Transfer of Heat and MHD Boundary Layer Flow of a Dusty Cu-H_2O Nanofluid Across an Exponentially Extending Sheet

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Abstract: The current study focused on heat transmission and MHD boundary layer flow over a growing exponentially sheet. Cu-H_2O nanofluid has dispersed conducting dust particles. The momentum and energy equations undergo transformation into nonlinear coupled ordinary differential equations using similarity transformations, and the MATLAB bvp4c approach is being utilized to solve these equations numerically. The visualization of the temperature and velocity profiles of the flow against non-dimensional parameters, such as the magnetic field parameter M, the Prandtl number Pr, the Eckert number Ec, the mass concentration of dust particles α, the fluid particle interaction parameter for the velocity β, the volume fraction of dust particles ϕ_d, and the volume fraction of nanoparticles ϕ, is discussed.

Keywords: *Dust and Nanoparticles, Exponentially Expanding Sheets, Nanofluid, Heat Transmission, and Boundary Layer Flow.*

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I. INTRODUCTION

Nanofluid plays an important role in current technological era. Nanofluid is a newly developed engineering fluid obtained by mixing 10^{-9} meter sized particles in to the base fluid. Larger surface areas of the nanoparticles improve heat transfer capabilities and stability of the suspension. It also alters fluid flow. Since materials of nano meter sizes have unique chemical and physical properties. It enhances thermal conductivity. It exhibit larger thermal properties than conventional coolants. It has several applications in various industries and in engineering fields. It is used in polymer industry, stretching of plastic sheets. It has cooling applications such as electronics cooling, vehicles cooling, cooling of microchips, transformer cooling, and computer cooling,Continuous casting, the production of paper, glass, food, and fiber, polymer drawing, and metal spinning are some of its uses. These days, a number of challenges with electronic equipment are caused by low heat conductivity in convectional fluids that include water, ethylene glycol, and oil. Over the past few decades, many researchers have focused on mixing nano or micrometer-sized particles in the base fluids in an effort to work around this problem to boost the thermal conductivity of convectional fluids.. Because of enormous applications of dusty nanofluid many researchers have shown interest in this research area. Cu-H_2O nanofluid embedded with conducting particles was employed in this investigation.

Sakiadis [1, 2] assessed the performance of boundary layers on continuous solid surfaces. He addressed boundary layer behavior for two-dimensional and asymmetric flow as well as continuous solid and continuous flat surfaces. The word "nanofluid" was initially adopted by Choi [3]. He developed the concept of nanofluids to suspend fluids containing ultra-fine particles (diameter less than 50 nm). He clarified that a substantial reduction in heat exchange pumping power is one advantage of nanofluid..Magyari and B. Keller [4] assessed heat and mass transmission in the boundary layers by taking into account a continuous exponentially extending surface.He explored a similarity solution that tackled the stable plane boundary layers on a continuous surface with an exponential temperature distribution and exponential stretch, and he discovered the solution both analytically and numerically. Heat transmission of a viscoelastic fluid over an exponentially expanding sheet with different thermal conductivity and radiation in a porous media was examined by Singh and Agarwal [5]. He scrutinized the heat transfer and boundary layer flow of second grade and second order fluids with different thermal conductivity and radiation across an exponentially stretched sheet in a porous medium. Nadeem and Lee [6] provided a justification of the boundary layer flow of nanofluid across an exponentially stretched surface. The homotopy analysis approach was employed to analyze this issue analytically, and Brownian motion and thermophoresis characteristics were taken into account in the transport equations. Gireesh et al. [7] provide a comprehensive analysis

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of boundary layer flow and heat transport of a dusty fluid across an exponentially growing sheet. He focused on the consequences of a magnetic field on a boundary layer flow and the heat transfer of a dusty fluid across an exponentially extended surface with an exponential temperature.

Singh and Agarwal [8] probed MHD flow and heat transmission for Maxwell fluid across an exponentially expanding sheet with varying thermal conductivity.They employ an implicit finite difference strategy called the Keller-Box approach. Nadeem and Lee [9] investigated nanofluid boundary layer flow on an increasingly stretched surface. They observed the continuous two-dimensional stagnation point flow of a water-based nanofluid over an exponentially growing and shrinking sheet in its own plane. Three different nanoparticles—copper (Cu), alumina (Al_2O_3), and titania (TiO_2)—in a water-based fluid with Prandtl number Pr=6.2 are employed in that analysis.

Liu et al. [10] tackled the heat transfer of three-dimensional flow across an exponentially growing surface. They formulated a numerical approach that combines the Ackroyd methodology with the Runge Kutta integration scheme to solve the momentum and energy equations. They figured out that the properties of heat transport were substantially affected by the stretching ratio, temperature exponent, and Prandtl number. Preeti Agarwala and Khare [12] investigated Cu-H2O nanofluid MHD flow on a stretched sheet with second order slip conditions.It has been noticed that the temperature elevates and the velocity drops as the slip and magnetic parameters increase.

The overall objective of the current study is to apply the findings of [7] to dusty nanofluid. We explored the significance of the magnetic field parameter M, Prandtl number Pr, Eckert number Ec, mass concentration of dust particle α, fluid particle interaction parameter for the velocity β, volume fraction of dust particle ϕ_d, and volume fraction of nano particle ϕ on the velocity and temperature profiles of the fluid phase and dust phase.

