

Thermophysical and Viscoelastic Effects on Magnetically Controlled Mixed Convection in Functional Porous Materials

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Abstract: The reliability of heat-induced flow in porous materials is crucial for geothermal energy retrieval, subterranean thermal storage, and many renewable energy systems. This research investigates how heat radiation, magnetic fields, porous-media properties, and viscoelastic features collectively affect the onset of mixed-convection instability in a horizontal porous layer filled with a Kelvin–Voigt viscoelastic fluid. These fluids are relevant to subsurface and energy systems because polymeric substances, suspended particulates, and complex fluid–rock interactions can lead to non-Newtonian behavior. Comprehending the factors that trigger flow instability is crucial for ensuring efficient heat transfer, minimizing unwanted thermal stratification, and enhancing the sustained performance of geothermal and porous thermal systems. A linear stability framework assesses key instability conditions by examining the effects of the Hartmann number, Darcy number, Richardson number, Prandtl number, thermal radiation parameter, and viscoelastic coefficient. The analysis shows that these dimensionless variables substantially alter the interplay among buoyancy, viscous forces, magnetic damping, thermal diffusion, and elastic resistance. Increasing magnetic field intensity strengthens the Lorentz force, reducing velocity fluctuations and delaying the onset of unstable convection. Thermal radiation alters energy transfer by improving effective heat diffusion and reducing sharp temperature gradients, thereby aiding flow stabilization. Fluctuations in the permeability of porous media, denoted by the Darcy number, strongly affect momentum transfer and the instability threshold. The Kelvin–Voigt viscoelastic behavior adds further elastic resistance to deformation and modifies the development of hydrodynamic perturbations. The Prandtl number influences the relative velocities of momentum and thermal diffusion, resulting in different stability responses based on the disturbance wavenumber and flow parameters. Conversely, increasing the Richardson number enhances buoyancy relative to forced convection, fostering natural-convection fluctuations and often reducing the critical stability limit. The findings indicate that magnetic influences, radiation, permeability, buoyancy, and fluid viscoelasticity significantly determine mixed convection in porous media. These results provide valuable theoretical insights for designing and enhancing geothermal reservoirs, solar-assisted porous heat exchangers, underground thermal energy storage systems, and associated renewable energy applications that require steady, efficient heat transfer.

Keywords: Hybrid Convection; Kelvin-Voigt Fluid; Permeable Substance; Thermal Emission; Electromagnetic Field; Thermal Volatility.

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I. INTRODUCTION

Transport in porous media has garnered significant interest because of its relevance to geothermal energy extraction, subterranean thermal-energy storage, groundwater hydrology, filtering mechanisms, chemical reactors, carbon dioxide sequestration, and several other technical applications [1]. In these systems, the interplay between a flowing fluid and a porous solid framework

governs momentum and heat-transfer properties. Darcy's law provides a fundamental description of fluid movement through porous media; however, its relevance diminishes when viscous shear effects are significant. The Brinkman model improves the traditional Darcy equation by adding a viscous diffusion term while preserving the permeability-linked resistance and pressure gradient. Thus, the Brinkman method is especially advantageous for highly porous media and scenarios where velocity fluctuations within the porous

medium are pronounced. When a permeable layer is heated, the resulting temperature variations may create density gradients and buoyancy effects. When an externally applied flow occurs concurrently, forced and natural convection combine to produce mixed convection. In some cases, minor perturbations can amplify and alter the base flow, leading to hydrodynamic and thermal instabilities that significantly affect total heat and mass transfer.

Researchers have therefore thoroughly examined the stability of mixed convection in channels, ducts, porous media, and other structures. Preliminary studies indicated that instability thresholds are strongly influenced by thermal forcing, fluid velocity, geometry, and disturbance direction. [2] examined mixed convection in a horizontal channel under a temperature gradient and identified key variables linked to the shift from stability to convective instability. [3] later investigated Poiseuille flow subjected to uniform heating and showed that the choice between transverse and longitudinal convective patterns may fluctuate with Reynolds number and wall conditions. The development of various roll structures is particularly significant since these modes alter heat and momentum distribution in fundamentally distinct ways. [4] also showed that density inversion and geometric properties may modify the rivalry between transverse and longitudinal modes in Poiseuille–Rayleigh–Bénard convection. These data suggest that mixed-convection instability cannot be defined solely by thermal forcing, since shear and geometric confinement also have a significant influence.

Further enquiries have examined more intricate heating configurations and boundary circumstances. [5] showed that non-continuous heating from underneath may produce stable transverse roll formations, even under conditions markedly different from those typical of classical Rayleigh–Bénard convection. Research by [6] also uncovered intricate neutral-stability curves and shifts between disturbance modes as control parameters change. These findings suggest that minor changes in flow configuration may influence the onset of convection. Permeable resistance further complicates this behavior. [7] noted that buoyancy, as indicated by the Richardson number, may amplify convective instability, whereas Darcy-number variations alter disturbance progression via porous resistance. [8] showed that through-flow in a Brinkman porous medium may inhibit or enhance convection, depending on the Rayleigh and Peclet number regimes. Their research underscored the need to integrate temperature and porous-medium factors in establishing essential stability criteria. Similarly, [9] examined the effects of thermal slip. They found that transverse flow might mitigate instability under certain conditions, but stronger thermal and inertial forces may promote disturbance growth. The impact of slip, temperature ratios, and the selection of critical wavenumber has been examined by [10] and associated research [11].

While several conventional stability assessments assume Newtonian fluids, many real energy and industrial applications involve fluids with complex rheological characteristics. Polymer solutions, suspensions, biological fluids, petroleum derivatives, and chemically reactive mixes

may exhibit elastic properties alongside standard viscosity. As a result, viscoelastic constitutive models have become important for studying transport in porous media. Prevalent models include Maxwell, Oldroyd-B, generalized Maxwell, Kelvin–Voigt, and advanced fluid representations. The Kelvin–Voigt model offers a straightforward approach to characterize a material that demonstrates concurrent viscous dissipation and elastic resistance. Incorporating this rheological behavior may markedly alter the amplification or attenuation of disturbances, since the fluid's response depends not only on immediate deformation but also on its elastic properties.

Numerous studies show that viscoelasticity can affect convection in multifaceted ways. [12] investigated thermosolutal convection exhibiting Kelvin–Voigt-type characteristics. They found that viscoelastic attributes can significantly alter oscillatory instability and critical parameters, especially when supplementary factors such as Soret effects, rotation, and friction are considered [13]. [14] examined Poiseuille flow in anisotropic porous media and revealed that instability arises only within certain ranges of viscoelastic and porous-medium characteristics. This demonstrates that elastic effects cannot be definitively categorized as solely stabilizing or destabilizing. Their impact depends on the disturbance, porous resistance, and the relative strength of viscous and elastic processes. Modal and non-modal analyses of Brinkman systems have similarly shown that Kelvin–Voigt parameters may alter the amplification of transient disturbances, even when the asymptotic flow is steady [15]. This ephemeral behavior is pertinent to real-world systems, since sufficiently robust short-term amplification may cause significant fluctuations in heat and momentum transfer before disturbances dissipate.

Researchers have investigated the influence of viscoelasticity in spontaneously induced convection. Research conducted by [16] examined convection inside vertical porous structures and revealed a significant interaction between rheological factors and heat diffusion as well as disturbance propagation. The Prandtl number is crucial since it quantifies the comparative velocities of momentum and heat diffusion. Thus, variations in the Prandtl number might modify the effect of viscoelastic resistance on both stationary and travelling perturbations. [17] investigated thermohaline convection in Navier–Stokes–Voigt fluids and showed that temperature-dependent phenomena might alter the nonlinear response and bifurcation dynamics. [18] examined penetrative convection and showed that internal heating and solutal influences might act in opposing ways. Subsequent studies have extended non-Newtonian models to intricate channels, cilia-mediated transport, and boundary-layer arrangements. [19] examined viscoelastic transport in geometrically intricate channels. In contrast, [20] explored viscoplastic boundary-layer dynamics and showed that rheological factors may significantly alter velocity and temperature profiles. Relevant advances in viscoelastic transport appear in References [21].

Another physically relevant process is magnetohydrodynamics. When a conductive fluid traverses a magnetic field, it may generate electric currents within the fluid. Their engagement with the magnetic field generates a Lorentz force that alters the momentum equilibrium. In several flow configurations, this force acts as a damping agent, reducing velocity variations orthogonal to the magnetic field. Consequently, magnetic fields provide a viable method for regulating convective instability without necessitating direct mechanical involvement. This attribute pertains to electrically conductive working fluids, ferrofluids, liquid-metal energy systems, magnetically sensitive suspensions, and advanced thermal-management technologies.

The interplay between magnetic fields and porous convection has thus garnered increasing scholarly interest. [22] investigated the magnetic and gravitational influences inside Darcy–Brinkman–Bénard setups that include Kelvin–Voigt rheology and localized thermal nonequilibrium. Their research showed that magnetic forces can significantly alter key parameters and the stability characteristics of the conduction state. Magnetic-force-influenced nonlinear thermosolutal convection has similarly shown strong parameter dependence [23]. In mixed-convection porous-channel systems, [24] observed reduced convective disturbances as magnetic influence intensified. These findings align with Lorentz damping, which dampens fluid dynamics and may delay instability emergence in response to increased thermal or hydrodynamic forces.

Recent quantitative studies have expanded magnetohydrodynamics research to include biofluids, nanofluids, phase-change substances, and intricate porous structures. [25] investigated the influence of electromagnetic phenomena on biofluid movement inside a converging setup. They recognized the crucial impact of both the Darcy and Hartmann numbers on velocity dynamics. [26] examined thermal transport inside an enclosure exposed to radiation and a periodic magnetic field, showing that magnetic influence can significantly modify heat and mass transfer. Related research has examined magnetohydrodynamic bioconvection, nano-enhanced phase-transition substances, hybrid ferrofluids, and oscillating magnetic fields. These studies indicate that magnetic influences remain significant even when multiple heat-transfer processes operate concurrently.

