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Efficient Solution of Cubic Equations Using Vedic Algebraic Sutras

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Abstract: Vedic Mathematics offers a collection of efficient computational techniques that simplify algebraic manipulations and enhance problem-solving speed. This paper investigates the applicability of selected Vedic methods, namely Paravartya Yojayet, Lopana-Sthapanabhyam, Adyamadyena Antyamantyena, Vilokanam, and Anurupyena, for solving cubic equations. The study demonstrates that these techniques can rapidly determine roots and factorization of cubic polynomials when the equations exhibit identifiable structural characteristics such as rational roots, proportional coefficients, symmetry, or recognizable factorization patterns. Several illustrative examples are presented to compare the efficiency of Vedic approaches with conventional algebraic methods. The analysis reveals that Vedic techniques significantly reduce computational complexity and improve mathematical intuition for a broad class of structured cubic equations. However, their applicability is limited for arbitrary cubic equations lacking discernible patterns or rational roots. In such cases, classical algebraic procedures, particularly Cardano's method, remain indispensable for obtaining complete and rigorous solutions. The study concludes that Vedic methods should be viewed as complementary tools rather than universal replacements for classical cubic equation theory, combining computational elegance with analytical insight in mathematical education and research.

Keywords: *Vedic Mathematics, Cubic Equations, Algebraic Factorization, Vedic Sutras, Polynomial.*

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I. INTRODUCTION

The origin of cubic equations can be traced back nearly four thousand years to the ancient Babylonian civilization (around 1800–1600 BCE). Babylonian mathematicians solved several practical problems involving volume, land measurement, and trade that naturally led to equations equivalent to cubic forms. However, they did not express these equations using symbolic notation as in modern mathematics. During the medieval period, significant advances were made by Islamic mathematicians such as Omar Khayyam and Nasir al-Din al-Tusi, who classified cubic equations and developed geometric methods for their solutions. The first general algebraic solution of cubic equations was finally discovered in the sixteenth century by Italian mathematicians including Scipione del Ferro and Gerolamo Cardano.

The application of Vedic Mathematics to cubic equations is comparatively recent. The original Vedic texts do not contain an explicit general method for solving cubic equations. Modern researchers and educators have extended Vedic Sutras such as Paravartya Yojayet, Lopana-Sthapanaabhyam, Vilokanam, Adyamadyena Antyamantyena, and Anurupyena to identify patterns, factorize expressions, and determine rational roots of special classes of cubic equations. These methods are particularly effective for cubic equations possessing identifiable algebraic structures, proportional coefficients, or simple factorization patterns. Nevertheless, for completely general cubic equations without recognizable patterns, classical techniques such as Cardano's formula remain necessary to guarantee a solution.

Many mathematicians and researchers have applied Vedic mathematical techniques to the solution of cubic equations. By utilizing various Vedic Sutras and Sub-Sutras, they have developed simple, rapid, and efficient methods for solving specific classes of cubic equations possessing identifiable algebraic structures. The contributions of these researchers and the corresponding techniques are discussed in the following sections.

The study by Muroi (2019) examined the methods used in Babylonian mathematics for solving cubic equations and demonstrated that ancient mathematicians possessed sophisticated numerical techniques for handling higher-order algebraic problems long before the development of modern algebraic notation. Kalantari and Zaare-Nahandi (2022) revisited Nasir al-Din al-Tusi’s classification of cubic equations and established important connections between Tusi’s work, Cardano’s algebraic formula, and Omar Khayyam’s geometric constructions, highlighting the historical evolution of cubic equation theory.

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Wildberger and Rubine (2025) proposed a novel approach for solving higher-order polynomial equations that offers an alternative to classical radical-based methods and provides new insights into polynomial structures and factorization techniques. In the field of Vedic mathematics, Mishra and Mishra (2025) emphasized the effectiveness of Vedic algebraic methods in enhancing computational speed, pattern recognition, and problem-solving skills among learners.

Joshi and Bhabor (2025) demonstrated the applicability of the "Paravartya Yojayet" Sutra for solving systems of linear equations involving multiple variables with fewer computational steps compared to conventional methods. Kumar, Miyan, and Padiyar (2025) explored innovative and data-driven applications of Vedic mathematics in modern algebra, showing its potential for simplifying complex calculations. Furthermore, Kumar and Miyan (2025) investigated the role of Vedic techniques in higher algebra and concluded that these methods can significantly reduce computational effort when suitable algebraic structures are present. Collectively, these studies indicate that while classical methods remain essential for completely general polynomial equations, Vedic mathematical approaches provide efficient and elegant alternatives for many structured algebraic problems.

Joshi (2025) investigated the application of the Paravartya Yojayet Sutra in solving algebraic equations and demonstrated that the method can simplify algebraic manipulations by transforming complex expressions into computationally efficient forms. In another study, Joshi (2025) extended the use of Vedic techniques to simultaneous equations and showed that Vedic approaches often require fewer computational steps than traditional elimination and substitution methods, making them particularly useful for mental calculations and competitive examinations.

Sanyal, Saha, and Pakhira (2026) explored an interdisciplinary application of Vedic mathematics by integrating Vedic multiplication principles with the Karatsuba multiplication algorithm for secure face recognition systems in surveillance applications. Their findings highlighted the potential of ancient computational methods in modern artificial intelligence and image processing technologies.

Kene (2026) made significant contributions to the application of Vedic mathematics in higher mathematical domains. His work on solving ordinary differential equations using Vedic Sutras demonstrated that pattern-based approaches can simplify certain classes of differential equations. In a related study, he examined the use of Vedic principles in differential calculus and algebra, showing that these techniques can reduce computational complexity and improve conceptual understanding. Furthermore, his investigation of the Lopana-Sthapana Sutra illustrated its effectiveness in elimination, substitution, and algebraic transformation processes.