II. MATHEMATICAL MODEL

An exponentially expanded sheet is traversed by an incompressible dusty nanofluid in a continuous two-dimensional laminar, MHD boundary layer flow. The surface is considered to be stretched with exponential velocity $U_w = U_0 e^{x/L}$, where U_0 reference velocity and L is is the reference length.The y axis is normal to the sheet, which is located along the plane y=0. Along the y direction, a constant magnetic field B is applied. At first, we assumed that both the fluid and the dust particles were static and Dust particles were of uniform size. Additionally, we took into account the volume fraction, dust particle number density, and spherical-shaped nanoparticles. $T_w = T_\infty + T_0 e^{x/2L}$ Is the temperature maintained close to the surface.

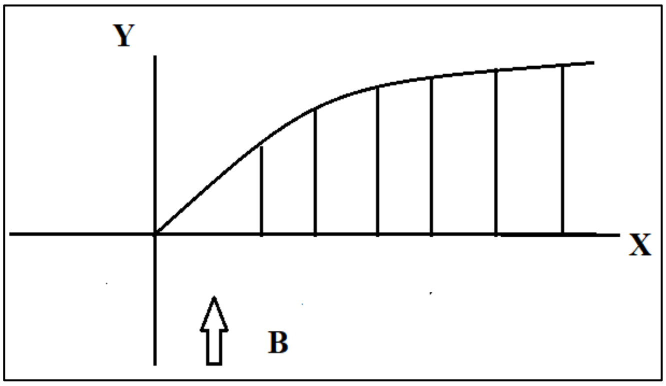

Fig 1 A Physical Representation of the Problem.

According to the preceding assumptions, the boundary layer equations that control the current flow are provided by:

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \tag{1}$$

$$(1-\phi_d)\rho_{nf}\left(u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y}\right) = (1-\phi_d)\mu_{nf}\frac{\partial^2 u_1}{\partial y^2} + KN(u_p - u_1) - \sigma B^2 u_1 , \tag{2}$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial u_p}{\partial y} = 0 , \tag{3}$$

$$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \frac{K(u_1 - u_p)}{m} \tag{4}$$

The current problem's boundary conditions are:

$$u_1 = u_w(x) \ , v_1 = 0 \text{ at } y=0 \text{ and } u \to 0, u_p \to 0, v_p \to v_1 \text{ as } y \to \infty \tag{5}$$

Where (u_1,v_1) and (u_p, v_p) are velocity components of nanofluid and dust particles in x and y directions respectively. ϕ is the volume fraction of nano particles, ϕ_d is the volume fraction of dust particles, μ_{nf} is the kinematic viscosity of the nanofluid, K is the Stokes resistance, m is the mass of the dust particles, N is the number density of the dust particles and $\rho_{nf}$is the density of nanofluid.

We introduce the following set of similarity variables involving dimensionless variable η as:

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$$u_1 = U_0 e^{\frac{x}{L}} f'(\eta),\ u_p = U_0 e^{\frac{x}{L}} F'(\eta),\ v_1 = -\sqrt{\frac{U_0 \upsilon}{2L}}\, e^{x/2L}\{f(\eta) + \eta f'(\eta)\}\ ,$$

$$v_p = -\sqrt{\frac{U_0 \upsilon}{2L}}\, e^{x/2L}\{F(\eta) + \eta F'(\eta)\},\quad \eta = \sqrt{\frac{U_0}{2\upsilon L}}\, e^{x/2L} y,\ B = B_0 e^{\frac{x}{2L}} \tag{6}$$

Where B_0 is magnetic field flux density.

Equation (1) and (3) are automatically satisfied. Equations (2) and (4) are converted into the following ordinary differential equations:

$$\frac{(1-\phi_d)}{(1-\phi)^{2.5}} f''' + (1-\phi_d)\left(1-\phi+\frac{\phi\rho_s}{\rho_f}\right)(ff'' - 2(f')^2) - Mf' + 2\alpha\beta(F'-f') = 0\ , \tag{7}$$

$$2(F')^2 - FF'' + 2\beta(F'-f') = 0\ . \tag{8}$$

Where ρ_s is the density of the nanoparticles of copper (cu), ρ_f is the density of base fluid water(H_2O) , $\alpha = \frac{Nm}{\rho_f}$ is the mass concentration of the dust particles, $M = \frac{2\sigma B^2 L}{U_0 \rho_f} e^{\frac{-x}{L}}$ is the magnetic field parameter, $\beta = \frac{L}{U_0 \tau_v} e^{\frac{-x}{L}}$ is the local fluid particle interaction parameter for velocity.

Equation (5) is transformed in to:

$$F'(\eta) = 1, f(\eta) = 0 \text{ at } \eta = 0 \text{ and } f'(\eta) \to 0,\ F'(\eta) \to 0,\ F(\eta) \to f(\eta) + \eta\, f'^{(\eta)} - \eta F'^{(\eta)} \text{ as } \eta \to \infty. \tag{9}$$

III. HEAT TRANSFER ANALYSIS

The governing heat transport equations are given by:

$$(\rho c_p)_{nf}\left(u_1 \frac{\partial T}{\partial x} + v_1 \frac{\partial T}{\partial y}\right) = k_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{N(c_p)_{nf}}{\tau_T}(T_p - T) + \frac{N(u_p - u_1)^2}{\tau_v}\ , \tag{10}$$

$$Nc_m\left(u_p \frac{\partial T_p}{\partial x} + v_p v \frac{\partial T_p}{\partial y}\right) = -\frac{N(c_p)_{nf}}{\tau_T}(T_p - T) \tag{11}$$

Where T and T_p are the temperature of the nanofluid and dust particles respectively. k_{nf} is the thermal conductivity of the nanofluid, $(c_p)_{nf}$,c_m are the specific heats of the nanofluid and dust particles respectively. τ_T is the thermal equilibrium time and τ_v is the relaxation time of the dust particles. $T_\omega = T_\infty + T_0 e^{x/2L}$ is the temperature distribution in the stretching surface, where T_0 is the reference temperature.

