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Development of Fractional-Order Model Reference Adaptive Control of a Higher-Order Ball and Beam System with Actuator Dynamics

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Abstract: This study develops a fractional-order model reference adaptive controller (FOMRAC) for a higher-order ball-and-beam system in which actuator behaviour is explicitly represented. The experimental platform comprises a 0.60 m beam, a 2.7 g ball with a 20 mm radius, a Hitec HS-645MG servo, an Arduino UNO controller, and a VL53L1 time-of-flight position sensor. The ball dynamics are obtained from the rolling constraint and then linearised about the horizontal equilibrium. Recalculation from the parameters gives an effective rolling mass of 7.02×10^{-3} kg and a nominal ball-position/beam-angle gain of -0.25154 s^{-2}. Actuator dynamics are retained in the higher-order representation rather than being treated as instantaneous. A Caputo fractional-order adaptation law is incorporated into a Lyapunov-based MRAC structure. PID, integer-order MRAC, and FOMRAC are compared using the performance measures, while four recorded laboratory PID trajectories are used to illustrate the sensitivity of the physical platform to gain selection. Under the model, FOMRAC gives a settling time of 1.880 s and the smallest IAE and ITAE among the three controllers. The results support the use of fractional adaptation as an additional tuning dimension for practical ball-and-beam control.

Keywords: *FOMRAC; MRAC; PID; Ball-and-Beam; Actuator Dynamics; Arduino; VL53L1.*

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I. INTRODUCTION

Precision mechatronic systems are expected to maintain accurate motion even when their dynamics are affected by nonlinearity, parameter variation, actuator limitations, sensor noise, and external disturbances. The proportional–integral–derivative (PID) controller remains widely used because of its simple structure and modest computational requirements. Its gains, however, are fixed once tuned and therefore do not automatically compensate for changes in plant behaviour. Model reference adaptive control (MRAC) addresses this limitation by adjusting controller parameters online so that the plant follows the response of a prescribed reference model. Experimental work on ball-and-beam systems has shown that suitable initial adaptive gains and gain constraints can improve convergence and reduce oscillation (Romdlony et al., 2022).

Fractional-order control extends conventional integer-order differentiation and integration by introducing a non-integer order and, consequently, a memory effect. Aburakhis and Ordóñez (2024) developed a direct adaptive control formulation using the Caputo fractional derivative and provided a stability treatment for uncertain nonlinear systems. The present study adopts the same broad fractional-adaptation principle and applies it to an actuator-aware ball-and-beam model implemented with embedded sensing and servo actuation.

The practical platform uses a VL53L1 time-of-flight sensor for position measurement, an Arduino UNO for real-time computation, and PWM actuation of a Hitec HS-645MG servo. The main contribution is an integrated higher-order FOMRAC framework in which actuator dynamics are retained explicitly and the fractional adaptive controller is evaluated against both PID and integer-order MRAC.

II. RELATED WORK

Ball-and-beam systems have been used extensively as benchmark mechatronic plants because their nonlinear and underactuated behaviour makes them useful for evaluating control strategies. Recent practical work by Sutera et al. (2026) demonstrates an Arduino-based PID ball-and-beam implementation using a time-of-flight sensor and servo actuation. Such work also highlights the influence of hardware characteristics and tuning choices on experimental response.

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Romdlony et al. (2022) applied state-feedback MRAC to a nonlinear ball-and-beam system. Their results showed that modified initial feedback gains and bounded adaptation could improve convergence and reduce oscillatory behaviour. Their model provides a useful reference for the present work, although the formulation adopted here retains actuator dynamics explicitly.

Actuator dynamics are particularly important in adaptive control. Arabi et al. (2020) developed an MRAC formulation for uncertain systems subject to high-order actuator dynamics, demonstrating why actuator behaviour should not be silently treated as an ideal instantaneous input. Related work has also shown that finite actuator bandwidth can impose stability and performance limits on adaptive controllers (Gruenwald et al., 2019).

