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Climate Risk and Optimal Sovereign Debt Ceilings for Emerging Economies: A Stochastic Control Approach under NGFS Scenarios

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Abstract: This paper investigates the impact of climate risk on optimal sovereign debt ceilings in emerging economies. Extending the stochastic debt management framework of Brachetta and Ceci (2022), public debt is modeled as a controlled geometric Brownian motion in which climate change affects debt accumulation through scenario-dependent adjustments to economic growth, fiscal pressure, and macroeconomic volatility. The government's debt management problem is formulated as an infinite-horizon stochastic optimal control problem and solved through the Hamilton–Jacobi–Bellman equation. The resulting free-boundary problem yields an endogenous debt ceiling representing the optimal threshold at which fiscal intervention becomes active. Climate risk is incorporated using the Network for Greening the Financial System (NGFS) climate scenarios, including Orderly Transition, Disorderly Transition, and Hot House World pathways. Numerical results demonstrate how worsening climate conditions reduce sustainable borrowing capacity and necessitate earlier fiscal adjustment. The findings provide a quantitative framework for climate-resilient debt management in emerging economies.

Keywords: *Sovereign Debt, Climate Risk, Stochastic Control, HJB Equation, Debt Sustainability, NGFS Scenarios.*

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I. INTRODUCTION

Public debt sustainability in emerging economies faces growing threats from climate change. Physical risks such as extreme weather events and chronic productivity losses, alongside transition risks from decarbonization policies, exert significant pressure on fiscal balances, economic growth, and debt dynamics. In many developing countries, particularly in Sub-Saharan Africa and other climate-vulnerable regions, these effects are amplified by limited daptive capacity and heavy reliance on climate-sensitive sectors.Conventional debt sustainability analyses often overlook these forward-looking climate risks, leading to potentially over-optimistic borrowing thresholds. This paper investigates the impact of climate risk on optimal sovereign debt ceilings in emerging economies. Extending the stochastic debt management framework of Brachetta and Ceci (2022), we model the debt-to-GDP ratio as a controlled Geometric Brownian Motion, augmented with a climate damage term (ϕC) calibrated to NGFS Phase V scenarios. The government's objective is to minimize the expected discounted cost of holding debt over an infinite horizon. This problem is solved via the Hamilton–Jacobi–Bellman (HJB) equation, yielding an endogenous optimal debt ceiling $\tilde{x}$ as the free boundary at which the government optimally switches between expansionary and contractionary fiscal policy.

- *Research Question:*

How do different NGFS climate scenarios (Orderly Transition, Disorderly Transition, and Hot House World) affect the optimal sovereign debt ceiling and the associated debt maintenance costs in emerging economies?

- *Key Findings:*

Our results show that climate risk systematically lowers the optimal debt ceiling. Under baseline conditions, the ceiling stands at approximately 53.86% of GDP. This declines modestly under orderly transition scenarios but drops more significantly under Hot House World conditions to around 51.72% of GDP. Climate change also raises the minimum debt maintenance cost, as reflected in the upward shift of the value function. These findings highlight the need for more conservative, climate-resilient debt targets and proactive fiscal management in vulnerable economies.

II. LITERATURE REVIEW

Climate change poses a systemic threat to public debt sustainability, particularly in emerging and developing

