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Normal Spaces on Vague Sets

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Abstract: The concept of vague sets provides an effective mathematical framework for representing uncertainty by considering both the evidence supporting and opposing the membership of an element. Integrating vague set theory with topology leads to the development of vague topological spaces, which generalize classical topological concepts under uncertain environments. In this paper, we introduce and investigate $\mathcal{V}_{sg}$- T_r, $\mathcal{V}_{sg}$-T_s, $\mathcal{V}_{sg}$-$T_{1/2}$ spaces and examines few theorems pertaining to them. Several fundamental properties and characterization theorems of vague normal spaces are established. The relationships between vague normality are examined, and the behaviour of vague normality under vague continuous mappings and subspaces is discussed. Furthermore, sufficient conditions for a vague topological space to be vague normal are presented, accompanied by suitable illustrative examples. The results obtained contribute to the theory of vague topology and provide a foundation for further research in uncertainty modelling, decision-making, information systems, and intelligent computing.

Keywords: $\mathcal{V}$ *– normal space*, $\mathcal{V}$ *– sg normal space*, $\mathcal{V}$ *– ultra normal space*, $\mathcal{V}$ *- sg ultra normal space*, $\mathcal{V}$ *- almost normal space*, $\mathcal{V}$ *-mildly normal space*, $\mathcal{V}$ *- sg almost normal space*, $\mathcal{V}$ *-sg mildly normal space*, $\mathcal{V}$ *- sg strongly normal space.*

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I. INTRODUCTION

Zadeh [27] developed the fuzzy set notion in 1965 and has since penetrated almost every area of mathematics. In 1967, Chang [8] developed the idea of fuzzy topological spaces. In 1970, Levine [16] started looking into generalized closed sets. Levine's semi-open sets [15] have been used instead of open sets to extend many topological concepts. Semi-generalized closed sets were implemented by Bhattacharyya and Lahiri [7] via semi-openness. In topological spaces, Arya and Nour [4] presented generalized semi-open sets. Atanassov created the intuitionistic fuzzy set (IFS) in 1986[5], which is another kind of fuzzy set that works better in practical situations. Dogan Coker [9] put forward an intuitionistic fuzzy topological space. Santhi R [19] presented intuitionistic fuzzy semi-generalized closed sets and their uses in 2010.

The topic of vague graphs with associated properties was covered by Rashmanlou, H., and Samanta S [18] in 2016.Singal and Singal [23] independently provide up with t he concept of mildly normal space.

Singal and Arya
[22] presented the fact that space is almost normal. In 2010, Sharma and Kumar [21] also looked at the idea of $\pi\beta$-normality. Nidhi Sharma [17] investigated almost normality in 2014. Hamant Kumar and Sharma [12] investigated into almost γ-normal spaces. Jothimani [13] presented and examined a number of $\pi g\beta$ normal spaces in intuitionistic fuzzy topological spaces in 2016.

Abdul Gawad [1] also created and investigated the concepts of normality and b- regularity in fuzzy intuitionistic topological spaces during 2019. Later in 2018, Abdul Gawad [2] introduced and studied the new kinds of fuzzy intuitionistic topological spaces that are β *and* $\beta*$ normal spaces. Vague sets were initially put forward by Gau and Buehrer [11] in 1993 as a consequence of fuzzy set theory. Actually, Vakithabegam R and Helen [24], [25] presented vague semi-generalized closed sets in vague topological spaces in 2022. Vague semi-generalized continuous and irresolute mappings in vague topological spaces were added in 2024.

Although being a theoretical idea in topology, vague normal spaces are useful in a wide range of fields, including bioinformatics, robotic navigation, data analysis, urban planning, and network performance optimization. Normal spaces enhance theoretical understanding and practical abilities to solve problems in practical situations by simplifying the process of separate closed sets [3, 6, 14, 26]. Vague sets have occupied a large amount of space in these articles.

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II. PREAMBLES

➢ *Definition 2.1:[11]*

Let us suppose that $\mathbb{Y}$ is a non-empty set. In the universe of discourse $\mathbb{Y}$, a *vague* set $\mathcal{B}_{v}$ in order to describe two membership functions are provided by,

- $T_{\mathcal{B}_{v}}: \mathbb{Y} \to [0,1]$ is a true membership function and
- $F_{\mathcal{B}_{v}}: \mathbb{Y} \to [0,1]$ is a false membership function

Whereas $T_{\mathcal{B}_{v}}(y)$ is called "evidence for y" and is lower bound on the grade of membership of y, $F_{\mathcal{B}_{v}}(y)$ is called "evidence against y" and is an upper bound on the negation of y which satisfies the condition $T_{\mathcal{B}_{v}}(y) + F_{\mathcal{B}_{v}}(y) \leq 1$. Therefore, a subinterval $\left(T_{\mathcal{B}_{v}}(y), 1 - F_{\mathcal{B}_{v}}(y)\right)$ of $[0,1]$ is the grade of membership of y in the *vague* set $\mathcal{B}_{v}$. This shows that $T_{\mathcal{B}_{v}}(y) \leq \mu_{v}(y) \leq 1 - F_{\mathcal{B}_{v}}(y)$, where $\mu_{v}(y)$ *is* grade of membership. Hence $\mathcal{B}_{v} = \{\langle y, (T_{\mathcal{B}_{v}}(y), 1 - F_{\mathcal{B}_{v}}(y))\rangle / y \in \mathbb{Y}\}$ is a *vague* in $\mathcal{B}_{v}$.

➢ *Definition 2.2:[11]*

Let us suppose that $\mathcal{B}_{v}$ and $\mathcal{C}_{v}$ is a *vague* set of the form $\mathcal{B}_{v} = \{\langle y, (T_{\mathcal{B}_{v}}(y), 1 - F_{\mathcal{B}_{v}}(y))\rangle / y \in \mathbb{Y}\}$ and $\mathcal{C}_{v} = \{\langle y, (T_{\mathcal{C}_{v}}(y), 1 - F_{\mathcal{C}_{v}}(y))\rangle / y \in \mathbb{Y}\}$.

Then,

- $\mathcal{B}_{v} \subseteq \mathcal{C}_{v}$ if and only if $T_{\mathcal{B}_{v}}(y) \leq T_{\mathcal{C}_{v}}(y)$ and $1 - F_{\mathcal{B}_{v}}(y) \leq 1 - F_{\mathcal{C}_{v}}(y)$ for all $y \in \mathbb{Y}$
- $\mathcal{B}_{v} = \mathcal{C}_{v}$ if and only if $\mathcal{B}_{v} \subseteq \mathcal{C}_{v}$ and $\mathcal{C}_{v} \subseteq \mathcal{B}_{v}$
- $\mathcal{B}_{v}^{c} = \left\{\langle y, \left(F_{\mathcal{B}_{v}}(y), 1 - T_{\mathcal{B}_{v}}(y)\right) / y \in \mathbb{Y}\rangle\right\}$
- $\mathcal{B}_{v} \cap \mathcal{C}_{v} = \left\{\langle y, \min\left(T_{\mathcal{B}_{v}}(y), T_{\mathcal{C}_{v}}(y)\right), \min\ (1 - F_{\mathcal{B}_{v}}(y), 1 - F_{\mathcal{C}_{v}}(y))\rangle / y \in \mathbb{Y}\right\}$
- $\mathcal{B}_{v} \cup \mathcal{C}_{v} = \left\{\langle y, \max\left(T_{\mathcal{B}_{v}}(y), T_{\mathcal{C}_{v}}(y)\right), \max\ (1 - F_{\mathcal{B}_{v}}(y), 1 - F_{\mathcal{C}_{v}}(y))\rangle / y \in \mathbb{Y}\right\}$

For simplicity, we shall use the notation $\mathcal{B}_{v} = \langle y, T_{\mathcal{B}_{v}}, 1 - F_{\mathcal{B}_{v}}\rangle$ instead of $\mathcal{B}_{v} = \{\langle y, (T_{\mathcal{B}_{v}}(y), 1 - F_{\mathcal{B}_{v}}(y))\rangle / y \in \mathbb{Y}\}$.

➢ *Definition 2.3:[10]*

An $Intuitionistic\ fuzzy\ topological\ space$ $(\mathbb{Y}, \xi_{v})$ is called normal if for every $Intuitionistic\ fuzzy\ closed\ sets\ U\ and\ V$ such that there exists disjoint $Intuitionistic\ fuzzy\ open\ sets\ U_1, V_1$ such that $U \subseteq U_1$ and $V \subseteq V_1$

➢ *Definition 2.4:[22]*

An $Intuitionistic\ fuzzy\ topological\ space$ $(\mathbb{Y}, \xi_{v})$ is called $mildly\ normal$ if for every two disjoint sets $regular\ closed\ sets\ U\ and\ V$ such that there exists disjoint $Intuitionistic\ fuzzy\ open\ sets\ U_1, V_1$ such that $U \subseteq U_1$ and $V \subseteq V_1$.

➢ *Definition 2.5:[23]*

An $Intuitionistic\ fuzzy\ topological\ space$ $(\mathbb{Y}, \xi_{v})$ is called $almost\ normal$ if for every $IFCS\ U, IF\ regular\ closed\ sets\ V$ such that $U \cap V = 0$, there exists disjoint $Intuitionistic\ fuzzy\ open\ sets\ U_1, V_1$ such that $U \subseteq U_1$ and $V \subseteq V_1$.