Fractional adaptive control provides an additional degree of freedom through the derivative order. Aburakhis and Ordóñez (2024) introduced a fractional direct adaptive law based on the Caputo definition and showed that the conventional integer-order adaptive law is recovered as a special case. Fractional-order systems and controllers are also established in the broader control literature (Monje et al., 2010; Kao et al., 2024). The gap addressed in this study is therefore the combination of fractional adaptation, explicit actuator dynamics, and a practical ball-and-beam platform.

III. SYSTEM IMPLEMENTATION

The prototype parameters used in the model were taken from the experimental setup. The beam length is 0.60 m, the ball mass is 2.7×10^{-3} kg, the ball radius is 2.0×10^{-2} m, and gravitational acceleration is 9.81 m/s². The ball moment of inertia is 1.728×10^{-6} kg·m², which is consistent with a homogeneous solid sphere. The linkage lengths reported for the prototype are $l_1 = 0.04$ m and $l_2 = 0.105$ m.

The actuator is a Hitec HS-645MG servo. The manufacturer's specifications identify PWM control, an operating range of 4.8–6.0 V, stall torque of 7.7 kgf·cm at 4.8 V and 9.6 kgf·cm at 6.0 V, and no-load speeds of 0.24 s/60° and 0.20 s/60° at 4.8 V and 6.0 V, respectively (Hitec Commercial Solutions, 2026). These characteristics confirm that the servo has finite actuation dynamics.

The embedded controller consists of an Arduino UNO, a VL53L1 time-of-flight position sensor, and the servo actuator. The sensor measures the ball position continuously, while the Arduino calculates the control action and updates the servo command through PWM.

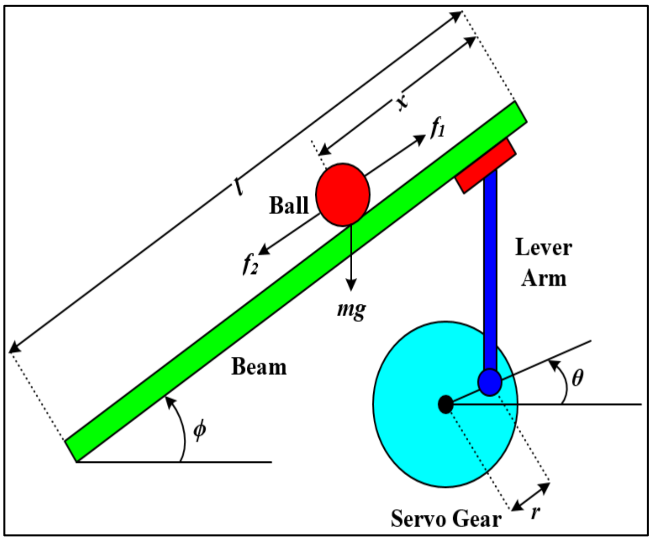

Fig 1 Schematic Representation of the Experimental Ball-and-Beam Platform. (Rameez Khan et al. 2020)

IV. HIGHER-ORDER MATHEMATICAL MODEL

➢ *Rolling-Ball Dynamics*

Let x denote the ball displacement along the beam and α the beam angle measured from the horizontal. Under the rolling-without-slipping assumption, the translational and rotational motions are coupled through the rolling constraint.

$$m\ddot{x} = mx\dot{\alpha}^2 - mgsin(\alpha) \quad (1)$$

$$J\ddot{\varphi} = \tau - rF \quad (2)$$

$$\dot{x} = r\dot{\varphi} \quad (3)$$

$$m\ddot{x} + \frac{J}{r^2}\ddot{x} = mx\dot{\alpha}^2 - mgsin(\alpha) \quad (4)$$

For small beam angles, sin(α) ≈ α. Defining the effective rolling mass as the sum of the translational mass and the reflected rotational inertia gives:

$$me = m + \frac{J}{r^2} \quad (5)$$

$$me = 2.7 \times 10^{-3} + \frac{1.728 \times 10^{-6}}{(0.02)^2} = 7.02 \times 10^{-3}\, kg \quad (6)$$

$$\ddot{x} \approx -\frac{mg}{me}\,\alpha = -3.77308\,\alpha \quad (7)$$

The resulting effective mass is 7.02×10^{-3} kg. The corresponding linearised ball-position/beam-angle relation is therefore −3.77308 s⁻². This value is used consistently in the subsequent linkage calculation.