Temperature boundary conditions in this problem are,

$$T = T_\omega \text{ at } y=0 \text{ and } T \to T_\infty, T_p \to T_\infty \text{ as } y \to \infty\ , \tag{12}$$

Where T_ω, T_∞ are the temperatures near the wall and far away from the wall respectively.

Non-dimensional variables to get the similarity solution of equations are,

$$\theta = \frac{T - T_\infty}{T_\omega - T_\infty},\ \theta_p = \frac{T_p - T_\infty}{T_\omega - T_\infty} \text{ where } T = T_\infty + T_0 e^{x/2L}\theta(\eta) \tag{13}$$

Equation (10) & (11) transformed in to ordinary differential equations as:

$$\theta'' + 2a_2 N P_r E_c\left(\frac{k_{nf}}{k_f}\right)(F'-f')^2 + 2a_1 N P_r \frac{(c_p)_f k_f}{k_{nf}}\left\{\frac{1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}}{1-\phi+\phi\frac{\rho_s}{\rho_f}}\right\}(\theta_p - \theta) P_r \left\{\frac{1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}}{\frac{k_{nf}}{k_f}}\right\}(f\theta' - f'\theta) = 0 \tag{14}$$

$$F'\theta_p - F\theta_p' + 2b_1\left\{\frac{1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}}{1-\phi+\phi\frac{\rho_s}{\rho_f}}\right\}(\theta_p - \theta) = 0 \tag{15}$$

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Equation (12) is transformed into

$$\theta(\eta) = 1 \text{ at } \eta = 0 \text{ and } \theta(\eta) \to 0\,,\ \theta_p \to 0 \text{ as } \eta \to \infty, \tag{16}$$

Where $P_{r=}(\mu c_p)_f/k_f$ is the Prandtl number , $E_c = \frac{U_0^2}{C_p T_0}$ is the Eckert number, N is the number density of the dust particles, $\tau_v = \frac{m}{k}$ is the relaxation time of the particle phase, $a_1 = \frac{L}{U_0 \rho \tau_T} e^{\frac{-x}{L}}$ and , $b_1 = \frac{c_p L}{U_0 c_m \tau_T} e^{\frac{-x}{L}}$ are the local fluid particle interaction parameter for temperature, $a_2 = \frac{L}{U_0 \rho \tau_v} e^{\frac{x}{2L}}$ is the local fluid particle interaction parameter for velocity.

The constants for the nanofluid are provided by:

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + (\phi)(\rho c_p)_s\ , \tag{17}$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\ , \tag{18}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}\ , \tag{19}$$

$$(\rho)_{nf} = (1-\phi)(\rho)_f + \phi\rho_s\,. \tag{20}$$

Where μ_{nf} is the dynamic viscosity of the nanofluid, k is the Stoke's resistance, ϕ is the volume fraction of the nano particles. The subscripts f and s refer to fluid and solid properties respectively.

IV. RESULTS AND DISCUSSIONS

The MATLAB bvp4c technique is used to numerically solve the coupled nonlinear ordinary differential equations (7), (8), (14), and (15) with boundary equations (9) and (16). With the exception of the different values displayed in the graphs and table, we maintained the following values for the numerical results throughout the investigation.

$$\phi_d = 0.1, \phi = 0.1, \alpha = 2, \beta = 2, N = 1, P_r = 1, E_c = 2, a_1 = 2, a_2 = 2\ ,\ b_1 = 1,\ M = 1, \eta = 5.$$

We considered Cu-H_2O nanofluid embedded with conducting dust particles.

Table 1 Thermo Physical Properties of Base Fluid Water (H_2O) and Nanoparticle Copper (Cu).

	ρ(Kgm^{-3})	**c_p(JKg^{-1}K^{-1})**	**k(Wm^{-1}K^{-1})**
H_2O	997.1	4179	0.613
Cu	8933	385	401

The effect of the magnetic field parameter M on the fluid and dust phase velocity and temperature profiles is depicted in Figures 2 and 3. Figure 2 makes it clear that fluid flow velocity diminishes with enhancing M. Figure 3 makes it evident that the temperature profile increases as M rises.

Figure 4 exhibits how the Prandtl number Pr influences fluid flow temperature profiles. It makes reasonable that the temperature of the fluid flow decreases with rising Pr.

The effect of Eckert number Ec on fluid flow temperature profiles is depicted in Figure 5. It has been found that raising the Ec value raises the fluid flow temperature.

Figures 6 and 7 demonstrate the manner in which dust particle α mass concentration affects fluid flow velocity and temperature curves. Figure 7 displays the way rising values of α raise the temperature of the fluid flow, whereas Figure 6 reveals that increasing α slows down the velocity of both the fluid phase and the dust phase.