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economies. Physical risks and transition risks (policy shifts toward low-carbon economies) increase fiscal pressures through multiple channels. (Volz2022) and (Kling et al. 2023) document that climate-related disasters lead to higher reconstruction expenditures, reduced tax revenues, and lower economic growth, thereby elevating debt-to-GDP ratios. The (IMF,2021) projects that climate change could add between 10 and 20 percentage points to public debt in low-income countries by 2050 under adverse scenarios. In Sub-Saharan Africa, where economies are heavily dependent on rain-fed agriculture and have limited fiscal buffers, these effects are particularly pronounced (African Development Bank, 2022). Stochastic optimal control has become a powerful framework for determining sustainable debt thresholds under uncertainty. (Brachetta and Ceci 2022) pioneered a debt-smoothing approach in which the debt-to-GDP ratio follows a controlled Geometric Brownian Motion. They solve the associated Hamilton-Jacobi Bellman (HJB) equation to derive an endogenous optimal debt ceiling, the point at which the government optimally switches from expansionary to contractionary fiscal policy. Their model minimizes the expected discounted quadratic deviation from a target debt ratio and provides analytical tractability through value-matching and smooth-pasting conditions. Subsequent contributions, such as (Cadenillas and Aguilar,2015) and Ferrari and Ferrari 2023), extended singular stochastic control techniques to debt reduction problems. These studies highlight the importance of dynamic fiscal policy in managing debt under uncertainty. Integrating Climate Risk into Stochastic Control Frameworks The integration of climate risk into stochastic optimal control models remains an emerging but rapidly developing area. Most early attempts used deterministic or scenario-based adjustments to growth or volatility parameters (NGFS, 2022; Battiston et al., 2021). A few recent studies have begun incorporating climate uncertainty more rigorously. For example, some authors introduce climate damage as a stochastic process affecting the drift of the debt dynamics (e.g., through an additional termϕCt) or as a separate state variable in multi-dimensional HJB equations. Notable contributions include works that link climate scenarios to debt sustainability analysis using stochastic programming (Monasterolo et al., 2021; Lamperti et al., 2021). However, applications of full stochastic optimal control (HJB-based free-boundary problems) to climate-augmented debt management are still scarce, especially in the context of Sub- Saharan African economies. Most existing climate-debt studies rely on reduced-form econometric approaches or deterministic stress-testing rather than deriving endogenous optimal debt ceilings under stochastic control. Despite growing recognition of climate risks, there remains a significant gap in applying rigorous stochastic optimal control techniques to derive climate-adjusted optimal debt in emerging markets. Existing stochastic control models of debt do not explicitly account for climate damage, while climate-focused debt studies rarely employ dynamic optimization with free-boundary solutions. This study addresses this gap by extending the (Brachetta and Ceci 2022) framework. We introduce climate risk through a scenariobased damage term (ϕC) calibrated to NGFS pathways and examine its impact on the endogenous optimal debt ceiling for Kenya. By doing so, we provide one of the first applications of stochastic optimal control to climate-resilient debt management in Sub-Saharan Africa, bridging the gap between theoretical stochastic control and policy-relevant climate risk analysis.

III. METHODOLOGY

➢ *Climate-Adjusted Stochastic Debt Dynamics*

Let Xt denote the public debt-to-GDP ratio at time t. Following the stochastic debt management framework outlined by (Brachetta and Ceci 2022), public debt is modeled as a controlled stochastic process whose evolution depends on macroeconomic conditions and government fiscal policy. To account for climate-related risks, we introduce a climate damage component that captures the adverse effects of climate change on economic growth and fiscal sustainability.

Climate risk is incorporated through the term ϕC_t ,

Where:

- $\phi > 0$ denotes the climate sensitivity parameter, measuring the impact of climate risk on debt accumulation. A larger value of ϕ implies that debt dynamics are more responsive to climate-related shocks.
- $C_t \geq 0$ represents the level of climate risk exposure at time t. This variable captures the economic consequences of both physical and transition climate risks. Physical risks include droughts, floods, storms, and other climate-related disasters, while transition risks arise from policy adjustments, carbon pricing mechanisms, and structural changes associated with the transition to a low-carbon economy.

The debt-to-GDP ratio is assumed to evolve according to the following stochastic differential equation:

$$dX_t = X_t [r - g(u_t) - u_t + \phi C_t] dt + \sigma X_t dW_t \quad (1)$$

Where:

- $r > 0$ is the effective real interest rate on government debt;
- $g(u_t)$ is the economic growth rate, which depends on fiscal policy;
- u_t is the fiscal control variable representing the government's primary balance effort;
- ϕC_t captures the deterioration in debt dynamics caused by climate-related damages;
- $\sigma > 0$ denotes the volatility parameter measuring macroeconomic uncertainty;
- W_t is a standard Brownian motion defined on a filtered probability space (Ω,F, P).