➢ *Definition 2.6:[20]*

An $Intuitionistic\ fuzzy\ topological\ space$ $(\mathbb{Y}, \xi_{v})$ is called $strongly\ normal$ if for every two disjoint sets $IFCS\ U\ and\ V,$ there exists disjoint $Intuitionistic\ fuzzy\ open\ sets\ U_1, V_1$ such that $U \subseteq U_1$ and $V \subseteq V_1$.

III. $\mathcal{V}_{sg}$- T_r, $\mathcal{V}_{sg}$-T_s, $\mathcal{V}_{sg}$-$T_{1/2}$ SPACE

➢ *Definition 3.1:*

A $\mathcal{VT}$ $(\mathbb{Y}, \xi_{v})$ is defined to be $\mathcal{V}_{sg}$- T_r space if every $\mathcal{V}$-$sg\mathcal{CS}$ is $\mathcal{V}$- $r\mathcal{CS}$ in $\mathbb{Y}$. The collection of all $\mathcal{V}$- $r\mathcal{OS}$ in $(\mathbb{Y}, \xi_{v})$ is denoted by $\mathcal{V}$- $r\mathcal{O}(\xi_{v})$.

➢ *Theorem 3.2:*

Every $\mathcal{V}_{sg}$- $T_{1/2}\ space$ is $\mathcal{V}_{sg}$- $T_s\ space$ but the converse need not true.

Proof: Assume that $(\mathbb{Y}, \xi_{v})$ is a $\mathcal{V}_{sg}$- $T_{1/2}$ space and let $\mathcal{B}_{v}$ be a $\mathcal{V}$- $sg\mathcal{CS}$. Let $\mathcal{B}_{v} \in \mathcal{V}_{sg}$- $T_{1/2}$ space. So $\mathcal{B}_{v}$ is $\mathcal{V}$-$sg\mathcal{CS}$. But since every $\mathcal{V}$- $\mathcal{CS}$ is $\mathcal{V}$- $s\mathcal{CS}$. Hence every $\mathcal{V}$-$sg\mathcal{CS}$ is $\mathcal{V}$- $s\mathcal{CS}$. Thus $\mathcal{B}_{v} \in \mathcal{V}_{sg}$- T_s space. Therefore, the theorem is proved.

➢ *Example 3.3:*

Let $\mathbb{Y} = \{r, s\}$ and let $\xi_{v} = \{0_{v}, 1_{v}, \mathcal{H}_{v}\}$ be a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{H}_{v} = \{\langle r, (0.4, 0.9)\rangle, \langle s, (0.5, 0.8)\rangle\}$. Let $\mathcal{B}_{v} = \{\langle r, (0.3, 0.4)\rangle, \langle s, (0.7, 0.9)\rangle\}$ be any $\mathcal{V}$- $\mathcal{S}$in $\mathbb{Y}$. Therefore $\mathcal{B}_{v}$ is $\mathcal{V}$- $sg\mathcal{CS}$ but $\mathcal{B}_{v}$ is not $\mathcal{V}$- $\mathcal{CS}$ in $\mathbb{Y}$ since $\mathcal{V}$- $cl\ (\mathcal{B}_{v}) = {\mathcal{H}_{v}}^{c} \neq \mathcal{B}_{v}$.

➢ *Theorem 3.4:*

Every $\mathcal{V}_{sg}$- T_r space is $\mathcal{V}_{sg}$- $T_{1/2}$ space but the converse need not true.

Proof: Suppose that $(\mathbb{Y}, \xi_{v})$ a $\mathcal{V}_{sg}$- T_r space and let $\mathcal{B}_{v} \in \mathcal{V} - sg\mathcal{CS}$. So $\mathcal{B}_{v}$ is $\mathcal{V}$ - $r\mathcal{CS}$. But every $\mathcal{V}$ - $r\mathcal{CS}$ is $\mathcal{V}$ - $\mathcal{CS}$. Hence every $\mathcal{V}$ - $sg\mathcal{CS}$ is $\mathcal{V}$ - $\mathcal{CS}$. Therefore $\mathcal{B}_{v} \in \mathcal{V}_{sg}$ - $T_{1/2}$ space. Thus, the theorem is proved.

➢ *Example 3.5:*

Let $\mathbb{Y} = \{r, s\}$ and let $\xi_{v} = \{0_{v}, 1_{v}, \mathcal{H}_{v}\}$ be a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{H}_{v} = \{\langle r, (0.2, 0.4)\rangle, \langle s, (0.5, 0.6)\rangle\}$. Let $\mathcal{B}_{v} = \{\langle r, (0.3, 0.4)\rangle, \langle s, (0.6, 0.8)\rangle\}$ be any $\mathcal{V}$-$\mathcal{S}$ in $\mathbb{Y}$. Therefore $\mathcal{B}_{v}$ is $\mathcal{V}$ - $sg\mathcal{CS}$ but $\mathcal{B}_{v}$ is not $\mathcal{V}$ - $r\mathcal{CS}$ in $\mathbb{Y}$ since $\mathcal{V}$- cl ($\mathcal{V}$ - int $(\mathcal{B}_{v})) = 0_{v} \neq \mathcal{B}_{v}$.

➢ *Theorem 3.6:*

Suppose that $(\mathbb{Y}, \xi_{v})$ be a $\mathcal{VT}$. In turn, the following assertions are true.

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- $\mathcal{V}$ - $\mathcal{O}$ $(\xi_v) \subset \mathcal{V}$ - $sg\mathcal{O}$ (ξ_v)
- $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}_{sg}$ - $T_{1/2}$ space if $\mathcal{V}$ - $\mathcal{O}$ $(\xi_v) = \mathcal{V}$ - $sg\mathcal{O}$ (ξ_v)
- *Proof*: Proof is obvious.

➢ *Theorem 3.7:*

Suppose that $(\mathbb{Y}, \xi_v)$ be a $\mathcal{VT}$. In turn, the following assertions are true.

- $\mathcal{V}$ - $r\mathcal{O}$ $(\xi_v) \subset \mathcal{V}$ - $sg\mathcal{O}$ (ξ_v)
- $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}_{sg}$ - T_r space if $\mathcal{V}$ - $r\mathcal{O}$ $(\xi_v) = \mathcal{V}$ - $sg\mathcal{O}$ (ξ_v)
- *Proof*: Proof is obvious.

➢ *Theorem 3.8:*

Suppose that $(\mathbb{Y}, \xi_v)$ be a $\mathcal{VT}$. In turn, the following assertions are true.

- $\mathcal{V}$ -$s\mathcal{O}$ $(\xi_v) \subset \mathcal{V}$ -$sg\mathcal{O}$ (ξ_v)
- $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}_{sg}$ -T_s space if $\mathcal{V}$ -$s\mathcal{O}$ $(\xi_v) = \mathcal{V}$ -$sg\mathcal{O}$ (ξ_v)
- *Proof*: Proof is clear.

IV. $\mathcal{V}$ -sg $\mathcal{N}$ORMAL $\mathcal{S}$PACE

➢ *Definition 4.1:*

A $\mathcal{VT}$ $(\mathbb{Y}, \xi_v)$ is called $\mathcal{V}$ *-normal space* ($\mathcal{V}$ -$\mathbb{NS}$ in short), if for every two disjoint, $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$ then there exists two disjoint, $\mathcal{V}$ -$\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ such that $\mathcal{B}_v \subseteq \mathcal{H}_v$ and $\mathcal{C}_v \subseteq \mathcal{J}_v$.

➢ *Definition 4.2:*

A $\mathcal{VT}$ $(\mathbb{Y}, \xi_v)$ is called $\mathcal{V}$ -sg *normal space* ($\mathcal{V}$ -sg $\mathbb{NS}$ in short), if for every two disjoint, $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$ then there exists two disjoint, $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ such that $\mathcal{B}_v \subseteq \mathcal{H}_v$ and $\mathcal{B}_v \subseteq \mathcal{J}_v$.

➢ *Theorem 4.3:*

Every $\mathcal{V}$ -$\mathbb{NS}$ is $\mathcal{V}$ -sg $\mathbb{NS}$. *Proof*: Assume that $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -$\mathbb{NS}$. So that for any two disjoint, $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$ in $\mathbb{Y}$, there exists disjoint $\mathcal{V}$ -$\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ such that $\mathcal{B}_v \subseteq \mathcal{H}_v$ and $\mathcal{C}_v \subseteq \mathcal{J}_v$. Since every $\mathcal{V}$ -$\mathcal{OS}$ is $\mathcal{V}$ -$sg\mathcal{OS}$. Hence $\mathcal{H}_v$ and $\mathcal{J}_v$ are $\mathcal{V}$ -$sg\mathcal{OS}$. Thus, for every two disjoint $\mathcal{V}$ - $\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$, two disjoint $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ exists such that $\mathcal{B}_v \subseteq \mathcal{H}_v$ and $\mathcal{C}_v \subseteq \mathcal{J}_v$. Therefore $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

The example is shown below since the previous theorem's reverse need not be true.

