A Reduced Differential Transform Framework for Fractional Partial Differential Equations: Accuracy and Convergence Comparison with ADM, HPM, RPS, and L-RPS

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Abstract: Due to the intricacy of fractional derivatives and boundary conditions, finding trustworthy solutions for fractional partial differential equations (FPDEs) continues to be a significant challenge. In this research, a framework for effectively solving FPDEs using the Reduced Differential Transform Method (RDTM) is presented. Based on a straightforward recursive formulation, the proposed RDTM offers rapidly convergent analytical–approximate solutions without the need for discretization, linearization, or intricate integral computations. The accuracy and convergence of RDTM are methodically compared with those of ADM, HPM, RPS, and L-RPS. According to numerical studies, RDTM maintains computational simplicity while achieving greater numerical stability, faster convergence, and higher accuracy. The durability and efficacy of the proposed framework are confirmed by applications to fractional wave, telegraph, and Poisson equations, indicating its potential as a dependable tool for intricate fractional models in mathematical physics and engineering.

Keywords: *Fractional PDEs; Reduced Differential Transform; Accuracy; Convergence*.

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I. INTRODUCTION

A vast range of physical phenomena, such as decay processes, rheology, diffusion, electrostatics, electrodynamics, fluid dynamics, and elasticity, can now be modeled and analyzed using fractional partial differential equations (FPDEs) [7–15]. Compared to conventional differential equations, FPDEs provide more realistic models of many natural systems by accounting for memory effects and nonlocal dynamics. Obtaining explicit analytical solutions for many fractional differential equations, especially linear partial cases, is often very difficult or even impossible. Therefore, it is essential to develop reliable numerical and approximation approaches [8–7].

Several semi-analytical methods, such as the Adomian Decomposition Method (ADM) [17], the Homotopy Perturbation Method (HPM) [10], the Variational Iteration Method (VIM) [10], and the Laplace Residual Power Series (L-RPS) method [9], have been widely used. Numerous FPDEs, including Navier–Stokes equations, wave equations, telegraph equations, and fractional Poisson equations, have been successfully solved using these methods. Nevertheless, the majority of these techniques require laborious and intricate calculations.

In this work, we present a novel and effective method for solving linear fractional partial differential equations: the Reduced Differential Transform Method (RDTM) framework. RDTM is a reliable alternative to current techniques such as RPS, VIM, HPM, NIM, and L-RPS due to its straightforward recursive formulation, ease of implementation in computational software, and excellent accuracy.

Comparative analyses validate the superior performance of the proposed RDTM framework in solving linear fractional partial differential equations, and the results demonstrate that it provides highly accurate approximate solutions with rapid convergence.

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Definition 1.The real function $f(x)$ for $x > 0$ is in the space C_α, $\mathbb{R}\epsilon\alpha$, If there exists a real number $p > \alpha$ such that $f(x) = x^p f_1(x)$,Then $f_1(x) \in C[0,\infty)$.

Definition 2. A real function $f(x)$ for $x > 0$ is said to be in space $C_\alpha^m, m \in N \bigcup \{o\}$ If $f^{(m)} \in C_\alpha$.

Definition 3. If $f \in C_\alpha$ and $\alpha > -1$ then the left-hand Riemann-Liouville integral of order μ is defined as follows:

$$J_t^\mu u(x,t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-\tau)^{\mu-1} u(x,\tau) d\tau, \tau > 0$$

Definition 4. If $f \in C_{-1}^m$ and $m\epsilon N \cup \{0\}$ then the Caputo left-hand derivative of order μ is defined as follows.

$$D_t^\mu u(x,t) = \frac{\partial^\mu u(x,t)}{\partial t^\mu} = \begin{cases} \dfrac{d^m u(x,t)}{dt^m}, \mu = m \\ \dfrac{1}{\Gamma(m-\mu)} \displaystyle\int_0^t (t-\tau)^{m-\mu-1} \frac{d^m u(x,\tau)}{dt^m} d\tau, m-1 < \mu \le m, m \in N \end{cases}$$

It is worth recalling that the following relations exist between the Caputo fractional derivative and the Riemann-Liouville fractional order integral:

$$J_t^\mu D_t^\mu u(x,t) = u(x,t) - \sum_{k=0}^{m-1} \frac{d^k u(x,0)}{dt^k} \frac{t^k}{\Gamma(k+1)}, m-1 < \mu \le m, m \in N$$
$$D_t^\mu J_t^\mu u(x,t) = u(x,t)$$
$$J_t^\mu t^\alpha = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+\mu+1)} t^{\alpha+\mu}$$
$$D_t^\mu t^\alpha = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-\mu+1)} t^{\alpha-\mu}, \alpha > \mu - 1$$