The term $(r-g(u_t)-u_t)$ represents the conventional implement fiscal consolidation measures. The additional climate damage term ϕC_t increaseses the drift of the debt process, reflecting the fact that climate change can reduce economic output, lower tax revenues, increase public expenditure on adaptation and disaster recovery, and consequently worsen fiscal balances. To capture the

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interaction between fiscal policy and economic growth, the growth rate is assumed to take the linear form.

$$g(u_t) = g_0 - \alpha u_t \qquad (2)$$

Where:

- $g_0 > 0$ denotes the baseline growth rate in the absence of fiscal intervention;
- $\alpha > 0$ measures the sensitivity of economic growth to fiscal adjustment. The parameter α reflects the short-run contractionary effects of fiscal consolidation. Higher fiscal surpluses may reduce aggregate demand and slow economic activity, resulting in lower growth.

Substituting the growth specification into the debt dynamics yields:

$$dX_t = X_t\,[r - g_0 + \alpha u_t - u_t + \phi C_t]dt + \sigma X_t dW_t \qquad (3)$$

The term $((\alpha - 1)u_t)$ captures the net effect of fiscal policy on debt dynamics. When $\alpha < 1$, fiscal tightening reduces debt accumulation despite its negative effect on economic growth. Conversely, if α approaches one, the effectiveness of fiscal adjustment is weakened by its adverse impact on economic activity. For computational tractability and to facilitate scenario analysis, climate exposure is assumed to be constant within each climate scenario, such that $C_t = C$. Consequently, the debt dynamics simplify to:

$$dX_t = X_t\,[r - g_0 + (\alpha - 1)u_t + \phi C]dt + \sigma X_t dW_t \qquad (4)$$

This specification forms the basis of the climate-adjusted stochastic debt management model.The parameter C will subsequently be calibrated using climate scenarios representing alternative future pathways, allowing the analysis of how different levels of climate risk affect optimal sovereign debt ceilings in emerging economies.

- *Government Optimization Model*

The government's aim is to minimize the objective cost given by:

$$J(x, u) = E\,[\int_0^\infty \exp\{-\alpha_t\}\, f(x, u)\, dt \mid X_0 = x]$$

The corresponding climate-risk adjusted HJB equation is:

$$\theta v(x) = \inf_{\{u\in[-U_1,U_2]\}} \{ (x - \bar{x})^2 + (r - g_0 + \alpha u - u + \phi C)\, x\, v'(x) + ½\sigma^2 x^2 v''(x) \}$$

Where v(x) is the value function.

- *Regime-Specific Debt Management Policies*

We assume that there exists an endogenous debt threshold, denoted by x, which represents the optimal sovereign debt ceiling. The debt ceiling divides the state space into two distinct policy regimes. When the debt-to-GDP ratio remains below the threshold, the government pursues an expansionary fiscal stance to support economic activity and growth. However, once debt exceeds the threshold, concerns regarding sustainability dominate, prompting the government to implement fiscal consolidation measures aimed at stabilizing debt dynamics.

The fiscal control is therefore defined as:

$$u = -U_1 \text{ for } x < \tilde{x}, \quad u = U_2 \text{ for } x \geq \tilde{x}$$

Where:

- $U_1 > 0$ denotes the magnitude of expansionary fiscal policy;
- $U_2 > 0$ denotes the magnitude of contractionary fiscal policy;
- $\tilde{x}$ is the optimal debt ceiling to be determined endogenously.
- The existence of the threshold $\tilde{x}$ reflects the government's optimal trade-off between supporting economic growth and maintaining debt sustainability under uncertainty.
- When the debt ratio exceeds the optimal debt ceiling, i.e., $x \geq \tilde{x}$, the government adopts a contractionary fiscal policy and sets $u = U_2$. Substituting this control into the Hamilton–Jacobi–Bellman equation yields.

$$\theta v_1(x) = (x - \bar{x})^2 + (r - g_0 + \alpha U_2 - U_2 + \phi C)\, x\, v_1'(x) + ½\sigma^2 x^2 v_1''(x) \qquad (5)$$

This regime characterizes periods of elevated debt during which policymakers actively seek to reduce debt accumulation through fiscal tightening.