➢ *Example 4.4:*

Let $\mathbb{Y} = \{r, s\}$ and let $\xi_v = \{0_v, 1_v, \mathcal{B}_v, \mathcal{C}_v\}$ be a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{B}_v = \{\langle r, (0.2, 0.3)\rangle, \langle s, (0.4, 0.9)\rangle\}$, $\mathcal{C}_v = \{\langle r, (0.6, 0.9)\rangle, \langle s, (0.4, 0.9)\rangle\}$. For the pair of disjoint, $\mathcal{V}$ -$\mathcal{CS}$ 0_v and $\mathcal{B}_v{}^c$ there exists a pair of disjoint, $\mathcal{V}$ -$sg\mathcal{OS}$ 0_v and $\mathcal{C}_v$ such that $0_v \subseteq 0_v$ and $\mathcal{B}_v{}^c \subseteq \mathcal{C}_v$. Hence $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -sg $\mathbb{NS}$. But for the pair of $\mathcal{V}$ - $\mathcal{CS}$ 0_v and $\mathcal{C}_v{}^c$, there is no distinct $\mathcal{V}$ -$\mathcal{OS}$ exists which satisfies the condition of $\mathcal{V}$ -$\mathbb{NS}$.Thus $\mathcal{V}$ -sg $\mathbb{NS}$ does not have to be equal to $\mathcal{V}$ - $\mathbb{NS}$.

➢ *Theorem 4.5:*

Suppose that $(\mathbb{Y}, \xi_v)$ a $\mathcal{V}$ -sg $\mathbb{NS}$ and $\mathcal{V}_{sg}$ -$T_{1/2}$ space then $(\mathbb{Y}, \xi_v)$ is a $\mathcal{V}$ -$\mathbb{NS}$. *Proof*: Assume that $(\mathbb{Y}, \xi_v)$ is a $\mathcal{V}$ -sg $\mathbb{NS}$. So that for every two disjoint, $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$, two disjoint, $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ exists such that $\mathcal{B}_v \subseteq \mathcal{H}_v$ and $\mathcal{C}_v \subseteq \mathcal{J}_v$. Every $\mathcal{V} - sg\mathcal{OS}$ is $\mathcal{V}$ -$\mathcal{OS}$, since $(\mathbb{Y}, \xi_v)$ is a $\mathcal{V}_{sg}$ -$T_{1/2}$ space. Thus $\mathcal{H}_v$ and $\mathcal{J}_v$ are $\mathcal{V}$ -$sg\mathcal{OS}$ in $\mathbb{Y}$. Hence $(\mathbb{Y}, \xi_v)$ is a $\mathcal{V}$ -$\mathbb{NS}$.

➢ *Theorem 4.6:*

Suppose that $(\mathbb{Y}, \xi_v)$ a $\mathcal{VT}$. Consequently, the following are similar to one another:

- $\mathbb{Y}$ is a $\mathcal{V}$ -sg $\mathbb{NS}$
- For every two $\mathcal{V}$ -$\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ whose union is 1_v , there exists $\mathcal{V}$ -$sg\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$ such that $\mathcal{B}_v \subseteq \mathcal{H}_v$ and $\mathcal{C}_v \subseteq \mathcal{J}_v$ and $\mathcal{B}_v \cup \mathcal{C}_v = 1_v$.
- For every $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{K}_v$ and every $\mathcal{V}$ -$\mathcal{OS}$ $\mathcal{L}_v$ containing $\mathcal{K}_v$, there exists a $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{H}_v$ such that $\mathcal{K}_v \subseteq \mathcal{H}_v \subseteq \mathcal{V}$ -sg $c\ell(\mathcal{H}_v) \subseteq \mathcal{L}_v$.
- For every pair of $\mathcal{V}$ -$sg\mathcal{CS}$ $\mathcal{K}_v$ and $\mathcal{L}_v$ of $\mathbb{Y}$, a $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{H}_v$ of $\mathbb{Y}$ exists that satisfies the condition $\mathcal{K}_v \subseteq \mathcal{H}_v$ and $\mathcal{V}$ -sg $c\ell(\mathcal{H}_v) \cap \mathcal{L}_v = 0_v$.
- For every pair of disjoint, $\mathcal{V}$ -$sg\mathcal{CS}$, $\mathcal{K}_v$ and $\mathcal{L}_v$ of $\mathbb{Y}$, there exists a $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ of $\mathbb{Y}$ such that $\mathcal{K}_v \subseteq \mathcal{H}_v$ and $\mathcal{L}_v \subseteq \mathcal{J}_v$ and $\mathcal{V}$ -sg $c\ell(\mathcal{H}_v) \cap \mathcal{V}$ -sg $c\ell(\mathcal{J}_v) = 0_v$.
- *Proof*: Proof is obvious.

➢ *Definition 4.7:*

Let ant two $\mathcal{VT}$ be $\mathbb{Y}$ and $\mathbb{Z}$. A $\mathcal{V}$ -*bijective mapping* $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is said to be $\mathcal{V}$ -$\mathcal{C}$ *homeomorphism*($\mathcal{V} - \mathcal{C}$ $\mathcal{H}om$ in short) if p and p^{-1} is $\mathcal{V}$ -$\mathcal{CTM}$.

➢ *Theorem 4.8:*

$\mathcal{V}$ - *normality* satisfies the property of topology.

Proof is followed by definition.

➢ *Theorem 4.9:*

If $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is a $\mathcal{V}$ -$\mathcal{CTM}$, $\mathcal{V}-$ *bijective mapping*, $\mathcal{V}$ - sg strongly open *m*apping from a $\mathcal{V}$ -sg $\mathbb{NS}$, $\mathbb{Y}$ *onto* $\mathbb{Z}$ then $\mathbb{Z}$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is followed by definition.

➢ *Theorem 4.10:*

If $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is a $\mathcal{V}$ -*bijective mapping*, $\mathcal{V}$ -sg strongly open mapping , $\mathcal{V}$ -*IR function* from $\mathcal{V}$ -sg $\mathbb{NS}$ $\mathbb{Y}$ *into a* $\mathcal{VT}$ $\mathbb{Z}$, then $\mathbb{Z}$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is obvious.

➢ *Definition 4.11:*

A function $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is called $\mathcal{V}$ -sg *strongly continuous*, p^{-1} $(\mathcal{J}_v)$ is $\mathcal{V}$ -$\mathcal{OS}$ in $\mathbb{Y}$ for every $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{J}_v$ in $\mathbb{Z}$.

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➢ *Definition 4.12:*

A function $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ is called $\mathcal{V}$ -closed $mapping$, if $\mathcal{p}(\mathcal{B}_{\mathcal{v}})$ is $\mathcal{V}$ -$\mathcal{CS}$ in $\mathbb{Z}$ for every $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_{\mathcal{v}}$ in $\mathbb{Y}$.

➢ *Theorem 4.13:*

If a $mapping$ $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ is $\mathcal{V}$ -sg $strongly$ $continuous$, $\mathcal{V}$ -closed $mapping$ and $\mathbb{Z}$ is $\mathcal{V}$ -sg $\mathbb{NS}$, then $\mathbb{Y}$ is $\mathcal{V}$ -$\mathbb{NS}$.

Proof: Let us assume that $E_{\mathcal{v}}$ and $F_{\mathcal{v}}$ be two disjoint $\mathcal{V}$ -$\mathcal{CS}$ in $\mathbb{Y}$. As $\mathcal{p}$ is $\mathcal{V}$ -closed $mapping$, $\mathcal{p}(E_{\mathcal{v}})$ and $\mathcal{p}(F_{\mathcal{v}})$ are two disjoint $\mathcal{V}$ -$\mathcal{CS}$ in $\mathbb{Z}$. Since $\mathbb{Z}$ is $\mathcal{V}$ -sg $\mathbb{NS}$, there exists disjoint $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{H}_{\mathcal{v}}$ and $\mathcal{J}_{\mathcal{v}}$ such that $\mathcal{p}(E_{\mathcal{v}}) \subseteq \mathcal{H}_{\mathcal{v}}$ and $\mathcal{p}(F_{\mathcal{v}}) \subseteq \mathcal{J}_{\mathcal{v}}$. Hence $E_{\mathcal{v}} \subseteq \mathcal{p}^{-1}(\mathcal{H}_{\mathcal{v}})$ and $F_{\mathcal{v}} \subseteq \mathcal{p}^{-1}(\mathcal{J}_{\mathcal{v}})$. As $\mathcal{p}$ is $\mathcal{V}$ -sg $strongly$ $continuous$, $\mathcal{p}^{-1}(\mathcal{H}_{\mathcal{v}})$ and $\mathcal{p}^{-1}(\mathcal{J}_{\mathcal{v}})$ are disjoint $\mathcal{V}$ -$\mathcal{OS}$ in $\mathbb{Y}$. Thus $\mathbb{Y}$ is $\mathcal{V}$ -$\mathbb{NS}$.

➢ *Definition 4.14:*

A function $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ is called $\mathcal{V}$ -sg $perfectly$ $continuous$, $\mathcal{p}^{-1}$ $(\mathcal{J}_{\mathcal{v}})$ is both $\mathcal{V}$ -$clopen$ in $\mathbb{Y}$ for every $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{J}_{\mathcal{v}}$ in $\mathbb{Z}$.

➢ *Definition 4.15:*

A space $(\mathbb{Y}, \xi_{\mathcal{v}})$ is called $\mathcal{V}$ -$ultra$ $normal$ $space$ if for any pair of disjoint $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}}$ in $\mathbb{Y}$ then two disjoint $\mathcal{V} - clopen$ sets $\mathcal{H}_{\mathcal{v}}$ and $\mathcal{J}_{\mathcal{v}}$ exists which satisfies $\mathcal{B}_{\mathcal{v}} \subseteq \mathcal{H}_{\mathcal{v}}$, $\mathcal{C}_{\mathcal{v}} \subseteq \mathcal{J}_{\mathcal{v}}$.

