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Numerical Solution of the System of Differential Equations of the SEIHFR Compartmental Model of the Dynamics of the Ebola Epidemic by the Fourth-Order Runge-Kutta Method

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Abstract: This document presents the detailed numerical solution of the SEIHFR (Susceptible, Exposed, Infectious, Hospitalized, Funeral, Recovered) model using the fourth-order Runge- Kutta (RK4) method. This model, dedicated to the transmission dynamics of the Ebola virus, is formulated as a system of six nonlinear ordinary differential equations. The study covers the mathematical formulation of the problem, the general principle of the RK4 method, its step-by-step application to the system, and a concrete numerical example of the first time step. The complete algorithm is presented in pseudocode, and the numerical properties of the solution, including the conservation of the total population and the convergence error, are analyzed.

Keywords: *SEIHFR Model, Ebola, 4th Order Runge- Kutta, Numerical Solution, Ordinary Differential Equations, Mathematical Epidemiology, Numerical Simulation.*

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I. INTRODUCTION

In this article, we present the detailed numerical resolution of the epidemiological model SEIHFR (Susceptible, Exposed, Infectious, Hospitalized, Funeral, Recovered) is calculated using the fourth-order Runge- Kutta (RK4) method. This model, introduced to describe the transmission dynamics of the Ebola virus, is a system of six coupled nonlinear ordinary differential equations. The objective of this article is to provide a comprehensive description of the RK4 algorithm applied to our system, without skipping any computational steps. We will detail:

- The mathematical formulation of the problem
- The general principle of the RK4 method
- Step-by-step application to the SEIHFR system
- A concrete digital example of the first time step
- The numerical properties of the solution

- *Reminder of the SEIHFR Model*

- *System of Differential Equations*

The SEIHFR model presented in this study follows on from the seminal work of Legrand et al. [7] on modeling the transmission dynamics of the Ebola virus. This compartmental model, based on the general principles of epidemiological modeling described by Anderson & May [5], is governed by the following system:

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$$\begin{cases} \frac{dS}{dt} = -\lambda(t)S(t) \\ \frac{dE}{dt} = \lambda(t)S(t) - \sigma E(t) \\ \frac{dI}{dt} = \sigma E(t) - (\alpha_I + \theta + \delta_I)I(t) \\ \frac{dH}{dt} = \theta I(t) - (\alpha_H + \delta_H)H(t) \\ \frac{dF}{dt} = \delta_I I(t) + \delta_H H(t) - \gamma F(t) \\ \frac{dR}{dt} = \alpha_I I(t) + \alpha_H H(t) + \gamma F(t) \end{cases} \quad (1)$$

Where the infection strength $\lambda(t)$is defined by:

$$\lambda(t) = \beta_I \frac{I(t)}{N} + \beta_H \frac{H(t)}{N} + \beta_F \frac{F(t)}{N} \quad (2)$$

- *Model Parameters*

Table 1 summarizes the meaning and reference values of the parameters. The influence of these parameters on the basic reproduction number R0 was analyzed in detail by Chowell et al. [6].

Table 1 SEIHFR Model Parameters and Reference Values

Paramètre	Signification	Valeur	Unité
β_I	Transmission communautaire	0,25	jour^{-1}
β_H	Transmission nosocomiale	0,13	jour^{-1}
β_F	Transmission funéraire	0,40	jour^{-1}
σ	Taux d'incubation ($1/\sigma = 7$ jours)	$1/7 \approx 0,143$	jour^{-1}
θ	Taux d'hospitalisation	0,30	jour^{-1}
α_I	Taux de guérison (communauté)	0,10	jour^{-1}
δ_I	Taux de mortalité (communauté)	0,30	jour^{-1}
α_H	Taux de guérison (hôpital)	0,20	jour^{-1}
δ_H	Taux de mortalité (hôpital)	0,15	jour^{-1}
γ	Taux d'inhumation sécurisée ($1/\gamma = 2$ jours)	0,50	jour^{-1}
N	Population totale	100 000	individus

- *Initial Conditions*

$$\mathbf{Y}(0) = \mathbf{Y}_0 = \begin{pmatrix} S(0) \\ E(0) \\ I(0) \\ H(0) \\ F(0) \\ R(0) \end{pmatrix} = \begin{pmatrix} N-1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 99999 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

- *Vector Formulation*

To apply the numerical methods, we write the system in compact vector form.