When the debt ratio remains below the optimal debt ceiling, i.e., $x < \tilde{x}$, the government adopts an expansionary fiscal policy and sets $u = -U_1$. The corresponding HJB equation becomes:

$$\theta v_2(x) = (x - \bar{x})^2 + (r - g_0 - \alpha U_1 + U_1 + \phi C)\, x\, v_2'(x) + ½\sigma^2 x^2 v_2''(x) \qquad (6)$$

This regime reflects periods of relatively low debt burden in which fiscal policy prioritizes economic expansion.

Define the boundary growth rates as $G_1 = -g_0 + (\alpha - 1)U_2 + \phi C$ and $G_2 = -g_0 - (\alpha - 1)U_1 + \phi C$. G_1 and G_2 summarize the combined effects of economic growth, fiscal policy, and climate damages on debt accumulation in the high-debt and low-debt regimes, respectively. The climate component ϕC enters both expressions additively, implying that higher climate risk increases debt accumulation pressure regardless of the fiscal regime.

Using these definitions, the HJB equations can be expressed more compactly as:

$$\theta v_1(x) = (x - \bar{x})^2 + (r + G_1)\, x\, v_1'(x) + ½\sigma^2 x^2 v_1''(x) \qquad (7)$$

And

$$\theta v_2(x) = (x - \bar{x})^2 + (r + G_2)\, x\, v_2'(x) + ½\sigma^2 x^2 v_2''(x) \qquad (8)$$

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These regime-specific equations form the basis for deriving the value functions and determining the climate-adjusted optimal debt ceiling.

In each regime, the HJB becomes a second-order linear ODE; thus we assume a general solution form:

$$v(x) = a_i x^2 + b_i x + c + d_i x^{y_i}, \quad i = 1, 2 \tag{9}$$

Substituting the assumed form and solving the resulting characteristic equation gives the coefficients:

$$a_i = 1 / [\theta - 2(G_i + r) - \sigma^2], \quad b_i = 2\bar{x} / (G_i + r - \theta), \quad c_i = \bar{x}^2 / \theta$$

$$y_1 = [-(2(G_1+r) - \sigma^2) - \sqrt{((2(G_1+r) - \sigma^2)^2 + 4\theta\sigma^2)}] / 2\sigma^2$$

$$y_2 = [-(2(G_2+r) - \sigma^2) + \sqrt{((2(G_2+r) - \sigma^2)^2 + 4\theta\sigma^2)}] / 2\sigma^2$$

At the optimal debt ceiling $\tilde{x}$, we impose continuity and smoothness (value-matching and smooth-pasting conditions):

- $v(\tilde{x})^+ = v(\tilde{x})^-$
- $v'(\tilde{x})^+ = v'(\tilde{x})^-$

These two conditions allow us to solve for the unknown constants d_1 and d_2, and most importantly, for the free boundary $\tilde{x}$ itself, using the numerical Newton–Raphson root-finding method.

Substituting the solution forms and solving these two equations yields:

$$d_1 = -(2a_1\tilde{x} + b_1) / (y_1\tilde{x}^{y_1-1})$$

$$d_2 = [2(a_1 - a_2)\tilde{x} - (y_1 - 1)(2a_1\tilde{x} + b_1)] / [y_2(y_2 - 1)\tilde{x}^{y_2-2}]$$

Hence we obtain the optimal debt threshold by solving for $\tilde{x}$ in:

$$a_1\tilde{x}^2 + b_1\tilde{x} + c_1 + d_1\tilde{x}^{y_1} = a_2\tilde{x}^2 + b_2\tilde{x} + c_2 + d_2\tilde{x}^{y_2}$$

IV. DATA CALIBRATION AND RESULTS

➢ *Calibration Strategy*

Parameter values are calibrated to reflect typical conditions in climate-vulnerable emerging and developing economies (EMDEs). To ensure representativeness, we draw on a broad sample of countries from Sub-Saharan Africa, South Asia, and Latin America. These are regions identified by the NGFS and IMF as facing the highest climate-related fiscal risks.

- *Data Sources:*

✓ World BankWorld Development Indicators (WDI) and International Debt Statistics for historical debt-to-GDP ratios and macroeconomic variables
✓ IMF World Economic Outlook (WEO) and Fiscal Monitor databases for growth, interest rates, and primary balance data
✓ NGFS Phase V Scenario Portal (2024) for climate-adjusted projections on GDP growth, volatility, and physical risk damages.