➢ *Definition 4.16:*

A space $(\mathbb{Y}, \xi_{\mathcal{v}})$ is called $\mathcal{V}$ -sg $ultra$ $normal$ $space$ if for any pair of disjoint $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}}$ in $\mathbb{Y}$ then two disjoint $\mathcal{V}$ -sg $clopen$ sets $\mathcal{H}_{\mathcal{v}}$ and $\mathcal{J}_{\mathcal{v}}$ exists which satisfies $\mathcal{B}_{\mathcal{v}} \subseteq \mathcal{H}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}} \subseteq \mathcal{J}_{\mathcal{v}}$.

➢ *Theorem 4.17:*

Every $\mathcal{V}$ -$ultra$ $normal$ $space$ is a $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is clear.

➢ *Theorem 4.18:*

If a mapping $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ is $\mathcal{V}$ -sg $perfectly$ $continuous$, $\mathcal{V}$ -injective, $\mathcal{V}$ -closed $mapping$ and $\mathbb{Z}$ is $\mathcal{V}$ -sg $\mathbb{NS}$, then $\mathbb{Y}$ is $\mathcal{V}$ -$ultra$ $normal$ $space$.

Proof is obvious.

➢ *Definition 4.19:*

Let $\mathbb{Y}$ and $\mathbb{Z}$ be any two $\mathcal{VT}$, a mapping $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ is called $\mathcal{V}$ -sg $contra$ $continuous$ if for every $\mathcal{V}$ -$\mathcal{CS}$ ($\mathcal{V}$ -$\mathcal{OS}$) $\mathcal{J}_{\mathcal{v}}$ in $\mathbb{Z}$, $\mathcal{p}^{-1}(\mathcal{J}_{\mathcal{v}})$ is $\mathcal{V}$ -$sg\mathcal{OS}$ ($\mathcal{V}$ -$sg\mathcal{CS}$) in $\mathbb{Y}$.

➢ *Theorem 4.20:*

If a $mapping$ $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ is $\mathcal{V}$ -sg $contra$ $continuous$, $\mathcal{V}$ -injective, $\mathcal{V}$ -closed $mapping$ and $\mathbb{Z}$ is a $\mathcal{V}$ -$ultra$ $normal$ $space$, then $\mathbb{Y}$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is straightforward.

➢ *Theorem 4.21:*

Let $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ be a $\mathcal{V} -$ $bijective$ $mapping$. If $\mathcal{p}$ is $\mathcal{V} - sg$ open $mapping$,

$\mathcal{V}$ -$\mathcal{CTM}$ and $(\mathbb{Y}, \xi_{\mathcal{v}})$ is a $\mathcal{V}$ -$\mathbb{NS}$, $\mathbb{Y}$ then $(\mathbb{Z}, \delta_{\mathcal{v}})$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is evident.

➢ *Theorem 4.22:*

Let $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ be a $\mathcal{V}$ -$bijective$ $mapping$. If $\mathcal{p}$ is $\mathcal{V}$ -closed $mapping$, $\mathcal{V}$ -sg $\mathcal{CTM}$ and $(\mathbb{Z}, \delta_{\mathcal{v}})$ is a $\mathcal{V}$ -$\mathbb{NS}$, $\mathbb{Y}$ then $(\mathbb{Y}, \xi_{\mathcal{v}})$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is apparent.

➢ *Theorem 4.23:*

Let $\mathcal{p} : (\mathbb{Y}, \xi_{\mathcal{v}}) \rightarrow (\mathbb{Z}, \delta_{\mathcal{v}})$ be a $\mathcal{V}$ -$injective$ $mapping$. If $\mathcal{p}$ is $\mathcal{V}$ -sg IR $function$, $\mathcal{V}$ -closed $mapping$ and $(\mathbb{Z}, \delta_{\mathcal{v}})$ is a $\mathcal{V}$ -sg $\mathbb{NS}$, $\mathbb{Y}$ then $(\mathbb{Y}, \xi_{\mathcal{v}})$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is clear.

➢ *Theorem 4.24:*

The image of a $\mathcal{V}$ -sg $\mathbb{NS}$ under a $\mathcal{V}$ -sg open $mapping$, $\mathcal{V}$ -$\mathcal{CTM}$, $\mathcal{V}$ - $injective$ $function$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is obvious.

V. $\mathcal{V}$ -sg $\mathcal{A}$LMOST NORMAL SPACE AND $\mathcal{V}$ -sg $\mathcal{M}$ILDLY NORMAL SPACE

➢ *Definition 5.1:*

A $space$ $(\mathbb{Y}, \xi_{\mathcal{v}})$ is said to be $\mathcal{V}$ -$almost$ $normal$ $space$ ($\mathcal{V}$ -$\mathbb{ANS}$ in short) if for each $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_{\mathcal{v}}$ and $\mathcal{V}$ -$r\mathcal{CS}$ $\mathcal{C}_{\mathcal{v}}$ such that $\mathcal{B}_{\mathcal{v}} \cap \mathcal{C}_{\mathcal{v}} = 0_{\mathcal{v}}$, there exists disjoint $\mathcal{V}$ -$\mathcal{OS}$ $\mathcal{H}_{\mathcal{v}}$ and $\mathcal{J}_{\mathcal{v}}$ such that $\mathcal{B}_{\mathcal{v}} \subseteq \mathcal{H}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}} \subseteq \mathcal{J}_{\mathcal{v}}$.

➢ *Definition 5.2:*

A space $(\mathbb{Y}, \xi_{\mathcal{v}})$ is called $\mathcal{V}$ -$mildly$ $normal$ $space$ ($\mathcal{V}$ -$\mathbb{MNS}$ in short) if for every two disjoint $\mathcal{V}$ -$r\mathcal{CS}$ $\mathcal{B}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}}$, two disjoint $\mathcal{V}$ -$\mathcal{OS}$ $\mathcal{H}_{\mathcal{v}}$ and $\mathcal{J}_{\mathcal{v}}$ exists which satisfies $\mathcal{B}_{\mathcal{v}} \subseteq \mathcal{H}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}} \subseteq \mathcal{J}_{\mathcal{v}}$.

➢ *Definition 5.3:*

A space $(\mathbb{Y}, \xi_{\mathcal{v}})$ is called $\mathcal{V}$ -sg $almost$ $normal$ $space$ ($\mathcal{V}$ -sg $\mathbb{ANS}$ in short) if for each $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_{\mathcal{v}}$ and $\mathcal{V}$ -$r\mathcal{CS}$ $\mathcal{C}_{\mathcal{v}}$ such that $\mathcal{B}_{\mathcal{v}} \cap \mathcal{C}_{\mathcal{v}} = 0_{\mathcal{v}}$, there exists disjoint $\mathcal{V}$ -sg $\mathcal{OS}$ $\mathcal{H}_{\mathcal{v}}$ and $\mathcal{J}_{\mathcal{v}}$ such that $\mathcal{B}_{\mathcal{v}} \subseteq \mathcal{H}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}} \subseteq \mathcal{J}_{\mathcal{v}}$.

➢ *Definition 5.4:*

A space $(\mathbb{Y}, \xi_{\mathcal{v}})$ is called $\mathcal{V}$ -sg $mildly$ $normal$ $space$ ($\mathcal{V}$ -sg $\mathbb{MNS}$ in short) if for every two disjoint $\mathcal{V}$ -$r\mathcal{CS}$ $\mathcal{B}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}}$, two disjoint $\mathcal{V}$ -sg $\mathcal{OS}$ $\mathcal{H}_{\mathcal{v}}$ and $\mathcal{J}_{\mathcal{v}}$ exists which satisfies $\mathcal{B}_{\mathcal{v}} \subseteq \mathcal{H}_{\mathcal{v}}$ and $\mathcal{C}_{\mathcal{v}} \subseteq \mathcal{J}_{\mathcal{v}}$.

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- *Theorem 5.5:*

Every $\mathcal{V}$ -$\mathbb{NS}$ is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$. *Proof*: Let us assume that $\mathcal{B}_{\mathscr{v}}$ be a $\mathcal{V}$ -$\mathcal{CS}$ and $\mathcal{C}_{\mathscr{v}}$ be a $\mathcal{V}$ -$\mathscr{r}\mathcal{CS}$ such that $\mathcal{B}_{\mathscr{v}} \cap \mathcal{C}_{\mathscr{v}} = 0_{\mathscr{v}}$. Every $\mathcal{V}$ -$\mathscr{r}\mathcal{CS}$ is $\mathcal{V}$ -$\mathcal{CS}$, which implies $\mathcal{C}_{\mathscr{v}}$ is a $\mathcal{V}$ -$\mathcal{CS}$. As ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathbb{NS}$, so that for any two disjoint $\mathcal{V}$ -$\mathcal{CS}$, there exists $\mathcal{V}$ -$\mathcal{OS}$ $\mathcal{H}_{\mathscr{v}}$ and $\mathcal{J}_{\mathscr{v}}$ such that $\mathcal{B}_{\mathscr{v}} \subseteq \mathcal{H}_{\mathscr{v}}$, $\mathcal{C}_{\mathscr{v}} \subseteq \mathcal{J}_{\mathscr{v}}$ and $\mathcal{H}_{\mathscr{v}} \cap \mathcal{J}_{\mathscr{v}} = 0_{\mathscr{v}}$. We know that, every $\mathcal{V}$ -$\mathcal{OS}$ is $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$. Hence, for the $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_{\mathscr{v}}$ and $\mathcal{V}$ -$\mathscr{r}\mathcal{CS}$ $\mathcal{C}_{\mathscr{v}}$ such that $\mathcal{B}_{\mathscr{v}} \cap \mathcal{C}_{\mathscr{v}} = 0_{\mathscr{v}}$, there exists a pair of disjoint $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ $\mathcal{H}_{\mathscr{v}}$ and $\mathcal{J}_{\mathscr{v}}$ in $\mathbb{Y}$, such that $\mathcal{B}_{\mathscr{v}} \subseteq \mathcal{H}_{\mathscr{v}}$, $\mathcal{C}_{\mathscr{v}} \subseteq \mathcal{J}_{\mathscr{v}}$ and $\mathcal{H}_{\mathscr{v}} \cap \mathcal{J}_{\mathscr{v}} = 0_{\mathscr{v}}$. Hence ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$.