Researchers have investigated convection initiation in rotating and magnetized porous systems using several non-Newtonian constitutive models. [27] investigated thermal convection inside a spinning Darcy–Brinkman porous medium infused with nanofluid. [28] examined porous convection with elasto-viscous fluids, rotation, and suspended particles, demonstrating the impact of rheology and porous resistance on instability thresholds. [29] examined thermal instability inside a Darcy–Brinkman layer infused with a Jeffrey nanofluid, while [30] investigated thermosolutal instability in a rotating, magnetized elasto-viscous fluid situated in a porous medium. Tayebi's investigations [31] delved further into local temperature

discrepancies, entropy production, magnetic influences, and porous thermal-energy retention. [32] broadened the mixed-convection examination to include magnetized hybrid nanofluids traversing a three-dimensional porous annular structure. Together, these works illustrate the growing importance of integrating porous-medium resistance, complex rheology, and electromagnetic effects in modelling contemporary thermal systems.

Thermal radiation is a significant process, particularly in high-temperature energy applications. Radiative heat transfer can alter effective thermal-energy conveyance and, in turn, affect the temperature gradient that drives buoyancy-induced convection. Researchers have examined its relationship with magnetic damping, viscous dissipation, Joule heating, suction or injection, and non-Newtonian characteristics in various mixed-convection and hybrid-fluid setups. Nonetheless, several current investigations emphasize heat-transfer rates after a flow structure has formed, rather than identifying the exact conditions under which small perturbations become unstable. Moreover, few studies combine Brinkman porous resistance, Kelvin–Voigt viscoelasticity, magnetic forces, and thermal radiation in a single linear stability framework.

This Study examines mixed-convection instability in a horizontal Brinkman porous medium saturated with a Kelvin–Voigt viscoelastic fluid, while also considering magnetic fields and radiative heat transfer. The formulation provides a framework for analyzing the competition among buoyancy, applied shear, porous drag, viscous diffusion, elastic resistance, magnetic damping, and thermal energy transfer. This Study places special emphasis on the conditions for instability and the development characteristics of the most unstable perturbations. Characteristics such as the Reynolds, Richardson, Darcy, Hartmann, and Prandtl numbers, along with radiation and viscoelastic effects, can be systematically analyzed to assess their individual and combined effects on stability. This Study pertains to geothermal reservoirs, subterranean heat-storage systems, permeable heat exchangers, solar-enhanced thermal technologies, and other renewable energy applications. A clearer understanding of these interrelated principles will enable better control of flow instability, more reliable heat extraction, less severe thermal stratification, and safer long-term operation of porous energy systems.

➤ *The Key Contributions of the Research are:*

- Construct an extensive Multiphysics framework that integrates the interrelated effects of viscoelasticity, Lorentz forces, heat radiation, and pore diffusion in a Brinkman-type porous medium.
- Examine the system's dynamic behavior by assessing key thermal instability thresholds across mixed-convection scenarios defined by the Reynolds, Hartmann, Darcy, Richardson, Prandtl, and Kelvin–Voigt numbers.
- Assess the individual and collective impacts of key factors such as the Prandtl number, magnetic field strength, viscoelasticity, effective permeability, and radiation coefficient on thermal stability.

- Ascertain the stabilizing and destabilizing processes linked to the interplay of viscoelastic, magnetic, and radiative influences, and identify behaviors that Newtonian or non-radiative frameworks cannot sufficiently forecast.
- Develop an enhanced predictive model for the design and optimization of thermal systems that include intricate fluid-saturated porous media, focusing specifically on geothermal applications, porous thermal storage substances, and biothermal reactors.

This Study is directly relevant to several practical engineering and energy applications where heat transfer and flow stability in porous materials are crucial. The findings may be used to design and improve renewable energy technologies, including solar thermal collectors, geothermal heat exchangers, and porous absorbers used in thermal energy storage systems. Examining viscoelastic fluids is particularly important for advanced cooling and heat transfer systems that use polymer-based or non-Newtonian working fluids to improve thermal efficiency. Furthermore, the synergistic effect of magnetic fields and thermal radiation analyzed in this Study has practical relevance for magnetically regulated cooling systems, high-temperature porous reactors, and energy conversion apparatus functioning in radiative settings. The current results provide valuable insights for managing convective instabilities in porous materials, which is crucial for improving the efficacy, dependability, and thermal management of contemporary renewable and sustainable energy systems.

The article is structured into many parts. The text begins with an overview of mixed convection in porous materials, the importance of viscoelastic fluids, and the rationale for integrating magnetic and radiative effects. Section 2 outlines the mathematical framework, including the Navier-Stokes, Brinkman, Maxwell, and energy equations. Section 3 presents the resolutions related to the fundamental process. Section 4 elucidates the linear stability methodology, the normal perturbations applied to the system, and the consequent eigenvalue equations used to define unstable modes. Section 5 describes the numerical approach using spectral collocation with Chebyshev polynomials. Section 6 offers quantitative validation and examines the findings by contrasting them with earlier documented research and boundary scenarios. Section 7 outlines the practical applications and implications of the current Study. Section 8 delineates the findings on instability and examines both the singular and collective effects of the controlling factors. Finally, Section 9 summarizes the Study's main findings and suggests avenues for further research.

II. MATHEMATICAL FORMULATION

This research is driven by a combined solar energy system that merges photovoltaic modules with thermal collectors to concurrently produce electricity and heated water, as seen in Fig. 1. During operation, the flow of heat-transfer fluids with polymer additives or agents that enhance heat transfer may exhibit non-Newtonian characteristics due to pumping, acceleration, and thermal exchange. These conditions may promote viscoelastic phenomena, notably relaxation and fluid-memory characteristics, in areas where temperature and velocity fluctuate rapidly, such as within pipes. Preliminary hydrodynamic perturbations might therefore affect the system's response and, under sufficiently intense conditions, lead to thermal instability. Solar absorption, fluid movement, thermal retention, and heat-exchange mechanisms play crucial roles in the system's overall efficacy.

Figure 2 provides a more comprehensive depiction of the solar panel's internal thermal mechanisms. These include solar radiation absorption, natural and forced convective heat transfer, and electrical current flow through the collector layers. To improve heat absorption and thermal conductivity, solar thermal collectors could use highly porous materials or metallic foams. This arrangement supports modeling the collector as a permeable medium, where internal resistance and heat dissipation become significant physical processes.

In sophisticated hybrid systems, electromagnetic phenomena could potentially influence the internal fluid dynamics. When the circulating heat-transfer fluid exhibits electrical conductivity, even at minimal levels, the magnetic fields generated by electrical currents in photovoltaic elements may influence fluid dynamics. In light of these physical factors, Fig. 3 presents an abstract mathematical framework including a horizontal Brinkman-type porous medium permeated by a viscoelastic fluid, influenced by descending heat radiation and a lateral magnetic field. The magnetic field generates an electromagnetic impedance that counteracts fluid movement. Consequently, the transition from Figs. 1 and 2 to Fig. 3 links the physical solar-energy setup to its mathematical representation, enabling examination of thermal stability by simultaneously accounting for porous-medium resistance, viscoelasticity, radiation, and magnetic effects.

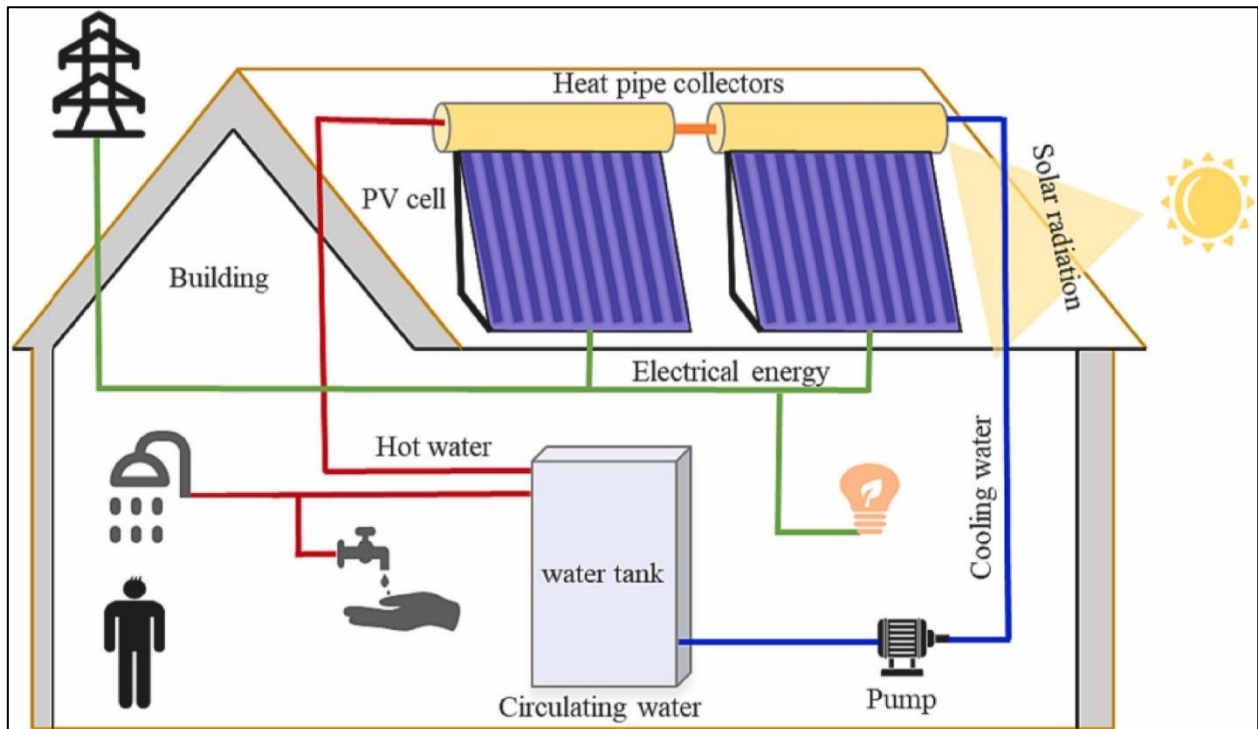


Fig 1 Diagram Illustrating the Comprehensive Use of Solar Energy for Generating Power and Heating Water.