Table 1 Reduced Differential Transformation

Original Function	**Fractional Reduced Differential Transformed Function**
$R_D[\omega(x,t)\varphi(x,t)]$	$\Omega_k \otimes \Phi_k = \sum_{l=0}^{k} \Omega_l(x)\Phi_{k-l}(x)$
$R_D[\alpha\omega(x,t) \pm \beta\varphi(x,t)]$	$\alpha\Omega(x) \pm \beta\Phi(x)$
$R_D\left[\frac{\partial^1}{\partial t^1}\omega(x,t)\right]$	$\frac{\Gamma(k+2)}{\Gamma(k+1)}\Omega_{k+1}(x)$
$R_D\left[\frac{\partial^{n\alpha}}{\partial t^{n\alpha}}\omega(x,t)\right]$	$\frac{\Gamma(k\alpha+n\alpha+1)}{\Gamma(k\alpha+1)}\Omega_{k+n}(x)$
$R_D\left[\frac{\partial^{n+s}}{\partial x^n \partial t^s}\omega(x,t)\right]$	$\frac{\Gamma(k+s+1)}{\Gamma(k+1)}\frac{\partial^n}{\partial x^n}\Omega_{k+s}(x)$
$R_D\left[\frac{\partial^n}{\partial x^n}\omega(x,t)\right]$	$\frac{\partial^n}{\partial x^n}\Omega_k(x)$
$R_D\left[x^m t^n \omega(x,t)\right]$	$x^m\Omega_{k-n}(x)$
$R_D\left[x^m t^n\right]$	$x^m\delta(k-n) = \begin{cases} 1, k=n \\ 0, k \ne n \end{cases}$

$R_D\left[e^{\lambda t}\right]$	$\frac{\lambda^k}{k!}$
$R\left[x^{\alpha}\right]$	$\delta_{k,1}=\delta(k-1)$

II. GENERAL REPRESENTATION OF THE EQUATION

For more information, the general case of a fractional-order linear partial differential equation is given by:

$$\frac{\partial^\alpha}{\partial t^\alpha}\varphi(x,t)=\sum_{v=0}^{\eta}\beta_v\frac{\partial^v\varphi(x,t)}{\partial t^v}+f(x,t),\eta\in\mathbb{N},m-1<\alpha\le m \tag{1}$$

With initial conditions:

$$\sum_{k=0}^{m-1}\frac{\partial^k\varphi(x,t)}{\partial t^k}=h_i(x,t),\qquad i=0,1,2,\dots m-1 \tag{2}$$

Given the initial conditions, we consider the solution series as follows:

$$\varphi(x,t)=\sum_{j=0}^{\infty}\phi_k t^{k\alpha}+\sum_{k=0}^{\infty}\Phi_k t^{k\alpha+1} \tag{3}$$

According to equation (2), we should assume the number of solution sequences equal to the number of initial conditions, and for each initial condition, only one solution sequence should be defined in such a way that they can be defined and combined together. In this paper, we work on fractional-order partial linear equations that often have two initial conditions; therefore, a reduced differential transformation is considered for each initial condition. Therefore, the hypothetical solution sequence given by equation (3) is presented.

In this research, according to the definition of initial conditions, our assumption in each equation is that $1<\alpha\le 2$.

Therefore, the initial conditions are defined according to equation (2) from equation (1) as follows:

$$\varphi_0(x,t)=h_0,\ for\ k=0 \tag{a.2}$$

$$and\ \frac{\partial\varphi(x,t)}{\partial t}=h_1,\ for\ k=1 \tag{b.2}$$

Equations (2.a) and (2.b) are the initial conditions for the solutions of $\sum_{j=0}^{\infty}\phi_k t^{k\alpha}$ and $\sum_{k=0}^{\infty}\Phi_k t^{k\alpha+1}$ in equation (1), respectively.

Now, by applying the reduced differential transformation to equations (2.a) and (2.b), respectively, we have:

$$R\left[\varphi_0(x,t)\right]=R\left[h_0\right],\ \Rightarrow\phi_0(x,t)=H_0,$$

$$and\ R\left[\frac{\partial\varphi(x,t)}{\partial t}\right]=R\left[h_1\right],\ \Rightarrow\Phi_0(x,t)=H_0^{'}$$

By applying differential transformation to equation (1), we have, in order of initial conditions:

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$$R\left(\frac{\partial^\alpha}{\partial t^\alpha}\varphi(x,t)\right)=R\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu\varphi(x,t)}{\partial t^\nu}\right)+R\left[f(x,t)\right]$$

$$\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)}\phi_{k+1}(x,t)=\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\phi_k(x,t)\right)+F(x,t)$$

$$\phi_{k+1}(x,t)=\frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)}\left[\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\phi_k(x,t)\right)+F(x,t)\right],\ for\ k=0,1,2\ldots$$

$$\phi_n(x,t)=\frac{\Gamma((n-1)\alpha+1)}{\Gamma(n\alpha+1)}\left[\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\phi_{n-1}(x,t)\right)+F(x,t)\right],\ n=1,2,3,\ldots \tag{4}$$

Also, for the initial condition (2.b), in equation (1), we have:

$$R\left(\frac{\partial^\alpha}{\partial t^\alpha}\varphi(x,t)\right)=R\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu\varphi(x,t)}{\partial t^\nu}\right)+R[f(x,t)]$$

$$\frac{\Gamma(k\alpha+\alpha+2)}{\Gamma(k\alpha+2)}\Phi_{k+1}(x,t)=\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\Phi_k(x,t)\right)+F(x,t)$$

By simplifying the above relationship, we have:

$$\Phi_{k+1}(x,t)=\frac{\Gamma(k\alpha+2)}{\Gamma(k\alpha+\alpha+2)}\left[\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\Phi_k(x,t)\right)+F(x,t)\right], for\ k=0,1,2\ldots$$