The example is shown below since the previous theorem's reverse need not be true.

- *Example 5.6:*

Assume $\mathbb{Y} = \{r, s\}$ and $\xi_{\mathscr{v}} = \{0_{\mathscr{v}}, 1_{\mathscr{v}}, \mathcal{H}_{\mathscr{v}}, \mathcal{B}_{\mathscr{v}}\}$ is a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{H}_{\mathscr{v}} = \{\langle r, (1, 0)\rangle, \langle s, (0.5, 0.5)\rangle\}$, $\mathcal{B}_{\mathscr{v}} = \{\langle r, (1, 0)\rangle, \langle s, (0.6, 0.8)\rangle\}$. For the $\mathcal{V}$ -$\mathcal{CS}$ $0_{\mathscr{v}}$ and $\mathcal{V}$ -$\mathscr{r}\mathcal{CS}$ ${\mathcal{H}_{\mathscr{v}}}^c$ such that $0_{\mathscr{v}} \cap {\mathcal{H}_{\mathscr{v}}}^c = 0_{\mathscr{v}}$, there exists a pair of disjoint, $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ $0_{\mathscr{v}}$ and $\mathcal{B}_{\mathscr{v}}$ such that $0_{\mathscr{v}} \subseteq 0_{\mathscr{v}}$ and ${\mathcal{H}_{\mathscr{v}}}^c \subseteq \mathcal{B}_{\mathscr{v}}$. Hence ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$. But for the $\mathcal{V}$ -$\mathcal{CS}$ $0_{\mathscr{v}}$ and ${\mathcal{B}_{\mathscr{v}}}^c$, there is no disjoint $\mathcal{V}$ -$\mathcal{OS}$ exists which satisfies $\mathcal{V}$ -$\mathbb{NS}$. Hence $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$ does not have to be equal to $\mathcal{V}$ - $\mathbb{NS}$.

- *Theorem 5.7:*

Every $\mathcal{V}$ -$\mathbb{NS}$ is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$. *Proof*: Let us assume that $\mathcal{B}_{\mathscr{v}}$ and $\mathcal{C}_{\mathscr{v}}$ be a $\mathcal{V}$ - $\mathscr{r}\mathcal{CS}$ such that $\mathcal{B}_{\mathscr{v}} \cap \mathcal{C}_{\mathscr{v}} = 0_{\mathscr{v}}$. Every $\mathcal{V}$ - $\mathscr{r}\mathcal{CS}$ is $\mathcal{V}$ -$\mathcal{CS}$, which implies $\mathcal{B}_{\mathscr{v}}$ and $\mathcal{C}_{\mathscr{v}}$ are $\mathcal{V}$ -$\mathcal{CS}$. As ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathbb{NS}$, so that for the pair of disjoint $\mathcal{V}$ -$\mathcal{CS}$, there exists $\mathcal{V}$ -$\mathcal{OS}$ $\mathcal{H}_{\mathscr{v}}$ and $\mathcal{J}_{\mathscr{v}}$ such that $\mathcal{B}_{\mathscr{v}} \subseteq \mathcal{H}_{\mathscr{v}}$, $\mathcal{C}_{\mathscr{v}} \subseteq \mathcal{J}_{\mathscr{v}}$ and $\mathcal{H}_{\mathscr{v}} \cap \mathcal{J}_{\mathscr{v}} = 0_{\mathscr{v}}$. We know that, every $\mathcal{V}$ -$\mathcal{OS}$ is $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$. Hence, for the pair of $\mathcal{V}$ - $\mathscr{r}\mathcal{CS}$ $\mathcal{B}_{\mathscr{v}}$ and $\mathcal{C}_{\mathscr{v}}$ such that $\mathcal{B}_{\mathscr{v}} \cap \mathcal{C}_{\mathscr{v}} = 0_{\mathscr{v}}$, there exists a pair of disjoint $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ $\mathcal{H}_{\mathscr{v}}$ and $\mathcal{J}_{\mathscr{v}}$ in $\mathbb{Y}$, such that $\mathcal{B}_{\mathscr{v}} \subseteq \mathcal{H}_{\mathscr{v}}$, $\mathcal{C}_{\mathscr{v}} \subseteq \mathcal{J}_{\mathscr{v}}$ and $\mathcal{H}_{\mathscr{v}} \cap \mathcal{J}_{\mathscr{v}} = 0_{\mathscr{v}}$. Hence ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$.

The example is shown below since the previous theorem's reverse need not be true.

- *Example 5.8:*

Assume $\mathbb{Y} = \{r, s\}$ and $\xi_{\mathscr{v}} = \{0_{\mathscr{v}}, 1_{\mathscr{v}}, \mathcal{H}_{\mathscr{v}}, \mathcal{B}_{\mathscr{v}}\}$ is a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{H}_{\mathscr{v}} = \{\langle r, (1, 0)\rangle, \langle s, (0.7, 0.7)\rangle\}$, $\mathcal{B}_{\mathscr{v}} = \{\langle r, (1, 0)\rangle, \langle s, (0.4, 0.4)\rangle\}$. For the pair of disjoint $\mathcal{V}$ - $\mathscr{r}\mathcal{CS}$ $0_{\mathscr{v}}$ and ${\mathcal{H}_{\mathscr{v}}}^c$ such that $0_{\mathscr{v}} \cap {\mathcal{H}_{\mathscr{v}}}^c = 0_{\mathscr{v}}$, there exists a pair of disjoint, $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ $0_{\mathscr{v}}$ and $\mathcal{B}_{\mathscr{v}}$ such that $0_{\mathscr{v}} \subseteq 0_{\mathscr{v}}$ and ${\mathcal{H}_{\mathscr{v}}}^c \subseteq \mathcal{B}_{\mathscr{v}}$. Hence ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$. But for the $\mathcal{V}$ -$\mathcal{CS}$ $0_{\mathscr{v}}$ and ${\mathcal{B}_{\mathscr{v}}}^c$, there is no disjoint $\mathcal{V}$ -$\mathcal{OS}$ exists which satisfies $\mathcal{V}$ -$\mathbb{NS}$. Thus $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$ does not have to be equal to $\mathcal{V}$ - $\mathbb{NS}$.

- *Theorem 5.9:*

Let $(\mathbb{Y}, \xi_{\mathscr{v}})$ be a $\mathcal{VT}$.

- Every $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{NS}$ is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$.
- Every $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{NS}$ is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$.
- Every $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$ is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$.

Proof is straightforward.

The example is shown below since the previous theorem's reverse need not be true.

- *Example 5.10:*

Suppose $\mathbb{Y} = \{r, s\}$ and $\xi_{\mathscr{v}} = \{0_{\mathscr{v}}, 1_{\mathscr{v}}, \mathcal{H}_{\mathscr{v}}, \mathcal{B}_{\mathscr{v}}, \mathcal{C}_{\mathscr{v}}, \mathcal{D}_{\mathscr{v}}, E_{\mathscr{v}}\}$ is a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{H}_{\mathscr{v}} = \{\langle r, (0.3, 0.7)\rangle, \langle s, (0.4, 0.8)\rangle\}$,$\mathcal{B}_{\mathscr{v}} = \{\langle r, (0, 1)\rangle, \langle s, (0.3, 0.4)\rangle\}$, $\mathcal{C}_{\mathscr{v}} = \{\langle r, (0.8, 0.9)\rangle, \langle s, (0.3, 0.6)\rangle\}$, $\mathcal{D}_{\mathscr{v}} = \{\langle r, (0.7, 0.8)\rangle, \langle s, (0.1, 0.5)\rangle\}$, $E_{\mathscr{v}} = \{\langle r, (1, 0)\rangle, \langle s, (0.7, 0.7)\rangle\}$. For the pair of disjoint $\mathcal{V}$ -$\mathcal{CS}$ $0_{\mathscr{v}}$ and $\mathcal{V}$- $\mathscr{r}\mathcal{CS}$ ${\mathcal{C}_{\mathscr{v}}}^c$ such that $0_{\mathscr{v}} \cap {\mathcal{C}_{\mathscr{v}}}^c = 0_{\mathscr{v}}$, there exists a pair of disjoint, $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ $0_{\mathscr{v}}$ and $\mathcal{D}_{\mathscr{v}}$ such that $0_{\mathscr{v}} \subseteq 0_{\mathscr{v}}$ and ${\mathcal{C}_{\mathscr{v}}}^c \subseteq \mathcal{D}_{\mathscr{v}}$. Hence ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$. But for the $\mathcal{V}$ -$\mathcal{CS}$ $0_{\mathscr{v}}$ and ${E_{\mathscr{v}}}^c$, there is no disjoint $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ exists which satisfies $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{NS}$. Thus $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$ does not have to be equal to $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{NS}$.