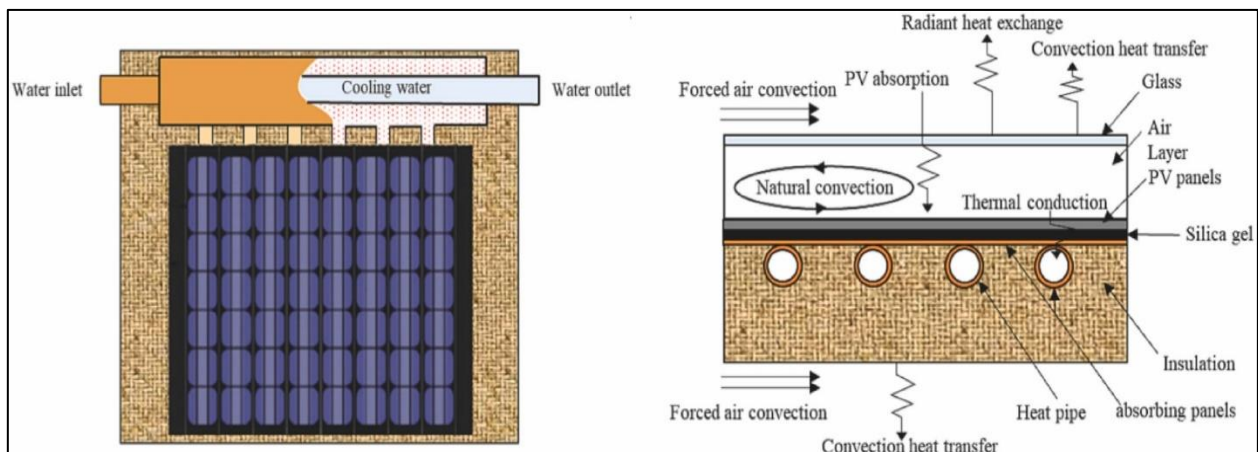


Fig 2 Schematic of Heat Transfer Mechanisms in the Layers of a Photovoltaic and Thermal Panel.

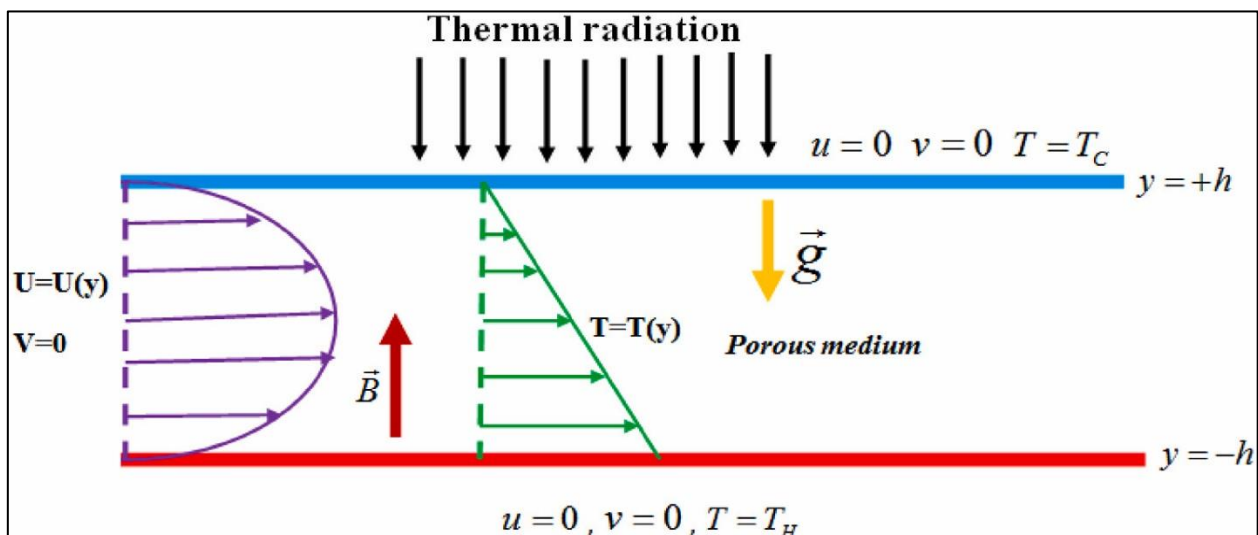


Fig 3 Physical Representation.

This Study examines mixed convection in an electrically conductive viscoelastic Kelvin–Voigt fluid flowing through a channel saturated with an isotropic Brinkman porous medium. The lower channel wall is maintained at a steady elevated temperature, TH. In contrast, the upper limit is maintained at a lower fixed temperature, TC, with TH exceeding TC. A uniform streamwise pressure gradient produces the fundamental fluid movement in the x-direction. A uniform transverse magnetic field is applied to the system, and heat radiation is also included, as shown in Fig. 3. The liquid is treated as viscous and incompressible, and both channel boundaries are assumed to be rigid. We model buoyancy-related density fluctuations using the Bossiness approximation.

The electromagnetic characteristics are expressed using the quasi-static magnetohydrodynamic framework, based on the assumptions of a small magnetic Reynolds number and negligible displacement current. Radiative heat transfer is characterized by the Rosseland diffusion approximation, in conjunction with the standard linearization of the temperature-dependent radiative term around a chosen reference temperature. Unless stated otherwise, the thermophysical characteristics of the fluid are presumed to be invariant. We neglect Joule heating and viscous dissipation in the energy balance.

Let $\vec{q}(x, y, z, t)$ denote the volume-averaged velocity according to the Darcy–Brinkman model, whilst $p(x, y, z, t)$ and $T(x, y, z, t)$ denote the pressure and temperature fields, respectively. The variables E, B, and J denote the electric field, magnetic field, and electric current density, respectively. The permeable material is defined by its permeability K. Additionally, $\vec{g}$ signifies gravitational acceleration, ρ represents fluid density, μ indicates dynamic viscosity, σ stands for electrical conductivity, k refers to thermal conductivity, C_p depicts specific heat capacity, and β symbolizes the thermal expansion coefficient. Using these premises and the Boussinesq approximation, we can establish the governing field equations for the system under Study.

➤ *Continuity Equation [33].*

$$\nabla \cdot \vec{q} = 0 \quad (1)$$

➤ *Navier-Stokes Equation [34].*

$$\rho \left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right) = -\nabla P + \nabla \tau + (\rho \beta)(T - T_c) \vec{g} - \frac{\mu}{k^*} \vec{q} + \vec{j} \wedge \vec{B} \quad (2)$$

In this research, the Kelvin-Voigt rheological model captures the fluid's viscoelastic characteristics by linearly combining viscous and elastic responses. In this context, the stress tensor is characterized as a combination of a Newtonian viscous component and an elastic component associated with the strain rate, allowing the fluid to exhibit both solid-like elasticity and fluid-like viscosity. This model is particularly suitable for flows with low deformation rates and minor disturbances, as often examined in linear stability analysis.

In equation (2), τ denotes the viscoelastic stress tensor, which is characterized in this research by the Kelvin-Voigt model [35]:

$$\tau = \rho \lambda \frac{\partial}{\partial t} (\nabla \vec{q} + (\nabla \vec{q})^T) + \mu (\nabla \vec{q} + (\nabla \vec{q})^T) \quad (2a)$$

➤ *Energy Equation [36].*

$$\rho C_p \left(\frac{\partial T}{\partial t} + \vec{q} \cdot (\nabla T) \right) = k \Delta T - \frac{\partial q_r}{\partial y} \quad (3)$$

In the current model, setting the Kelvin-Voigt parameter to zero ($\lambda = 0$) makes the fluid behave as a Newtonian fluid, effectively disregarding viscoelastic effects.

➤ *Maxwell Equations [37]*

$$\text{div}(\vec{E}) = 0 \quad (4)$$

$$\text{div}(\vec{B}) = 0 \quad (5)$$

$$r \partial_t(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \quad (6)$$

$$r \partial_t(\vec{B}) = \mu_0 (\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}) \quad (7)$$

➤ *Ohm's Law [38].*

$$\vec{j} = \sigma(\vec{E} + \vec{q} \wedge \vec{B}) \quad (8)$$

The viscoelastic fluid's heat capacity and thermal expansion coefficient are specified. μ_0 and ϵ_0 denote the medium's magnetic and electric permittivity, respectively. And t signifies the temporal dimension.

The radiative flow is described by Reference [39]:

$$q_r = -\frac{4\sigma^* \partial T^4}{3\beta_r \partial y} \quad (9)$$

The fluid's absorption coefficient is denoted by β_f , while the Stefan-Boltzmann constant is represented by σ^* .

In this case, T^4 is expressed as a Taylor series [40].

$$T^4 \approx 4T_c^3 T - 3T_c^3 \quad (10)$$

The radiative flow transforms into:

$$T^4 \approx 4T_c^3 T - 3T_c^3 \quad (11)$$

The radiative flow transforms into:

Based on equation (11) and the Boussinesq approximation equations [41], the challenge in Cartesian coordinates is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (12)$$

$$\rho[(1-\lambda)\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}] = -\frac{\partial P}{\partial x} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) - \frac{\mu}{k}u - \sigma B_0^2 u, \quad (13)$$

$$\rho[(1-\lambda)\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}] = -\frac{\partial P}{\partial y} + \mu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + (\rho\beta)g(T - T_c) - \frac{\mu}{k}v \quad (14)$$

$$\rho C_p(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) + \frac{16T_c^3 \sigma^*}{3\beta_r} \frac{\partial^2 T}{\partial y^2} \quad (15)$$

With the issue equations expressed in Cartesian coordinates, we will now provide their dimensionless representation. To do this, we provide the following variables:

$$\tilde{u} = \frac{u}{U}, \tilde{v} = \frac{v}{U}, t = \frac{tU}{h}, \tilde{P} = \frac{P}{\rho U^2}, \theta = \frac{T - T_c}{T_H - T_c} \quad (16)$$

Employing equation (16), the dimensional equations pertinent to the issue are:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \quad (17)$$

$$(1 - E\Delta)\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{\partial \tilde{P}}{\partial \tilde{x}} + \frac{\Delta \tilde{u}}{Re} - \frac{\tilde{u}}{DaRe} - \frac{M^2}{Re}\tilde{u} \quad (18)$$

$$(1 - E\Delta)\frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{\partial \tilde{P}}{\partial \tilde{y}} + \frac{\Delta \tilde{v}}{Re} - \frac{\tilde{v}}{DaRe} + R_i \tilde{\theta} \quad (19)$$

$$\frac{\partial \tilde{\theta}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{\theta}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{\theta}}{\partial \tilde{y}} = \frac{\Delta \tilde{\theta}}{PrRe} + \frac{R_d}{PrRe} \frac{\partial^2 \tilde{\theta}}{\partial \tilde{y}^2} \quad (20)$$

These dimensionless variables define the comparative impact of inertial, viscous, porous, magnetic, buoyant, thermal, radiative, and viscoelastic phenomena inside the system. Their differences provide a robust basis for assessing flow dynamics, thermal transfer properties, and the fluid's overall stability.

