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Gas-Light Interaction Intensity Limits: A Framework for Quantum Internet Architecture

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Abstract: The realization of a scalable Quantum internet requires coherent quantum-state transduction between superconducting microwave qubits operating at milli-kelvin temperatures and optical telecommunication band channels. Despite substantial advances in electro-optic and optomechanical conversion devices, the thermodynamic constraints governing coherence preservation during this translation remain insufficiently understood. In this work, a predictive thermodynamic design framework is established based on the scale-invariant Concentration–Coupling Law, Γ. Here, Γ is explicitly defined as the Gas-Light Interaction Intensity, a dimensionless order parameter where both the optical field congestion and the ambient gas pressure scale are directly related. This framework eliminates abstract mathematical ambiguity by showing that any escalation in photon occupancy or residual molecular pressure directly drives a proportional increase in the interaction magnitude and physical phase noise. The framework is applied to an ultra-high lithium-niobate (LiNbO3) standing-wave nanocavity (V_{cav} = 5 x 10^8 nm^3) operating across a nine-order of magnitude power spectrum (1 pW to 1 mW). A coupled formulation that integrates the Pauli–Fierz Hamiltonian, Quantum-Electrodynamic Density Functional Theory (QED-DFT), and Lorentz–Lorenz dielectric-density relations was used to derive the analytical boundary for the Critical Quantum Preservation Threshold (Γ_q = 3.81 x 10^{-7}). This threshold represents the absolute physical ceiling and maximum permissible interaction intensity before stochastic environmental collisions permanently disrupt phase coherence over a mandatory $\tau_{transduct}$ 10 < microseconds transduction window .Comparative multi-regime evaluation reveals a strict linear trade-off driven by this direct proportionality. Scaling the optical drive from a dilute Pico-Regime (1 pW) to a congested, saturated Milli-Regime (1 mW) boosts photon crowding, which forces a corresponding nine-order-of-magnitude reduction in background gas parameters to restrict $\Gamma < \Gamma_q$. Consequently, maintaining the interaction intensity within the safe quantum domain plunges the critical pressure baseline from 1.48 x 10^6 Pa down to an ultra-strict 1.48 x 10^{-3} Pa. These results provide an intuitive, physics-driven baseline indicating that gas-light interaction dynamics function as a dynamic primary variable in distributed Quantum Internet routing and hardware architectures.

Keywords: *Quantum Internet, Gas-Light Interaction, Phase Coherence, Lithium-Niobate, Resonant Volume.*

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I. INTRODUCTION

The development of a global Quantum Internet is widely recognized as a key objective of modern quantum science and engineering. Quantum communication networks require the faithful transfer of quantum information between stationary quantum processors and photonic transmission channels. While superconducting qubits currently represent one of the most mature platforms for quantum computation, they operate in the microwave frequency regime and require cryogenic environments. Optical telecommunications infrastructure, in contrast, operates in the near-infrared domain and provides long-distance, low-loss transmission at room temperature. Bridging these two domains remains one of the principal technological challenges in quantum networking [1-3].

Current approaches rely primarily on electro-optic, optomechanical, and piezoelectric transducers engineered to maximize conversion efficiency and phase matching. However, most existing analyses assume that the background environment behaves as an inert vacuum reservoir. Under realistic operating conditions, intense optical pump fields coexist with residual molecular gases inside confined resonant volumes, producing previously neglected thermodynamic interactions capable of degrading phase coherence [4, 5].

The present study investigates these interactions through a universal concentration-coupling framework derived from the fundamental resonant-volume invariant $\underline{V_{res}}$ = $\underline{286.398\ nm^3}$, derivation of this fundamental resonant volume is presented in detail in [6] and part of it is shown in the appendix. The objective of the study is to establish a quantitative relationship between photon occupancy,

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background molecular density, and quantum-state preservation during microwave-to-optical conversion.

The principal contribution of this work is the derivation of a thermodynamic design criterion linking photon concentration and molecular gas pressure through a single scale-invariant order parameter, Γ, formulated with direct positive scaling in the numerator. This framework enables the prediction of exact coherence-preservation limits by treating environmental collisions and field crowding as cooperative mechanisms that amplify phase noise in quantum transducers.