- *Example 5.11:*

Let $\mathbb{Y} = \{r, s\}$ and let $\xi_{\mathscr{v}} = \{0_{\mathscr{v}}, 1_{\mathscr{v}}, \mathcal{B}_{\mathscr{v}}, \mathcal{C}_{\mathscr{v}}, \mathcal{D}_{\mathscr{v}}\}$ be a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{B}_{\mathscr{v}} = \{\langle r, (0.5, 0.8)\rangle, \langle s, (0,2, 0.6)\rangle\}$, $\mathcal{C}_{\mathscr{v}} = \{\langle r, (1, 0)\rangle, \langle s, (0.1, 0.7)\rangle\}$, $\mathcal{D}_{\mathscr{v}} = \{\langle r, (0.5, 0.5)\rangle, \langle s, (0.6, 0.9)\rangle\}$. For the pair of disjoint $\mathcal{V}$ -$\mathscr{r}\mathcal{CS}$ $0_{\mathscr{v}}$ and ${\mathcal{B}_{\mathscr{v}}}^c$ such that $0_{\mathscr{v}} \cap {\mathcal{B}_{\mathscr{v}}}^c = 0_{\mathscr{v}}$, there exists a pair of disjoint, $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ $0_{\mathscr{v}}$ and $\mathcal{D}_{\mathscr{v}}$ such that $0_{\mathscr{v}} \subseteq 0_{\mathscr{v}}$ and ${\mathcal{B}_{\mathscr{v}}}^c \subseteq \mathcal{D}_{\mathscr{v}}$. Hence ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$. But for the pair of disjoint $\mathcal{V}$ -$\mathcal{CS}$ $0_{\mathscr{v}}$ and ${\mathcal{C}_{\mathscr{v}}}^c$, there is no disjoint $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ exists which satisfies $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{NS}$. Thus $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$ does not have to be equal to $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{NS}$.

- *Example 5.12:*

Let $\mathbb{Y} = \{r, s\}$ and let $\xi_{\mathscr{v}} = \{0_{\mathscr{v}}, 1_{\mathscr{v}}, \mathcal{H}_{\mathscr{v}}, \mathcal{B}_{\mathscr{v}}\}$ be a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{H}_{\mathscr{v}} = \{\langle r, (1,\ 0)\rangle, \langle s, (0.1,\ 0.7)\rangle\}$, $\mathcal{B}_{\mathscr{v}} = \{\langle r, (1,\ 0)\rangle, \langle s, (0.6,\ 0.8)\rangle\}$. For the pair of disjoint $\mathcal{V}$ -$\mathscr{r}\mathcal{CS}$ $0_{\mathscr{v}}$ and ${\mathcal{H}_{\mathscr{v}}}^c$ such that $0_{\mathscr{v}} \cap {\mathcal{H}_{\mathscr{v}}}^c = 0_{\mathscr{v}}$, there exists a pair of disjoint $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ $0_{\mathscr{v}}$ and $\mathcal{B}_{\mathscr{v}}$ such that $0_{\mathscr{v}} \subseteq 0_{\mathscr{v}}$ and ${\mathcal{H}_{\mathscr{v}}}^c \subseteq \mathcal{B}_{\mathscr{v}}$. Hence ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{NS}$. But for the $\mathcal{V}$ -$\mathcal{CS}$ $0_{\mathscr{v}}$ and $\mathcal{V}$ -$\mathscr{r}\mathcal{CS}$ ${\mathcal{B}_{\mathscr{v}}}^c$ there is no disjoint $\mathcal{V}$ -$\mathscr{sg}\mathcal{OS}$ exists which satisfies$\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$. Hence $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{MNS}$ does not have to be equal to $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$.

- *Theorem 5.13:*

$\mathcal{V}$ - *almost normality* satisfies the property of topology.

Proof: Comparable to theorem 4.8

- *Theorem 5.14:*

$\mathcal{V}$ -*mild normality* satisfies the property of topology.

Proof: Comparable to theorem 4.8

- *Theorem 5.15:*

Suppose that ($\mathbb{Y}$, $\xi_{\mathscr{v}}$) a $\mathcal{VT}$. Consequently, the following are similar to one another:

- $(\mathbb{Y}, \xi_{\mathscr{v}})$ is $\mathcal{V}$ -$\mathscr{sg}$ $\mathbb{ANS}$

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- For each two of $\mathcal{VS}$ $\mathcal{M}_v$ and $\mathcal{N}_v$,one set is $\mathcal{V}$ -$\mathcal{OS}$ and other set is $\mathcal{V}$ - $r\mathcal{OS}$ that satisfies $\mathcal{M}_v \cup \mathcal{N}_v = 1_v$, a $\mathcal{V}$ -sg $\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$ exists such that $\mathcal{B}_v \subseteq \mathcal{M}_v$ and $\mathcal{C}_v \subseteq \mathcal{N}_v$ and $\mathcal{B}_v \cup \mathcal{C}_v = 1_v$.
- For every $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_v$ and every $\mathcal{V}$ - $r\mathcal{OS}$ $\mathcal{C}_v$ containing $\mathcal{B}_v$, there exists a $\mathcal{V}$ -sg $\mathcal{OS}$ $\mathcal{P}_v$ such that $\mathcal{B}_v \subseteq \mathcal{P}_v \subseteq \mathcal{V} - sg\ c\ell\ (\mathcal{P}_v) \subseteq \mathcal{C}_v$.

Proof: Comparable to theorem 4.6

- *Theorem 5.16:*

Let $(\mathbb{Y}, \xi_v)$ is a $\mathcal{V}$ -sg $\mathbb{MNS}$ and $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is a $\mathcal{V}$ -sg open mapping, $\mathcal{V}$ - $\mathcal{CTM}$ and $\mathcal{V}$ - injective mapping. Then $p(\mathbb{Y})$ is a $\mathcal{V}$ -sg $\mathbb{MNS}$.

Proof: Let us assume that $\mathcal{B}_v$ and $\mathcal{C}_v$ be any two $\mathcal{V}$ - $r\mathcal{CS}$ of $p(\mathbb{Y})$ such that $\mathcal{B}_v \cap \mathcal{C}_v = 0_v$. So p^{-1} $(\mathcal{B}_v)$ and $p^{-1}(\mathcal{C}_v)$ are $\mathcal{V}$ - $r\mathcal{CS}$ of $\mathbb{Y}$. As $\mathbb{Y}$ is $\mathcal{V}$ -sg $\mathbb{MNS}$, there are two disjoint $\mathcal{V}$ -$sg\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ in $\mathbb{Y}$, such that p^{-1} $(\mathcal{B}_v) \subseteq \mathcal{H}_v$, p^{-1} $(\mathcal{C}_v) \subseteq \mathcal{J}_v$ and $\mathcal{H}_v \cap \mathcal{J}_v = 0_v$. Hence $\mathcal{B}_v \subseteq p(\mathcal{H}_v)$ and $\mathcal{C}_v \subseteq p(\mathcal{J}_v)$. Since p is $\mathcal{V}$ -sg open mapping and $\mathcal{V}$ -injective mapping, $p(\mathcal{H}_v)$ and $p(\mathcal{J}_v)$are $\mathcal{V}$ -$sg\mathcal{OS}$ in $\mathbb{Z}$. Then $p(\mathbb{Y})$ is a $\mathcal{V}$ -sg $\mathbb{MNS}$.

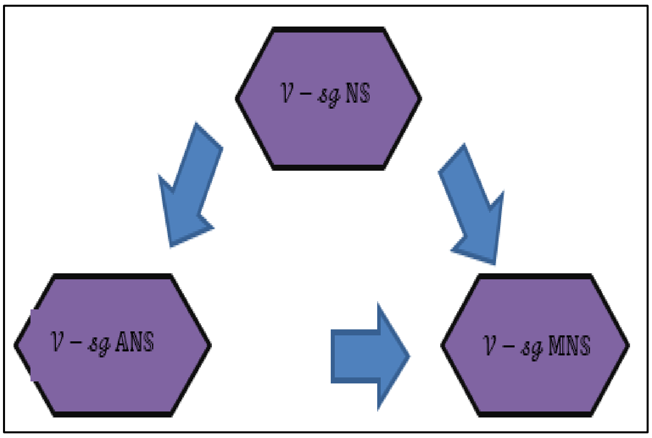

Fig 1 Shows the Inter-Relationship Between $\mathcal{V}$ -sg $\mathbb{NS}$, $\mathcal{V}$ -sg $\mathbb{ANS}$, $\mathcal{V}$ -sg $\mathbb{MNS}$.