Let the state vector be $Y(t) \in R^6$:

$$\mathbf{Y}(t) = \begin{pmatrix} S(t) \\ E(t) \\ I(t) \\ H(t) \\ F(t) \\ R(t) \end{pmatrix} \quad (4)$$

System (1) takes the canonical form of a Cauchy problem:

$$\boxed{\frac{d\mathbf{Y}}{dt} = \mathbf{f}(t, \mathbf{Y}), \quad \mathbf{Y}(0) = \mathbf{Y}_0} \quad (5)$$

Where the function $f : \mathbb{R} \, x \, \mathbb{R}^6 \rightarrow \mathbb{R}^6$is defined by its six components:

$$\mathbf{f}(t, \mathbf{Y}) = \begin{pmatrix} f_1(\mathbf{Y}) \\ f_2(\mathbf{Y}) \\ f_3(\mathbf{Y}) \\ f_4(\mathbf{Y}) \\ f_5(\mathbf{Y}) \\ f_6(\mathbf{Y}) \end{pmatrix} = \begin{pmatrix} -\lambda S \\ \lambda S - \sigma E \\ \sigma E - (\alpha_I + \theta + \delta_I)I \\ \theta I - (\alpha_H + \delta_H)H \\ \delta_I I + \delta_H H - \gamma F \\ \alpha_I I + \alpha_H H + \gamma F \end{pmatrix} \quad (6)$$

$$\text{avec } \lambda = \frac{\beta_I I + \beta_H H + \beta_F F}{N} \quad (7)$$

Note: The function f does not explicitly depend on time t. The system is said to be autonomous. Nevertheless, we retain the general notation f (t; Y) for the sake of compatibility with the standard formulation of Runge-Kutta methods.

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26may1726

➢ *The 4th-Order Runge-Kutta Method*

- *General Principle*

The fourth-order Runge-Kutta (RK4) method is a one-step numerical method for solving ordinary differential equations with high accuracy [1, 3]. It belongs to the family of Runge-Kutta methods, which generalize Euler's method by performing multiple evaluations of the function f within each time step.

- *Intuitive Idea*

Euler's explicit method approximates the solution using only the slope at the beginning of the interval:

$$\mathbf{Y}_{n+1} = \mathbf{Y}_n + h\,\mathbf{f}(t_n, \mathbf{Y}_n) \tag{8}$$

This approximation is only of order 1. To improve accuracy, RK4 uses a weighted average of four slope estimates calculated at different points in the interval [t_n ; t_{n+1}]:

- ✓ An estimate at the beginning of the interval (k1)
- ✓ Two estimates in the middle of the interval (k2 and k3)
- ✓ An estimate at the end of the interval (k4)

The slopes estimated in the middle of the interval receive double weight because they are more representative of the average slope over the interval.