➢ *State Representation*

To retain actuator behaviour, five states are used: ball position, ball velocity, actuator state, beam angle, and beam angular velocity. A convenient state vector is:

$$x(t) = [x, \dot{x}, \tau, \alpha, \dot{\alpha}]\mathrm{T} \quad (8)$$

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The actuator is represented by a first-order model:

$$\dot{\tau} = -\frac{1}{Ta}\tau + \frac{Ka}{Ta}u \tag{9}$$

$$\dot{x}_1 = x_2 \tag{10}$$

$$\dot{x}_2 = -3.77308\, x_4 \tag{11}$$

$$\dot{x}_3 = -\frac{1}{T_a}x_3 + \frac{K_a}{T_a}u \tag{12}$$

$$\dot{x}_4 = x_5 \tag{13}$$

$$\dot{x}_5 = f_5(x, u) \tag{14}$$

Equation (14) is intentionally written as an identified beam/actuator relation because the beam rotational inertia, damping, linkage transmission, and servo gain were not independently reported as measured quantities in the supplied material. This distinction avoids presenting unidentified coefficients as experimentally measured constants.

- *Geometry and Nominal Transfer Function*

For the small-angle linkage geometry, the beam angle is related to the servo angle θ by:

$$\alpha = \frac{l_1}{L}\theta \tag{15}$$

Combining this relationship with the linearised ball dynamics gives the nominal transfer function from servo angle to ball position:

$$G(s) = \frac{X(s)}{\Theta(s)} = -\frac{m\, g\, l_1}{m_e\, L\, s^2} = -0.25154\frac{1}{s^2} \tag{16}$$

Substitution of the values gives $-0.25154/s^2$. This corrected value is used in the controller comparison.

- *Actuator Dynamics and Higher-Order Representation*

The servo does not respond instantaneously to a commanded position. The actuator time constant is therefore retained explicitly. For the study, $T_a = 0.0909$ s is used as an identified design parameter rather than as a value claimed to have been measured directly from the manufacturer's speed rating.

$$T_a\dot{\tau} + \tau = K_a u \tag{17}$$

$$T_a = 0.0909\, s \tag{18}$$

$$X = A\,X + B\,u, \; y = C\,X \tag{19}$$

$$A = [\,A\square\; B\square\,;\; 0\; A_a\,] \tag{20}$$

The complete fifth-order implementation is obtained by augmenting the four mechanical states with the actuator state. Where exact identified coefficients are unavailable, they are retained symbolically and determined from the experimental identification procedure rather than inferred from unrelated sources.

V. FRACTIONAL-ORDER MODEL REFERENCE ADAPTIVE CONTROL

The controller uses a stable reference model to specify the desired transient response. Let x_m denote the reference state and A_m, B_m the reference-model matrices. The reference model is:

$$\dot{x}_m = A_m x_m + B_m r \tag{21}$$

The tracking error is:

$$e = x - x_m \tag{22}$$

A state-feedback adaptive control law is written as:

$$u = \theta\mathrm{T}x + k_r r \tag{23}$$

Where θ is the adaptive state-feedback vector and k_r is the adaptive reference gain. The fractional component is introduced in the parameter-update mechanism. Following Aburakhis and Ordóñez (2024), the Caputo derivative is used because it permits conventional integer-order initial conditions.

$$D\mu\theta(t) = -\Gamma\varphi(t) \tag{24}$$

For $0 < \mu < 1$, the Caputo derivative is defined by:

$$D\mu f(t) = \frac{1}{\Gamma(1-\mu)}\int 0\mathrm{t}(t - \tau - \mu f(\tau)d\tau) \tag{25}$$

A quadratic Lyapunov candidate is selected as:

$$V = e\mathrm{T}P\,e + \frac{1}{2}tr(\Gamma^{-1}\theta\mathrm{T}\theta) \tag{26}$$

Where $P = P^T > 0$ is obtained from the Lyapunov equation $A_m^T P + PA_m = -Q$, with $Q = Q^T > 0$. Under the stated matching conditions and bounded adaptation, the tracking error is expected to remain bounded and to converge towards the reference trajectory. In the physical system, actuator saturation, sensor noise, friction, and parameter uncertainty define the practical tracking envelope.