The temperature profile and velocity of the fluid flow are influenced by the fluid particle interaction parameter for the velocity β, as shown in Figures 8 and 9. Figure 8 exhibits that as β is elevated, the fluid phase temperature rises while the dust phase temperature is reduced. Figure 9 illustrates how a decreasing fluid flow temperature is caused by increases in β.

Figures 10 and 11 highlight the way the volume percentage of dust particles ϕ_d impacts fluid flow velocity and temperature profiles. The fluid phase and dust phase velocities also decrease with hiking, as seen in Figure 10.d. Figure 11 indicates that when ϕ_d increases, the fluid flow's temperature increases.

Figures 12 and 13 demonstrate the manner in which the volume percentage of nanoparticles ϕ affects fluid flow velocity and temperature profiles. It is apparent from Figures 12 and 13 that the fluid flow's temperature is higher and its velocity drops as ϕ increases. Table 2exhits the impact of different values of $M, \alpha, \beta, \phi, \phi_d$on friction factor and rate of heat transfer.Table 3 depicts the influence of different values of Prandtl number Pr on rate of heat transfer.

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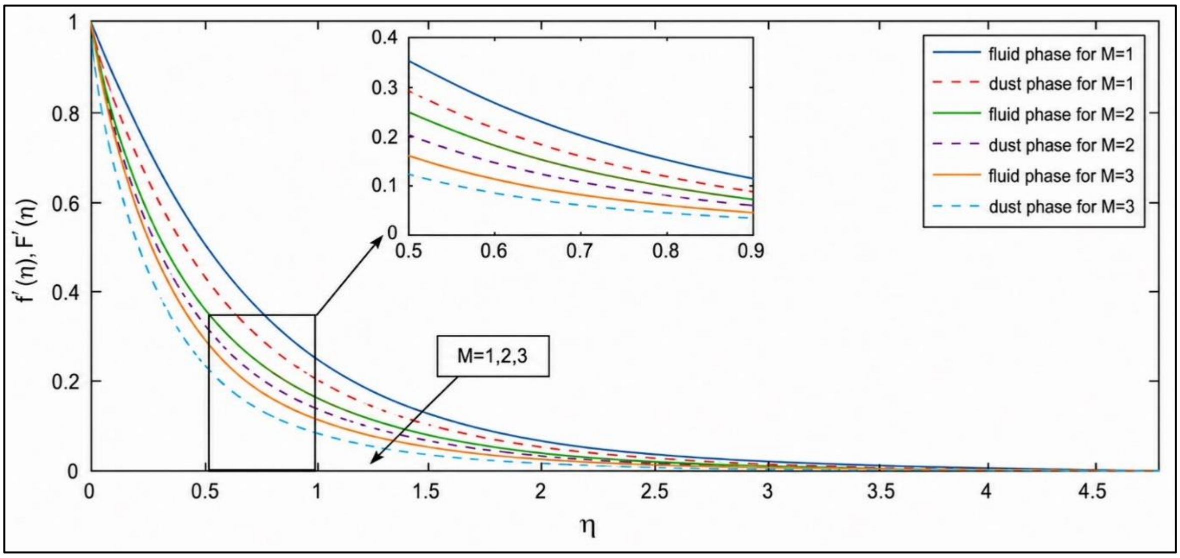

Fig 2 Profiles of Velocity for a Range of M.

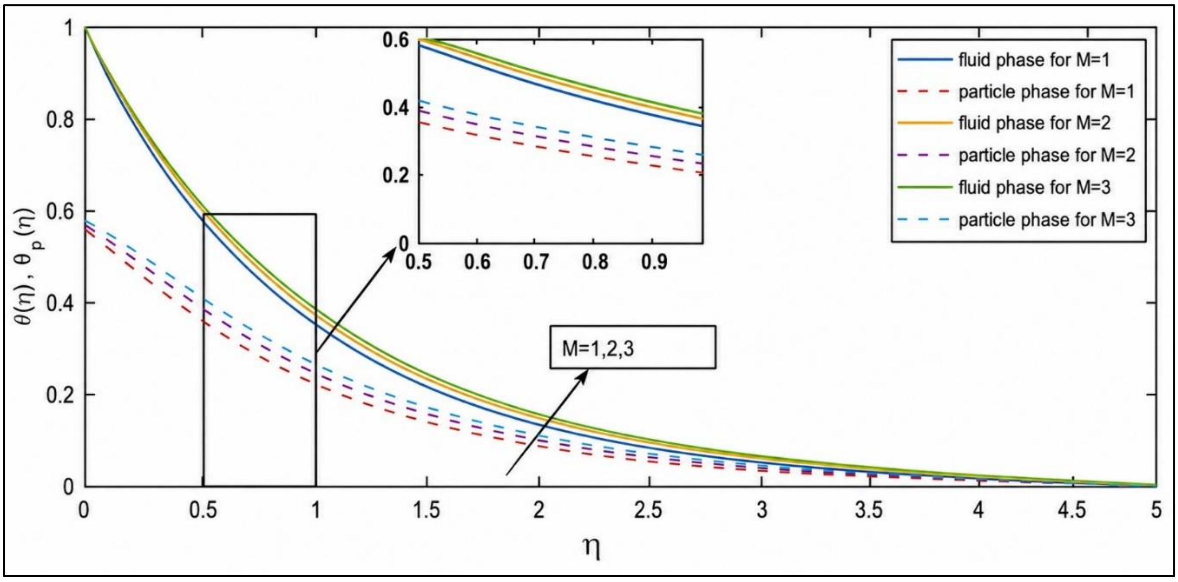

Fig 3 Profiles of Temperatures for a Range of M.