- *Geometric Interpretation*

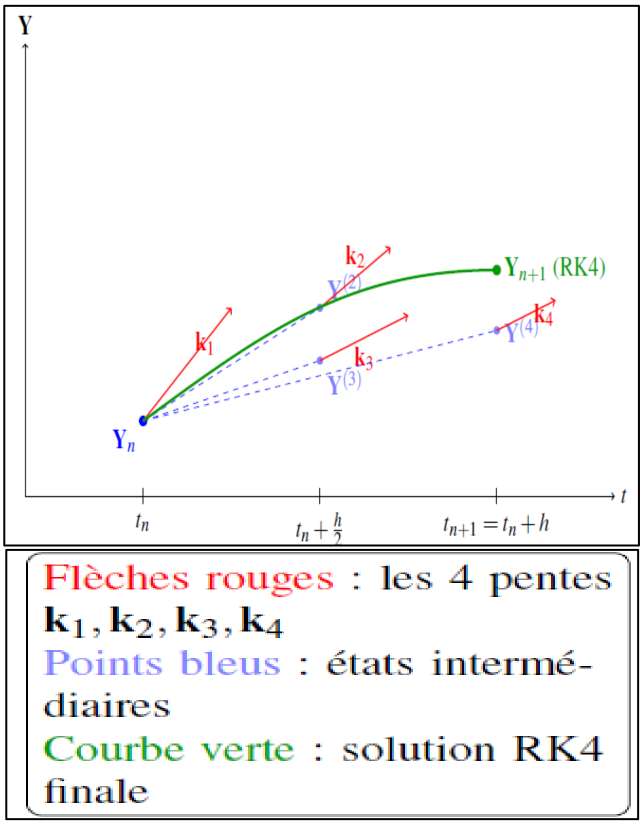

Fig 1 Geometric Interpretation of the RK4 Method

- *Formulas of the RK4 Scheme*

For a Cauchy problem $\frac{dY}{dt} = f(t, \boldsymbol{Y})$with initial condition Y (t0) = Y0 , the RK4 method calculates the solution at time $t_{n+1} = t_n + h$ using the formula:

$$\boxed{\mathbf{Y}_{n+1} = \mathbf{Y}_n + \frac{h}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)} \tag{9}$$

Where the four slope vectors are defined successively by:

$$\mathbf{k}_1 = \mathbf{f}(t_n,\ \mathbf{Y}_n) \tag{10}$$

$$\mathbf{k}_2 = \mathbf{f}\left(t_n + \frac{h}{2},\ \mathbf{Y}_n + \frac{h}{2}\mathbf{k}_1\right) \tag{11}$$

$$\mathbf{k}_3 = \mathbf{f}\left(t_n + \frac{h}{2},\ \mathbf{Y}_n + \frac{h}{2}\mathbf{k}_2\right) \tag{12}$$

$$\mathbf{k}_4 = \mathbf{f}(t_n + h,\ \mathbf{Y}_n + h\,\mathbf{k}_3) \tag{13}$$

- *Justification of the Coefficients*

Formula (7) may seem arbitrary. It actually follows from a Taylor series expansion of the exact solution up to order 4 [4]. Let us require that the numerical solution Y _{n+1} coincides with the Taylor series expansion of the exact solution Y(t _n + h) up to order h⁴:

$$\mathbf{Y}(t_n + h) = \mathbf{Y}(t_n) + h\mathbf{Y}'(t_n) + \frac{h^2}{2!}\mathbf{Y}''(t_n) + \frac{h^3}{3!}\mathbf{Y}'''(t_n) + \frac{h^4}{4!}\mathbf{Y}^{(4)}(t_n) + O(h^5) \tag{14}$$

By replacing the derivatives of Y with their expressions in terms of f and its partial derivatives (via the chain rule), and seeking a linear combination of the form:

$$\mathbf{Y}_{n+1} = \mathbf{Y}_n + h\,(w_1\mathbf{k}_1 + w_2\mathbf{k}_2 + w_3\mathbf{k}_3 + w_4\mathbf{k}_4) \tag{15}$$

With the k _i evaluated at points t _n + ci_h for states Y _n + h $\sum a_{ij} K_i$, we obtain a system of nonlinear equations for the coefficients w _i, ci _i, and a _{ij} . The classical solution of this system gives:

$$\begin{array}{c|ccc}
0 & 0 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 \\
1 & 0 & 0 & 1 \\
\hline
\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6}
\end{array}$$