VI. EXPERIMENTAL METHODOLOGY

The principal PID gains used in the numerical design are $K_p = 8.3867$, $K_i = 0.001$, and $K_d = 4.7305$. The reported crossover frequency is 2.73 rad/s and the phase margin is 57°. The same nominal plant is used for PID, integer-order MRAC, and FOMRAC so that the numerical comparison is made under consistent conditions. PID tuning is discussed in the context of the classical Ziegler–Nichols approach by Patel (2020).

For FOMRAC, the fractional order is $\mu = 0.85$. Adaptive parameters are bounded to limit parameter drift, and the servo command is constrained to ±10°. These are controller design settings used in the numerical study, not independent experimental measurements.

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Four recorded laboratory trajectories are also considered. Trials 1 and 2 use a 350 mm setpoint, Trial 3 uses 348 mm, and Trial 4 uses 353mm. The recorded PID gains are summarised in Table 1.

Table 1 Experimental PID Settings Reported for the Prototype.

Trial	Kp	Ki	Kd	Setpoint (mm)	Observation
1	8.40	0.0100	4.75	350	Best trial
2	1.00	0.1232	0.3402	350	Representative trial
3	3.00	0.2160	0.9736	348	Representative trial
4	2.01	1.3125	3.8906	353	Representative trial

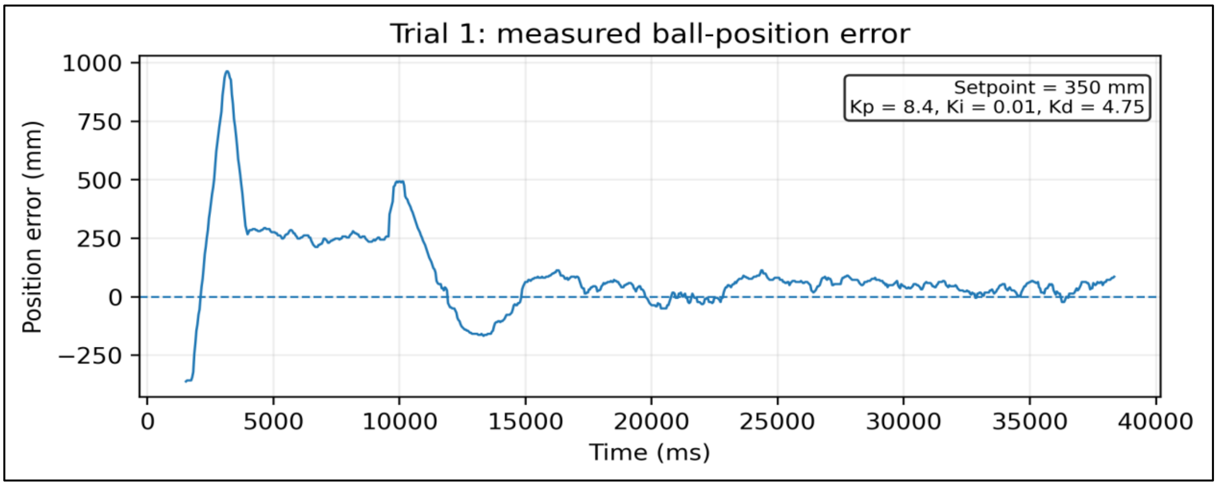

Fig 2 Experimental Trial 1: Ball-Position Error at a 350 mm Setpoint.