II. METHODOLOGY

➢ *Hybrid-Domain Coordinates and Nanocavity Characterization*

- *Spatial Confinement and Resonant Occupancy*

To investigate the thermodynamic limits of microwave-to-optical quantum transduction, a lithium-niobate (LiNbO3) standing-wave photonic nanocavity is adopted as a representative hybrid platform. Lithium niobate is widely regarded as one of the most promising materials for integrated electro-optic conversion due to its strong nonlinear response, low optical loss and compatibility with photonic integration technologies.

The transducer operates at the telecommunications wavelength $\lambda = 1550$ nm and is characterized by an ultra-high optical quality factor. These parameters place the device within a unique operating regime where quantum-scale confinement and photonic-scale transport coexist within the same physical structure.

Using the resonance-occupancy framework established in this work, the stored photon population is evaluated and mapped onto the universal resonance scale. The resulting coordinate identifies the nanocavity as occupying a region immediately above the single-photon quantum limit, thereby establishing it as a quantum-photonic hybrid system rather than a purely quantum or purely photonic device The Resonant-volume framework is explained in details hereby:

The proposed geometric framework assumes that the effective orbital radius of the hydrogen ground state is not identified with the Bohr radius. Instead, the model introduces a geometric reference radius defined as: $r_1 = 1$ nm.

✓ *Energy-Area postulate*

A principal postulate is that energy E_n is proportional to certain effective resonant area A_{eff} scanned by electron or photon:

$$E_n = k \, . \, A_{eff} \quad (1)$$

Where k (proportionality constant) is a geometric energy factor. For the ground state,

$$E_1 = k \, . \, pi. \, r_1^2 \quad (2)$$

Thus,

$$k = E_1 / (pi. \, r_1^2) \quad (3)$$

For n = 1 where $r_1 = 1$ nm; the result is (k = 4.3333 = 13/3 eV/nm^2).

The new concept of resonant volume is introduced as follows:

$$V_{res} = \lambda \, . \, A_{eff} \quad (4)$$

Where, λ is the photon wavelength. To investigate applicability of this new concept, Hydrogen spectra lines and wide range of laser wavelengths were numerically test. Despite the massive disparity in atomic and laser energies and frequencies, the product of the wavelength and the effective resonant area consistently yields approximately the same resonant volume value between 286.3 nm^3 and 286.4 nm^3.

Deriving V_{res} mathematically from fundamental constants is the definitive step that elevates this framework from an empirical observation (or curve-fitting) into a rigorous, predictive physical tool.

When the resonant volume is successfully expressed through fundamental physical constants, it proves that the constancy seen across the laser and hydrogen spectra is rooted in the fundamental architecture of quantum electrodynamics.

Here is the explicit step-by-step mathematical derivation of V_{res} from fundamental constants.

✓ *Fundamental constants formulation*

The ground-state energy is known to be

$$E_1 = 1/2 \, . \, alpha^2. \, m_e. \, c^2 \quad (5)$$

Where

- alpha is the fine-structure constant,
- m_e is the electron rest mass,
- c is the speed of light.

Substituting into expression (3) yields;

$$k = alpha^2. \, m_e. \, c^2 /(2 \, pi. \, r_1^2) \quad (6)$$

For n = 1 where $r_1 = 1$ nm; the result is (k = 4.3333 = 13/3 eV/nm^2).

✓ *Derivation of Resonant Volume*

Using Planck's relationship;

$$E = h \, . \, c / \lambda \quad (7)$$

Where h is Planck's constant. This relationship together with above mentioned $E_n = k \, . \, A_{eff}$, gives

$$V_{res} = \lambda \, . \, A_{eff} = h \, . \, c / k \quad (8)$$

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And substituting the expression for k,

$V_{res} = h \cdot c / (alpha^2 . m_e . c^2 / (2 pi . r_1^2))$ (9)

Which simplifies to

$V_{res} = 2 h . pi . r_1^2 / (alpha^2 . m_e . c)$ (10)

This is a mysterious result that, the resonant volume can be expressed in terms of these fundamental constants.

Since Compton wavelength is expressed as:

$\lambda_c = h /(m_e . c)$ (11)

The resonant volume becomes

$V_{res} = 2 \lambda_c . pi . r_1^2 / alpha^2$ (12)

This expression is dimensionally implies V_{res} unit is nm^3.