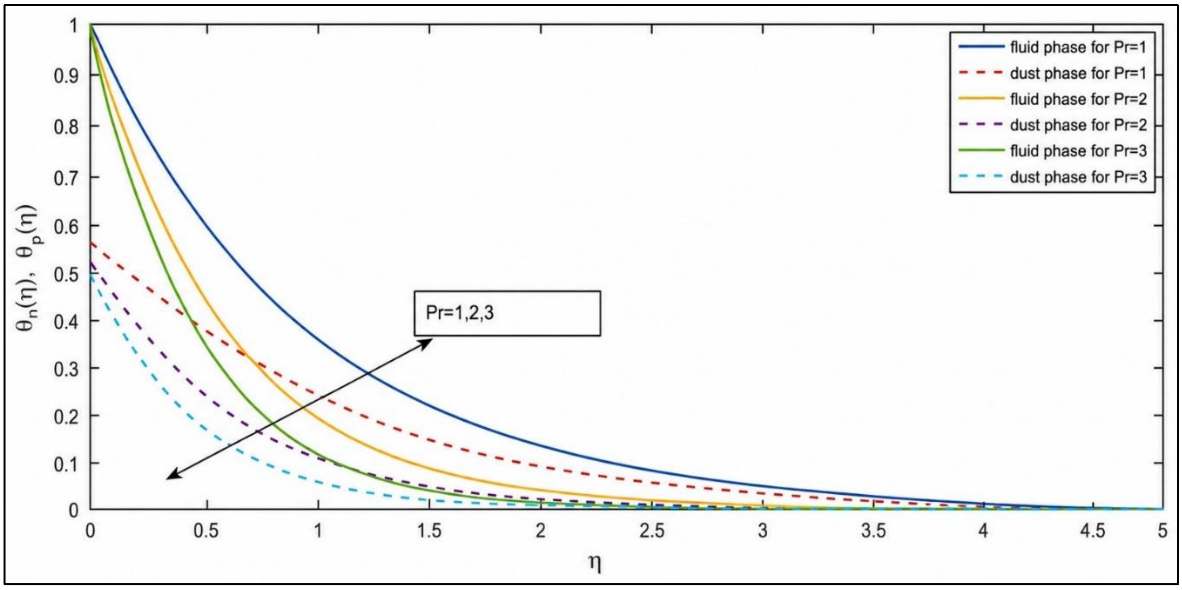

Fig 4 Profiles of Temperatures for a Range of Pr.

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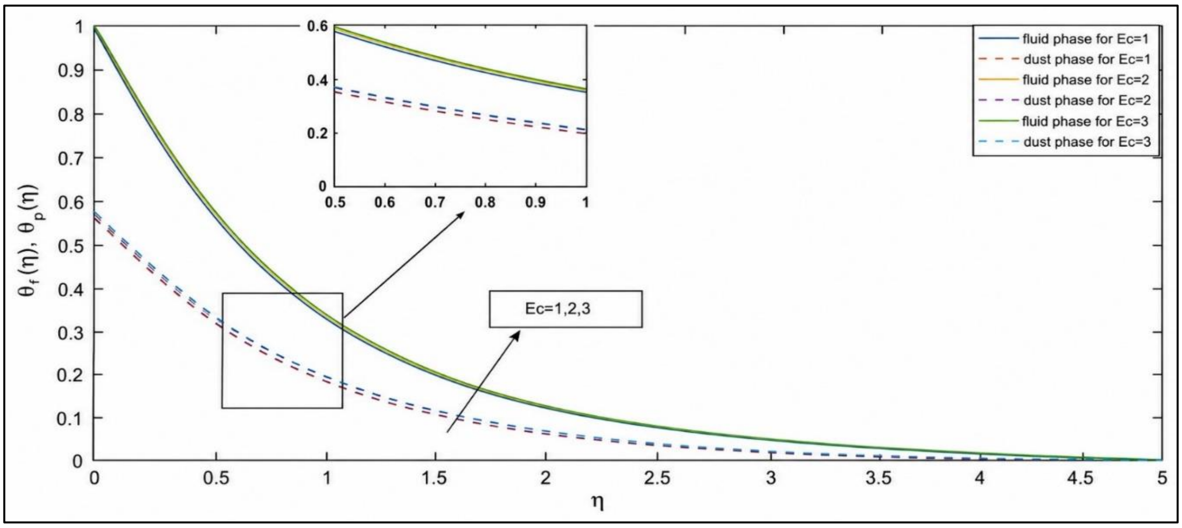

Fig 5 Profiles of Temperatures for a Range of Ec.