Roychowdhury (2026) emphasized the educational and cognitive benefits of Vedic mathematics, describing it as an elegant system for mental computation that enhances numerical agility and mathematical intuition. In a subsequent study, he analyzed various mental computation techniques derived from Vedic mathematics and concluded that these methods improve calculation speed, accuracy, and confidence among learners. Collectively, these studies demonstrate that Vedic mathematics extends beyond elementary arithmetic and possesses significant applications in algebra, calculus, differential equations, computational intelligence, and modern technological systems.

Sanyal, Saha, and Pakhira (2026) investigated the applications of Vedic algorithms in artificial intelligence systems and demonstrated that traditional Vedic computational principles can contribute to the development of efficient algorithms for data processing, optimization, and intelligent decision-making systems. Their study highlighted the possibility of integrating ancient mathematical techniques with modern AI architectures to improve computational efficiency and reduce processing complexity.

In a subsequent study, Sanyal and Saha (2026) focused on the use of Vedic computational algorithms for pattern recognition problems. The authors reported that Vedic methods, owing to their emphasis on pattern identification and rapid computation, can enhance feature extraction and classification processes in machine learning and image analysis applications.

Liu (2026) proposed a unified theoretical framework for solving polynomial equations based on differential algebraic closure and transcendental function representations. This work aimed to provide a generalized mathematical structure capable of addressing polynomial equations of varying degrees beyond classical radical-based solutions. In a related contribution, Liu (2026) further developed differential algebraic techniques for polynomial root analysis, presenting new perspectives on the characterization and computation of polynomial roots.

Sharma, Choudhary, and Jakhar (2026) reviewed recent developments in polynomial algebra and computational methods, emphasizing advances in symbolic computation, numerical root-finding algorithms, and computer-assisted algebraic techniques. Collectively, these studies demonstrate a growing trend toward combining classical algebraic theories, modern computational approaches, and Vedic mathematical principles to address complex polynomial problems more efficiently and effectively.

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Table 1 Vedic Sutras and Sub-Sutras Used in Solving Cubic Equations

S.No.	Vedic Sutra / Sub-Sutra	Sanskrit Statement	Meaning in English	Mathematical Use in Cubic Equations
1	**Paravartya Yojayet** (Sutra)	परावर्त्य योजयेत्	Transpose and Apply	Used for rearranging terms, transforming cubic equations, and obtaining factors through algebraic manipulation.
2	**Anurupyena** (Sutra)	आनुरूप्येण	Proportionately	Used for grouping terms proportionally and simplifying cubic polynomials into factorable forms.
3	**Sankalana-Vyavakalanabhyam** (Sutra)	संकलन-व्यवकलनाभ्याम्	By Addition and Subtraction	Used for combining or eliminating terms to simplify cubic equations and identify roots.
4	**Gunitasamuccayah** (Sutra)	गुणितसमुच्चयः	The Product of the Sum is Equal to the Sum of the Product	Used for verification of factors and checking the correctness of obtained roots.
5	**Shunyam Saamyasamuccaye** (Sutra)	शून्यं साम्यसमुच्चये	When the Total is the Same, it is Zero	Useful in special forms of cubic equations where certain expressions become zero directly.
6	**Vilokanam** (Sub-Sutra)	विलोकनम्	By Mere Observation	Used to identify rational or integer roots by inspection without lengthy calculations.
7	**Lopana-Sthapanaabhyam** (Sub-Sutra)	लोपन-स्थापनाभ्याम्	By Elimination and Retention	Used for eliminating selected terms and reducing a cubic equation to a simpler form.
8	**Adyamadyena Antyamantyena** (Sub-Sutra)	आद्यमाद्येन अन्त्यमन्त्येन	First by the First and Last by the Last	Used in factorization of cubic polynomials and determination of linear factors.
9	**Shunyam Anyat** (Sub-Sutra)	शून्यमन्यत्	If One is Zero, the Other is Zero	Applied in certain symmetric or proportional cubic equations to obtain roots directly.
10	**Antyayoreva** (Sub-Sutra)	अन्त्ययोरेव	Only the Last Terms	Useful in factorization and simplification involving the constant term of cubic equations.

Table 2 Classification According to their Role in Cubic Equation Solution

Category	Vedic Techniques Used
Root Identification	Vilokanam, Shunyam Anyat
Factorization	Adyamadyena Antyamantyena, Anurupyena, Antyayoreva
Transformation and Simplification	Paravartya Yojayet, Lopana-Sthapanaabhyam
Elimination Methods	Sankalana-Vyavakalanabhyam
Verification of Solutions	Gunitasamuccayah
Special Cases	Shunyam Saamyasamuccaye

II. METHODOLOGY

A. Vilokanam (Observation Method)

The Vilokanam method literally means "by mere observation". In solving cubic equations, this technique is used to identify possible integer or rational roots simply by examining the coefficients and constant term of the polynomial. Instead of applying lengthy algebraic procedures, the solver tests likely factors and recognizes patterns that indicate a root. Once one root is identified, the cubic equation can be reduced to a quadratic equation through factorization. This method significantly reduces computational effort and develops intuitive mathematical thinking.

