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Theoretical Investigation of Intrinsic and Extrinsic Plasmon Effects in the X-Ray Spectra of Aluminum

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Abstract: The origin of non-diagram X-ray satellite structures in aluminum (Al) is investigated theoretically by extending the Bohm-Pines Hamiltonian within a many-body framework. By rigorously separating the contributions of intrinsic and extrinsic plasmon-coupling mechanisms, explicit expressions for the non-diagram-to-parent-diagram X-ray line yield ratios and relative intensities are derived. The calculated energy separations (ΔE) and relative intensity profiles (I/I_0) for the high-energy plasmon satellites associated with the $K\alpha$ emission spectra show exceptional concordance with the experimental findings reported by Mauron and Dousse. These results confirm that collective plasma oscillations play a dominant role in governing the satellite line shapes and intensity distributions of light metals.

Keywords: *Bohm-Pines Hamiltonian, Many-Body Theory, Bulk Plasmon, Surface Plasmon, Non-Diagram Lines, Relative Intensity.*

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I. INTRODUCTION

Characteristic X-ray spectra comprise two distinct categories of spectral features: conventional diagram lines and non-diagram lines, commonly referred to as satellites or second-order lines [1-2]. While diagram lines correspond precisely to electronic transitions within standard atomic energy level models, non-diagram lines deviate from this conventional framework [3-4]. Typically characterized by weak intensities, satellites emerge immediately adjacent to more prominent parent lines, classified as high-energy satellites (HES) when they appear on the higher-energy side or low-energy satellites (LES) when observed on the lower-energy side [5-6]. The pioneering identification of these satellite features in K-spectra covering elements from Chromium ($Z = 24$) to Germanium ($Z = 32$) was conducted by Siegbahn and Stenstroem, with subsequent observations extended to L-spectra by Coster and Richtmyer, and to M-spectra by Hjalmar, Hindberg, and Hirsch.

Various theoretical models have been advanced to elucidate the genesis of these satellite emissions [7-8]. Among them, plasmon theory originally formulated by Bohm and Pines and later generalized by Houston, Ferrell, Noziers, and Pines is widely regarded as the most robust framework. According to this model, low-energy plasmon satellites are emitted when a valence electron excites a collective plasma oscillation during the annihilation of a core hole, whereas high-energy satellites occur when a pre-existing plasmon's energy is superimposed onto that of the primary diagram line [9-14].

In recent decades, the investigation of high-energy electronic excitations in condensed matter has experienced substantial advancements, driven by both experimental and theoretical breakthroughs [15-17]. Advanced spectroscopic analysis and rigorous theoretical models are crucial for elucidating the fundamental properties of materials, spanning both bulk systems and surface regions [18-22]. The theoretical framework employed in this study investigates the genesis of non-diagram X-ray spectra arising from excitation mechanisms involving both core and surface electrons. Specifically, by extending the Bohm-Pines Hamiltonian within a many-body theoretical framework, we evaluate the distinct contributions of intrinsic and extrinsic plasmon-coupling mechanisms [23-29]. This approach enables the quantitative derivation of non-diagram-to-parent-diagram X-ray line yield ratios for high-energy plasmon satellite profiles

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associated with the $K\alpha$ main lines in aluminum (Al) [30-32]. Our findings demonstrate that the experimental high-energy X-ray satellite line shapes reported by Mauron and Dousse originate directly from collective plasmon oscillations [33]. A comprehensive comparison between our theoretical predictions and the relative satellite yields obtained by these authors exhibits exceptional concordance, thereby validating the proposed microscopic mechanism.