The calibration is informed by historical data from a panel of 20 emerging economies (e.g.Kenya, Nigeria, Ghana, Ethiopia, India, Bangladesh, Indonesia, Brazil, and Colombia) over the period 1994–2023. Debt volatility (σ) is estimated from log returns of debt-to-GDP series across this sample, while baseline growth and interest rates reflect long-term averages adjusted for climate vulnerability

Table 1 NGFS Scenarios Table

NGFS Scenario	g_0	α	σ	ϕC	Interpretation
Baseline	0.049	0.70	0.120	0.000	Current conditions
Net Zero 2050	0.0455	0.75	0.135	0.009	Orderly transition
Below 2°C	0.0435	0.78	0.155	0.013	Delayed action
Delayed Transition	0.0410	0.82	0.180	0.019	Disorderly transition
Hot House World	0.0360	0.88	0.225	0.027	High physical risk

➢ *Baseline Parameters*

Table 2 Baseline Parameters.

r	g_0	σ	U_1	U_2	θ	$\bar{x}$	α
0.06	0.049	0.10	0.06	0.025	2.0	0.5	0.7

➢ *NGFS Scenario Calibration*

Climate effects are incorporated through NGFS Phase V scenarios by adjusting key parameters and introducing a climate damage term ϕC in the debt drift. The calibrations are informed by NGFS projections and regional studies on climate impacts in emerging economies. ϕC is derived by mapping NGFS-projected cumulative GDP losses for climate-vulnerable emerging economies into an equivalent increase in the debt drift:

- *Baseline Debt Ceiling*

Under baseline parameters, the model yields an optimal debt ceiling of 53.86% of GDP. At this threshold, the value function attains its minimum value of 0.0006. This represents the debt level at which the government should optimally switch from expansionary to contractionary fiscal policy to minimize long-term debt-smoothing costs. The convex U-shaped value function confirms the validity of the free-boundary solution.

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- *Orderly Transition Scenario*

Net Zero 2050 and Below 2°C produce relatively modest declines in the optimal ceiling(53.35% and 53.03%, respectively). These reflect limited physical risks under ambitious mitigation pathways, though higher volatility and small positive ϕC still require more conservative debt targets than the baseline.

- *Disorderly Transition Scenario*

The optimal ceiling drops to 52.59%. Delayed policy action leads to higher volatility and larger climate damage (ϕC =0.019), amplifying fiscal pressure and forcing earlier consolidation.

- *Hot House World Scenario*

Under the severe Hot House World scenario, the optimal debt ceiling drops to 51.72% of GDP , a reduction of 2.14 percentage points from the baseline. This substantial decline reflects the combined effects of significantly lower growth, higher volatility, and persistent climate damage, which together erode fiscal space and increase the long-term cost of high debt levels.