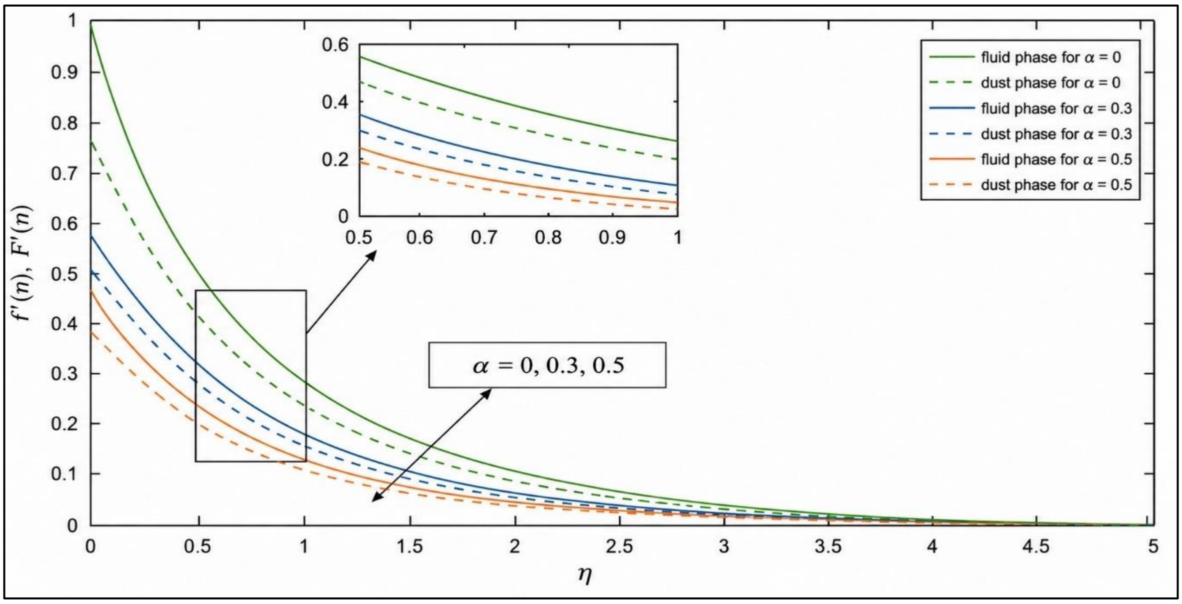

Fig 6 Profiles of Velocity for a Range of α.

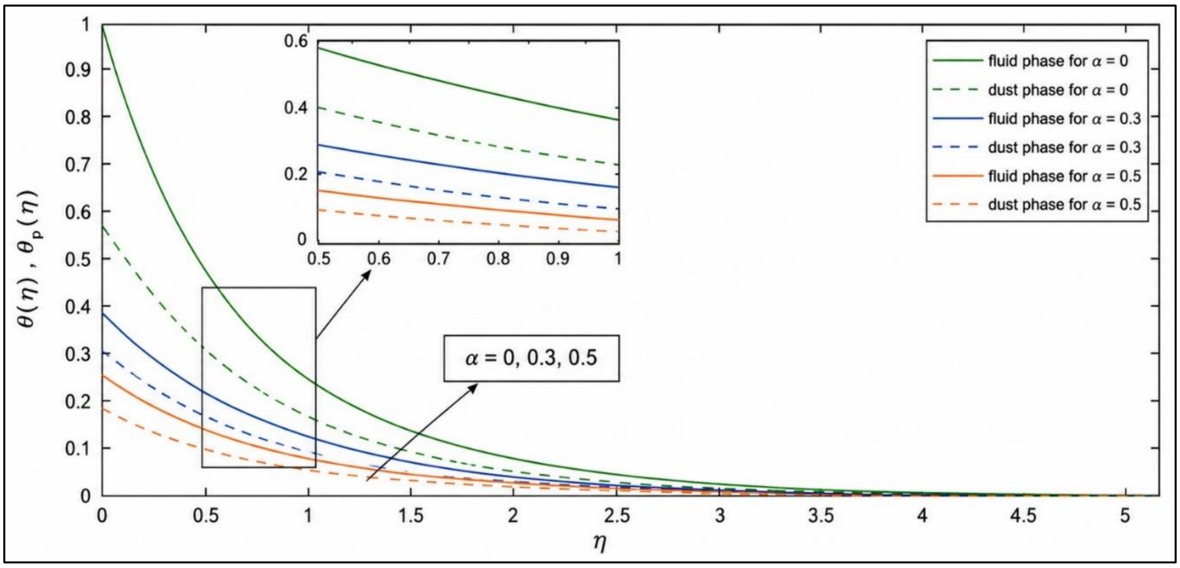

Fig 7 Profiles of Temperatures for a Range of α.