III. BASIC STATE

We determine the unaltered steady state by assuming a constant, fully established base flow [42]. Given these premises, Equations (17), (18), (19), and (20) simplify to

$$\frac{d^2 U_b}{dy^2} - \Omega^2 U_b = -A \text{ and } \frac{d^2 \theta_b}{dy^2} = 0 \quad (21)$$

where the amounts presented maintain the definitions and SI units established in Section 2 by $\Omega = \sqrt{\frac{1}{Da} + M^2}$ and $A = -Re \frac{\partial P}{\partial x}$.

The relevant boundary conditions are

$$U_b(\pm 1) = 0, \theta_b(-1) = 1 \text{ and } \theta_b(1) = 0 \quad (22)$$

Implemented at the two limiting surfaces of the layer/channel, as previously indicated.

Solving Equation (21) with boundary conditions (22) yields the fundamental fields.

$$U_b(y) = \frac{A}{\Omega^2} \left[\frac{\cosh(\Omega) - \cosh(\Omega y)}{\cosh(\Omega)} \right] \quad (23)$$

$$\theta_b(y) = \frac{1}{2}(1 - y) \quad (24)$$

These formulations satisfy equation (21) and the boundary conditions (22). They are used to formulate the perturbation decomposition for the next stability assessment.

IV. STABILITY ANALYSIS

According to linear stability theory [43], minuscule perturbations are applied to the base flow. Consequently, each flow variable is expressed as the sum of a constant laminar component and a small disturbance component, as shown in the following form following form [44].

$$\tilde{u} = U_b + u, \tilde{v} = v, \tilde{P} = P_b + P, \tilde{\theta} = \theta_b + \theta \quad (25)$$

Inserting Eq. (25) into the controlling Eqs. (17), (18), (19), and (20), while preserving just the linear terms concerning the injected perturbations, results in the following linearized system of equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (26)$$

$$(1 - E\Delta)\frac{\partial u}{\partial t} + U_b \frac{\partial u}{\partial x} + v \frac{\partial U_b}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\Delta u}{Re} - \frac{u}{DaRe} - \frac{M^2}{Re}u \quad (27)$$

$$(1 - E\Delta)\frac{\partial v}{\partial t} + U_b \frac{\partial v}{\partial x} = -\frac{\partial P}{\partial y} + \frac{\Delta v}{Re} - \frac{v}{DaRe} + R_i \theta \quad (28)$$

$$\frac{\partial \theta}{\partial t} + U_b \frac{\partial \theta}{\partial x} + v \frac{\partial \theta_b}{\partial y} = \frac{\Delta \theta}{PrRe} + \frac{R_d}{PrRe} \frac{\partial^2 \theta}{\partial y^2} \quad (29)$$

The constraints related to this system are specified by

$$u = v = \theta = 0 \text{ at } y = \pm 1 \quad (30)$$

These align with the previously mentioned physical limitations for the lower and higher thresholds.

In a normal-mode analysis, References [45] delineate the disturbances.

$$\{u, v, P, \theta\} = \{\hat{u}, \hat{v}, \hat{P}, \hat{\theta}\}(y)e^{i(\alpha x - \omega t)} \quad (31)$$

Where α signifies the wave number, $\omega = \alpha C$ indicates the frequency, and $C = Cr + iCi$ characterises the complex wave speed. According to the sign convention outlined under Eq. (31), temporal stability is ascertained by the imaginary

part (Ci). The flow is deemed stable if $Ci > 0$, unstable if $Ci < 0$, and neutrally stable if ($Ci = 0$).

$$i\alpha\hat{u} + \frac{d\hat{v}}{dy} = 0 \quad (32)$$

Substituting the normal-mode representation in Eq. (31) into Eqs. (26)– (29) produces the ensuing eigenvalue system:

$$i\alpha\{U_b - [1 - E(D^2 - \alpha^2)]C\}\hat{u} + \frac{dU_b}{dy}\hat{v} = -i\alpha\hat{P} + \frac{(D^2 - \alpha^2)\hat{u}}{Re} - \frac{\hat{u}}{DaKR_e} - \frac{M^2}{Re}\hat{u} \quad (33)$$

$$i\alpha\{U_b - [1 - E(D^2 - \alpha^2)]C\}\hat{v} = -\frac{d\hat{P}}{dy} + \frac{(D^2 - \alpha^2)\hat{v}}{Re} - \frac{\hat{v}}{DaRe} + Ri\hat{\theta} \quad (34)$$

$$i\alpha(U_b - C)\hat{\theta} + \frac{d\theta_b}{dy}\hat{v} = \frac{1}{PrRe}(D^2 - \alpha^2)\hat{\theta} + \frac{Rd}{PrRe}D^2\hat{\theta} \quad (35)$$

Removing the pressure variable from equations (33)– (34) using the conventional procedure outlined in the text yields the simplified system.

$$(U_b + i\frac{D^2 - \alpha^2}{\alpha Re})(D^2 - \alpha^2)\hat{v} - \frac{i}{\alpha DaRe}(K^{-1}D^2 - \alpha^2)\hat{v} - i\alpha Ri\hat{\theta} = (\frac{d^2 U_b}{dy^2} + \frac{iM^2}{\alpha Re}D^2)\hat{v} + C[1 - E(D^2 - \alpha^2)](D^2 - \alpha^2)\hat{v} \quad (36)$$

$$i\alpha(U_b - C)\hat{\theta} + \frac{d\theta_b}{dy}\hat{v} = \frac{1}{PrRe}(D^2 - \alpha^2)\hat{\theta} + \frac{Rd}{PrRe}D^2\hat{\theta} \quad (37)$$

Contingent upon the boundary conditions

$$\hat{v}(\pm 1) = 0 \text{ and } \hat{\theta}(\pm 1) = 0 \quad (38)$$

This system has no closed-form analytical solution. The following section delineates its numerical solution methodology.

V. NUMERICAL APPROACH

After formulating the eigenvalue system in the preceding section, we use the Chebyshev collocation technique to obtain a numerical solution. In equations (36)– (38), the unidentified fields are expressed as a finite sequence of Chebyshev polynomials [46], with their respective expansions articulated as follows:

$$\hat{v}(y) = \sum_{i=0}^N a_i T_i(y) \text{ and } \hat{\theta}(y) = \sum_{i=0}^N b_i T_i(y) \quad (39)$$

Where T_i is the Chebyshev polynomial of degree i , which is defined as?

$$T_i(\cos \theta) = \cos i\theta \quad (40)$$

The variables (a_i) and (b_i) denote the projection coefficients, while (N) denotes the truncation order, as in the notation above, indicating the total number of Chebyshev polynomials included in the expansion.

$$A_{11} = (U_b - \frac{i}{\alpha DaRe})(D^2 - \alpha^2) + \frac{i}{\alpha Re}(D^2 - \alpha^2)^2 + 2\frac{d^2 U_b}{dy^2} - \frac{iM^2}{\alpha Re}D^2 \quad (44)$$

$$A_{12} = -i\alpha Ri, A_{21} = \frac{1}{i\alpha} \frac{d\theta_b}{dy}, A_{22} = U_b + \frac{i}{\alpha PrRe}(D^2 - \alpha^2) + \frac{iRd}{\alpha PrRe}D^2 \quad (45)$$

$$B_{11} = [I - E(D^2 - \alpha^2)](D^2 - \alpha^2), B_{22} = I, B_{12} = B_{21} = 0 \quad (46)$$

The collocation method is executed at the Gauss–Lobatto [47] points, which are characterized as follows:

$$y_i = \cos(\frac{\pi i}{N}), -1 \leq y \leq 1 \text{ and } i = 0, \dots, N \quad (41)$$

Following the aforementioned protocol, the boundary conditions in Eqs. (36)– (38) are applied at the extremities of the collocation nodes. Inserting the expansion presented in Eq. (39) into Eqs. (36)– (38) and assessing the resultant expressions at the collocation sites produces the generalized eigenvalue problem.

$$A\Psi = CB\Psi \quad (42)$$

$\Psi = (a_0, a_1, \dots, a_N, b_0, b_1, \dots, b_N)^T$ represents the eigenvector, whereas C denotes the eigenvalue. The matrices A and B are specified to be of dimensions $2(N + 1) \times 2(N + 1)$.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \text{ and } B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad (43)$$

Matrix A and matrix B are both composed of block matrices of dimensions $(N + 1) \times (N + 1)$ that are specified by:

We analyze the convergence of the numerical solution by incrementally increasing the truncation order until fluctuations in the principal eigenvalues are minimal. This method ensures sufficient spectral resolution and improves the reliability of the stability assessments. The derived projection coefficients characterize the disturbance fields and enable precise reconstruction of the associated eigenfunctions over the computational domain.

VI. VALIDATION OF NUMERICAL CODE

The precision and dependability of the established numerical method are confirmed by juxtaposition with previously published benchmark data. Table 1 compares the calculated complex eigenvalues with the reference outcomes under the same flow conditions and parameter values. The strong alignment between the two datasets demonstrates the reliability of the numerical formulation and validates the convergence of the chosen computational method. This validation instills confidence in using the current methodology to investigate the impacts of Reynolds number, magnetic field, thermal radiation, porous resistance, and viscoelasticity on flow stability.

Table 1 Numerical Comparison of the Dominant Instability Mode for Selected Reynolds and Wave Numbers.