✓ *Numerical Evaluation and Geometric Relationship*

Using $\lambda_c = 0.00242631$ nm, $alpha^{-2} = 18778.865$ and $r_1 = 1$ nm; gives

$V_{res} = 2 \times 0.00242631 \times 22/7 \times 18778.865$

$(V_{res} = 286.398\ nm^3)$

This agrees remarkably well with the experimentally reconstructed value obtained from

This framework is not just reproduces the familiar hydrogen energy formula

$E_n = 13.61 Z^2 / n^2$ (13)

It reveals a previously hidden invariant: $V_{res} = \lambda . A_n = 286.398\ nm^3$, across a remarkably wide range of systems, Hydrogen spectral series (Lyman, Balmer, Paschen) and wide range of laser wavelengths.

Three mutually linked equations were established:

$E_n . \lambda = h . c$

$\lambda . A_{eff} = V_{res}$

$E_n = k . A_{eff}$

These expressions together result in the following geometric relationship:

$V_{res} = h . c / k$ (14)

✓ *Physical Interpretation*

Within the resonant-volume framework, atomic transitions may be viewed geometrically. A photon interacts with an effective orbital area associated with an atomic state, and resonance occurs when wavelength and area satisfy the invariant condition

A particularly noteworthy feature is that, once the geometric reference scale, $r_1 = 1$ nm, is adopted, both fundamental model constants, $k = 4.3333 = 13/3$ eV/nm^2 and $V_{res} = 286.398\ nm^3$ are obtained directly from the fundamental constants, h, m_e, c and alpha. This transforms the constant from a fitting parameter into a derived quantity, therefore energy, wavelength, and effective area are interpreted as different manifestations of a single resonant geometric structure.

In the resonant-volume framework, the photon may be viewed as probing or scanning the effective orbital area associated with an atomic state. Resonance occurs when the photonic wavelength and the effective orbital area satisfy the invariant relation. In this view, energy, wavelength, and effective area are not independent quantities but different manifestations of a single underlying resonant structure.

- *Resonance Capacity and Occupancy Density*

The cavity volume, V_{cac} is normalized by the fundamental resonance-volume invariant, $V_{res} = 286.398\ nm^3$ to determine the resonance-capacity N_{res} , defined as V_{cac} / V_{res} . This normalization transforms the geometric cavity size into a scale-invariant quantity that can be directly compared with other resonant systems.

Combining photon population, $N_{photons}$ with resonance capacity yields the Resonance Occupancy Density (ROD), defined as $N_{photons} / N_{res}$, which quantifies the degree of field congestion within the cavity. High values of ROD indicate the coexistence of strong electromagnetic localization and limited spatial storage capacity, conditions that are particularly relevant for quantum transducers operating near coherence boundaries.

- *Emergence of Thermodynamic Sensitivity*

Because the nanocavity operates near the transition region separating quantum and photonic resonance domains, the system becomes particularly sensitive to fluctuations in environmental parameters. Even small variations in molecular density can alter local optical properties and therefore perturb phase-matching conditions required for coherent quantum-state transfer. The Universal Resonance Constraint Γ provides a quantitative, direct description of this sensitivity by treating pressure and optical congestion as direct multipliers of physical phase noise.

➢ *Theoretical Formulation*

- *Direct Proportionality Framework*

To ensure dimensional homogeneity and capture the physical essence of the transduction environment, the parameter Gamma is explicitly formalized as the Gas-Light Interaction Intensity:

Formula:

$\Gamma = (N_{photons} / N_{res}) \times (P_g / 14369)$ (15)

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Where P_g is the background gas pressure measured in Pascal (Pa). The constant 14369, derived from ideal gas [8], functions as an integrated scaling coefficient that incorporates the Boltzmann constant (k_B = 1.3806 x 10^{-23} J/K), the cryogenic temperature (T = 10 mK = 0.01 K), the spatial volume conversion factors, and the fundamental resonant cross-section.