Therefore, for $k=0,1,2\ldots$ we have:

$$\Phi_2(x,t)=\frac{\Gamma(\alpha+2)}{\Gamma(2\alpha+2)}\left[\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\Phi_1(x,t)\right)+F(x,t)\right]$$

$$\Phi_3(x,t)=\frac{\Gamma(2\alpha+2)}{\Gamma(3\alpha+2)}\left[\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\Phi_2(x,t)\right)+F(x,t)\right]$$

$$\Phi_3(x,t)=\frac{\Gamma(3\alpha+2)}{\Gamma(4\alpha+2)}\left[\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\Phi_3(x,t)\right)+F(x,t)\right]$$

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$$\Phi_n(x,t)=\frac{\Gamma((n-1)\alpha+2)}{\Gamma(n\alpha+2)}\left[\left(\sum_{\nu=0}^{\eta}\beta_\nu\frac{\partial^\nu}{\partial t^\nu}\Phi_{n-1}(x,t)\right)+F(x,t)\right], n=1,2,3,\ldots \tag{5}$$

Therefore, we consider the final solution of equations (1) and (2) according to definition (3) as follows:

$$\varphi(x,t)=\sum_{j=0}^{m-1}\phi_k t^{k\alpha}+\sum_{k=0}^{\infty}\Phi_k t^{k\alpha+1}$$

By substituting relations (4) and (5) into the above equation, we have:

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$$\varphi(x,t) = \sum_{j=0}^{m-1} \phi_k t^{k\alpha} + \sum_{k=0}^{\infty} \Phi_k t^{k\alpha+1}$$

$$= \sum_{n=1}^{\infty} \frac{\Gamma((n-1)\alpha+1)}{\Gamma(n\alpha+1)} \left[\left(\sum_{\nu=0}^{\eta} \beta_\nu \frac{\partial^\nu}{\partial t^\nu} \phi_{n-1}(x,t) \right) + F(x,t) \right] t^{k\alpha}$$

$$+ \sum_{n=1}^{\infty} \frac{\Gamma((n-1)\alpha+2)}{\Gamma(n\alpha+2)} \left[\left(\sum_{\nu=0}^{\eta} \beta_\nu \frac{\partial^\nu}{\partial t^\nu} \Phi_{n-1}(x,t) \right) + F(x,t) \right] t^{k\alpha+1}, for\ k = 0,1,.. \quad (6)$$

III. APPLICATIONS AND EXAMPLES

➢ *Example 1 Consider the Following Fractional Wave Equation in Homogeneous Space-Time*:

$$\frac{\partial^\alpha}{\partial t^\alpha} \varphi(x,t) = \frac{1}{2} x^2 \frac{\partial^2}{\partial x^2} \varphi(x,t), x \in \mathbb{R}, t \geq 0,\ \ 0 < \alpha \leq 2 \quad (1.1)$$

$$\varphi_0(x,0) = x, \varphi_t(x,0) = x^2 \quad (1.2)$$

Where

$$\beta_0 = 0, \beta_1 = 0, \beta_2 = \frac{1}{2} x^2, f(x,t) = 0$$

With initial conditions[7]

$$\varphi_0(x,0) = x \text{Þ} \ \ \phi_0(x,t) = x \quad (1.3)$$

Equation (5.2) can be written as follows:

$$\phi_{k+1}(x,t) = \frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)} \left[\frac{x^2}{2} \frac{\partial^2}{\partial x^2} \varphi(x,t) \right], for\ k = 0,1,2,.. \quad (1.4)$$

Now we obtain the differential transformation of equation (5.1) as follows:

$$\varphi_0(x,0) = x \Rightarrow \phi_0(x,t) = x$$

$$\phi_1(x,0) = \frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)} \left[\frac{x^2}{2} \frac{\partial^2}{\partial x^2} \phi_0(x,t) \right] = 0$$

$$\phi_2(x,0) = \frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)} \left[\frac{x^2}{2} \frac{\partial^2}{\partial x^2} \phi_1(x,t) \right] = 0$$

$$\phi_3(x,0) = \phi_4(x,0) = \ldots = \phi_n(x,0) = 0 \quad (1.5)$$

Now, according to definition (2.b), initial conditions (1.2) and relation (1.1), we have:

$$\varphi_t(x,t) = x^2 \Rightarrow \Phi_0(x,t) = x^2$$

$$\Phi_{k+1}(x,t) = \frac{\Gamma(k\alpha+2)}{\Gamma(k\alpha+\alpha+2)} \left[\frac{x^2}{2} \frac{\partial^2}{\partial x^2} \Phi_k(x,t) \right], for\ k = 0,1,2,\ldots$$

$$\Phi_1(x,t) = \frac{\Gamma(2)}{\Gamma(\alpha+2)} \left(\frac{x^2}{2} \frac{\partial^2}{\partial x^2} \Phi_0(x,t) \right) = \frac{x^2}{\Gamma(\alpha+2)}$$

$$\Phi_2(x,t) = \frac{\Gamma(\alpha+2)}{\Gamma(2\alpha+2)} \left(\frac{x^2}{2} \frac{\partial^2}{\partial x^2} \Phi_0(x,t) \right) = \frac{x^2}{\Gamma(2\alpha+2)}$$

$$\Phi_3(x,t) = \frac{\Gamma(2\alpha+2)}{\Gamma(3\alpha+2)}\left(\frac{x^2}{2}\frac{\partial^2}{\partial x^2}\Phi_0(x,t)\right) = \frac{x^2}{\Gamma(3\alpha+2)} \tag{6}$$