Reynolds Number (Re)	Wave Number (α)	[48], $C = C_r + iC_i$	Present Study, $C = C_r + iC_i$
10,000	1.000	0.23752649 + 0.00373966i	0.27752649 + 0.00373967i
20,000	1.095	0.215981 – 0.00412403i	0.215981 – 0.00412404i

The numerical reliability of the current stability assessment is also supported by the comparisons shown in Tables 2 and 3. The anticipated critical Reynolds numbers and associated critical wave numbers align closely with reported benchmark values under the same limiting conditions. This agreement shows that the computational

model effectively detects the onset of instability. Furthermore, the validation confirms that the spectrum discretization, numerical execution, and established boundary conditions provide sufficiently precise results for the subsequent examination of various controlling factors.

Table 2 A Comparative Evaluation of the Key Reynolds and Wave Numbers from the Studies by [49], and the Current Research.

E	α [50]	Re [51]	α [52]	Re [53]	α (Current Work)	Re (Current Work)
0	1.0205	5772.2218	1.0205	5772.2218	1.0205	5772.2218
10^{-10}	—	—	1.0205	5772.6550	1.0205	5772.6548
10^{-8}	1.0205	5772.65470	1.0205	5773.0793	1.0205	5773.0784
10^{-6}	1.019037	5815.883702	1.0190	5816.1701	1.0190	5816.5049
10^{-5}	1.0049	6246.32355	1.0049	6246.8426	1.0049	6246.5587

Table 3 Evaluation of the Critical Reynolds Number and Wavenumber Across Various Permeability Parameter Values.

S	α [54]	Re [55]	C_r [56]	α (Current work)	Re (Current work)	C_r (Current work)
0.0	1.0205	5772.2218	0.263999	1.02052	5772.2283	0.263997
0.2	1.0180	6015.5071	0.258340	1.01781	6015.0334	0.258313
0.4	1.0108	6787.7731	0.242404	1.01074	6787.3070	0.242499
0.6	0.9997	8228.0747	0.219401	0.99966	8227.4284	0.219445
0.8	0.9861	10587.966	0.192761	0.98608	10586.3898	0.192782

The convergence outcomes in Table 4 further assess the numerical precision of the spectral method used. Increasing the number of collocation points yields increasingly small variations in the computed eigenvalue, indicating that the solution becomes independent of spatial resolution. This

consistent behavior indicates satisfactory convergence of the Chebyshev discretization and confirms that the chosen numerical resolution is sufficient for reliable stability forecasts.

Table 4 Convergence Assessment of the Dominant Unstable Mode Using the Chebyshev Approximation at Re = 10,000, $\alpha = 1$, M = E = 0, S = 0, Ri = Rd = 0, and Pr $\rightarrow \infty$.

Number of Collocation Points, N	Complex Phase Velocity, $C = C_r + iC_i$
10	0.23713751 + 0.00563644i
20	0.23752676 + 0.00373427i
30	0.23752648 + 0.00373967i
40	0.23752649 + 0.00373967i
50	0.23752649 + 0.00373967i
60	0.23752649 + 0.00373967i

VII. RESULTS AND DISCUSSION

A methodical parametric analysis is conducted to elucidate the impact of porous resistance, magnetic phenomena, buoyancy, viscoelastic properties, thermal radiation, and thermal diffusion on the flow's linear stability characteristics. The examination primarily focuses on fluctuations in the Reynolds number and wave number, while individual control parameters are varied to assess their distinct effects on instability onset. The neutral stability boundary serves as a useful standard for distinguishing stable from unstable flow conditions. In contrast, the imaginary part of the complex phase velocity indicates whether small disturbances amplify or decay over time. The Darcy numbers $Da = 0.045, 0.048, 0.13$, and 1 are examined to investigate variations related to porous-media permeability. The Study of magnetic effects is conducted by analyzing Hartmann numbers $M = 0, 5, 8$, and 101 , allowing assessment of flow responses to varying magnetic-field intensities. Similarly, Ri values of $0, 0.1, 0.2$, and 0.3 are used to assess the impact of buoyancy compared with forced convection. Viscoelastic properties are defined by Kelvin–Voigt parameters $E = 0, 10^{-7}, 10^{-6}$, and 10^{-5} to evaluate the impact of increased elastic response on disturbance progression. Radiative heat transfer is analyzed for Rd values of $3, 6, 9$, and 12 , with Pr values of $6.2, 8.2, 10.2$, and 12.2 used to assess the influence of momentum and thermal diffusivity. Furthermore, wave numbers of $1, 1.02, 2$, and 3 are deemed essential for capturing disturbances with varying spatial attributes. Together, these modifications provide a comprehensive framework for detecting changes in key stability conditions and analyzing the interplay among hydrodynamic, porous,

magnetic, thermal, buoyancy, and viscoelastic processes within the interconnected system [57].

➤ Effect of the Darcy Number Da (Figs. 4 and 5)

Fig. 5(a) and Fig. 5(b) show how the Darcy number affects the system's stability characteristics. As shown in Fig. 5(a), increasing Da shifts the neutral stability boundary and reduces the Reynolds-number interval over which disturbances undergo temporal amplification. The shift of the neutral curves to higher Reynolds numbers indicates that more inertial forces are necessary to trigger instability as Da rises. This reply emphasizes the relationship among permeability, momentum transfer, viscous influences, and disturbance enhancement in the porous medium. As wave numbers increase, the stability curves converge, indicating a steady reduction in the flow's sensitivity to Da .

Additionally, Fig. 5(b) shows a more complex stability response in buoyancy-assisted scenarios. For $Ri = 0.01$, fluctuations in Da may cause consecutive shifts from a stable condition to an unstable phase, followed by destabilization. This behavior illustrates the competition between porous-media effects and buoyancy-enhanced shear processes. Other studies have documented comparable stability patterns linked to the Darcy number [58]. These findings show that permeability strongly influences disturbance evolution and defines the key criteria for instability. Consequently, regulating the Darcy number may improve flow management, reduce variations, and increase operational efficiency in porous heat-transfer systems, geothermal operations, thermal-energy storage designs, and related engineering applications.

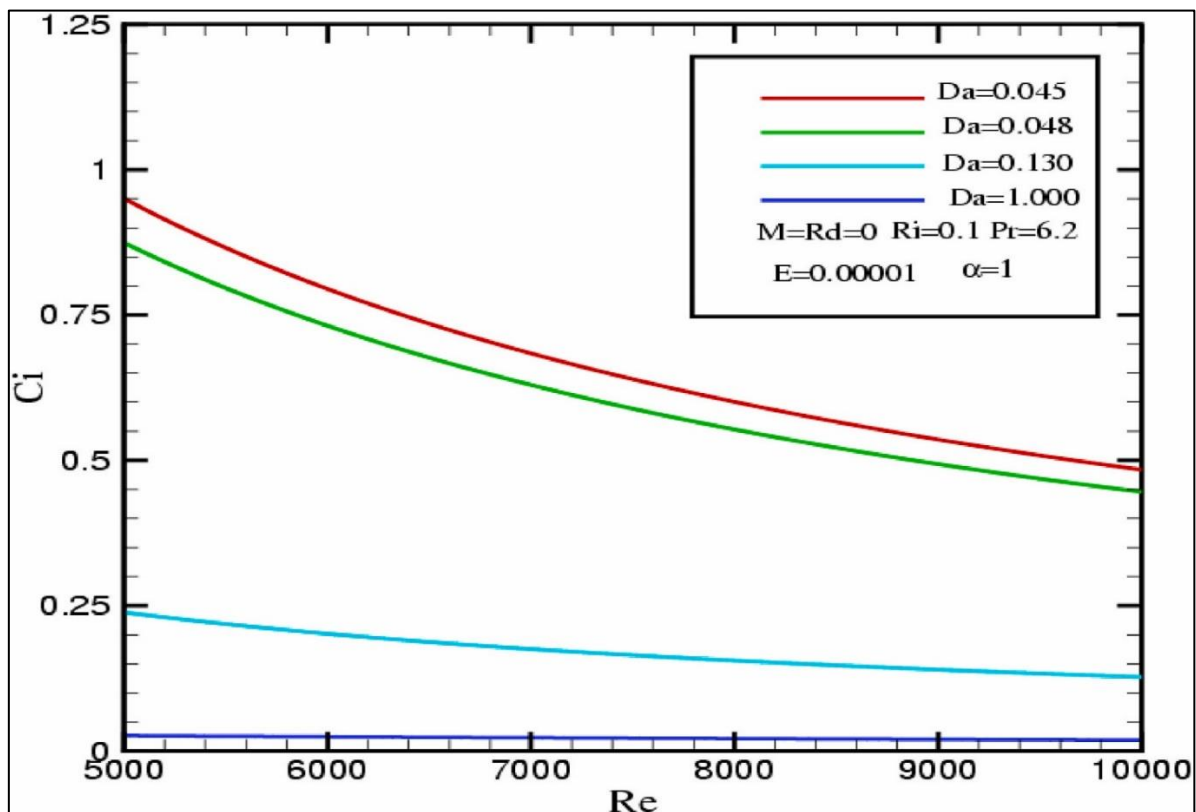


Fig 4 Influence of the Darcy Number on the Rate of Expansion.

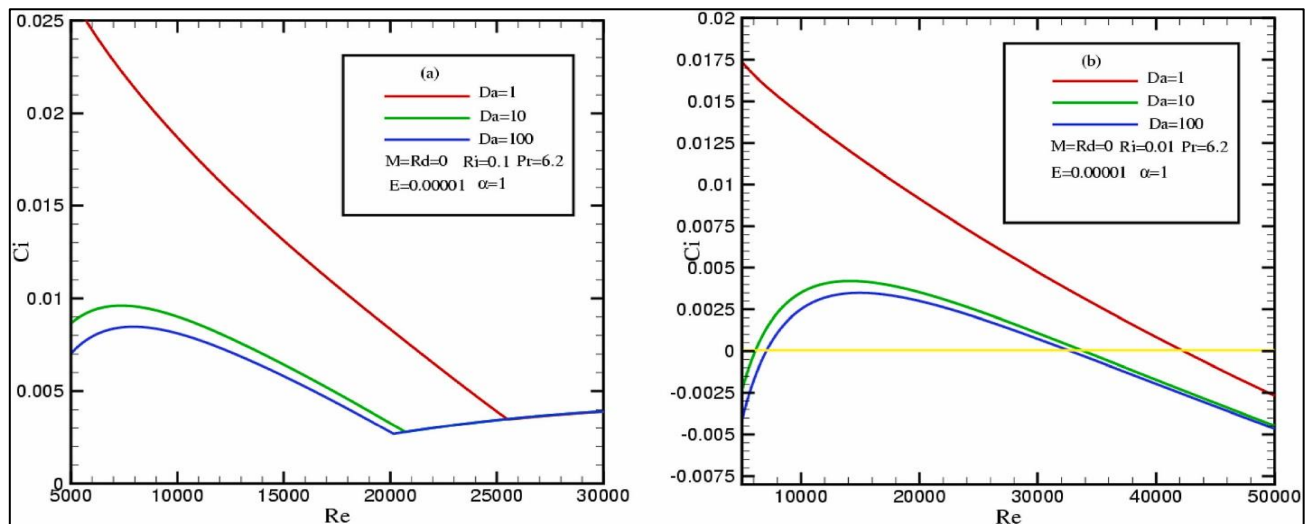


Fig 5 The Impact of the Darcy Number on the Rate of Increase.

