minimum and maximum electrical capacity limits of HFC
	u_t^{HFC}	Binary operational status variable for HFC (0 or 1)
	$P_t^{CHP} / P_{\omega t}^{CHP}$	Electrical power output generated by CHP
	$Q_t^{CHP} / Q_{\omega t}^{CHP}$	Thermal power output generated by CHP
	$P_{min}^{CHP} / P_{max}^{CHP}$	Minimum and maximum electrical capacity limits of CHP
	u_t^{CHP}	Binary operational status variable for CHP (\$0\$ or \$1\$)
	$Q_t^{GB} / Q_{\omega t}^{GB}$	Thermal output generated by Gas Boiler
	Q_{max}^{GB}	Maximum thermal capacity limit of Gas Boiler
	SOC_t	State of charge of BESS at hour t
	SOC_{min}, SOC_{max}	Minimum and maximum SOC limits for BESS
	$P_t^{BESS, ch} / P_{\omega t}^{BESS, ch}$	Charging power rate of BESS

	$P_t^{BESS,dis} / P_{\omega,t}^{BESS,dis}$	Discharging power rate of BESS
	$P_{min}^{ch} / P_{max}^{dis}$	Maximum charge and discharge rate limits of BESS
	u_t^{BESS}	Binary charging/discharging state indicator for BESS
	H_t^{TES}	Sensible heat energy state stored in TES tank at hour t
	$H_{min}^{TES}, H_{max}^{TES}$	Minimum and maximum thermal storage capacity limits of TES
	$Q_t^{TES,ch}, Q_{\omega,t}^{TES,ch}$	Heat charging rate into TES
	$Q_t^{TES,dis}, Q_{\omega,t}^{TES,dis}$	Heat discharging rate from TES
	$P^{PV} / P_{\omega,t}^{PV}$	Solar PV generation power
	$P^{WT} / P_{\omega,t}^{WT}$	Wind turbine generation power
	$p^{load} / P_{\omega,t}^{load}$	Electrical load demand
	$Q^{load} / Q_{\omega,t}^{load}$	Thermal load demand
	λ^m	Wholesale electricity market price vector
	X	Vector of stochastic variables ($X = [X_1, X_2, \dots, X_d]^T$)
	$F(x_1, \dots, x_d)$	Joint Cumulative Distribution Function
	$F_i(x_i)$	Continuous marginal CDF of the i -th variable
	$u_i / u_{s,i}$	Uniformly transformed random variables
	C / C_R	Copula function / Gaussian copula function
	$\Phi(\cdot), \Phi^{-1}(\cdot)$	Standard univariate normal CDF and its inverse
	$\Phi_R(\cdot)$	Standard multivariate normal CDF with correlation matrix R
	R	Pairwise rank correlation matrix ($R \in \mathbb{R}^{d \times d}$)
	$Z, Z_{s,i}$	Matrix/sample of correlated standard normal variables
	$X_{s,i}$	Physical domain transformed sample
	$DT(S, K)$	Geometric distance measure between scenario trajectories S and K
	$P_t^{grid,DA}$	Day-ahead contracted power injection/purchase schedule with macro-grid
	$\Delta P_{\omega,t}^{grid} / \Delta P_{\omega,t}^{grid}$	Real-time power balance deviation from day-ahead schedule
	$\Delta P_{\omega,t}^{shed} / \Delta P_{\omega,t}^{shed}$	Amount of emergency electrical load shed
	F^{DA}	Total first-stage (day-ahead) operating cost
	F_{ω}^{RT}	Second-stage (real-time) operating cost under scenario ω
	$y_t^i / y_t^{CHP}, y_t^{HFC}$	Binary start-up indicator variable for asset i
	ζ	Decision variable representing Value-at-Risk (VaR)
	$\varphi\omega$	Linear slack variable capturing tail losses exceeding VaR under scenario ω
	P_{max}^{grid}	Substation transformer power capacity limit

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