II. NON-DIAGRAM-TO-PARENT-DIAGRAM X-RAY LINE YIELD RATIO (RELATIVE INTENSITY) IN X-RAY EMISSION SPECTRUM

In summary, when the primary diagram line is normalized to unit intensity, the relative yield of the n-th plasmon loss peak corresponds directly to the coefficient of x^n in the series expansion of the combined interaction function:

$$e^{\beta x}(1-\alpha x)^{-1} \quad (1)$$

Within this formulation, the exponential prefactor accounts for intrinsic excitation mechanisms, whereas the algebraic term within the parentheses characterizes the extrinsic contribution. Consequently, the net contribution of both intrinsic and extrinsic plasmon-coupling pathways—expressed as the relative intensity (α_n) of high-energy plasmon satellites in the X-ray emission spectrum—is evaluated using established theoretical formalisms.

$$\frac{I}{I_0} = \alpha^n \sum_{m=0}^{n} \frac{(\beta/\alpha)^m}{m!} \quad (2)$$

The dimensionless coupling parameters β and α (where $\alpha = K_C/K_F$) are defined as functions of the inter-electronic spacing parameter r_s:

$$\beta = 0.12\, r_s \quad (3)$$

$$\alpha = 0.47\, r_s^{1/2} \quad (4)$$

Here, r_s represents the dimensionless inter-electronic spacing parameter in units of the Bohr radius, defined respectively for bulk and surface plasmons by:

$$r_s = \left(\frac{47.11}{\hbar\omega_p}\right)^{2/3} \quad \text{(for bulk plasmons)} \quad (5)$$

$$r_s = \left(\frac{47.11}{\hbar\omega_s}\right)^{2/3} \quad \text{(for surface plasmons)} \quad (6)$$

The bulk plasmon energy ($\hbar\omega_p$) of a metallic system is conventionally expressed as:

$$\hbar\omega_p = 28.8\left(\frac{Z'\rho}{W}\right)^{1/2} \quad (7)$$

In this relation, Z' denotes the effective number of valence or conduction electrons participating in collective plasma oscillations, ρ represents the mass density (specific gravity), and W corresponds to the molecular weight. Furthermore, adhering to standard electrodynamic formalisms for planar specimen geometries, the corresponding surface plasmon energy ($\hbar\omega_s = \frac{\hbar\omega_P}{\sqrt{2}}$) simplifies in accordance with boundary conditions.

III. RESULTS

Table 1 Energy - Separation (ΔE) for low & high energy plasmon satellites of Copper (Cu)

Sr. No.	Satellites of Al ($K_\alpha L^{(1)}$)	Z'	σ	W	Author's calculated values of ΔE (eV)		Observed energy separation by Mauron and Dousse (ΔE), eV
					ΔE = $\hbar\omega_p$ eV	**ΔE** = $\hbar\omega_s$ eV	
1.	I	1	2.70	26.9815	-	6.44	6.5
2.	II	1	2.70	26.9815	9.11 ($\hbar\omega_p$)	-	10.2
3.	III	1	2.70	26.9815	-	12.88 ($2\hbar\omega_s$)	12.2

Table 2 Relative Intensity 'I/I$_o$' of Low-Energy $K_{\beta'}$ & High-Energy $K_{\beta 2''}$ Satellites in K_β X-ray spectrum of Copper (Cu)

Sr. No	Satellites of Al ($K_\alpha L^{(1)}$)	r_s= (47.11/ΔE eV)$^{2/3}$	β= 0.12 r_s	α= 0.47 (r_s)½	Intensity Assignment	'I/I$_o$' in present work	I/I$_o$ by Mauron and Dousse	
							Obs.	Calc.
						Calc.		
1.	**I:1492.8 eV**	3.7683	0.4522	0.9124	$\beta^3/6\alpha^2$	0.019	0.016	0.010
2.	**II: 1496.5 eV**	2.9904	0.3588	0.8128	$\beta^3/6\alpha^2 + \beta^2/2\alpha$	0.091	0.088	0.095
3.	**III:1498.5 eV**	2.3739	0.2849	0.7242	$\beta^2/2\alpha$	0.056	0.050	0.040

IV. CONCLUSION

In conclusion, this study successfully investigates the origin of high-energy non-diagram X-ray satellite structures in aluminum by extending the Bohm-Pines Hamiltonian within a many-body theoretical framework. By rigorously evaluating the distinct contributions of intrinsic and extrinsic plasmon-coupling mechanisms, the non-diagram-to-parent-diagram X-ray line yield ratios and relative intensity profiles have been quantitatively derived. The theoretical findings demonstrate that the high-energy satellite line shapes observed in aluminum originate directly from collective bulk

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and surface plasma oscillations. Furthermore, a comprehensive comparison between the calculated values and the experimental data reported by Mauron and Dousse [33] exhibits exceptional concordance, thereby validating the proposed microscopic mechanism and underscoring the critical role of electron-electron interactions in X-ray emission spectra.