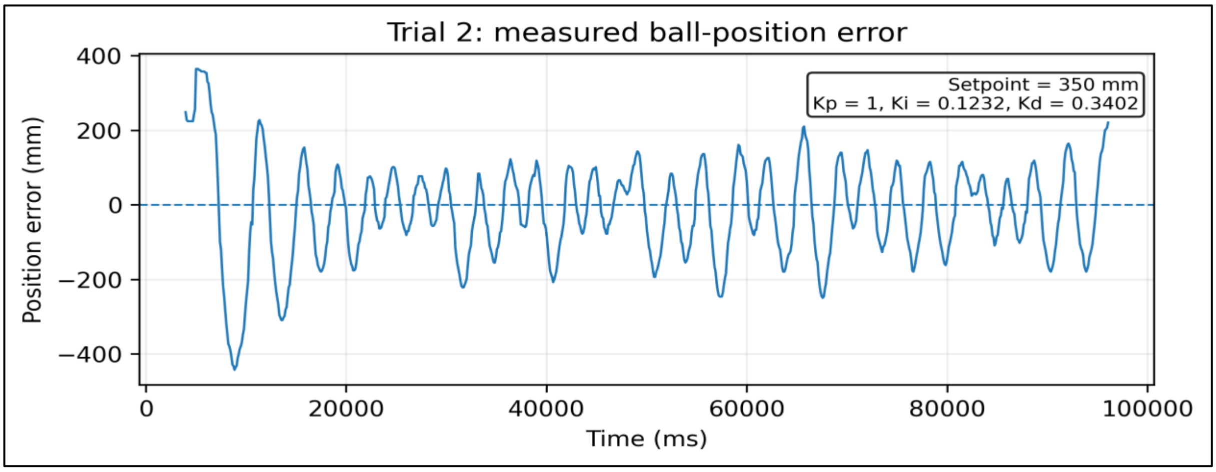

Fig 3 Experimental Trial 2: Ball-Position Error at a 350 mm Setpoint.

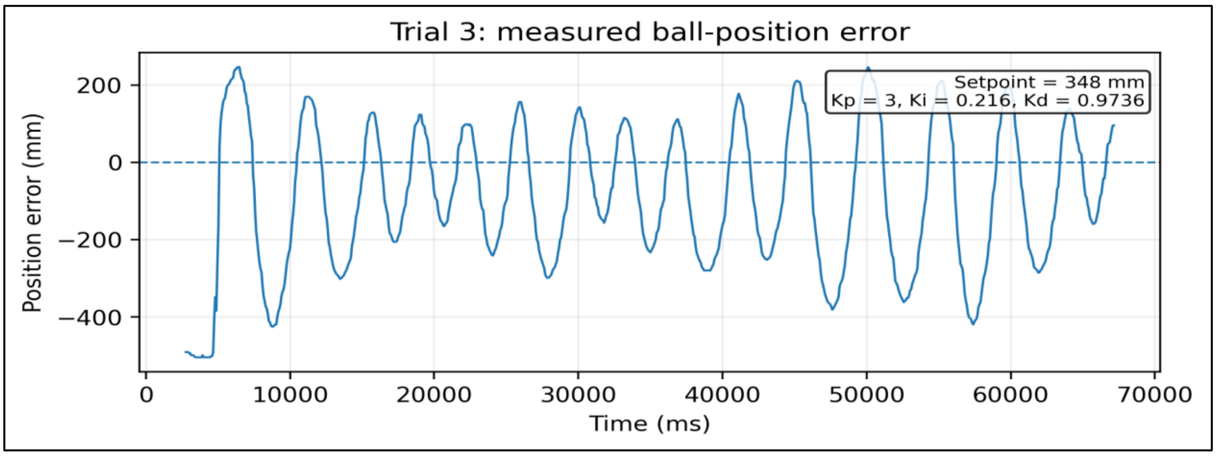

Fig 4 Experimental Trial 3: Ball-Position Error at a 348 mm Setpoint.