- *Theorem 5.17:*

Suppose that $(\mathbb{Y}, \xi_v)$ a $\mathcal{VT}$. Consequently, the following are similar to one another:

- $(\mathbb{Y}, \xi_v)$ is a $\mathcal{V}$ -sg $\mathbb{MNS}$
- For each two $\mathcal{V}$ - $r\mathcal{CS}$ $\mathcal{M}_v$ and $\mathcal{N}_v$that satisfies $\mathcal{M}_v \cup \mathcal{N}_v = 1_v$, ,there exists $\mathcal{V}$ -sg $\mathcal{OS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$ such that $\mathcal{B}_v \subseteq \mathcal{M}_v$ and $\mathcal{C}_v \subseteq \mathcal{N}_v$ and $\mathcal{B}_v \cup \mathcal{C}_v = 1_v$.
- For any $\mathcal{V}$ - $r\mathcal{CS}$ $\mathcal{B}_v$ and every $\mathcal{V}$ - $r\mathcal{OS}$ $\mathcal{C}_v$ containing $\mathcal{B}_v$, there exists a $\mathcal{V}$ -sg $\mathcal{OS}$ $\mathcal{P}_v$ such that $\mathcal{B}_v \subseteq \mathcal{P}_v \subseteq \mathcal{V}$ -$sg\ c\ell\ (\mathcal{P}_v) \subseteq \mathcal{C}_v$.
- For every pair of disjoint $\mathcal{V}$ -$r\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$, there exists a pair $\mathcal{V}$ -sg $\mathcal{OS}$ $\mathcal{M}_v$ and $\mathcal{N}_v$ such that $\mathcal{B}_v \subseteq \mathcal{M}_v$ and $\mathcal{C}_v \subseteq \mathcal{N}_v$ and $\mathcal{V}$ -$sg\ c\ell\ (\mathcal{M}_v) \cap \mathcal{V}$ -$sg\ c\ell\ (\mathcal{N}_v) = 0_v$.

Proof: Comparable to theorem 4.6

- *Definition 5.18:*

Let $\mathbb{Y}$ and $\mathbb{Z}$ be any two $\mathcal{VT}$, then a mapping $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is called $\mathcal{V}$ - completely continuous mapping($\mathcal{V}$ - completely $\mathcal{CTM}$ in short), if $p^{-1}(\mathcal{J}_v)$ is $\mathcal{V}$ - $r\mathcal{CS}$ in $(\mathbb{Y}, \xi_v)$ for every $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{J}_v$ in $(\mathbb{Z}, \delta_v)$.

- *Theorem 5.19:*

If $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is $\mathcal{V}$ -completely $\mathcal{CTM}$, $\mathcal{V}$ -bijective mapping, $\mathcal{V}$ -sg open mapping and $\mathbb{Y}$ is a $\mathcal{V}$ - $\mathbb{MNS}$ then $\mathbb{Z}$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is straightforward.

- *Definition 5.20:*

A mapping $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is called $\mathcal{V}$ - rc preserving mapping if $p(\mathcal{B}_v)$ is $\mathcal{V}$ - $r\mathcal{CS}$ in $\mathbb{Z}$ for every $\mathcal{V}$ - $r\mathcal{CS}$ $\mathcal{B}_v$ in $\mathbb{Y}$.

- *Definition 5.21:*

A mapping $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is called $\mathcal{V}$ - almost closed mapping, if $p(\mathcal{B}_v)$ is $\mathcal{V}$ -$\mathcal{CS}$ in $\mathbb{Z}$ for every $\mathcal{V}$ -$\mathcal{CS}$ $\mathcal{B}_v$ in $\mathbb{Y}$.

- *Theorem 5.22:*

If $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is $\mathcal{V}$ -sg $\mathcal{CTM}$, $\mathcal{V}$ - almost closed mapping, $\mathcal{V}$ -surjective mapping and $\mathbb{Z}$ is $\mathcal{V}$ - $\mathbb{NS}$ then $\mathbb{Y}$ is $\mathcal{V}$ -sg $\mathbb{MNS}$.

Proof is straightforward.

- *Theorem 5.23:*

If $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ is $\mathcal{V}$ -sg $\mathcal{CTM}$, $\mathcal{V}$ – rc preserving mapping, $\mathcal{V}$ - injective mapping and $\mathbb{Z}$ is $\mathcal{V}$ - $\mathbb{MNS}$ then $\mathbb{Y}$ is $\mathcal{V}$ -sg $\mathbb{MNS}$.

Proof is straightforward.

VI. $\mathcal{V}$ -sg STRONGLY NORMAL SPACE

- *Definition 6.1:*

A space $(\mathbb{Y}, \xi_v)$ is called $\mathcal{V}$ -sg strongly normal space ($\mathcal{V}$ -sg $\mathbb{SNS}$ in short) if for any pair of disjoint $\mathcal{V}$ -sg $\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$, there exists disjoint $\mathcal{V}$ -sg $\mathcal{OS}$ $\mathcal{H}_v$ and $\mathcal{J}_v$ such that $\mathcal{B}_v \subseteq \mathcal{H}_v, \mathcal{C}_v \subseteq \mathcal{J}_v$.

- *Proposition 6.2:*

Let $(\mathbb{Y}, \xi_v)$ be a $\mathcal{VT}$.

- Every $\mathcal{V}$ -sg $\mathbb{SNS}$ is $\mathcal{V}$ -$\mathbb{NS}$.
- Every $\mathcal{V}$ -sg $\mathbb{SNS}$ is $\mathcal{V}$ -sg $\mathbb{MNS}$.
- Every $\mathcal{V}$ -sg $\mathbb{SNS}$ is $\mathcal{V}$ -sg $\mathbb{ANS}$.
- Every $\mathcal{V}$ -sg $\mathbb{SNS}$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof: (i) Let $(\mathbb{Y}, \xi_v)$ be a $\mathcal{V}$ -sg $\mathbb{SNS}$. Let us assume that $\mathcal{B}_v$ and $\mathcal{C}_v$ be two disjoint $\mathcal{V}$ -$\mathcal{CS}$ in $\mathbb{Y}$. Every $\mathcal{V}$ -$\mathcal{CS}$ is $\mathcal{V}$ -sg $\mathcal{CS}$, which implies $\mathcal{B}_v$ and $\mathcal{C}_v$ are $\mathcal{V}$ -sg $\mathcal{CS}$ in $\mathbb{Y}$ and they are disjoint. As $(\mathbb{Y}, \xi_v)$ is a $\mathcal{V}$ -sg $\mathbb{SNS}$, for each disjoint pair of $\mathcal{V}$ -sg $\mathcal{CS}$ $\mathcal{B}_v$ and $\mathcal{C}_v$, there exists a pair of disjoint $\mathcal{V}$

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-$sgOS$ $\mathcal{H}_v$ and $\mathcal{J}_v$ such that $\mathcal{B}_v \subseteq \mathcal{H}_v$, $\mathcal{C}_v \subseteq \mathcal{J}_v$. Hence $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -$\mathbb{NS}$.

Proof of (ii) to (iv) is straightforward.

The example is shown below since the previous theorem's reverse need not be true.

➢ *Example 6.3:*

Suppose $\mathbb{Y} = \{r, s\}$ and $\xi_v = \{0_v, 1_v, \mathcal{B}_v, \mathcal{C}_v, \mathcal{D}_v\}$ is a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{B}_v = \{\langle r, (0.2, 0.6)\rangle, \langle s, (0.5, 0.8)\rangle\}$, $\mathcal{C}_v = \{\langle r, (1, 0)\rangle, \langle s, (0.2, 0.3)\rangle\}$, $\mathcal{D}_v = \{\langle r, (0.6, 0.9)\rangle, \langle s, (0.5, 0.8)\rangle\}$. For the pair of disjoint $\mathcal{V}$ -$\mathcal{CS}$ 0_v and $\mathcal{D}_v{}^c$, there exists a pair of disjoint $\mathcal{V}$ -$sgOS$ 0_v and $\mathcal{B}_v$ such that $0_v \subseteq 0_v$ and $\mathcal{D}_v{}^c \subseteq \mathcal{B}_v$. Hence $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -sg $\mathbb{NS}$. But for the pair of disjoint $\mathcal{V}$ -sg $\mathcal{CS}$ 0_v and $\mathcal{C}_v{}^c$, there is no disjoint $\mathcal{V}$ -$sgOS$ exists which satisfies $\mathcal{V}$ -sg $\mathbb{SNS}$. Thus $\mathcal{V}$ -sg $\mathbb{NS}$ does not have to be equal to $\mathcal{V}$ -sg $\mathbb{SNS}$.

➢ *Example 6.4:*

Suppose $\mathbb{Y} = \{r, s\}$ and $\xi_v = \{0_v, 1_v, \mathcal{H}_v, \mathcal{B}_v\}$ is a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{H}_v = \{\langle r, (0.4, 0.8)\rangle, \langle s, (0.2, 0.7)\rangle\}$, $\mathcal{B}_v = \{\langle r, (0.5, 0.8)\rangle, \langle s, (0.3, 0.9)\rangle\}$. For the pair of disjoint $\mathcal{V}$ -$r\mathcal{CS}$ 0_v and $\mathcal{H}_v{}^c$, there exists a pair of disjoint $\mathcal{V}$ -$sgOS$ 0_v and $\mathcal{B}_v$ such that $0_v \subseteq 0_v$ and $\mathcal{H}_v{}^c \subseteq \mathcal{B}_v$. Hence $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -sg $\mathbb{MNS}$. But for the pair of disjoint $\mathcal{V}$ -sg $\mathcal{CS}$ 0_v and $\mathcal{B}_v{}^c$, there is no disjoint $\mathcal{V}$ -$sgOS$ exists which satisfies $\mathcal{V}$ -sg $\mathbb{SNS}$. Thus $\mathcal{V}$ -sg $\mathbb{MNS}$ does not have to be equal to $\mathcal{V}$ -sg $\mathbb{SNS}$.