This table by Butcher [1] characterizes the classic RK4 scheme and shows that:

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26may1726

- ✓ k_1 is evaluated at point t_n (coefficient $c_1 = 0$)
- ✓ k2 and k3 are evaluated at the midpoint $t_n + (\frac{h}{2}$ coefficient $c_2 = c_3 = \frac{1}{2})$
- ✓ k_4 is evaluated at the end $t_n + h$ (coefficient $c_4 = 1$)
- ✓ The weights are $w1 = \frac{1}{6}, w2 = \frac{1}{3}, w3 = \frac{1}{3}, w4 = \frac{1}{4}$

- *Order of Convergence*

Theorem 0.3.1 (Order of the RK4 method). Let f be a C^4 function on a neighborhood of the exact solution. Then the RK4 method is of order 4, that is, there exists a constant C > 0 independent of h such that:

$$\|\mathbf{Y}(t_n) - \mathbf{Y}_n\| \leq Ch^4 \tag{16}$$

For all $t_n \leq T$, where Y(tn) is the exact solution and Yn is the numerical approximation [3].

Idea of the proof. The demonstration relies on the Taylor series expansion of the exact solution and the numerical solution. It is shown that the expansions coincide up to and including order 4. The terms of order 5 differ, resulting in a local truncation error of $O(h^5)$. Since there are N = T/h time steps, the overall accumulated error is $O(h^4)$.

Note 0.3.2. For h = 0.01 day, the error is on the order of $(0.01)4 = 10^{-8}$, which is perfectly negligible compared to the uncertainties on the biological parameters (typically on the order of 10^{-2} to 10^{-1}).

II. APPLICATION OF RK4 TO THE SEIHFR SYSTEM

➢ *Notations*

- *Auxiliary Constants:*

$$\Sigma = \alpha_I + \theta + \delta_I = 0,70, \qquad \Psi = \alpha_H + \delta_H = 0,35$$

Compact Function f:

$$\mathbf{f}(\mathbf{Y}) = \begin{pmatrix} -\lambda S \\ \lambda S - \sigma E \\ \sigma E - \Sigma I \\ \theta I - \Psi H \\ \delta_I I + \delta_H H - \gamma F \\ \alpha_I I + \alpha_H H + \gamma F \end{pmatrix}$$

- *Unfolding of an RK4 Step*

Let Yn be at time tn. Calculation of Y_{n+1} at $t_{n+1} = t_n + h$.

Step 1: k1 (start)

Infection strength:

$$\lambda^{(1)} = \frac{\beta_I I_n + \beta_H H_n + \beta_F F_n}{N}$$

$$\mathbf{k}_1 = \begin{pmatrix} -\lambda^{(1)} S_n \\ \lambda^{(1)} S_n - \sigma E_n \\ \sigma E_n - \Sigma I_n \\ \theta I_n - \Psi H_n \\ \delta_I I_n + \delta_H H_n - \gamma F_n \\ \alpha_I I_n + \alpha_H H_n + \gamma F_n \end{pmatrix}$$

Step 2: State $Y^{(2)}$

$$\mathbf{Y}^{(2)} = \mathbf{Y}_n + \frac{h}{2}\mathbf{k}_1$$

Step 3: k_2 (first middle slope)

$$\lambda^{(2)} = \frac{\beta_I I^{(2)} + \beta_H H^{(2)} + \beta_F F^{(2)}}{N}$$

$$\mathbf{k}_2 = \begin{pmatrix} -\lambda^{(2)} S^{(2)} \\ \lambda^{(2)} S^{(2)} - \sigma E^{(2)} \\ \sigma E^{(2)} - \Sigma I^{(2)} \\ \theta I^{(2)} - \Psi H^{(2)} \\ \delta_I I^{(2)} + \delta_H H^{(2)} - \gamma F^{(2)} \\ \alpha_I I^{(2)} + \alpha_H H^{(2)} + \gamma F^{(2)} \end{pmatrix}$$