Using equation (6), we have:

$$\varphi(x,t) = \sum_{k=0}^{m-1}\phi_k t^{k\alpha} + \sum_{k=0}^{\infty}\Phi_k(x,t)t^{k\alpha+1} = \phi_0 + \Phi_0 t + \Phi_1 t^{\alpha+1} + \Phi_2 t^{2\alpha+1}\ldots$$

By substituting the above relations, we have:

$$\varphi(x,t) = x + x^2\left[t + \frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)} + \frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)} + \ldots\right] \tag{1.7}$$

Therefore:

$for\ \alpha = 2$

$\varphi(x,t) = x + x^2\left[t + \frac{t^3}{3!} + \frac{t^5}{5!} + \frac{t^7}{7!} + \ldots\right] = x + x^2\, sinh(t)$(1.8)

Hence, this solution is identical to that obtained using the Laplace Residual Power Series Method (L-RPSM) and the Residual Power Series Method (RPSM) as follows [7–9].

$$\varphi(x,t) = \sum_{n=0}^{\infty}\varphi_n(x,t) = \varphi_0(x,t) + x^2\left(\sum_{n=0}^{\infty}\frac{t^{n\alpha+1}}{\Gamma(2+n\alpha)}\right) = x + x^2\left(\sum_{n=0}^{\infty}\frac{t^{n\alpha+1}}{\Gamma(2+n\alpha)}\right),$$

Therefore, the exact solution of equations (1.1) and (1.2)in closed form and using elementary functions will be as follows:

$\varphi(x,t) = x + x^2\sinh(t).$

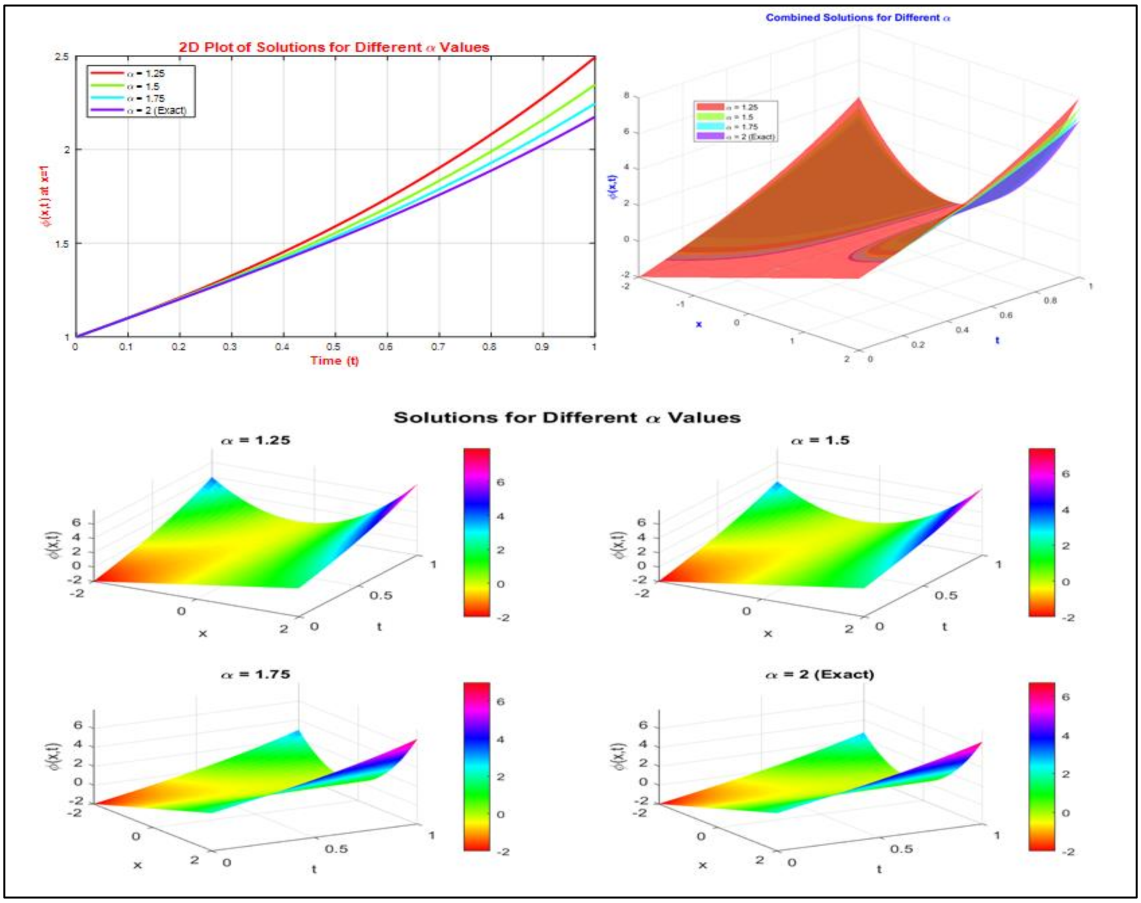

Fig 1 Corresponding to Example 1, for the Exact Solution with (α = 1.25,1.5) and (1.75).