➢ *Example 6.5:*

Suppose $\mathbb{Y} = \{r, s\}$ and $\xi_v = \{0_v, 1_v, \mathcal{H}_v, \mathcal{B}_v, \mathcal{C}_v, \mathcal{D}_v, E_v\}$ is a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{H}_v = \{\langle r, (1, 0)\rangle, \langle s, (0.3, 0.3)\rangle\}$, $\mathcal{B}_v = \{\langle r, (0.3, 0.7)\rangle, \langle s, (0.4, 0.9)\rangle\}$, $\mathcal{C}_v = \{\langle r, (0.5, 0.8)\rangle \langle s, (0.2, 0.8)\rangle\}$, $\mathcal{D}_v = \{\langle r, (0.1, 0.5)\rangle, \langle s, (0.2, 0.4)\rangle\}$, $E_v = \{\langle r, (0.4, 0.7)\rangle, \langle s, (0.5, 0.5)\rangle\}$. For the disjoint $\mathcal{V}$ -$\mathcal{CS}$ 0_v and $\mathcal{V}$ -$r\mathcal{CS}$ $\mathcal{B}_v{}^c$, there exists a pair of disjoint, $\mathcal{V}$ $sgOS$ 0_v and $\mathcal{C}_v$ such that $0_v \subseteq 0_v$ and $\mathcal{B}_v{}^c \subseteq \mathcal{C}_v$. Hence $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -sg $\mathbb{ANS}$. But for the pair of disjoint $\mathcal{V}$ -sg $\mathcal{CS}$ 0_v and $\mathcal{H}_v{}^c$, there is no disjoint $\mathcal{V}$ -$sgOS$ exists which satisfies $\mathcal{V}$ -sg $\mathbb{SNS}$. Thus $\mathcal{V}$ -sg $\mathbb{ANS}$ does not have to be equal to $\mathcal{V}$ -sg $\mathbb{SNS}$.

➢ *Example 6.6:*

Suppose $\mathbb{Y} = \{r, s\}$ and $\xi_v = \{0_v, 1_v, \mathcal{B}_v, \mathcal{C}_v\}$ is a $\mathcal{VT}$ on $\mathbb{Y}$, where $\mathcal{B}_v = \{\langle r, (0.3, 0.5)\rangle, \langle s, (0.2, 0.5)\rangle\}$, $\mathcal{C}_v = \{\langle r, (0.5, 0.9)\rangle, \langle s, (0.6, 0.8)\rangle\}$. For the pair of disjoint $\mathcal{V}$ -$\mathcal{CS}$ 0_v and $\mathcal{C}_v{}^c$, there exists a pair of disjoint, $\mathcal{V}$ -$sgOS$ 0_v and $\mathcal{B}_v$ such that $0_v \subseteq 0_v$ and $\mathcal{C}_v{}^c \subseteq \mathcal{B}_v$. Hence $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -sg $\mathbb{NS}$. But for the pair of disjoint $\mathcal{V}$ -sg $\mathcal{CS}$ 0_v and $\mathcal{B}_v{}^c$, there is no disjoint $\mathcal{V}$ -$sgOS$ exists which satisfies $\mathcal{V}$ -sg $\mathbb{SNS}$. Thus $\mathcal{V}$ -sg $\mathbb{NS}$ does not have to be equal to $\mathcal{V}$ -sg $\mathbb{SNS}$.

➢ *Theorem 6.7:*

$\mathcal{V}$ -$\mathbb{SNS}$ satisfies the property of topology.

Proof: Comparable to theorem 4.8

➢ *Theorem 6.8:*

Assume $(\mathbb{Y}, \xi_v)$ is a $\mathcal{VT}$. If $(\mathbb{Y}, \xi_v)$ is

- $\mathcal{V}_{sg}$ -$T_{1/2}$ space and $\mathcal{V}$ -sg $\mathbb{NS}$, then it is $\mathcal{V}$ -sg $\mathbb{SNS}$.
- $\mathcal{V}_{sg}$ -T_r and $\mathcal{V}$ -sg $\mathbb{MNS}$, then it is $\mathcal{V}$ -sg $\mathbb{SNS}$.
- $\mathcal{V}_{sg}$ -T_r, $\mathcal{V}_{sg}$ -$T_{1/2}$space and $\mathcal{V}$ -sg $\mathbb{ANS}$, then it is $\mathcal{V}$ -sg $\mathbb{SNS}$.
- $\mathcal{V}_{sg}$ -$T_{1/2}$ space and $\mathcal{V}$ -$\mathbb{NS}$, then it is $\mathcal{V}$ -sg $\mathbb{SNS}$.

Proof is obvious.

➢ *Theorem 6.9:*

Let $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ be a $\mathcal{V}$ -$bijective\ mapping$. If p is a$\mathcal{V}$ -sg $strongly\ open\ mapping$, $\mathcal{V}$ - CTM and $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -sg $\mathbb{SNS}$ then $(\mathbb{Z}, \delta_v)$ is $\mathcal{V}$ -sg $\mathbb{NS}$.

Proof is straightforward.

➢ *Theorem 6.10:*

If there is a $\mathcal{V}$ -$s$urjective $f$unction, $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ such that p is $\mathcal{V}$ -sg $perfectly\ continuous$, $\mathcal{V}$ -sg $strongly\ open\ mapping$ then $\mathbb{Z}$ is $\mathcal{V}$ -sg $\mathbb{SNS}$.

Proof is straightforward.

➢ *Theorem 6.11:*

Let $p : (\mathbb{Y}, \xi_v) \to (\mathbb{Z}, \delta_v)$ be a $\mathcal{V}$ -$injective\ mapping$, $\mathcal{V}$ -sg $open\ mapping$. If p is a $\mathcal{V}$ -sg $irresolute\ mapping$ and $(\mathbb{Y}, \xi_v)$ is $\mathcal{V}$ -sg $\mathbb{SNS}$ then $(\mathbb{Z}, \delta_v)$ is $\mathcal{V}$ -sg $\mathbb{SNS}$.

Proof is straightforward.

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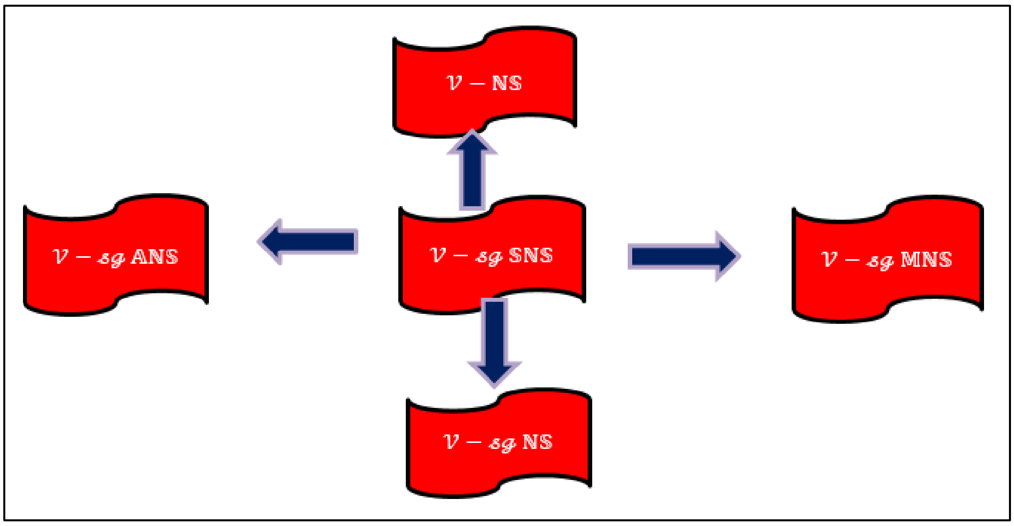

Fig 2 Shows Inter-Relationship Between $\mathcal{V}$ -$\mathcal{sg}$ $\mathbb{SNS}$, $\mathcal{V}$ -$\mathbb{NS}$, $\mathcal{V}$ -$\mathcal{sg}$ $\mathbb{NS}$, $\mathcal{V}$ -$\mathcal{sg}$ $\mathbb{ANS}$, $\mathcal{V}$ -$\mathcal{sg}$ $\mathbb{MNS}$.

VII. CONCLUSION

Here $\mathcal{V}_{\mathcal{sg}}$- T_r, $\mathcal{V}_{\mathcal{sg}}$-$T_s$, $\mathcal{V}_{\mathcal{sg}}$-$T_{1/2}$ spaces are introduced and few theorems related to these spaces are examined in this study. Additionally, a number of theorems pertaining to the above-mentioned spaces are introduced and investigated, including vague normal spaces and vague semi generalized normal spaces. We also look into $\mathcal{V}$ -$\mathcal{sg}$ $\mathcal{s}$lightly $\mathcal{n}$ormal $\mathcal{s}$paces, $\mathcal{V}$-$\mathcal{sg}$ almost $\mathcal{n}$ormal $\mathcal{s}$paces and $\mathcal{V}$ -$\mathcal{sg}$ $\mathcal{s}$trongly $\mathcal{n}$ormal $\mathcal{s}$paces in vague topological spaces, as well as their properties. As we construct a comprehensive framework for practical applications, we hope that the findings in this document will aid numerous researchers in their ongoing exploration of new topological spaces.

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