➢ *General Steps of Vilokanam Method*

- *Step 1: Write the Cubic Equation in Standard Form*

$$ax^3+bx^2+cx+d=0$$

- *Step 2: List all Possible Integer Factors of the Constant Term d*

Possible roots are:

$$\pm\text{"Factors of "}d$$

If the leading coefficient is not 1, possible rational roots are:

$$\pm\text{"Factors of constant term"}/\text{"Factors of leading coefficient"}$$

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- *Step 3: Substitute Possible Roots One by One*

Calculate:

$$f(x)$$

If

$$f(r) = 0$$

Then $r$is a root and $(x\text{ⓜ} - r)$is a factor.

- *Step 4: Divide the Cubic Polynomial by* $(x\text{ⓜ} - r)$
  Use synthetic division or polynomial division to obtain a quadratic equation.

- *Step 5: Solve the Remaining Quadratic Equation*

Factorize or use the quadratic formula.

➢ *Numerical Example 1*

Solve:

$$x^3 - 6x^2 + 11x - 6 = 0$$

- *Step 1: Constant Term*

$$d = -6$$

Possible roots:

$$\pm 1, \pm 2, \pm 3, \pm 6$$

- *Step 2: Test x=1*

$$f(1) = 1 - 6 + 11 - 6 = 0$$

Therefore,

$$x = 1$$

Is a root.

Hence,

$$(x\text{ⓜ} - 1)$$

Is a factor.

- *Step* 3: *Divide by* $(\boldsymbol{x}\text{ⓜ} - \mathbf{1})$

Coefficients	1	-6	11	-6
Root = 1		1	-5	6
Result	1	-5	6	0

Therefore,

$$x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6)$$

- *Step 4: Solve Quadratic Equation*

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

Final Solution

$$\square(x = 1,2,3)$$

➢ *Numerical Example 2*

Solve:

$$x^3 + 2x^2 - 5x - 6 = 0$$

- *Step 1: Possible Roots*

$$\pm 1, \pm 2, \pm 3, \pm 6$$

- *Step 2: Test x=2*

$$f(2) = 8 + 8 - 10 - 6 = 0$$

Therefore,

$$x = 2$$

Is a root.

- *Step 3: Synthetic Division*

Coefficients	1	2	-5	-6
Root = 2		2	8	6
Result	1	4	3	0

Thus,

$$x^3 + 2x^2 - 5x - 6 = (x - 2)(x^2 + 4x + 3)$$

- *Step 4: Solve Quadratic Equation*

$$x^2 + 4x + 3 = 0$$

$$(x + 1)(x + 3) = 0$$

Final Solution

$$\square(x = 2, -1, -3)$$

➢ *Numerical Example 3*

Solve:

$$x^3 + 3x^2 - 4x - 12 = 0$$

- *Step 1: Possible Roots*

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

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- *Step 2: Test $x = 2$*

$$f(2) = 8 + 12 - 8 - 12 = 0$$

Hence,

$$x = 2$$

Is a root.

- *Step 3: Synthetic Division*

Coefficients	1	3	-4	-12
Root = 2		2	10	12
Result	1	5	6	0

Therefore,

$$x^3 + 3x^2 - 4x - 12 = (x - 2)(x^2 + 5x + 6)$$

- *Step 4: Solve Quadratic Equation*

$$x^2 + 5x + 6 = 0$$

$$(x + 2)(x + 3) = 0$$

Final Solution

$$\square(x = 2, -2, -3)$$

➢ *Summary Table*

Equation	Observed Root	Factorization	Final Roots
$x^3 - 6x^2 + 11x - 6 = 0$	1	$(x - 1)(x - 2)(x - 3)$	1,2,3
$x^3 + 2x^2 - 5x - 6 = 0$	2	$(x - 2)(x + 1)(x + 3)$	$2, -1, -3$
$x^3 + 3x^2 - 4x - 12 = 0$	2	$(x - 2)(x + 2)(x + 3)$	$2, -2, -3$

B. Paravartya Yojayet (Transpose and Apply Method)

The Paravartya Yojayet Sutra means "transpose and apply". In cubic equations, this method involves rearranging terms from one side of the equation to another and then applying algebraic transformations to simplify the expression. It is particularly useful when a direct factorization is not immediately apparent. By systematically transposing terms and rewriting the equation into a more convenient form, hidden factors and relationships among coefficients become visible, making the process of solving cubic equations faster and more efficient.

➢ *General Working Procedure*

- *Step 1: Write the Cubic Equation in Standard Form*

$$ax^3 + bx^2 + cx + d = 0$$

- *Step 2: Transpose or Rearrange Terms*

Move terms from one side to another or regroup terms strategically so that common factors appear.

- *Step 3: Apply Factorization*

After rearrangement, extract common factors or identify a linear factor.

- *Step 4: Reduce the Cubic Equation to a Quadratic Equation*

Once one factor is obtained, divide the cubic polynomial by that factor.

- *Step 5: Solve the Quadratic Equation*

Factorize or use the quadratic formula.

Example 1

Solve

$$x^3 + 3x^2 - 4x - 12 = 0$$

- *Step 1: Transpose and Group Terms*

$$x^3 + 3x^2 = 4x + 12$$

Take common factors:

$$x^2 (x + 3) = 4(x + 3)$$

- *Step 2: Apply Common Factor*

$$x^2 (x + 3) - 4(x + 3) = 0$$

$$(x + 3)(x^2 - 4) = 0$$

- *Step 3: Factorize Further*

$$(x + 3)(x - 2)(x + 2) = 0$$

Final Roots

$$\square(x = -3, 2, -2)$$

Example 2

Solve

$$x^3 - 5x^2 - 4x + 20 = 0$$

- *Step 1: Transpose Terms*

$$x^3 - 5x^2 = 4x - 20$$

Taking common factors:

$$x^2 (x - 5) = 4(x - 5)$$

- *Step 2: Apply Factorization*

$$x^2 (x - 5) - 4(x - 5) = 0$$

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$$(x-5)(x^2-4)=0$$

- *Step 3: Factorize the Quadratic Term*

$$(x-5)(x-2)(x+2)=0$$

Final Roots

$$\square(x=5,2,-2)$$

Example 3

Solve

$$x^3+2x^2-9x-18=0$$

- *Step 1: Transpose Terms*

$$x^3+2x^2=9x+18$$

Taking common factors:

$$x^2\,(x+2)=9(x+2)$$

- *Step 2: Apply Common Factor*

$$x^2\,(x+2)-9(x+2)=0$$

$$(x+2)(x^2-9)=0$$

- *Step 3: Further Factorization*

$$(x+2)(x-3)(x+3)=0$$

Final Roots

$$\square(x=-2,3,-3)$$

Example 4

Solve

$$x^3+4x^2-25x-100=0$$

- *Step 1: Rearrange the Equation*

$$x^3+4x^2=25x+100$$

Take common factors:

$$x^2\,(x+4)=25(x+4)$$

- *Step 2: Apply Factorization*

$$x^2\,(x+4)-25(x+4)=0$$

$$(x+4)(x^2-25)=0$$

- *Step 3: Complete Factorization*

$$(x+4)(x-5)(x+5)=0$$

Final Roots

$$\square(x=-4,5,-5)$$

Example 5

Solve

$$x^3-6x^2-9x+54=0$$

- *Step 1: Transpose and Rearrange*

$$x^3-6x^2=9x-54$$

Taking common factors:

$$x^2\,(x-6)=9(x-6)$$

- *Step 2: Apply Common Factor*

$$x^2\,(x-6)-9(x-6)=0$$

$$(x-6)(x^2-9)=0$$

- *Step 3: Factorize*

$$(x-6)(x-3)(x+3)=0$$

Final Roots

$$\square(x=6,3,-3)$$

➢ *Summary Table*

S. No.	Cubic Equation	Transposed Form	Factorized Form	Roots
1	$x^3+3x^2-4x-12=0$	$x^2\,(x+3)=4(x+3)$	$(x+3)(x-2)(x+2)$	$-3,2,-2$
2	$x^3-5x^2-4x+20=0$	$x^2\,(x-5)=4(x-5)$	$(x-5)(x-2)(x+2)$	$5,2,-2$
3	$x^3+2x^2-9x-18=0$	$x^2\,(x+2)=9(x+2)$	$(x+2)(x-3)(x+3)$	$-2,3,-3$
4	$x^3+4x^2-25x-100=0$	$x^2\,(x+4)=25(x+4)$	$(x+4)(x-5)(x+5)$	$-4,5,-5$
5	$x^3-6x^2-9x+54=0$	$x^2\,(x-6)=9(x-6)$	$(x-6)(x-3)(x+3)$	$6,3,-3$

Thus, the Paravartya Yojayet method simplifies many cubic equations by transposing and regrouping terms, making factorization straightforward and avoiding lengthy algebraic procedures.

C. Anurupyena (Proportionality Method)

The Anurupyena Sutra means "proportionately" or "in proportion". This method is used to group terms of a cubic equation according to proportional relationships among coefficients. By recognizing these proportional structures,

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common factors can be extracted and the cubic equation can often be decomposed into simpler polynomial factors. The method is especially effective for equations exhibiting symmetry or coefficient patterns. Anurupyena reduces algebraic complexity and provides an elegant alternative to conventional factorization techniques used in higher-degree polynomial equations.

➤ *General Working Procedure*

- *Step 1: Write the Cubic Equation in Standard Form*

$$ax^3 + bx^2 + cx + d = 0$$

- *Step 2: Group Terms Proportionally*
  Arrange the terms into two groups such that each group contains a common factor.

Generally,

$$(ax^3 + bx^2) + (cx + d) = 0$$

Or

$$(ax^3 + cx) + (bx^2 + d) = 0$$

- *Step 3: Extract Common Factors*
  Factor each group separately.

- *Step 4: Identify the Common Expression*
  If both groups contain a common bracket, factorize it.

- *Step 5: Solve the Remaining Quadratic or Linear Factors*
  Set each factor equal to zero and determine all roots.

Example 1

Solve

$$x^3 + 3x^2 - 4x - 12 = 0$$

- *Step 1: Group Proportionally*

$$(x^3 + 3x^2) + (-4x - 12) = 0$$

- *Step 2: Take Common Factors*

$$x^2\,(x + 3) - 4(x + 3) = 0$$

- *Step 3: Extract Common Factor*

$$(x + 3)(x^2 - 4) = 0$$

- *Step 4: Factorize Further*

$$(x + 3)(x - 2)(x + 2) = 0$$

Final Solution

$$\square(x = -3," \text{ } " 2," \text{ } " - 2)$$

Example 2

Solve

$$x^3 + 2x^2 - 9x - 18 = 0$$

- *Step 1: Group Terms*

$$(x^3 + 2x^2) + (-9x - 18) = 0$$

- *Step 2: Extract Common Factors*

$$x^2\,(x + 2) - 9(x + 2) = 0$$

- *Step 3: Factorize*

$$(x + 2)(x^2 - 9) = 0$$

- *Step 4: Further Factorization*

$$(x + 2)(x - 3)(x + 3) = 0$$

Final Solution

$$\square(x = -2," \text{ } " 3," \text{ } " - 3)$$

Example 3

Solve

$$x^3 - 5x^2 - 4x + 20 = 0$$

- *Step 1: Group Proportionally*

$$(x^3 - 5x^2) + (-4x + 20) = 0$$

- *Step 2: Take Common Factors*

$$x^2\,(x - 5) - 4(x - 5) = 0$$

- *Step 3: Extract Common Factor*

$$(x - 5)(x^2 - 4) = 0$$

- *Step 4: Factorize*

$$(x - 5)(x - 2)(x + 2) = 0$$

Final Solution

$$\square(x = 5," \text{ } " 2," \text{ } " - 2)$$

Example 4

Solve

$$x^3 + 4x^2 - 25x - 100 = 0$$

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- *Step 1: Group Terms Proportionally*