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➢ *Example 2 Consider the Following Spatial Fractional Homogeneous Telegraph Equation [14-17]*

$$D_x^\alpha \varphi(x,t) = \frac{\partial^2 \varphi(x,t)}{\partial t^2} + \frac{\partial \varphi(x,t)}{\partial t} - x^2 - t + 1, \quad x \in ¡\ , \ t \geq 0, \quad 1 < \alpha \leq 2 \tag{2.1}$$

Where

$$\beta_0 = 0,\ \beta_1 = 1,\ \beta_2 = 1,\ f(x,t) = -\ x^2 -\ t + 1$$

With initial conditions:

$$\varphi(0,t) = t, \quad \frac{\partial \varphi(0,t)}{\partial x} = 0, \tag{2.2}$$

Applying the differential transformation in the initial conditions, we have:

$$\phi_0(x,t) = t,$$

$$\phi_{k+1}(x,t) = \frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)}\left[\frac{\partial^2}{\partial t^2}\phi_k(x,t) + \frac{\partial}{\partial t}\phi_k(x,t) - \delta(k-2) - \delta(k)(t-1)\right]$$

$$for \ \ k = 0,1,2,...$$

$$\phi_1(x,t) = \frac{1}{\Gamma(\alpha+1)}(2-t);$$

$$\phi_2(x,t) = \frac{-1}{\Gamma(2\alpha+1)}, \quad \phi_3(x,t) = \frac{1}{\Gamma(3\alpha+1)}, \quad \phi_4(x,t) = \phi_5(x,t) = 0,$$

Since the second part of the initial conditions is zero, its reduced differential transform also becomes zero, resulting in the solution as follows:

$$\varphi(x,t) = \sum_{k=0}^{\infty} \phi_k(x,t) x^{k\alpha} = t + (2-t)\frac{x^\alpha}{\Gamma(\alpha+1)} - \frac{x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{x^{3\alpha}}{\Gamma(3\alpha+1)}, \tag{2.3}$$

$$for \ \ \alpha=2$$

$$\varphi(x,t) = \sum_{k=0}^{\infty} \phi_k(x,t) x^{k\alpha} = t + (2-t)\frac{x^2}{2!} - \frac{x^4}{4!} - \frac{x^6}{6!} \tag{2.4}$$

The error in the reduced differential transformation method is the power of x^2 that appears in the equation, since the power of x cannot be a factor for the original structure of the solution series ($x^2 = x^{0.\alpha+2}$), the error in the last term for the reduced differential method has arisen. However, the solution obtained from the reduced differential transformation method compared to the exact solution has an absolute error of (0.0541) and a relative error on the scale of approximately (3.2%), which is insignificant compared to the speed of reaching the solution in other methods.

Of course, the solution obtained by the Residual Power Series (RPS) method is given as follows [7]:

$$\varphi(x,t) = t + (2-t)\frac{x^\alpha}{\Gamma(\alpha+1)} - 2\frac{x^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{x^{2\alpha}}{\Gamma(2\alpha+1)}.$$

$$for \ \alpha=2$$

$$\varphi(x,t) = t + (2-t)\frac{x^2}{2!} - \frac{x^4}{8}$$

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In this equation, the RPS method performed better than the methods mentioned in this study.

But the solution obtained by method (ADM) is as follows[17]

$$\varphi(x,t) = t + (1-t)\frac{x^{\alpha}}{\Gamma(1+\alpha)} + (1+t)\frac{x^{\alpha}}{\Gamma(1+\alpha)} - 2\frac{x^{\alpha+2}}{\Gamma(3+\alpha)} - t\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}.$$

➢ *Example 3 Consider the Following Spatial Fractional Homogeneous Telegraph Equation*[9].

$$D_x^{\alpha}\varphi(x,t) = \frac{\partial^2\varphi(x,t)}{\partial t^2} + \frac{\partial\varphi(x,t)}{\partial t} - x^{\alpha} - t + 1, \quad x,t \geq 0,\ 1<\alpha \leq 2 \qquad (3.1)$$