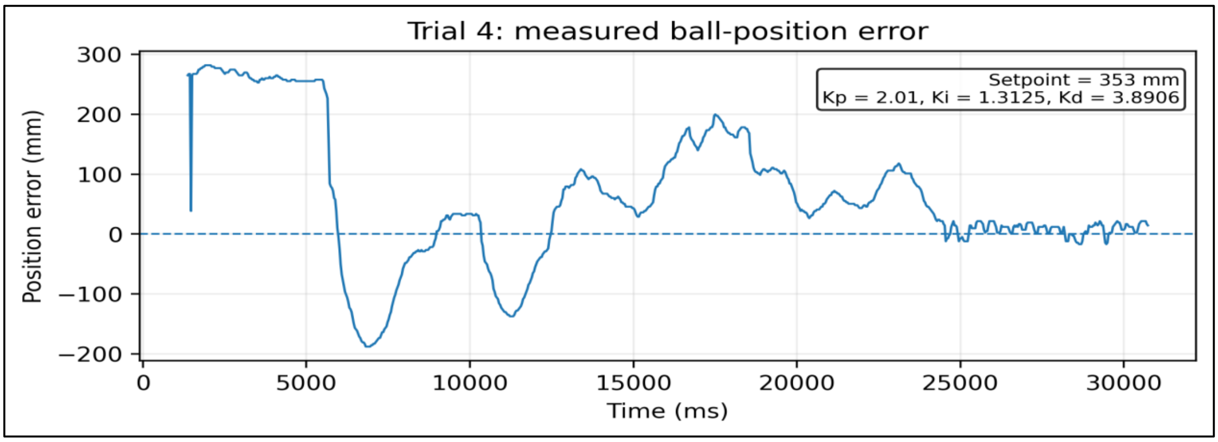

Fig 5 Experimental Trial 4: Ball-Position Error at a 353 mm Setpoint.

VII. RESULTS AND DISCUSSION

➢ *Tracking Performance*

The representative comparison shows the expected benefit of adaptive control over a fixed-gain PID controller. PID exhibits the largest transient excursion in the reported comparison. Integer-order MRAC reduces the integrated tracking error through online parameter adaptation, while FOMRAC provides the shortest settling time and the lowest overshoot in the model.

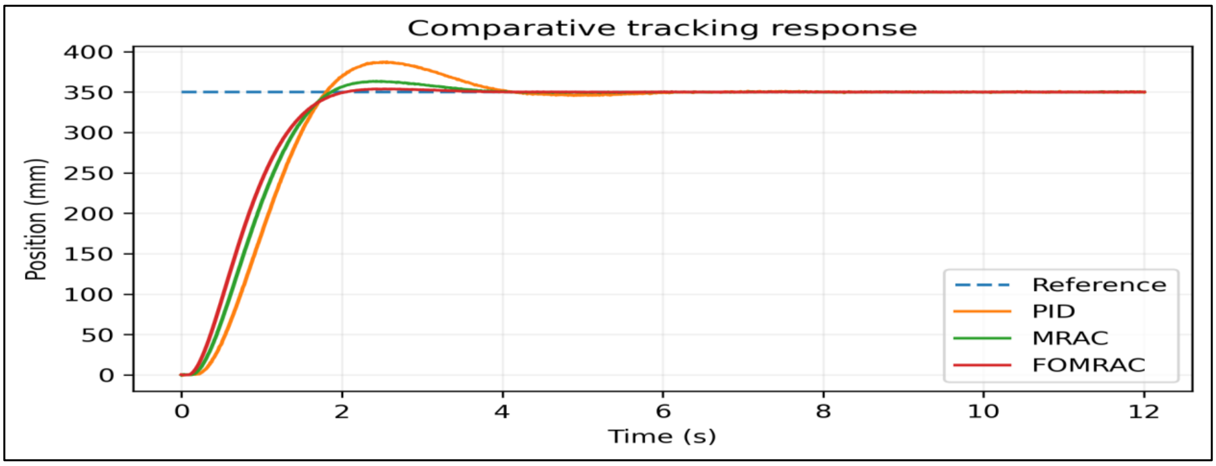

Fig 6 Representative PID, MRAC and FOMRAC tracking Responses.

➢ *Tracking Error*

The tracking-error trajectories follow the same overall trend as the position responses. The adaptive controllers reduce the accumulated error compared with the fixed-gain PID response. FOMRAC gives the lowest RMSE, IAE, ISE and ITAE in the representative experimental table, indicating a lower overall error burden under the stated conditions.