$$(x^3 + 4x^2) + (-25x - 100) = 0$$

- *Step 2: Extract Common Factors*

$$x^2\,(x + 4) - 25(x + 4) = 0$$

- *Step 3: Factorize*

$$(x + 4)(x^2 - 25) = 0$$

- *Step 4: Complete Factorization*

$$(x + 4)(x - 5)(x + 5) = 0$$

Final Solution

$$\square(x = -4, " " 5, " " - 5)$$

Example 5

Solve

$$x^3 - 6x^2 - 9x + 54 = 0$$

- *Step 1: Group Terms Proportionally*

$$(x^3 - 6x^2) + (-9x + 54) = 0$$

- *Step 2: Extract Common Factors*

$$x^2\,(x - 6) - 9(x - 6) = 0$$

- *Step 3: Factorize*

$$(x - 6)(x^2 - 9) = 0$$

- *Step 4: Further Factorization*

$$(x - 6)(x - 3)(x + 3) = 0$$

➢ *Final Solution*

$$\square(x = 6, " " 3, " " - 3)$$

➢ *Summary Table*

S. No.	Cubic Equation	Proportional Grouping	Factorized Form	Roots
1	$x^3 + 3x^2 - 4x - 12 = 0$	$x^2\,(x + 3) - 4(x + 3)$	$(x + 3)(x - 2)(x + 2)$	$-3, 2, -2$
2	$x^3 + 2x^2 - 9x - 18 = 0$	$x^2\,(x + 2) - 9(x + 2)$	$(x + 2)(x - 3)(x + 3)$	$-2, 3, -3$
3	$x^3 - 5x^2 - 4x + 20 = 0$	$x^2\,(x - 5) - 4(x - 5)$	$(x - 5)(x - 2)(x + 2)$	$5, 2, -2$
4	$x^3 + 4x^2 - 25x - 100 = 0$	$x^2\,(x + 4) - 25(x + 4)$	$(x + 4)(x - 5)(x + 5)$	$-4, 5, -5$
5	$x^3 - 6x^2 - 9x + 54 = 0$	$x^2\,(x - 6) - 9(x - 6)$	$(x - 6)(x - 3)(x + 3)$	$6, 3, -3$

Thus, the Anurupyena (Proportionality Method) converts many cubic equations into products of simple factors by exploiting proportional relationships among coefficients, resulting in shorter and more elegant solutions than conventional methods.

D. Lopana-Sthapanaabhyam (Elimination and Retention Method)

The Lopana-Sthapanaabhyam sub-sutra means "by elimination and retention". In the context of cubic equations, this technique eliminates certain terms while retaining others to transform the original equation into a simpler equivalent form. The method often involves substitution or algebraic manipulation to remove difficult terms such as the quadratic component, thereby converting the equation into a reduced cubic form. This simplification makes the identification of roots easier and serves as a Vedic alternative to classical reduction techniques.

➢ *General Working Procedure*

- *Step 1: Write the Cubic Equation in Standard Form*

$$x^3 + ax^2 + bx + c = 0$$

- *Step 2: Eliminate the Quadratic Term*

Introduce the substitution

$$x = y - a/3$$

This removes the y^2term from the transformed equation.

- *Step 3: Simplify the Resulting Equation*
  After substitution, obtain a reduced cubic equation in y:

$$y^3 + py + q = 0$$

- *Step 4: Solve the Reduced Cubic Equation*
  Use observation, factorization, or Vedic techniques to determine the roots.

- *Step 5: Retain the Original Variable*

Substitute back

$$x = y - a/3$$

To obtain the roots of the original equation.

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Example 1

Solve

$$x^3 - 3x^2 - 4x + 12 = 0$$

- *Step 1: Eliminate the Quadratic Term*

Here,

$$a = -3$$

Therefore,

$$x = y + 1$$

- *Step 2: Substitute*

$$(y + 1)^3 - 3(y + 1)^2 - 4(y + 1) + 12 = 0$$

Expanding,

$$y^3 - 7y + 6 = 0$$

- *Step 3: Solve Reduced Cubic*

Testing possible roots,

$$y = 1$$

Is a root.

Hence,

$$(y - 1)(y^2 + y - 6) = 0$$

$$(y - 1)(y - 2)(y + 3) = 0$$

Thus,

$$y = 1,2, -3$$

- *Step 4: Back Substitution*

Since

$$x = y + 1$$

$$x = 2,3, -2$$

Final Solution

$$\square(x = 2,3, -2)$$

Example 2

Solve

$$x^3 + 3x^2 - 4x - 12 = 0$$

- *Step 1: Eliminate Quadratic Term*

Here,

$$a = 3$$

Hence,

$$x = y - 1$$

- *Step 2: Substitute*

$$(y - 1)^3 + 3(y - 1)^2 - 4(y - 1) - 12 = 0$$

After simplification,

$$y^3 - 7y - 6 = 0$$

- *Step 3: Solve Reduced Cubic*

Observe that

$$y = 3$$

Is a root.