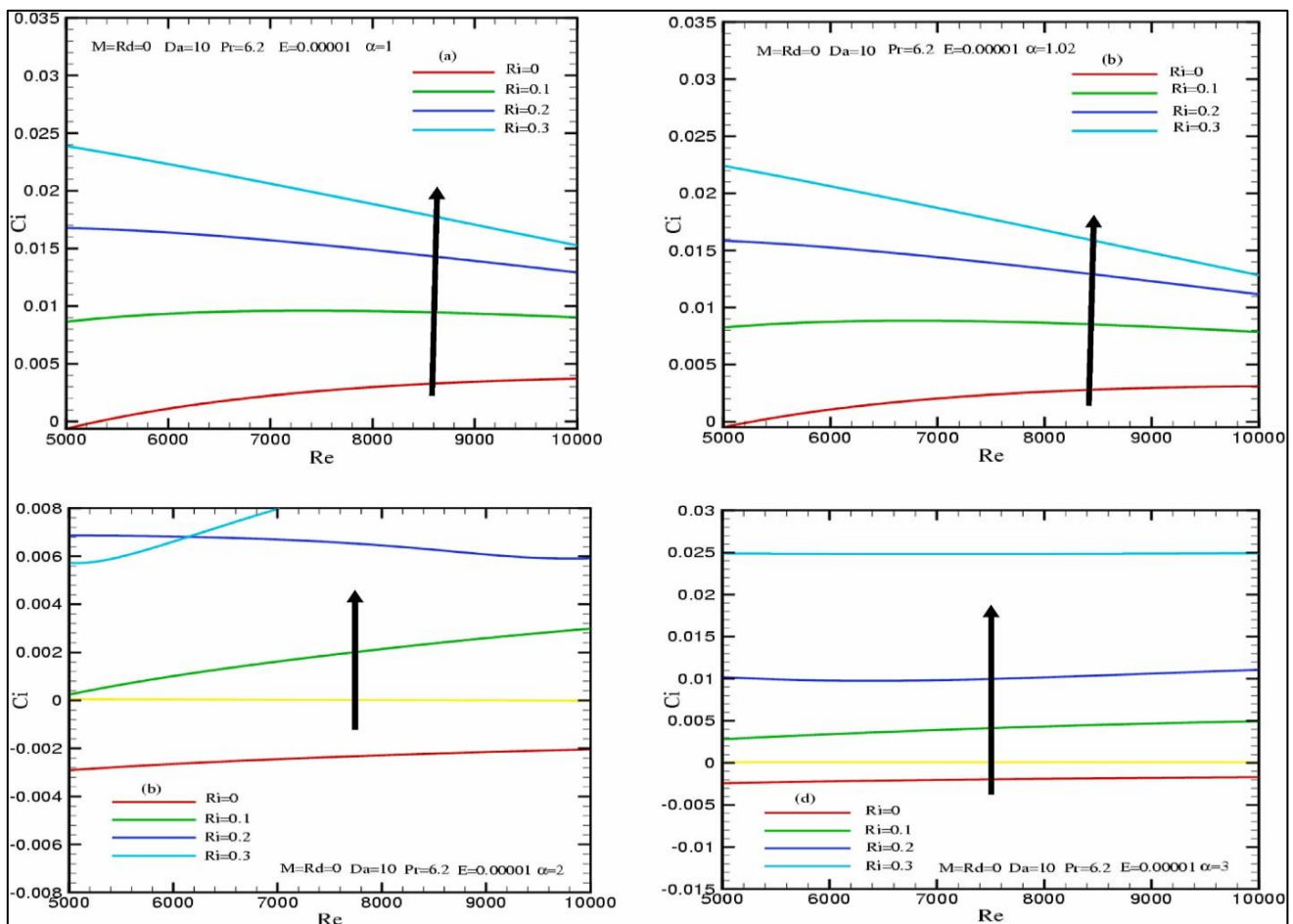


Fig 6 Influence of the Richardson Number on the Rate of Growth.

Fig. 4 shows the impact of buoyancy on the stability response under the reference conditions outlined in the accompanying description. The system shows either a fully stable state or a limited unstable region, depending on the wave number. At lower wave numbers, increasing the Reynolds number broadens the unstable spectrum and increases the disturbance growth rate C_i , thereby lowering the instability initiation threshold. This conduct suggests that buoyancy may enhance convective movements that interact

with the basal shear flow. At elevated wave numbers, the effect persists; however, changes in the neutral stability boundary diminish and are confined to a narrower segment of the spectrum.

The findings shown in Fig. 4 plainly indicate a reliance on the Richardson number. In some parameter combinations, the flow remains stable at $Ri = 0$ but becomes unstable as Ri increases, whereas other scenarios show instability over the

entire examined range of Ri . Overall, the rise in C_i with Ri illustrates buoyancy's destabilizing effect, aligning with the findings of [59]. As Richardson numbers increase, buoyancy effects become more pronounced than shear, fostering more intense convective disturbances and the formation of thermal structures. In a Kelvin–Voigt viscoelastic fluid, the interplay between temperature fluctuations and elastic properties may also alter their temporal progression. Such instability may lead to unfavorable variations in flow and temperature inside actual systems, such as heat exchangers, anaerobic digesters, and thermal-energy storage units. Consequently, understanding the impact of Ri is crucial for ensuring controlled heat transfer, stable operation, and reliable performance in mixed-convection systems.

➤ Effect of the Hartmann Number M (Fig. 7)

The stability characteristics shown in Fig. 7 further clarify the Hartmann number's influence on disturbance evolution in the electrically conducting viscoelastic fluid. At lower wave numbers, variations in M significantly affect the location and contour of the neutral stability barrier. In the presence of very weak magnetic influences, a limited unstable area appears, suggesting that perturbations may gain sufficient energy from the base flow to undergo temporal amplification. As M rises, this unstable area progressively diminishes, and the associated disturbance growth rate C_i declines. Thus, a higher Reynolds number could be necessary to maintain unstable modes, illustrating the stabilizing effect of the applied magnetic field. In certain parameter combinations, Fig. 7 illustrates sequential transitions among stable, unstable, and destabilized states, demonstrating the

interplay of inertial, viscous, buoyancy, porous, and electromagnetic forces. At higher wave numbers, the stability curves become tightly clustered or almost intersect, indicating that the effect of M on short-wavelength perturbations diminishes significantly.

This stabilization is largely due to the Lorentz force produced by the interplay between the imposed magnetic field and the conductive fluid. This force opposes fluid movement and generates electromagnetic resistance that dissipates kinetic energy from escalating disturbances. As magnetic strength increases, lateral fluid motion is constrained, velocity variations diminish, and the development of pronounced shear gradients and vortical structures is curtailed. These effects reduce the energy available for modal amplification, thereby promoting a more stable flow arrangement. The stabilizing tendency seen in Fig. 7 aligns with earlier documented results regarding magnetically driven convection. From an engineering standpoint, magnetic control provides an effective way to manage electrically conductive fluxes without physical mechanical involvement. Proper calibration of M may help mitigate undesirable oscillations, reduce localized thermal fluctuations, and maintain more consistent transport properties. These benefits are especially relevant to porous thermal-storage systems, geothermal heat-exchange apparatus, catalytic reactors, and other magnetohydrodynamic energy systems that require steady, interlinked momentum and heat transfer for reliable operation and improved process control.