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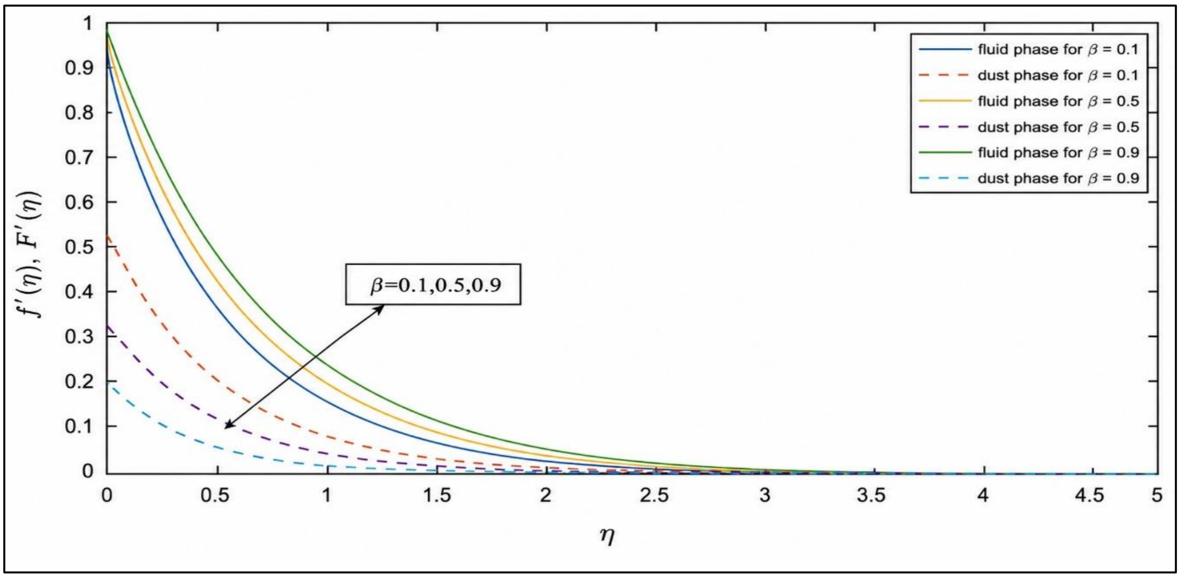

Fig 8 Profiles of Velocity for a Range of β.

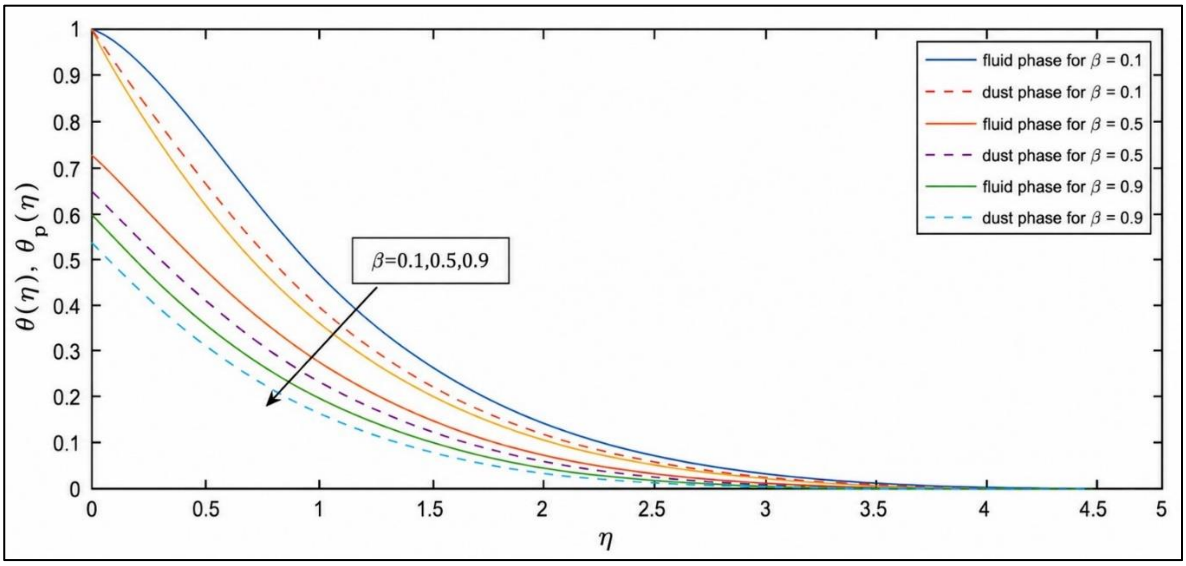

Fig 9 Profiles of Temperatures for a Range of β.

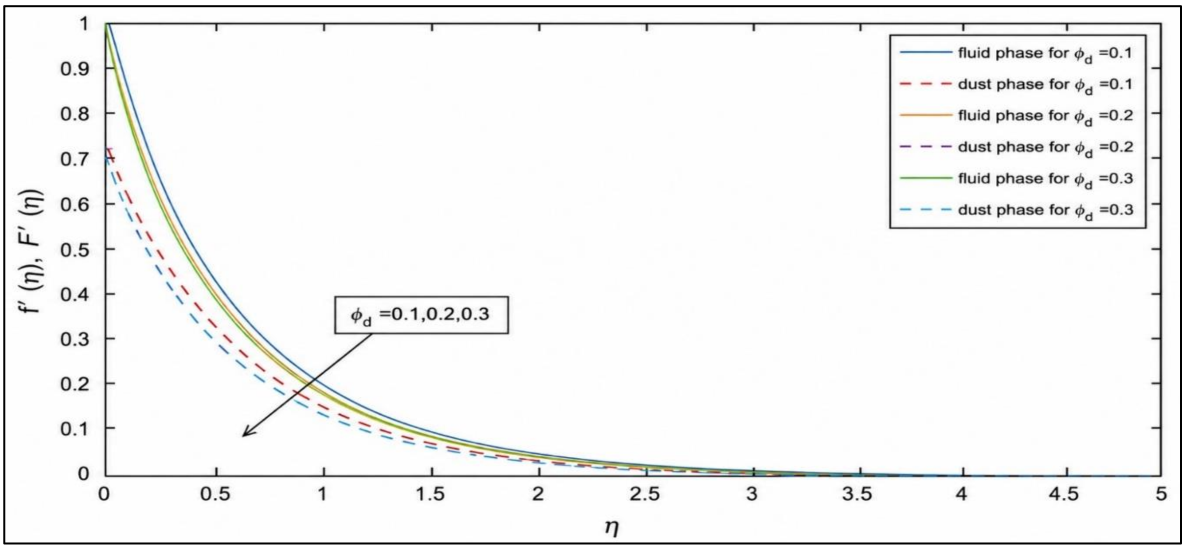

Fig 10 Profiles of Velocity for a Range of ϕ_d.