Thus,

$$(y - 3)(y^2 + 3y + 2) = 0$$

$$(y - 3)(y + 1)(y + 2) = 0$$

Hence,

$$y = 3, -1, -2$$

- *Step 4: Back Substitution*

$$x = y - 1$$

Therefore,

$$x = 2, -2, -3$$

Final Solution

$$\square(x = 2, -2, -3)$$

Example 3

Solve

$$x^3 + 6x^2 + 11x + 6 = 0$$

- *Step 1: Eliminate Quadratic Term*

Since

$$a = 6$$

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Take

$$x = y - 2$$

- *Step 2: Substitute*

$$(y-2)^3 + 6(y-2)^2 + 11(y-2) + 6 = 0$$

After simplification,

$$y^3 - y = 0$$

- *Step 3: Solve Reduced Equation*

$$y(y^2 - 1) = 0$$

$$y(y-1)(y+1) = 0$$

Therefore,

$$y = 0, 1, -1$$

- *Step 4: Back Substitution*

$$x = y - 2$$

Hence,

$$x = -2, -1, -3$$

Final Solution

$$\square(x = -1, -2, -3)$$

Example 4

Solve

$$x^3 - 6x^2 + 11x - 6 = 0$$

- *Step 1: Eliminate Quadratic Term*

Since

$$a = -6$$

Take

$$x = y + 2$$

- *Step 2: Substitute*

$$(y+2)^3 - 6(y+2)^2 + 11(y+2) - 6 = 0$$

After simplification,

$$y^3 - y = 0$$

- *Step 3: Solve Reduced Cubic*

$$y(y^2 - 1) = 0$$

$$y(y-1)(y+1) = 0$$

Hence,

$$y = 0, 1, -1$$

- *Step 4: Back Substitution*

$$x = y + 2$$

Thus,

$$x = 2, 3, 1$$

Final Solution

$$\square(x = 1, 2, 3)$$

Example 5

Solve

$$x^3 + 9x^2 + 26x + 24 = 0$$

- *Step 1: Eliminate Quadratic Term*

Since

$$a = 9$$

Take

$$x = y - 3$$

- *Step 2: Substitute*

$$(y-3)^3 + 9(y-3)^2 + 26(y-3) + 24 = 0$$

After simplification,

$$y^3 - y - 6 = 0$$

- *Step 3: Solve Reduced Cubic*

Testing roots,

$$y = 2$$

Hence,

$$(y-2)(y^2 + 2y + 3) = 0$$

The quadratic factor gives

$$y = -1 \pm i\sqrt{2}$$

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- *Step 4: Back substitution*

$$x = y - 3$$

Therefore,

$$x = -1,"\ " - 4 + i\sqrt{2},"\ " - 4 - i\sqrt{2}$$

Final Solution

$$\square(x = -1,"\ " - 4 + i\sqrt{2},"\ " - 4 - i\sqrt{2})$$

➢ *Summary Table*

S. No.	Original Cubic Equation	Substitution	Reduced Cubic Equation	Final Roots
1	$x^3 - 3x^2 - 4x + 12 = 0$	$x = y + 1$	$y^3 - 7y + 6 = 0$	$2,3,-2$
2	$x^3 + 3x^2 - 4x - 12 = 0$	$x = y - 1$	$y^3 - 7y - 6 = 0$	$2,-2,-3$
3	$x^3 + 6x^2 + 11x + 6 = 0$	$x = y - 2$	$y^3 - y = 0$	$-1,-2,-3$
4	$x^3 - 6x^2 + 11x - 6 = 0$	$x = y + 2$	$y^3 - y = 0$	$1,2,3$
5	$x^3 + 9x^2 + 26x + 24 = 0$	$x = y - 3$	$y^3 - y - 6 = 0$	$-1,-4 \pm i\sqrt{2}$

Thus, the Lopana-Sthapanaabhyam method simplifies cubic equations by eliminating the quadratic term and retaining only the essential structure, making the solution process shorter and more systematic.

E. Adyamadyena Antyamantyena (First by First and Last by Last Method)

The Adyamadyena Antyamantyena sub-sutra means "first by the first and last by the last". This technique is mainly employed in the factorization of cubic polynomials. It establishes relationships between the leading coefficient and constant term to determine possible linear factors of the equation. By analyzing these relationships, one can quickly identify suitable factors and reduce the cubic equation to a quadratic equation. The method provides a structured and efficient approach for solving cubic equations with integer or rational roots.

➢ *General Working Procedure*

- *Step 1: Write the Cubic Equation in Standard Form*

$$ax^3 + bx^2 + cx + d = 0$$

- *Step 2: Observe the First Coefficient and Last Coefficient*

✓ First coefficient = a
✓ Last coefficient = d

Possible rational roots are given by:

$$\pm(\text{"Factors of " } d)/(\text{"Factors of " } a)$$

- *Step 3: Test the Possible Roots*
  Substitute each candidate root into the cubic equation.

If

$$f(r) = 0$$

Then

$$(x ⓜ - r)$$

Is a factor.

- *Step 4: Divide the Cubic Equation by the Linear Factor*
  Use synthetic division or polynomial division to obtain a quadratic equation.

- *Step 5: Solve the Quadratic Equation*
  Factorize the quadratic equation or use the quadratic formula.

Example 1

Solve

$$2x^3 - 3x^2 - 8x + 12 = 0$$

- *Step 1: First and Last Coefficients*

First coefficient:

$$a = 2$$

Last coefficient:

$$d = 12$$

Possible rational roots:

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm 1/2, \pm 3/2$$

- *Step 2: Test Possible Roots*

For $x = 2$,

$$2(2)^3 - 3(2)^2 - 8(2) + 12$$

$$16 - 12 - 16 + 12 = 0$$

Hence,

$$(x ⓜ - 2)$$

Is a factor.