➢ *Author Contribution:*

Original draft writing, methodology, and conceptualization: Gyandeep Tripathi, Vipin Sharma, Editing: Prince, Supervision: Rajeev Kumar Srivastava and Lalit K. Dwivedi.

DECLARATIONS

- Competing interests: The authors declare no competing interests.
- Ethics Approval: This theoretical study does not require ethical approval.
- Conflict of Interest: The authors declare no competing interests.

REFERENCES

[1]. C. Denton, J.L. Gervasoni et al., "Plasmon Excitation by Charged Particles Moving Near A Solid Surface", Phys. Rev. A57, N6, 4498 (1998).

[2]. M. Polasik, "Influence of changes in the Valence Electronic Configuration on the Kβ - To - Kα X-Ray Intensity Ratios of 3d Transitions Metals", Phys. Rev. A58, 1840 (1998).

[3]. R.C. Shiell et al., "Characteristics of Correlation Satellites below 25eV in Xenon Probed by Pulsed-Field-Ionization-Zero-Kinetic-Energy Photoelectron Spectroscopy", Phys. Rev. A59, N4,2903 (1999).

[4]. C. Sternemann et al., "Evolution of the Germanium Kβ X-ray Satellites from Threshold to Saturation", Phys. Rev. A61, 020501 (R) (2000).

[5]. N. Stolterfoht et al., "Plasmon Production by the decay of hollow Ne atoms near an Al surface", Phys. Rev. A61, 052902 (2000).

[6]. O. Mauron et al., "*L*-shell shake processes resulting from 1*s* photoionization in elements $11 \leq Z \leq 17$", Phys. Rev. A 62, 062508 (2000).

[7]. R. Diamant et al., "Structure of the W X-ray Spectrum", Phys. Rev. A 63, 022508 (2001).

[8]. J.B. Greenwood et al., "Experimental Investigation of the Processes Determining X-ray Emission Intensities from Charge-Exchange Collisions" Phys. Rev. A 63, 062707 (2001).

[9]. L.G. Gerchikov, C. Guet and A.N. Ipatov, "Multiple Plasmons and Anharmonic Effects in Small Metallic Clusters", Phys. Rev. A66, 053202 (2002).

[10]. Alok Mishra et al., "Low energy plasmon satellite in X-ray excited Auger electron spectra of Ag, Pd, Rn, In, Sn & Sb", IJERA 2, N 2, 115-118 (2012).

[11]. M. Guzzo et al., "Plasmon Satellites in Valence-Band Photoemission Spectroscopy" Eur. Phys. J. B, 85:326 (2012).

[12]. C.T. Chantler et al., "Anomalous Satellite Intensity Discrepancy in Copper X-Ray Lines", Phys. Rev. A85, 032513 (2012).

[13]. C.T. Chantler et al., "High-Accuracy Reconstruction of Titanium X-Ray Emission Spectra, Including Relative Intensities, Asymmetry and Satellites, and Ab Initio Determination of Shake Magnitudes for Transition Metals" J. Phys. B: At. Mol. Opt. Phys. 46, 015002 (2013).

[14]. J. Lischner et al., "Physical Origin of Satellites in Photoemission of Doped Graphene: An Ab Initio GW Plus Cumulant Study", Phys. Rev. Lett. 110, 146801 (2013).