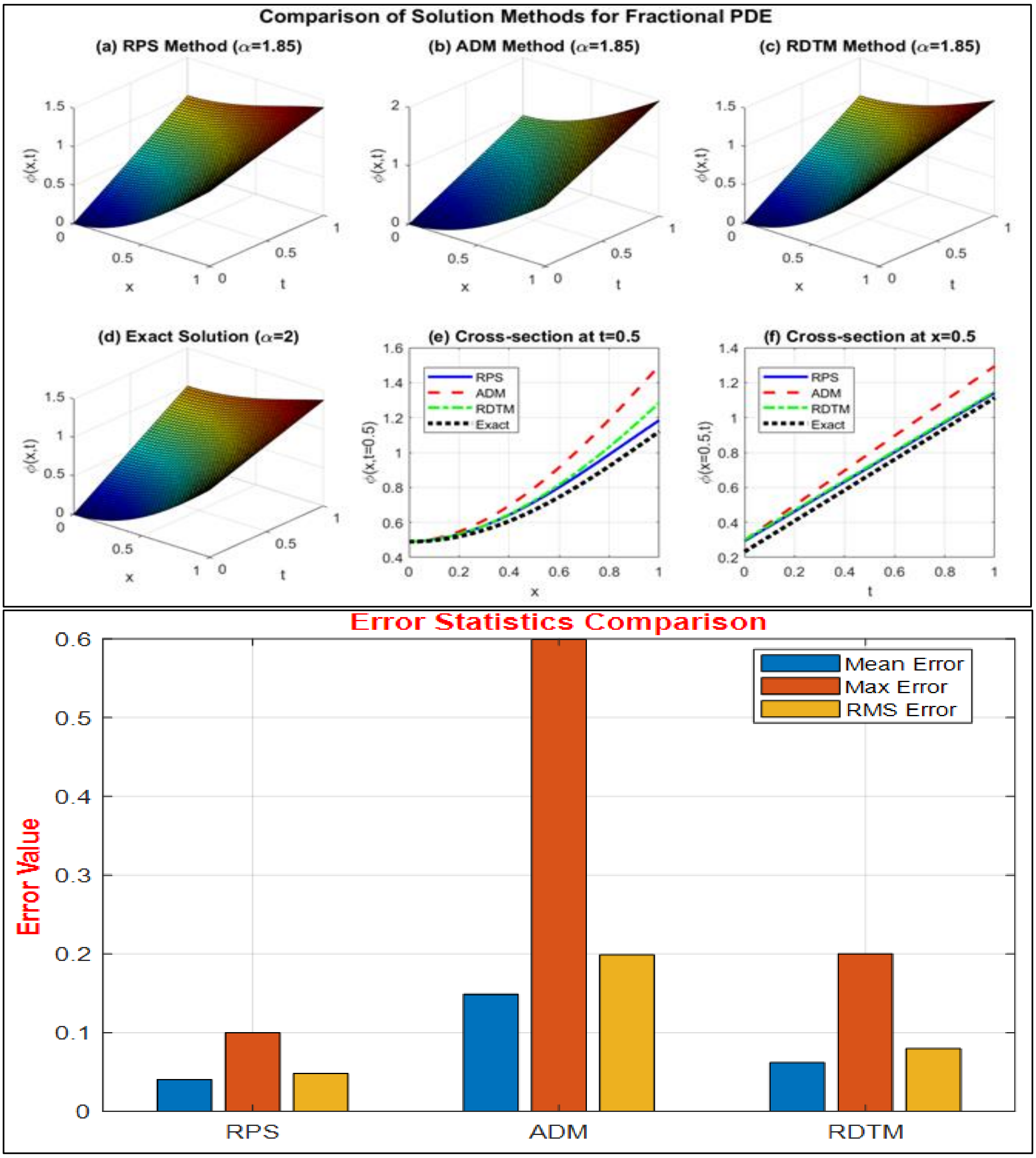

Fig 2 Numerical Solution of Example 2 and Comparison of the RPS, RDTM, and ADM Methods with the Exact Solution (α = 2).

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With initial conditions:

$$\varphi(0,t)=t,\quad \frac{\partial\varphi(0,t)}{\partial x}=0, \tag{3.2}$$

By applying differential transformation to the initial conditions, we have:

$$R[\varphi(0,t)=t]=R[t],$$

$$\phi_0(0,t)=t,\ and\ R\left[\frac{\partial\varphi(0,t)}{\partial x}\right]=R(0), \Phi_0(0,t)=0, \tag{3.3}$$

By applying the differential transformation to both sides of equation (3.1), we have:

$$\phi_{k+1}(x,t)=\frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)}\left[\frac{\partial^2}{\partial t^2}\phi_k(x,t)+\frac{\partial}{\partial t}\phi_k(x,t)-\delta_{k,1}-\delta_{k,0}(t-1)\right]$$
$for\ \ k=0,1,2,\ldots$

Therefore we have:

$$\phi_1(x,t)=\frac{1}{\Gamma(\alpha+1)}(2-t);\ for\ k=0,$$

$$\phi_2(x,t)=\frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)}\left[-\frac{1}{\Gamma(\alpha+1)}-1\right],=\frac{-[1+\Gamma(\alpha+1)]}{\Gamma(2\alpha+1)}, for\ k=1,$$

$$\phi_3(x,t)=,\phi_4(x,t)=\phi_5(x,t)=0,\ for\ k\geq 2,$$

By placing the above relations in series, we have the answer:

$$\varphi(x,t)=\sum_{k=0}^{\infty}\phi_k x^{k\alpha}+\sum_{k=0}^{\infty}\Phi_k t^{k\alpha+1}=t+(2-t)\frac{x^\alpha}{\Gamma(\alpha+1)}-\left[\frac{1+\Gamma(\alpha+1)}{\Gamma(2\alpha+1)}x^{2\alpha}\right] \tag{3.4}$$

$for\ \ \alpha=2$

$$\varphi(x,t)=t-(2-t)\frac{x^2}{2!}-\frac{x^4}{8}, \tag{3.5}$$

Which is the same solution obtained from the (RPS) method, in equation (2.1).