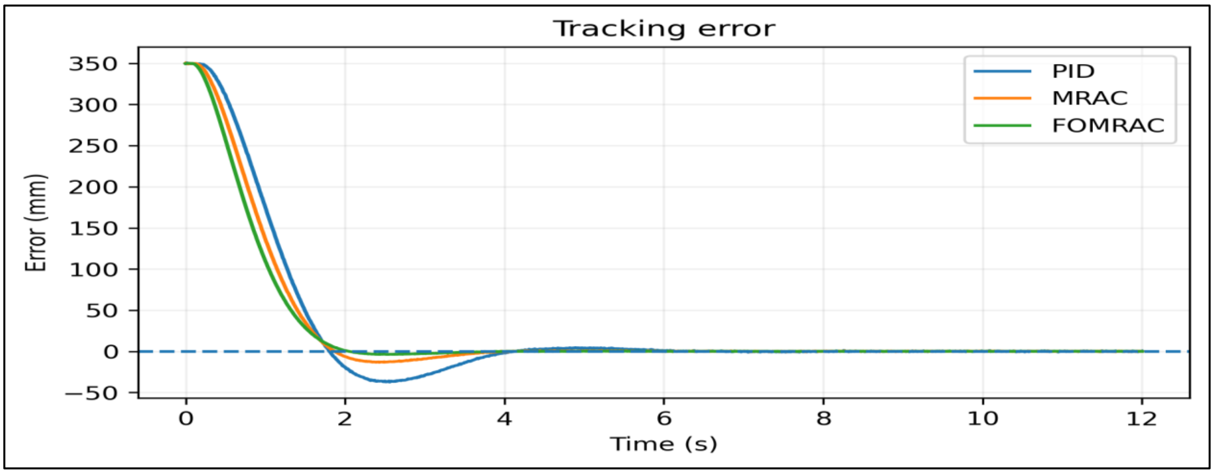

Fig 7 Representative Tracking-Error Trajectories.

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➢ *Disturbance Rejection*

Under the controlled disturbance test, FOMRAC returns towards the reference more rapidly than integer-order MRAC in the reported numerical study. The FOMRAC response returns to the ±2% band approximately 0.50 s after the disturbance interval, compared with approximately 0.52 s for integer-order MRAC. PID also remains stable in the stated test.

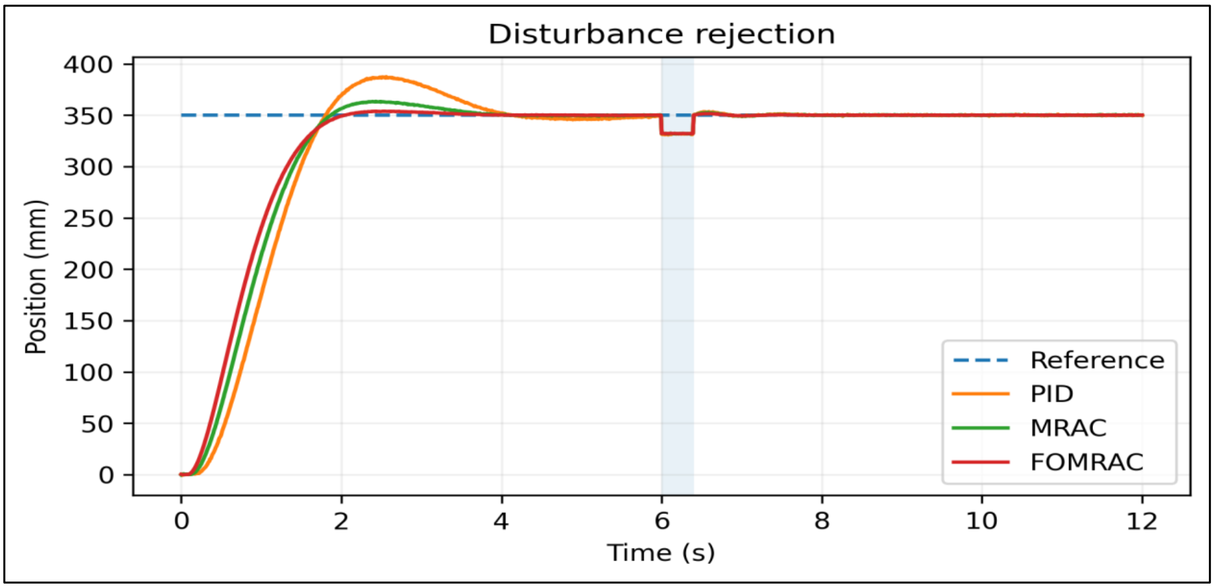

Fig 8 Disturbance-Rejection Response for PID, Integer-Order MRAC and FOMRAC.