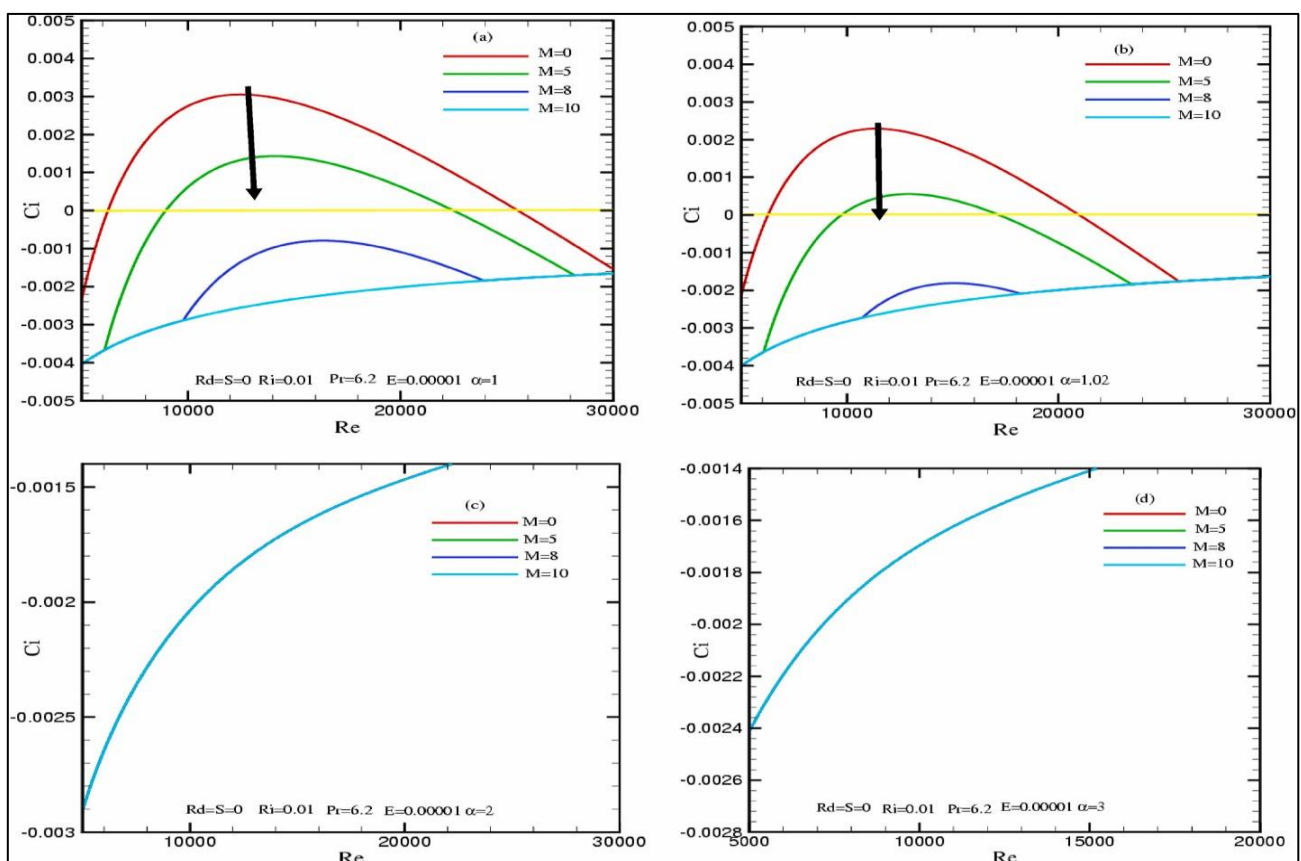


Fig 7 Influence of the Hartmann Number on the Pace of Growth.

➤ *Effect of the Thermal-Radiation Parameter R_d (Fig. 8)*

The stability patterns in Fig. 8(a–d) show that the thermal radiation parameter, R_d , strongly influences the evolution of disturbances in the mixed-convection system. At low wave numbers, as seen in Fig. 8(a) and (b), the stability spectrum reveals a distinctive pattern of stability, instability, and eventual restabilization. Fluctuations in R_d cause only minor changes in the peak value of C_i , but many neutral stability curves remain tightly clustered. This behavior suggests that long-wavelength fluctuations are only weakly influenced by radiative heat exchange under these conditions. Nonetheless, increasing R_d progressively reduces the unstable region, indicating a gradual attenuation of the thermal processes that enhance disturbances. A stronger effect appears at higher wave numbers in Fig. 8(c) and (d), where increasing R_d markedly reduces C_i and accelerates disturbance attenuation across the examined Reynolds-number spectrum. Enhanced thermal radiation physically reallocates energy in the fluid-saturated porous media and

diminishes pronounced temperature gradients that may amplify buoyancy-driven convection. The resulting more uniform thermal field reduces the energy available to developing thermal disturbances and lowers the likelihood of forming strong convective structures. This radiative stabilization works with porous resistance and viscoelastic dissipation, creating a synergistic damping effect on developing disturbances. As R_d grows, the system therefore approaches a more consistent and stable thermal-flow arrangement. This has practical significance because extreme temperature gradients can induce localized hot spots, oscillatory flow, and thermal strains in engineering apparatus. Consequently, effective regulation of radiative heat transfer could improve temperature consistency, reduce disturbance propagation, and bolster the operational reliability of porous heat exchangers, thermal storage systems, geothermal installations, chemical reactors, and various thermal processes that incorporate mixed convection.

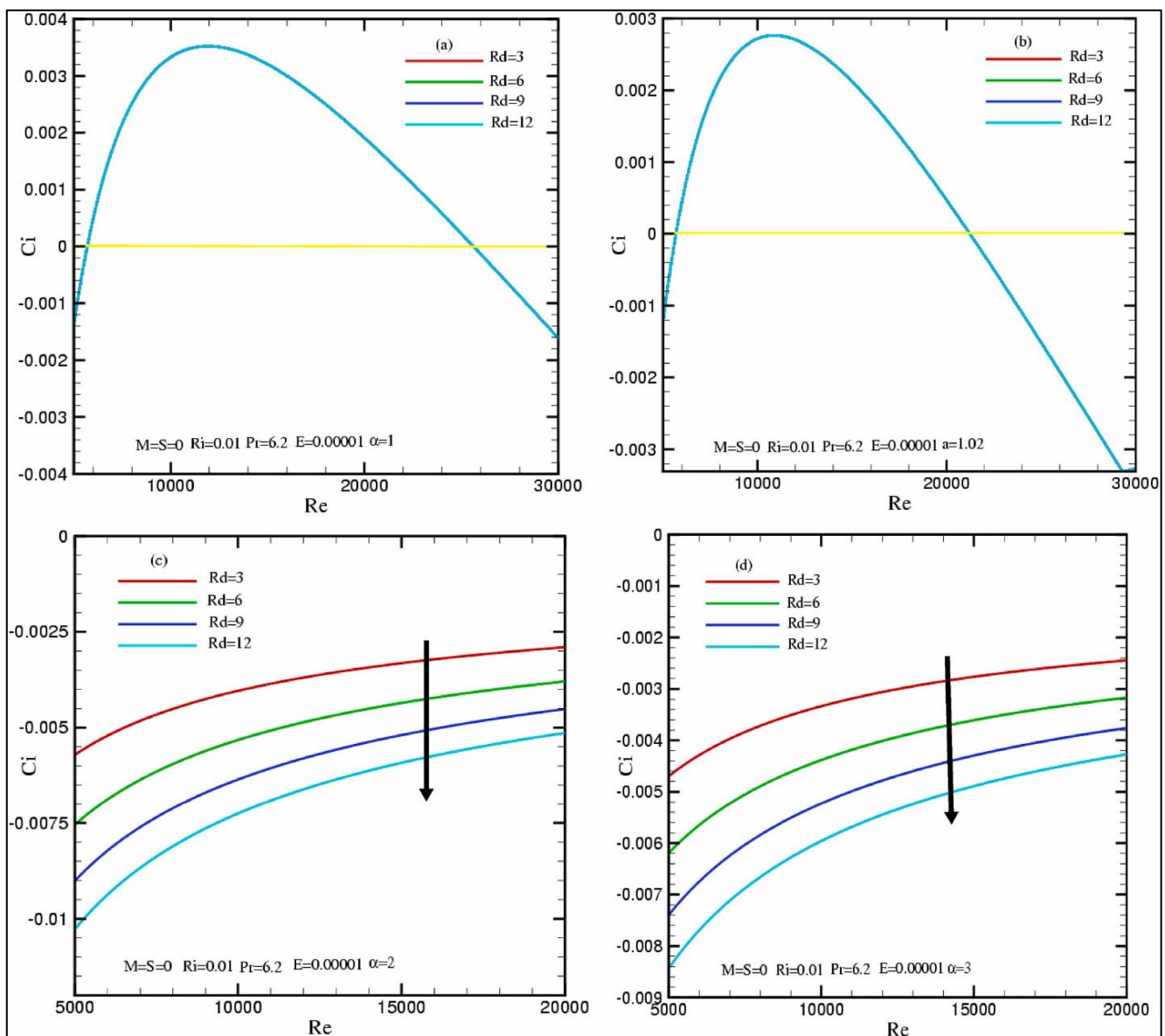


Fig 8 Influence of Heat Radiation Parameter on Growth Rate.

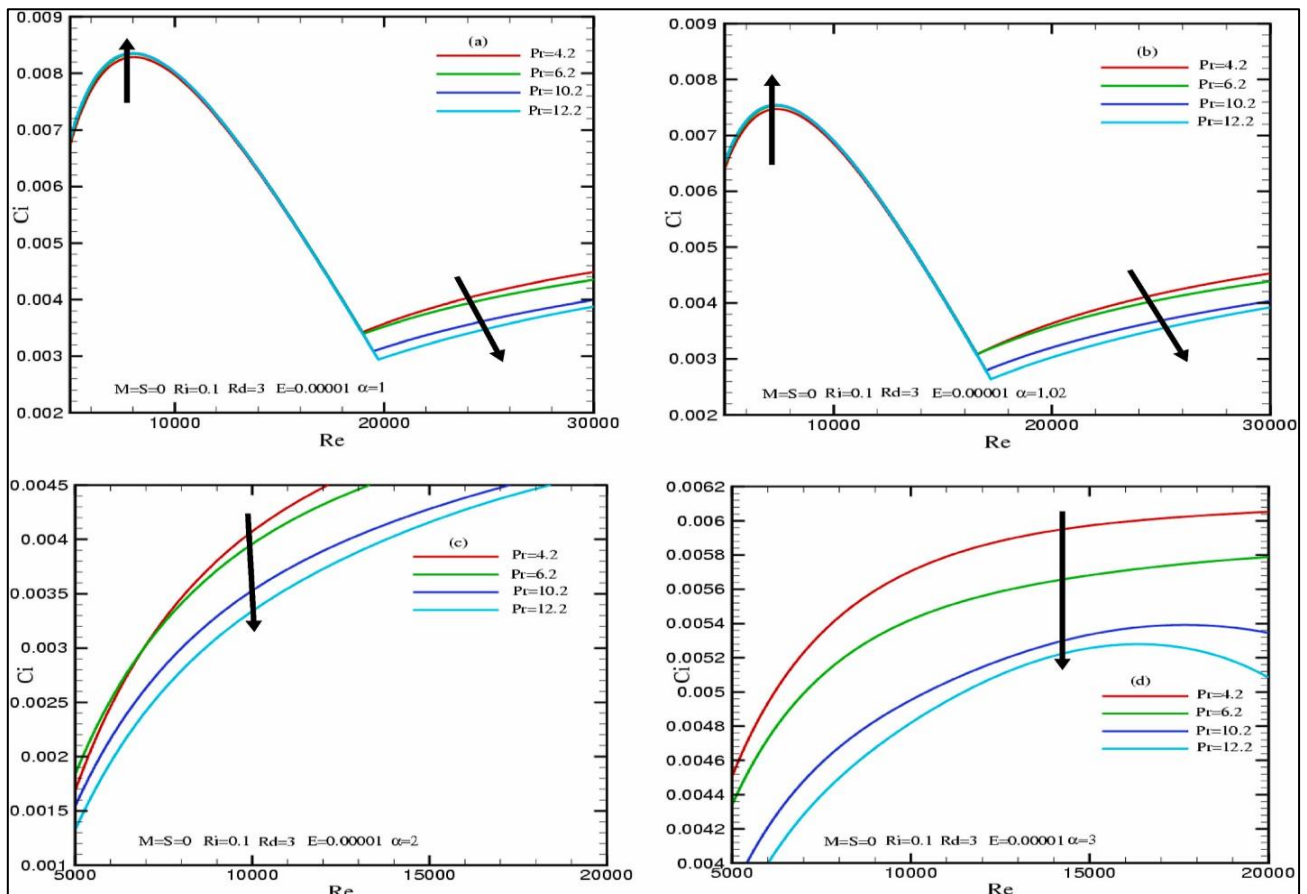


Fig 9 How the Prandtl Number Influences the Pace of Growth.