➢ *Example 4 Consider the following homogeneous fractional-space telegraph equation [14-17-9]*

$$D_x^\alpha\varphi(x,t)=\frac{\partial^2\varphi(x,t)}{\partial t^2}+\frac{\partial\varphi(x,t)}{\partial t}+\varphi(x,t),\quad x\in R,\ t\geq 0,\quad 0<\alpha\leq 2 \tag{4.1}$$

Where:

$$\beta_0=1,\ \beta_1=1,\ \beta_2=1,\ f(x,t)=0$$

With initial conditions:

$$\varphi(0,t)=Exp(-t),\quad \frac{\partial\varphi(0,t)}{\partial x}=Exp(-t)$$
$$\phi_0(x,t)=pxE(-t), \Phi_0(x,t)=pxE(-t) \tag{4.2}$$

According to the definition of relation (1), and according to the initial conditions of relation (3.1), we have:

$$\phi_{k+1}(x,t)=\frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)}\left[\frac{\partial^2\phi_k(x,t)}{\partial t^2}+\frac{\partial\phi_k(x,t)}{\partial t}+\phi_k(x,t)\right], for\ k=0,1,2,...$$

Hence,

$$\phi_1(x,t)=\frac{1}{\Gamma(\alpha+1)}Exp(-t),\ \phi_2(x,t)=\frac{1}{\Gamma(2\alpha+1)}Exp(-t)$$

$$\phi_3(x,t)=\frac{1}{\Gamma(3\alpha+1)}Exp(-t),\phi_4(x,t)=\frac{1}{\Gamma(4\alpha+1)}Exp(-t)$$

Therefore, by continuing this process, we have

$$\phi_n(x,t)=\frac{1}{\Gamma\left(n\alpha+1\right)}Exp(-t) \tag{4.3}$$

Similarly, for the initial condition $\Phi_0(x,t)=pxE(-t)$, we have:

$$\Phi_{k+1}(x,t)=\frac{\Gamma(k\alpha+2)}{\Gamma(k\alpha+\alpha+2)}\left[\frac{\partial^2\Phi_k(x,t)}{\partial t^2}+\frac{\partial\Phi_k(x,t)}{\partial t}+\Phi_k(x,t)\right], for\ k=0,1,2,...$$

$$\Phi_1(x,t)=\frac{\Gamma(k\alpha+2)}{\Gamma(k\alpha+\alpha+2)}\left(\frac{\partial^2\Phi_0(x,t)}{\partial t^2}+\frac{\partial\Phi_0(x,t)}{\partial t}+\Phi_0(x,t)\right), for\ k=0$$

$$\Phi_1(x,t)=\frac{\Gamma(2)}{\Gamma(\alpha+2)}\left(\Phi_0(x,t)\right)=\frac{1}{\Gamma(\alpha+2)}Exp(-t),\ for\ k=0$$

$$\Phi_2(x,t)=\frac{1}{\Gamma(2\alpha+2)}Exp(-t),\ \Phi_3(x,t)=\frac{1}{\Gamma(3\alpha+2)}Exp(-t),\ for\ k=,1,2$$

$$\Phi_4(x,t)=\frac{1}{\Gamma(4\alpha+2)}Exp(-t),\ for\ k=3$$

So for the last relation we have:

$$\Phi_n(x,t)=\frac{1}{\Gamma(n\alpha+2)}Exp(-t),\ for\ k=0,1,2,...,n \tag{4.5}$$

Therefore, the answer is presented as follows:

$$\varphi(x,t)=\sum_{j=0}^{m-1}\phi_k x^{k\alpha}+\sum_{k=0}^{\infty}\Phi_k x^{k\alpha+1}$$
$$=Exp(-t)\sum_{k=0}^{\infty}\left[\frac{x^{k\alpha}}{\Gamma\left(k\alpha+1\right)}+\frac{x^{k\alpha+1}}{\Gamma(k\alpha+2)}\right] \tag{4.6}$$

$for\quad \alpha=2,$

$$\varphi(x,t)=Exp(-t)\left[\sum_{n=0}^{\infty}\frac{x^{2n}}{\Gamma\left(2n+1\right)}+\frac{x^{2n+1}}{\Gamma(2n+2)}\right] \tag{4.7}$$

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According to the initial conditions in the (RPS) and (ADM) methods, for $\alpha = 2$ it results in the exact solution which is obtained as follows [7-9-17]

$$\varphi(x,t) = Exp(-t)\sum_{n=0}^{\infty}\left[\frac{x^{2n}}{\Gamma(2n+1)} + \frac{x^{2n+1}}{\Gamma(2n+2)}\right]$$
$$= Exp(-t)(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+...) = Exp(-t)Exp(x) = Exp(x-t)$$

- *Example 5 Consider the following inhomogeneous fractional-space Poisson equation [7].*

$$D_x^{\alpha}\varphi(x,t) = -\frac{\partial^2\varphi(x,t)}{\partial t^2} + x^{\alpha-1}Exp(t), \quad x,\ t \ge 0, \quad 1 < \alpha \le 2 \tag{5.1}$$