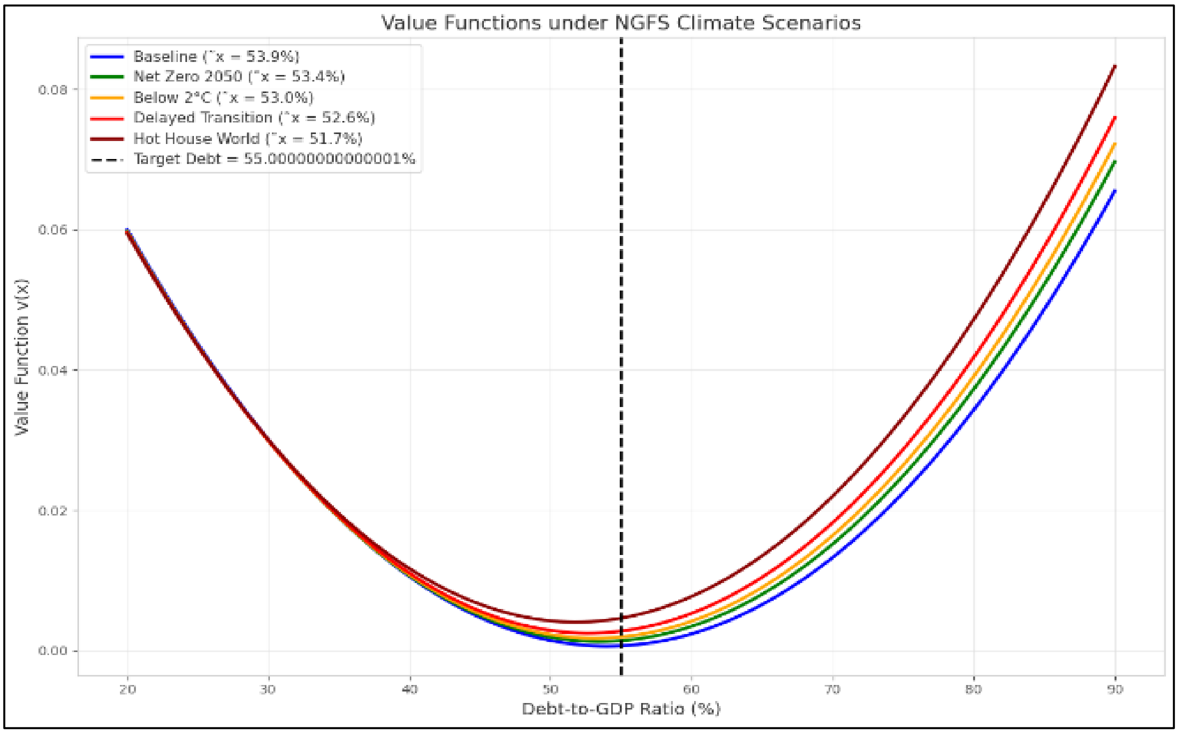

Fig 1 Value Function Curves Under NGFS Scenario.

➢ *Impact on Value Function*

In addition to shifts in the optimal debt ceiling, climate risk also affects the minimum welfare cost faced by the government, as captured by the value function v(x̃) evaluated at the optimal ceiling.

Table 3 Impact on Value Function

Scenario	Optimal Debt Ceiling (%)	V(x̃) At debt ceiling
Baseline	53.86	0.000808
Net Zero 2050	53.35	0.001522
Below 2 C	53.03	0.0002031
Delayed House world	53.59	0.002789
Hot House World	51.72	0.004445

This dual impact is particularly concerning as climate change not only forces governments to adopt lower debt levels but also raises the baseline cost of maintaining debt even when optimally managed. The convex U-shaped pattern of the value function is evident across all scenarios, but the curve shifts upward and becomes steeper under severe climate pathways, indicating greater sensitivity to deviations from the optimal ceiling through climate cost. These findings underscore that climate risk imposes a double burden on emerging economies: it compresses fiscal space while simultaneously increasing the economic cost of any given debt level.

V. CONCLUSION

This study extends the stochastic optimal control framework for debt optimization by incorporating climate risk through NGFS scenario-based adjustments. The results demonstrate that climate change systematically lowers optimal sovereign debt ceilings in emerging economies. Even under moderate scenarios, the optimal threshold declines,

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while severe climate futures (Hot House World) require substantially more conservative debt targets. The findings underscore the government's active role in debt management. Through dynamic fiscal policy responses, authorities can steer debt toward climate-resilient levels. For emerging economies, maintaining debt ratios closer to 50–53% of GDP under increasing climate risk appears prudent to preserve fiscal space and minimize long-term welfare costs.

➢ *Policy Recommendations*

Integrate NGFS-style climate stress testing into national debt sustainability frameworks. Adopt climate-adjusted debt ceilings that are more conservative than conventional targets. Prioritize fiscal consolidation and green investment to mitigate both physical and transition risks. Strengthen international cooperation on climate finance and debt relief for vulnerable economies

➢ *Limitations*

This study has a few limitations. First, the calibration of NGFS Phase V scenarios to model parameters is partially subjective. Although grounded in NGFS projections and regional data, the mapping to growth, volatility, and the climate damage coefficient ϕC involves modeling assumptions. Secondly ,the model assumes constant climate exposure within each scenario and abstracts from stochastic climate damages, endogenous default risk,and sudden capital flow reversals. Despite these limitations, the main qualitative findings remain robust under sensitivity analysis. Future work could incorporate stochastic climate processes and multi-dimensional HJB models.

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