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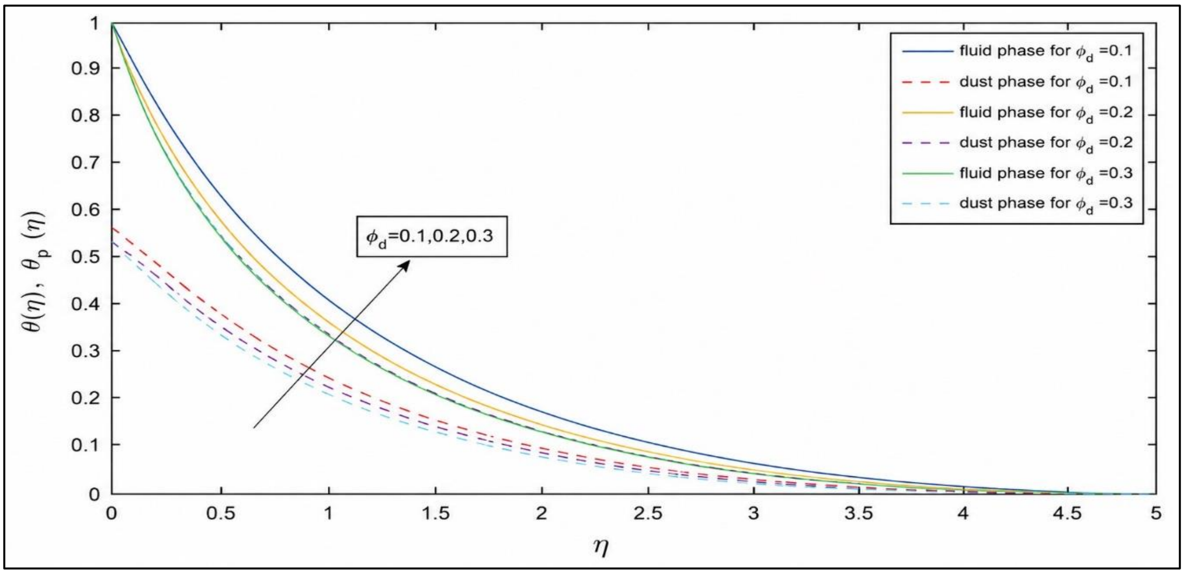

Fig 11 Profiles of Temperatures for a Range of ϕ_d.

Table 2 Variation in $f''(0)$ and $-\theta'(0)$.

M	α	β	Φ	Φ_d	f''(0)	$-\theta'(0)$
1					-2.217848908324490	0.939052628644068
2					-2.403820165281620	0.909190099111826
3					-2.576056394610110	0.882666726788899
	0.1				-1.795109357677720	0.998654429625400
	0.3				-1.844179305592640	0.991694968261334
	0.5				-1.891977388548490	0.984915098731943
		0.1			-1.849704674646180	0.072379567189720
		0.5			-2.017133370842080	0.647450122587729
		0.9			-2.102462357333120	0.808117420821501
			0.1		-2.217848938084240	0.939034489773991
			0.2		-2.158868886749980	0.715384726507798
			0.3		-2.025316977281410	0.561273710958247
				0.1	-2.217848908276500	1.067564573203050
				0.2	-2.291212127078240	1.056459287529190
				0.3	-2.382205439916200	1.042762942850320

Table 3 Temperature Gradient – $\theta'(0)$ for Different Values of Prandtl Number Pr with $M = \alpha = \beta = \phi = \phi_d = N = 0$.

P_r	Magyari et al [4]	Gireesha et al [7]	Present study
0.5	0.59434	0.59447	0.619972313635805
1	0.95478	0.95481	0.957645556228163
3	1.86908	1.86907	1.86854354710454
5	2.50014	2.49992	2.49972602501410
8	3.24213	3.24177	3.24177599270223
10	3.66038	3.66037	3.66004811879046

V. CONCLUSION

The fluid flow and heat transmission of a dusty nanofluid on an exponentially expanding sheet are simulated numerically using Cu-H2O nanofluid scattered with conductive dust particles. The governing equations of heat transfer and dusty nanofluid flow are transformed using similarity analysis. The simplified equations are solved using the MATLAB bvp4c approach. The effects of the fluid and dust phases' temperature and velocity profiles in respect to different non-dimensional parameter values are shown and analyzed.

- The temperature gradient and friction factor are reduced by the magnetic field parameter.
- As the mass concentration of dust particles increases, the temperature gradient and friction factor decrease.
- The temperature gradient rises as the fluid particle interaction parameter β grows, yet the friction factor decreases.
- The temperature gradient decreases as the volume fraction of nanoparticles (φ) increases. the element of friction.
- As the volume percent of dust particles $\boldsymbol{\phi_d}$ increases, both the temperature gradient and the friction factor decrease.
- Current results are compatible with the temperature gradient for different Prandtl number Pr values.

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