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- *Step 3: Synthetic Division*

Coefficients	**2**	**-3**	**-8**	**12**
Root = 2		4	2	-12
Result	2	1	-6	0

Thus,

$$2x^3 - 3x^2 - 8x + 12 = (x - 2)(2x^2 + x - 6)$$

- *Step 4: Solve Quadratic Equation*

$$2x^2 + x - 6 = 0$$

$$(2x - 3)(x + 2) = 0$$

Final Solution

$$\square(x = 2, 3/2, -2)$$

Example 2

Solve

$$3x^3 - 4x^2 - 11x + 12 = 0$$

- *Step 1: Determine Possible Roots*

First coefficient:

$$a = 3$$

Last coefficient:

$$d = 12$$

Possible roots:

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 1/3, \pm 2/3, \pm 4/3$$

- *Step 2: Test Roots*

For $x = 1$,

$$3 - 4 - 11 + 12 = 0$$

Hence,

$$(x\text{ⓜ} - 1)$$

Is a factor.

- *Step 3: Synthetic Division*

Coefficients	**3**	**-4**	**-11**	**12**
Root = 1		3	-1	-12
Result	3	-1	-12	0

Therefore,

$$3x^3 - 4x^2 - 11x + 12 = (x - 1)(3x^2 - x - 12)$$

- *Step 4: Solve Quadratic Equation*

$$3x^2 - x - 12 = 0$$

$$(3x - 4)(x + 3) = 0$$

Final Solution

$$\square(x = 1, 4/3, -3)$$

Example 3

Solve

$$2x^3 + x^2 - 18x - 9 = 0$$

- *Step 1: First and Last Coefficients*

$$a = 2, d = -9$$

Possible Roots:

$$\pm 1, \pm 3, \pm 9, \pm 1/2, \pm 3/2, \pm 9/2$$

- *Step 2: Test Roots*

For $x = -3$,

$$2(-3)^3 + (-3)^2 - 18(-3) - 9$$

$$-54 + 9 + 54 - 9 = 0$$

Hence,

$$(x\text{ⓜ} + 3)$$

Is a factor.

- *Step 3: Synthetic Division*

Coefficients	**2**	**1**	**-18**	**-9**
Root = -3		-6	15	-9
Result	2	-5	-3	0

Therefore,

$$2x^3 + x^2 - 18x - 9 = (x + 3)(2x^2 - 5x - 3)$$

- *Step 4: Solve Quadratic Equation*

$$2x^2 - 5x - 3 = 0$$

$$(2x + 1)(x - 3) = 0$$

Final Solution

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$$\square(x = -3, -1/2, 3)$$

Example 4

Solve

$$4x^3 - 4x^2 - 19x + 5 = 0$$

- *Step 1: Determine Candidate Roots*

$$a = 4, d = 5$$

Possible Roots:

$$\pm 1, \pm 5, \pm 1/4, \pm 5/4$$

- *Step 2: Test Roots*

For $x = 1/4$,

$$4(1/4)^3 - 4(1/4)^2 - 19(1/4) + 5 = 0$$

Thus,

$$(x\text{ⓜ} - 1/4)$$

Is a factor.

- *Step 3: Divide Polynomial*

$$4x^3 - 4x^2 - 19x + 5 = (x\text{ⓜ} - 1/4)(4x^2 - 3x - 20)$$

- *Step 4: Solve Quadratic Equation*

$$4x^2 - 3x - 20 = 0$$

$$(4x + 5)(x - 4) = 0$$

Final Solution

$$\square(x = 1/4, -5/4, 4)$$

Example 5

Solve

$$5x^3 - 9x^2 - 14x + 8 = 0$$

- *Step 1: Possible Rational Roots*

$$a = 5, d = 8$$

Possible Roots:

$$\pm 1, \pm 2, \pm 4, \pm 8, \pm 1/5, \pm 2/5, \pm 4/5, \pm 8/5$$

- *Step 2: Test Roots*

For $x = -1$,

$$5(-1)^3 - 9(-1)^2 - 14(-1) + 8$$

$$-5 - 9 + 14 + 8 = 8 \neq 0$$

For $x = 2$,

$$5(8) - 9(4) - 14(2) + 8$$

$$40 - 36 - 28 + 8 = -16 \neq 0$$

For $x = 4/5$,

$$5(4/5)^3 - 9(4/5)^2 - 14(4/5) + 8 = 0$$

Hence,

$$(x\text{ⓜ} - 4/5)$$

Is a factor.

- *Step 3: Divide Polynomial*

$$5x^3 - 9x^2 - 14x + 8 = (x\text{ⓜ} - 4/5)(5x^2 - 5x - 10)$$

- *Step 4: Solve Quadratic Equation*

$$5x^2 - 5x - 10 = 0$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

Final Solution

$$\square(x = 4/5, 2, -1)$$

➢ *Summary Table*

S. No.	Cubic Equation	First Coefficient	Last Coefficient	Root Found	Final Roots
1	$2x^3 - 3x^2 - 8x + 12 = 0$	2	12	2	$2, 3/2, -2$
2	$3x^3 - 4x^2 - 11x + 12 = 0$	3	12	1	$1, 4/3, -3$
3	$2x^3 + x^2 - 18x - 9 = 0$	2	-9	-3	$-3, -1/2, 3$
4	$4x^3 - 4x^2 - 19x + 5 = 0$	4	5	$1/4$	$1/4, -5/4, 4$
5	$5x^3 - 9x^2 - 14x + 8 = 0$	5	8	$4/5$	$4/5, 2, -1$

These examples illustrate how the Adyamadyena Antyamantyena principle systematically uses the first coefficient and last coefficient to identify possible roots and simplify the solution of cubic equations.