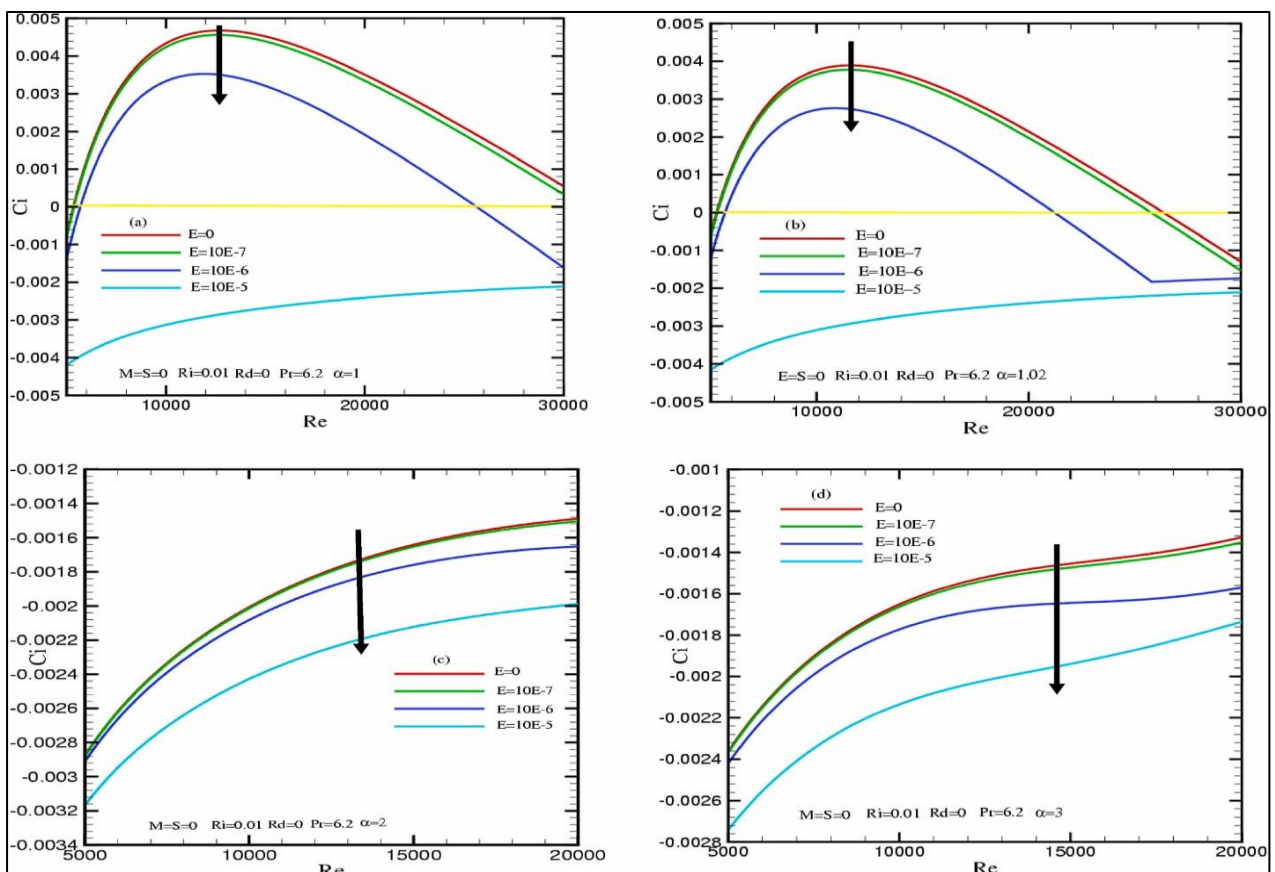


Fig 10 Influence of the Number of Kelvin-Voigt Parameters on the Rate of Growth.

➤ Effect of the Prandtl Number Pr (Fig. 9)

The stability characteristics shown in Fig. 7 indicate that the Prandtl number strongly affects disturbance evolution, but its impact varies with wave number. At lower wave numbers, variations in Pr may amplify or diminish C_i , reflecting a competition among heat diffusion, momentum diffusion, buoyancy, and shear. The proximity of several neutral stability curves indicates that the flow has restricted sensitivity to Pr under certain circumstances. As the wave number escalates, the reaction tends to stabilize, and a higher Pr typically reduces the rate of disturbance accumulation. This decrease in C_i signifies less temporal amplification and a gradual transition to a more stable flow regime. The fluctuating Reynolds-number range may also contract as diffusive influences progressively limit disturbance evolution. Consequently, the stability response cannot be ascribed solely to Pr , but depends on its interaction with the specific disturbance scale and Reynolds number. These findings underscore the significance of Pr in regulating the interrelated momentum and heat transmission in porous viscoelastic flows, especially when buoyancy and shear forces act concurrently.

➤ Effect of the Kelvin–Voigt Parameter E (Fig. 10)

Fig. 8 shows the impact of the Kelvin–Voigt parameter, E , on the stability attributes. At comparatively low wave numbers and near-zero E values, the neutral stability curve encloses a clearly defined unstable area within the spectrum. As E rises, the disturbance growth rate C_i decreases, steadily contracting the unstable zone. When E exceeds a certain threshold, this unstable pocket may vanish entirely, signifying the mitigation of the instability. As a result, the pattern of stability $\rightarrow$ instability $\rightarrow$ restabilization seen in weak viscoelasticity diminishes in prominence and may ultimately disappear with an increase in the elastic component. At elevated wave numbers, the system remains stable for the examined values of E , although the spectral curves further diminish as viscoelastic influences increase.

This behavior aligns with the Kelvin–Voigt model, where the viscoelastic reaction further counteracts rapid deformation. Increasing E limits disturbances linked to rapid fluctuations in strain rate and reduces the energy available to amplify perturbations. In the revised Orr–Sommerfeld framework, the viscoelastic component therefore adds extra damping that rapidly reduces fluctuating disturbances and constrains the unstable bandwidth.

The comprehensive parametric evaluation also illustrates the conflicting processes that dictate the system's stability. Within the examined parameter range, increasing M , Rd , Da , or E mitigates instability via magnetic damping, heat redistribution, porous-medium effects, and viscoelastic resistance, respectively, whereas increasing Ri enhances buoyancy-induced destabilization. Sensitivity to these characteristics often diminishes at higher wave numbers because dissipative and diffusive processes grow in importance. At lower wave numbers, energetic mixed-convection modes generate wider unstable zones and heightened parameter sensitivity. Numerous stability transitions arise when stabilizing influences contend with

shear-induced buoyancy, resulting in two neutral intersections in Re . As the stabilizing force intensifies, these intersections may converge, amalgamate, and eventually vanish, resulting in a stable spectrum. These stability maps therefore provide a valuable foundation for comprehending how various physical processes alter the key conditions for disturbance proliferation within the confines of linear stability analysis.

VIII. CONCLUSION

The current stability analysis provides a clearer understanding of the processes that regulate mixed convection in a Brinkman porous medium infused with a Kelvin–Voigt viscoelastic fluid, influenced by magnetic and radiative factors. The findings indicate that instability initiation is governed by a multifaceted interplay of inertial forces, buoyancy, momentum diffusion, porous-medium resistance, electromagnetic damping, thermal transport, and fluid elasticity. Rather than acting independently, these systems interact to determine the critical Reynolds number, the rate of disturbance growth, the unstable bandwidth, and the configuration of the neutral stability boundary. The numerical results show that changes in the Darcy number alter momentum transfer within the porous medium and generally stabilize the system across the examined parameter range. Conversely, increasing the Richardson number enhances buoyancy relative to forced convection, supplying more energy to amplify disturbances and fostering unstable mixed-convection patterns. The applied magnetic field creates a significant stabilizing force via the Lorentz effect. As the Hartmann number increases, electromagnetic damping constrains lateral fluid motion, reduces velocity fluctuations, and lowers the kinetic energy available for perturbation growth. Thermal radiation offers an alternative way to manage instability by redistributing thermal energy and reducing temperature gradients that may drive buoyancy-induced disturbances. Its stabilizing effect is especially apparent at higher wave numbers, where the unstable region gradually shrinks. The influence of the Prandtl number is intricate because it depends on the interplay between momentum and thermal diffusion. At low wave numbers, reactions may either stabilize or destabilize depending on the Reynolds number range, whereas a higher Prandtl number often promotes stability at higher wave numbers. Viscoelasticity strongly affects disturbance evolution. Increasing the Kelvin–Voigt parameter increases resistance to rapid deformation and reduces the growth of temporal disturbances, ultimately contracting or eliminating unstable zones under sufficiently strong viscoelastic influence. The emergence of stability–instability–restabilization processes at certain parameter combinations further highlights the conflicting nature of the physical mechanisms at play. As stabilizing forces intensify, the related neutral crossings may converge and eventually disappear, resulting in a completely stable spectrum within the examined range. These findings validate the efficacy of linear stability analysis in identifying pivotal operational parameters before disturbances escalate substantially. The integrated mathematical and numerical framework may therefore facilitate the understanding of transport dynamics in fluid-saturated porous systems that

include several interconnected physical processes. This expertise is relevant to advancing geothermal heat-transfer apparatus, porous thermal-energy storage units, chemical and catalytic reactors, heat exchangers, and related thermal technologies where mitigating unwanted flow oscillations, maintaining temperature consistency, and ensuring reliable heat-transfer performance are critical.

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