Step 4: State $Y^{(3)}$

$$\mathbf{Y}^{(3)} = \mathbf{Y}_n + \frac{h}{2}\mathbf{k}_2$$

Step 5: k_3 (2nd middle slope)

$$\lambda^{(3)} = \frac{\beta_I I^{(3)} + \beta_H H^{(3)} + \beta_F F^{(3)}}{N}$$

$$\mathbf{k}_3 = \begin{pmatrix} -\lambda^{(3)} S^{(3)} \\ \lambda^{(3)} S^{(3)} - \sigma E^{(3)} \\ \sigma E^{(3)} - \Sigma I^{(3)} \\ \theta I^{(3)} - \Psi H^{(3)} \\ \delta_I I^{(3)} + \delta_H H^{(3)} - \gamma F^{(3)} \\ \alpha_I I^{(3)} + \alpha_H H^{(3)} + \gamma F^{(3)} \end{pmatrix}$$

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26may1726

Step 6: State Y $^{(4)}$

$$\mathbf{Y}^{(4)} = \mathbf{Y}_n + h\,\mathbf{k}_3$$

Step 7: k $_4$ (end)

$$\lambda^{(4)} = \frac{\beta_I I^{(4)} + \beta_H H^{(4)} + \beta_F F^{(4)}}{N}$$

$$\mathbf{k}_4 = \begin{pmatrix} -\lambda^{(4)} S^{(4)} \\ \lambda^{(4)} S^{(4)} - \sigma E^{(4)} \\ \sigma E^{(4)} - \Sigma I^{(4)} \\ \theta I^{(4)} - \Psi H^{(4)} \\ \delta_I I^{(4)} + \delta_H H^{(4)} - \gamma F^{(4)} \\ \alpha_I I^{(4)} + \alpha_H H^{(4)} + \gamma F^{(4)} \end{pmatrix}$$

Step 8: Final Assembly

$$\boxed{\mathbf{Y}_{n+1} = \mathbf{Y}_n + \frac{h}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)}$$

Weight summary

Table 2 Evaluations and Weights in RK4

Pente	Temps	État	Poids
$\mathbf{k}_1$	t_n	$\mathbf{Y}_n$	$1/6$
$\mathbf{k}_2$	$t_n + h/2$	$\mathbf{Y}_n + \frac{h}{2}\mathbf{k}_1$	$2/6 = 1/3$
$\mathbf{k}_3$	$t_n + h/2$	$\mathbf{Y}_n + \frac{h}{2}\mathbf{k}_2$	$2/6 = 1/3$
$\mathbf{k}_4$	$t_n + h$	$\mathbf{Y}_n + h\mathbf{k}_3$	$1/6$

Somme des poids : $\frac{1}{6} + \frac{1}{3} + \frac{1}{3} + \frac{1}{6} = 1.$

III. DETAILED NUMERICAL EXAMPLE: FIRST TIME STEP

Initial data

Application of RK4 from t $_0$ = 0 to t $_1$ = h = 0.01 day.

Initial conditions:

$$\mathbf{Y}_0 = \begin{pmatrix} S_0 \\ E_0 \\ I_0 \\ H_0 \\ F_0 \\ R_0 \end{pmatrix} = \begin{pmatrix} 99\,999 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Settings:

$\beta_I = 0,25,\ \beta_H = 0,13,\ \beta_F = 0,40,\ \sigma = 1/7 \approx 0,142857$

$\theta = 0,30,\ \alpha_I = 0,10,\ \delta_I = 0,30,\ \alpha_H = 0,20,\ \delta_H = 0,15,\ \gamma = 0,050\ \lambda^{(3)} = 2,4992 \times 10^{-6}$

Constants:

$$\Sigma = \alpha_I + \theta + \delta_I = 0,70, \qquad \Psi = \alpha_H + \delta_H = 0,35$$