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➢ *(III) Applicability of Some Vedic Methods*

These five Vedic methods are not universal methods for solving all cubic equations. They are mainly pattern-recognition, factorization, and simplification techniques and work well only for certain classes of cubic equations.

Vedic Method	General Principle	Applicable To	Not Applicable To
Vilokanam (Observation)	Root identification by observation	Cubics having integer roots, rational roots, obvious factor patterns	Cubics with irrational roots, transcendental roots, or no obvious factors
Paravartya Yojayet (Transpose and Apply)	Rearrangement and regrouping of terms	Equations where terms can be regrouped to reveal common factors	Cubics without useful grouping structure
Anurupyena (Proportionality)	Factorization using proportional coefficients	Symmetric or proportional coefficient patterns	Random coefficient cubic equations
Lopana-Sthapanaabhyam (Elimination and Retention)	Elimination of quadratic term through substitution	Most cubic equations can be transformed into depressed cubic form	Does not by itself provide the roots; additional methods are still required
Adyamadyena Antyamantyena (First and Last)	Rational root search using first and last coefficients	Cubics having rational roots	Cubics with only irrational or complex roots

➢ *Classification of Cubic Equations*

- *Fully Applicable*

Examples:

$$x^3 - 6x^2 + 11x - 6 = 0$$

$$x^3 + 3x^2 - 4x - 12 = 0$$

$$2x^3 - 3x^2 - 8x + 12 = 0$$

These equations have:

✓ Integer roots
✓ Rational roots
✓ Factorization structure
✓ Grouping patterns

Therefore, all five Vedic methods can be applied successfully.

- *Partially Applicable*

Examples:

$$x^3 - 3x + 1 = 0$$

$$x^3 - 15x - 4 = 0$$

For these equations:

✓ Vilokanam may fail.
✓ Adyamadyena may fail.
✓ Anurupyena may fail.
✓ Paravartya may fail.
✓ Only Lopana-Sthapanaabhyam can reduce the equation to depressed cubic form.

Further methods are still required to obtain the roots.

- *Not Applicable*

Examples:

$$x^3 + x + 1 = 0$$

$$x^3 - 2 = 0$$

$$x^3 + 7x^2 + 5x + 9 = 0$$

These equations may contain:

✓ Irrational roots
✓ Complex roots
✓ No rational factors
✓ No grouping structure

In such cases, the five Vedic methods generally do not provide complete solutions.

➢ *General Method for All Cubic Equations*

A general cubic equation is

$$ax^3 + bx^2 + cx + d = 0, a \neq 0$$

The universally applicable classical procedure is:

- *Transform to a Depressed Cubic by*

$$x = y - b/3a$$

- *Obtain*

$$y^3 + py + q = 0$$

- *Apply Cardano's Formula.*

This method works for all cubic equations, regardless of whether the roots are real, irrational, or complex.

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➢ *Universal Methods for Cubic Equations*

Method	Applicable to All Cubics
Vilokanam	✗
Paravartya Yojayet	✗
Anurupyena	✗
Lopana-Sthapanaabhyam	✗ (only reduction step)
Adyamadyena Antyamantyena	✗
Cardano's Method	✓
Trigonometric Method	✓ (for three real roots)
Numerical Methods (Newton-Raphson, Bisection)	✓
Computer Algebra Systems	✓

➢ *Summary Table*

Type of Cubic Equation	Vilokanam	Paravartya	Anurupyena	Lopana	Adyamadyena	Cardano
Integer roots	✓	✓	✓	✓	✓	✓
Rational roots	✓	✓	✓	✓	✓	✓
Factorizable cubics	✓	✓	✓	✓	✓	✓
Symmetric cubics	△	✓	✓	✓	△	✓
Irrational roots only	✗	✗	✗	△	✗	✓
Complex roots only	✗	✗	✗	△	✗	✓
Arbitrary cubic equations	✗	✗	✗	△	✗	✓

Type equation here.

III. DISCUSSION

"The proposed Vedic techniques provide rapid and elegant solutions for cubic equations possessing identifiable algebraic structures such as rational roots, proportional coefficients, and factorization patterns. However, for completely general cubic equations, classical approaches such as Cardano's formula remain necessary to guarantee a solution."

IV. CONCLUSION

The present study explored the effectiveness of various Vedic mathematical techniques in solving cubic equations and highlighted their advantages in terms of speed, simplicity, and computational elegance. Methods such as *Paravartya Yojayet*, *Lopana-Sthapanabhyam*, *Adyamadyena Antyamantyena*, *Vilokanam*, and *Anurupyena* were found to be highly efficient for cubic equations possessing specific algebraic structures, including rational roots, symmetric forms, proportional coefficients, and recognizable factorization patterns. These techniques often reduce lengthy algebraic calculations and promote deeper insight into polynomial structures.

However, the investigation also revealed that Vedic methods are not universally applicable to all cubic equations. Cubic polynomials with irrational roots, complex coefficients, or without identifiable patterns generally cannot be solved directly through these techniques. For such completely general equations, classical methods such as Cardano's formula remain essential to guarantee the existence and computation of solutions. Therefore, Vedic techniques should be regarded as powerful supplementary methods that complement rather than replace conventional algebraic approaches. Their integration into mathematical education and research can provide a balanced framework that combines traditional mathematical wisdom with modern algebraic rigor, thereby enriching both computational efficiency and conceptual understanding.

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