[15]. L. K. Dwivedi, "Plasmon Effects in Non-diagram X-ray Spectra of Ar", in Proceedings of National Conference on Recent Advances in Chemical and Materials Science (NCRACMS-15), MMMUT, Gorakhpur, U.P., 64-65 (23-24 Feb., 2015).

[16]. J. Lischner et al., "Satellite band Structure in Silicon Caused by Electron-Plasmon Coupling", Phys. Rev. B91, 205113 (2015).

[17]. Sameer Sinha et al., "High Energy Double Plasmon Satellites in Transition Metal", Journal of Progressive Science Vol. 11, No. 1&2, 43-46 (2020).

[18]. V. Sharma & L. K. Dwivedi, "Plasmons in Non-Diagram X-ray Spectra of Scandium and Titanium Compounds", International Journal of Scientific Research in Science and Technology (IJSRST) Vol.10, Issue 1, 276-281, (2023).

[19]. V. Sharma & L. K. Dwivedi, "Plasmon Effects in Non-Diagram X- Ray Spectra of Calcium (Ca) and Cobalt (Co)", International Journal of Scientific Research in Science and Technology (IJSRST) Vol. 10, Issue 3, 315-319 (2023).

[20]. A. Garg, V. Sharma et al., "Theoretical Determination of Intensity of Satellite Structure of the Tungsten (W) Lα1,2 X-Ray Spectrum", International Journal of Scientific Research in Science and Technology (IJSRST) Vol.11, Issue 6, 108-112 (2024).

[21]. Sanjeet P. Singh et al., "Low-energy Kα, Single Plasmon Satellites of Cr and Its Compounds", International Journal of Contemporary Research in Multidisciplinary Vol. 5(1), 146-149 (2026).

[22]. A. Garg, V. Sharma, Sameer Sinha et al., "Theoretical Determination of Satellite Intensity Discrepancy in Copper Kβ X-Ray Lines", International Journal of Innovative Science and Research Technology, 11(9), 3179-3182 (2026).

[23]. A.M. Bradshaw et al., "Plasmon Coupling to Core Hole Excitations in Carbon", J. Phys. C. Solid State Phys.7, 4503-4505 (1974).

[24]. D.C. Langreth, "Theory of plasmon effects in high energy spectroscopy", in Collective Properties of Physical Systems, edited by B. Lundqvist and S. Lundqvist, Academic Press, New York (1974).

[25]. 25. M. Deutsch, et al., "Kα and Kβ X-ray Emission Spectra of Copper", Phys. Rev. A51, N1, 283–296 (1995).

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26nov261

[26]. 26. J.J. Chang and D.C. Langreth, “Deep-Hole Excitations in Solids II. Plasmons and Surface Effects in X-Ray Photoemission”, Phys. Rev. B8, 4638 (1973).
[27]. W.J. Pardee et al., “Analysis of Surface- and Bulk-Plasmon Contributions to X-Ray Photoemission Spectra”, Phys. Rev. B11, 3614 (1975).
[28]. O. K. Harsh and B. K. Agarwal, “Double Plasmon Oscillations during X-ray Emission”, Physica B+C Vol.145, Issue 1, 45-49 (1987).
[29]. O.K. Harsh, “Surface Plasmon satellite in the X-ray emission spectra of Magnesium”, J. Phys. Chem. Solids 58, N9, 1433 (1997).
[30]. C. Biswas et al., “Plasmons in core-level Photoemission Spectra of Al (111)”, Phys. Rev. B67, 165416 (2003).
[31]. Sanjeet P. Singh et al., “Generalized Mechanism of Plasmon Excitations in Non-diagram X-ray spectra”, Indian J. Phys. 85, N 12, 1897-1904 (2011).
[32]. Mohammad Balal et al., “Intrinsic and Extrinsic Plasmons in the Hard X-Ray Photoelectron Spectra of Nearly Free Electron Metals”, Phys. Rev. B109, 205419 (2024).
[33]. O. Mauron, J.-Cl. Dousse, “Double KL ionization in Al, Ca, and Co targets bombarded by low-energy electrons”, Phys. Rev. A 62, 062508 (2000).