Calculation of k $_1$

Initial infection strength:

$$\lambda^{(1)} = \frac{0,25 \times 1}{100\,000} = 2,5 \times 10^{-6}$$

Components:

$$k_{1,S} = -2,5 \times 10^{-6} \times 99\,999 = -0,2499975$$
$$k_{1,E} = 0,2499975 - 0 = 0,2499975$$
$$k_{1,I} = 0 - 0,70 \times 1 = -0,70$$
$$k_{1,H} = 0,30 \times 1 - 0 = 0,30$$
$$k_{1,F} = 0,30 \times 1 + 0 = 0,30$$
$$k_{1,R} = 0,10 \times 1 = 0,10$$

Intermediate state Y $^{(2)}$

$$\mathbf{Y}^{(2)} = \mathbf{Y}_0 + 0,005 \times \mathbf{k}_1$$

$$S^{(2)} = 99\,998,9987$$
$$E^{(2)} = 0,001250$$
$$I^{(2)} = 0,9965$$
$$H^{(2)} = 0,0015$$
$$F^{(2)} = 0,0015$$
$$R^{(2)} = 0,0005$$

Calculation of k $_2$

$$k_{2,S} \approx -0,249917$$
$$k_{2,E} \approx 0,249738$$
$$k_{2,I} \approx -0,697371$$
$$k_{2,H} \approx 0,298425$$
$$k_{2,F} \approx 0,298425$$
$$k_{2,R} \approx 0,100700$$

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26may1726

Intermediate state $Y^{(3)}$

$$S^{(3)} = 99\,998,998750$$
$$E^{(3)} = 0,001249$$
$$I^{(3)} = 0,996513$$
$$H^{(3)} = 0,001492$$
$$F^{(3)} = 0,001492$$
$$R^{(3)} = 0,000504$$

Calculation of k_3

$$\lambda^{(3)} = 2,49919 \times 10^{-6}$$
$$k_{3,S} \approx -0,249917$$
$$k_{3,E} \approx 0,249738$$
$$k_{3,I} \approx -0,697380$$
$$k_{3,H} \approx 0,298432$$
$$k_{3,F} \approx 0,298432$$
$$k_{3,R} \approx 0,100696$$

Estimated final state $Y^{(4)}$

$$\mathbf{Y}^{(4)} = \mathbf{Y}_0 + 0,01 \times \mathbf{k}_3$$

$$S^{(4)} = 99\,998,997501$$
$$E^{(4)} = 0,002497$$
$$I^{(4)} = 0,993026$$
$$H^{(4)} = 0,002984$$
$$F^{(4)} = 0,002984$$
$$R^{(4)} = 0,001007$$

Calculation of k_4

$$\lambda^{(4)} = 2,49809 \times 10^{-6}$$
$$k_{4,S} \approx -0,249734$$
$$k_{4,E} \approx 0,249377$$
$$k_{4,I} \approx -0,694761$$
$$k_{4,H} \approx 0,296863$$
$$k_{4,F} \approx 0,296863$$
$$k_{4,R} \approx 0,101369$$

Final assembly

$$\mathbf{Y}_1 = \mathbf{Y}_0 + \frac{h}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)$$

Weighted averages:

$$\bar{k}_S = -0,249900$$
$$\bar{k}_E = 0,249721$$
$$\bar{k}_I = -0,697377$$
$$\bar{k}_H = 0,298430$$
$$\bar{k}_F = 0,298430$$
$$\bar{k}_R = 0,100694$$

$$\boxed{\mathbf{Y}_1 = \begin{pmatrix} 99\,998,9975 \\ 0,002497 \\ 0,993026 \\ 0,002984 \\ 0,002984 \\ 0,001007 \end{pmatrix}}$$

- *Numerical Properties of the Solution*

Conservation of N two. Therefore:

Proposition 0.6.1. For all $n \geq 0$, RK4 satisfies $N_n+1 = N_n$.

$$N_{n+1} = N_n + \frac{h}{6}(0 + 2\cdot 0 + 2\cdot 0 + 0) = N_n$$

Proof. Each k_j has a sum of zero components because the terms cancel each other out in pairs.