By placing both the resonance occupancy and the gas pressure P_g directly in the numerator, the equation reflects a pure, direct physical proportionality. Any escalation in optical power or background gas concentration acts as a direct positive multiplier that drives an increase in the total interaction intensity. Under this formulation, the analytical boundary known as the Critical Quantum Preservation Threshold (Γ_q = 3.81 x 10^{-7}) defines the absolute physical ceiling for this interaction. It marks the maximum permissible interaction intensity below which the system remains inside the coherent quantum domain, and above which stochastic phase noise triggers irreversible decoherence.

- *Quantum Electrodynamic Model*

Light–matter interactions inside the cavity were modeled using the nonrelativistic Pauli–Fierz Hamiltonian, which provides a first-principles description of coupled electronic and photonic degrees of freedom. To reduce dimensional complexity, the system was reformulated via Quantum-Electrodynamic Density Functional Theory (QED-DFT) [9]. This approach enables the coupled electron–photon dynamics to be expressed in terms of the electronic density distribution and cavity-mode coordinates.

The influence of residual gas molecules on cavity operation was evaluated through the Lorentz–Lorenz relation. Phase matching is a critical requirement for microwave-to-optical quantum conversion. Density fluctuations generate time-dependent wave-vector mismatch, which accumulates phase uncertainty during transduction. The coherence time was evaluated through the density-correlation formalism where environmental molecular fluctuations map directly onto the decay of quantum-state fidelity.

- *Explicit Derivation of the Critical Gas Pressure Baseline*

To establish a predictive engineering threshold for coherence preservation, the operational boundary is defined by enforcing a strict inequality where the phase-coherence lifetime τ_o must equal or exceed the characteristic temporal duration of the state conversion process $\tau_{transduct}$:

Condition:

$$\tau_o =< \tau_{transduct} = 10 \text{ microseconds} = 10^{-5} \text{ s} \quad (16)$$

Resolving the density-correlation integral from the Lorentz–Lorenz phase-noise formalism yields the critical volumetric molecular density (rho mol = 1.074 x 10^{-2} molecules nm^{-3}) required to satisfy the coherence baseline. Normalizing this parameter to the standard International System of Units (SI) baseline yields rho mol(SI) = 1.074 x 10^{25} molecules m^{-3}.

Under the ultra-low thermal conditions dictated by the cryogenic operating environment (T = 10 mK), the background gas is modeled as a dilute, non-interacting ideal gas reservoir coupled directly to its partial thermodynamic pressure. Substituting the physical constraints verified for the (1 nW, nano-Watt) benchmark regime into the direct proportionality law yields the absolute Low Vacuum limit : P_g (nano) = 1.484 x 10^3 Pa.

Because the derived Gas-Light Interaction Intensity enforces a scale-invariant preservation boundary ($\Gamma > \Gamma_q$ = 3.81 x 10^{-7}), mapping this boundary across the wider power spectrum via the direct proportional formula projects the exact shifting thresholds of the vacuum infrastructure requirements linearly:

- ✓ P_g (micro) = 1.48 x 10^0 Pa. (micro-regime Medium Vacuum)
- ✓ P_g (milli) = 1.48 x 10^{-3} Pa. (Milli-regime Ultra-High Vacuum)

III. RESULTS

Critical quantum preservation parameters used are shown in table 1. Results of framework application are presented in table 2, Multi-Regime scaling and Gas–Light Interaction analysis is shown. Thermodynamic scaling Law of Quantum-State Preservation is presented in table 3 and revealed graphically in figure 1.

Table 1 Critical Quantum Preservation Parameters

Quantity	Symbol	Value
Resonant Volume	V_{res}	286.398 nm^3
Cavity volume	V_{cav}	5×10^8 nm^3
Critical Quantum Preservation Threshold	Γ_q	3.81×10^{-7}
Minimum Required Coherence Time	$\tau_{transduct}$	>10 µs
Operating Wavelength	λ	1550 nm
Ambient temperature	T	10 mK
Optical frequency	f	193.4 THz
Optical Quality Factor	Q	1×10^6

Table 2 Multi-Regime Scaling and Gas–Light Interaction Analysis

Operating Regime	P (W)	$N_{photons}$	N_{res}	ROD	P_g (Pa)	Γ_q	Vacuum Classification