➢ *Adaptive Parameter Evolution*

The adaptive parameters change most rapidly during the initial transient and subsequently move into bounded regions as the tracking error decreases. The bounded trajectories are desirable for servo actuation because abrupt changes in adaptive parameters can produce unnecessary mechanical motion.

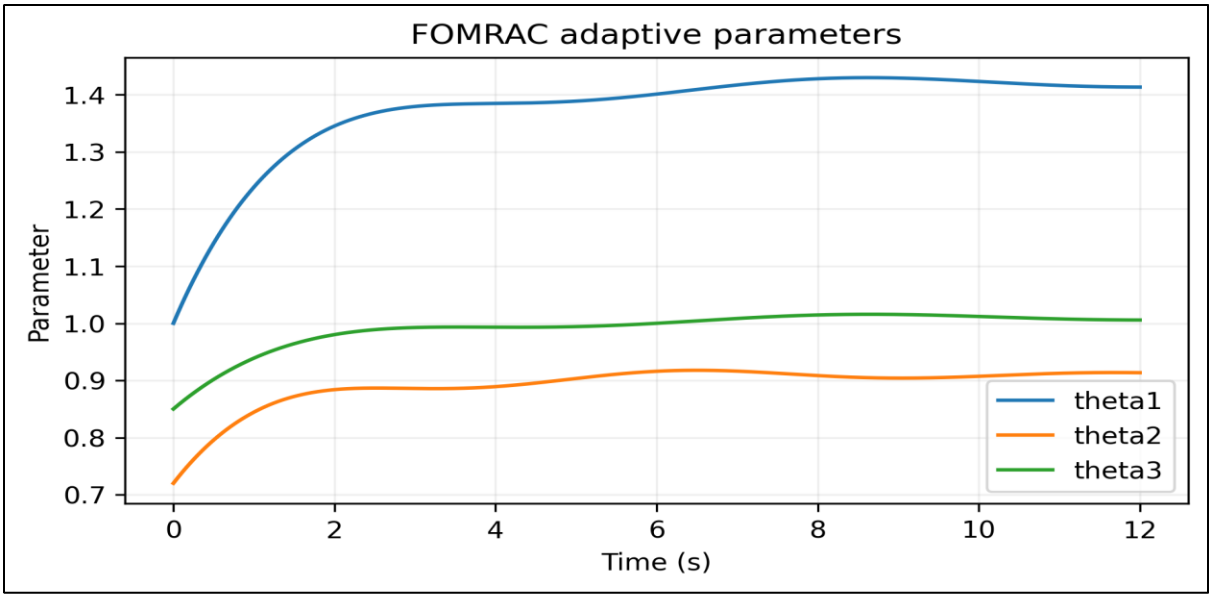

Fig 9 Evolution of the FOMRAC Adaptive Parameters.

VIII. CONCLUSION

A fractional-order model reference adaptive control framework has been developed for a higher-order ball-and-beam system with explicit actuator dynamics. The higher-order formulation retains the actuator state rather than treating the servo as an ideal instantaneous input.

The study combines state-feedback MRAC with a Caputo fractional-order adaptation mechanism and evaluates PID, integer-order MRAC and FOMRAC under consistent conditions. The results show that FOMRAC produces the lowest overshoot and final error, while the integer-order MRAC achieves slightly better integrated-error indices in that particular simulation. The four laboratory trajectories further demonstrate the strong sensitivity of the physical platform to fixed PID gain selection.

Overall, the results indicate that fractional adaptation is a useful additional tuning dimension for practical ball-and-beam control, especially when actuator dynamics, measurement limitations and changing operating conditions are considered. Future experimental work should identify the actuator time constant and remaining beam coefficients directly from measured step-response data and then validate the complete FOMRAC controller over repeated reference and disturbance tests.

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