Local truncation error

Proposal 0.6.2.

$$\|\mathbf{Y}(t_{n+1}) - \tilde{\mathbf{Y}}_{n+1}\| = O(h^5)$$

Global error

Proposal 0.6.3.

$$\|\mathbf{Y}(t_n) - \mathbf{Y}_n\| \leq Ch^4$$

This estimation of the overall error is demonstrated in reference works on numerical analysis [3, 4]. The constant C depends on the regularity of the function f and the time interval considered.

Choosing the time step

With h = 0.01 day:

ISSN No:-2456-2165 https://doi.org/10.38124/ijisrt/26may1726

- Not for 300 days: 30000
- Estimated error: ≈10^{-8}

Table 3 Convergence test for I_{max}

Pas h	$I_{\max}$	Écart
0,1	1245,3	–
0,05	1238,7	0,53%
0,02	1237,2	0,12%
0,01	**1236,9**	**0,02%**
0,005	1236,8	0,008%

The convergence observed with h = 0.01 corresponds to the theoretical results expected for the RK4 method [1].
Algorithmic complexity

Number of operations per step

- For each step, RK4 performs:
- 4 ratings of **f**
- By evaluation:
- 1 calculation of 1 (3 multiplications, 2 additions, 1 division)
- 6 components: ≈12 multiplications and 12 additions
- Final assembly: 6 multiplications and 12 additions

Total: ≈4 x 15 + 18 = 78 operations per step.

Total complexity

For N_{steps} ≈30,000 steps:

Calculations: ≈30000 x 78 = 2.34 x $^{10^{-6}}$

That's about 2.3 ms on a 109 processor

op/s. In practice (MATLAB/Python): 0.1 to 1 second.

Note 0.7.1. With Euler and h = 0.0001 for the same precision: 3000000 steps ≈3 seconds, i.e. 3 to 30 times more.

For a more detailed analysis of the complexity of Runge- Kutta methods , see [1] and [3].

Digital validation

Validation criteria

- Conservation of N:$|N(t) - N_0| < 10^{-10}$
- Positivity: all variables ≥ 0
- Convergence: independence of the step size for h ≤0, 01

These validation criteria are standard in numerical simulation of differential equations. [4].

IV. RESULTS

Table 4 Validation of the RK4 Implementation

Critère	Résultat
Erreur max sur $N(t)$	$< 10^{-12}$
Minimum de $S(t)$	≥ 0
Minimum des autres compartiments	≥ 0
Écart $I_{\max}$ ($h = 0{,}01$ vs $0{,}005$)	$< 0{,}01\%$

The results obtained confirm the robustness of the RK4 implementation for the model SEIHFR.

V. CONCLUSION

In this study, we have comprehensively presented the numerical solution of the SEIHFR model using the fourth-order Runge- Kutta method [1, 3]. This model, inspired by the work of Legrand et al. [7] on Ebola dynamics, is part of the general theory of epidemiological modeling [5]. We have:

- Formulate the problem in canonical vector form
- Detailed explanation of the mathematical principle of RK4 (Taylor series expansion, Butcher table)
- Explain each step of the calculation for the SEIHFR system
- Provided a concrete numerical example of the first time step
- Demonstrated accurate conservation of total population
- Analyzed convergence properties and algorithmic complexity

size h = 0.01 days offers an excellent compromise between accuracy (error ≈10^{-8}) and speed (computation time < 1 second). The convergence results presented in Table 1 confirm the expected order of the method, in accordance with theoretical analyses [4]. Finally, the influence of the model parameters on the basic reproduction number, analyzed by Chowell et al. [6], could be the subject of future studies based on this numerical implementation.

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