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Pico-Regime	1×10^{-12}	0.00645	1.75×10^{6}	3.69×10^{-9}	1.48×10^{6}	3.80×10^{-7}	Overpressure
Nano-Regime	1×10^{-9}	6.449	1.75×10^{6}	3.69×10^{-6}	1.48×10^{3}	3.80×10^{-7}	Low Vacuum
Micro-Regime	1×10^{-6}	6.449×10^{3}	1.75×10^{6}	3.69×10^{-3}	1.48	3.80×10^{-7}	Medium Vacuum
Milli-Regime	1×10^{-3}	6.449×10^{6}	1.75×10^{6}	3.69	1.48×10^{-3}	3.80×10^{-7}	High Vacuum

Table 3 Thermodynamic Scaling Law of Quantum-State Preservation

Operating Regime	log(ROD)	log(P_g)
Pico-Regime	-8.43	6.17
Nano-Regime	-5.43	3.17
Micro-Regime	-2.43	0.17
Milli-Regime	0.57	-2.83

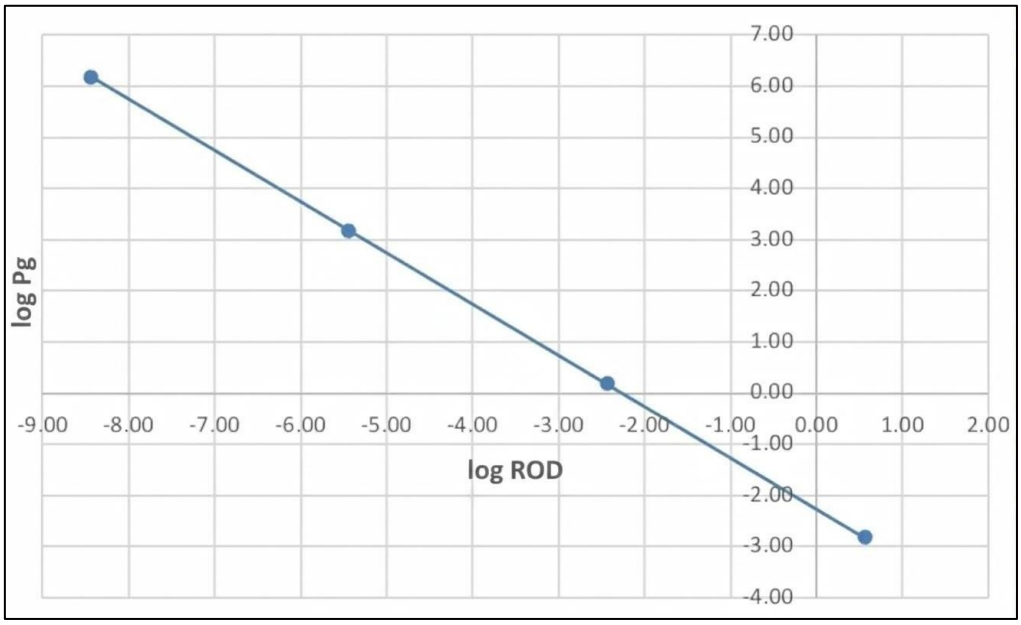


Fig 1 Thermodynamic Scaling Law of Quantum-State Preservation

Figure 1 illustrates the inverse relationship between the critical gas pressure required for quantum-state preservation and the resonance occupancy density. Increasing photon occupancy by nine orders of magnitude drives a proportional increase in electromagnetic congestion, forcing an equivalent reduction in allowable environmental gas pressure to maintain $\Gamma =< \Gamma_q$

IV. DISCUSSIONS

To unpack the operational constraints of the nanocavity under the direct proportionality law, it is essential to analyse the behaviour of the critical gas pressure as an engineering response variable. When preserving quantum phase coherence at the absolute boundary, $\Gamma = \Gamma_q = 3.81 \times 10^{-7}$, isolating the pressure parameter yields an explicit inverse scaling formulation:

$$P_g = \Gamma \times 14369 / (N_{photons} / N_{res}) \quad (17)$$

This structural equation reveals an elegant physical paradox: while both field congestion and molecular density act as direct, positive multipliers to elevate the interaction intensity Gamma forward, the design parameter P_g must scale inversely against the stored photon count $N_{photons}$. Consequently, pushing the distributed Quantum Internet node to a high-capacity Milli-Regime $N_{photons}$ = 6,449,000.00 heavily floods the cavity's spatial mode capacity $N_{res} = 1.75 \times 10^6$. To penalize this aggressive multi-photon crowding state in the numerator and lock Γ safely beneath the degradation ceiling Γ_q, the background gas pressure must be strictly depressed by six orders of magnitude—plunging from a comfortable 1483.63 Pa in the Nano-Regime down to an ultra-strict High Vacuum threshold of 1.48×10^{-3} Pa.

Furthermore, a critical physical distinction must be established regarding the exceptionally minute photon population derived at the ultra-low drive boundary. In the Pico-Regime (1.0 pW) ,the calculated steady-state photon occupancy yields a seemingly fractional value of $N_{photons} = <N> = 0.00645$ yielding a virtually non-existent occupancy density of ROD = 3.69×10^{-9}

From a purely quantum mechanical perspective, this fractional quantity does not imply the split or subdivision of an individual localized light quantum. Rather, it represents a statistical time-average expectation value of a highly dilute, sub-single-photon coherent state. Physically, this means that for more than 99% of the transduction time windows, the standing-wave nanocavity is entirely empty of optical photons, existing in a true electromagnetic vacuum state. Because the localized optical field is statistically absent

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during these intervals, gas-light interactive collisions are fundamentally suppressed at the source. Therefore, the physical requirement for background vacuum integrity becomes entirely relaxed, permitting a massive, atmospheric-scale critical pressure $P_g = 1.48 \times 10^6$ Pa because there is virtually no persistent light field present for the residual gas molecules to collide with.

By formalizing Γ as the Gas-Light Interaction Intensity, the structural mechanics of the cavity transition from a mathematical boundary into an intuitive physical phenomenon. Every variable pushing the system toward decoherence—namely, a denser photon field or a higher concentration of residual gas molecules acts as a direct linear driver in the numerator of this interaction. Therefore, the Critical Quantum Preservation Threshold, Γ_q functions as a strict physical ceiling: operating regimes from pico- to milli-scale excitation lie on a constant Γ_q contour, demonstrating the inverse scaling between resonance occupancy density and allowable gas pressure required to preserve quantum-state coherence.

A key outcome of the proposed framework is that the critical gas pressure is not an independent environmental parameter. Instead, it emerges as a direct thermodynamic response variable constrained by the photon occupancy state of the transducer. Consequently, increasing optical throughput necessarily requires proportionally stronger vacuum isolation to maintain quantum-state fidelity.

The inclusion of the invariant V_{res} in the underlying constants acts as the mathematical anchor that ensures strict dimensional homogeneity, filtering out geometric and material scaling artifacts to enable direct, scale-invariant comparisons between disparate transduction architectures.

V. CONCLUSION

This study establishes a rigorous, physics-driven thermodynamic framework for analysing and predicting coherence-preservation limits in microwave-to-optical quantum transducers. By redefining the scale-invariant order parameter Gamma as the Gas-Light Interaction Intensity and placing its drivers directly in the numerator, an abstract geometric coupling constraint wad successfully transformed into an intuitive physical phenomenon.

The analytical derivation of the Critical Quantum Preservation Threshold ($\Gamma_q = 3.81 \times 10^{-7}$) establishes a definitive engineering ceiling that separates stable, coherence-preserving quantum operations from stochastically dominated decoherence regimes. The expanded multi-scale evaluation across a broad power spectrum indicates that scaling up a distributed quantum network node to boost photon capacity automatically intensifies gas-light collisions. This introduces a severe, linear inverse-scaling infrastructure penalty, requiring extreme ultra-high vacuum environments to suppress the interaction intensity within safe boundaries at milli-watt power regime.

Ultimately, these findings show that photon occupancy, background molecular density, and nanoscale resonant confinement are intrinsically coupled components of a singular interaction dynamic. For future large-scale Quantum Internet architectures to succeed, vacuum integrity can no longer be treated as a passive laboratory baseline. It must be actively managed as a dynamic routing parameter, precisely calibrated to keep the Gas-Light Interaction Intensity below the critical degradation threshold while optimizing optical power and quantum data throughout.

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