According to the definition of relation (1) and the initial conditions of equation (4.1), we have:

$$\beta_0 = 0,\ \beta_1 = 0,\ \beta_2 = -1,\ f(x,t) = x^{\alpha-1}Exp(t)$$

With initial conditions:

$$\varphi(x,0) = e^t, \varphi_x(x,0) = e^t$$
$$\phi_0(x,t) = e^t,\ \Phi_0(x,t) = e^t \tag{5.2}$$

By applying differential transformation to equation (5.1), we have

$$\phi_{k+1}(x,t) = \frac{-\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)}\left[\frac{\partial^2\phi_k(x,t)}{\partial t^2} + \delta(k-\alpha+1)e^t\right],\ for\ k = 0,1,2,...$$
$$\phi_1(x,t) = \frac{-\Gamma(\alpha+1)}{\Gamma(\alpha+1)}\left(\frac{\partial^2\phi_0(x,t)}{\partial t^2}\right) = \frac{-1}{\Gamma(\alpha+1)}Exp(t),$$
$$\phi_2(x,t) = \frac{1}{\Gamma(2\alpha+1)}\left(\frac{\partial^2\phi_1(x,t)}{\partial t^2}\right) = \frac{1}{\Gamma(2\alpha+1)}Exp(t) \tag{5.3}$$

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$$\phi_n(x,t) = \frac{(-1)^n}{\Gamma(n\alpha+1)}Exp(t).$$

But the right-hand side of the equation appears as $x^{\alpha-1}$, which should be to the power of $1,\alpha,\alpha+1$. To fix this defect, the bounds of the solution series for the initial condition $\Phi_0(x,t) = e^t$ must be formed as follows:

$$\varphi(x,t) = \sum_{n=0}^{\infty}\phi_k t^{k\alpha} + \sum_{k=0}^{n-2}\Phi_k t^{k\alpha+1}$$

Therefore, by inserting relation (5.3) into the above series, we have:

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$$\varphi(x,t)=\sum_{n=0}^{\infty}\phi_k x^{n\alpha}+\sum_{k=0}^{n-2}\Phi_k x^{k\alpha+1}$$

$$=Exp(t)\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n\alpha+1)}x^{n\alpha}\right]+\sum_{k=0}^{n-2}\Phi_k x^{k\alpha+1}$$

$$=Exp(t)\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n\alpha+1)}x^{k\alpha}\right]+\Phi_0 x^1$$

$$=Exp(t)\left[\sum_{n=0}^{¥}\frac{(-1)^n}{\Gamma(n\alpha+1)}x^{n\alpha}+\right]+xExp(t)$$

$$=Exp(t)\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n\alpha+1)}x^{n\alpha}+x\right]=xExp(t)+Exp(t)\sum_{n=0}^{\infty}\left[\frac{(-1)^n}{\Gamma(n\alpha+1)}x^{n\alpha}\right]$$

Therefore, from the above relation we have:

$$\varphi(x,t)=xExp(t)+Exp(t)\sum_{n=0}^{\infty}\left[\frac{(-1)^n}{\Gamma(n\alpha+1)}x^{n\alpha}\right] \quad (5.4)$$

While in the power series method, the residual is obtained as follows:

$$\varphi(x,t)=e^t\left[x+\sum_{n=0}^{\infty}(-1)^n\frac{x^{n\alpha}}{\Gamma(n\alpha+1)}\right]$$

For $\alpha = 2$, the exact solution of the equation is obtained as follows[7]:

$$\varphi(x,t)=Exp(t)(x+\cos x)$$

- *Example* 6 *Consider the following inhomogeneous fractional-time Poisson equation*[7]

$$D_t^{\alpha}\varphi(x,t)=p+\frac{\partial^2\varphi(x,t)}{\partial x^2}+\frac{1}{x}\frac{\partial\varphi(x,t)}{\partial x},\quad p\in ¡\ ,\quad x,t\geq 0,\quad 0<\alpha\leq 1 \tag{6.1}$$

Where

$$\beta_0=0,\ \beta_1=\frac{1}{x},\ \beta_2=1,\ f(x,t)=p$$

With initial conditions:

$$\varphi(x,0)=1-x^2,$$
$$\Rightarrow\ \phi_0(x,t)=1-x^2 \tag{6.2}$$

By applying the differential transformation in (6.1), we have:

$$\phi_{k+1}(x,t)=\frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\alpha+1)}\left[\frac{\partial^2\phi_k(x,t)}{\partial x^2}+\frac{1}{x}\frac{\partial\phi_k(x,t)}{\partial x}+p\delta(k)\right],\ for\ k=0,1,2,...$$

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$$\phi_1(x,t)=\frac{1}{\Gamma(\alpha+1)}\left[\frac{\partial^2\phi_0(x,t)}{\partial x^2}+\frac{1}{x}\frac{\partial\phi_0(x,t)}{\partial x}+p\delta(0)\right]=\frac{p-4}{\Gamma(\alpha+1)},\ for\ k=0$$

$$\phi_2(x,t)=\frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)}\left[\frac{\partial^2\phi_1(x,t)}{\partial x^2}+\frac{1}{x}\frac{\partial\phi_1(x,t)}{\partial x}+p\delta(1)\right]=0,\ for\ k=1$$

$$\phi_3(x,t)=\frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)}\left[\frac{\partial^2\phi_2(x,t)}{\partial x^2}+\frac{1}{x}\frac{\partial\phi_2(x,t)}{\partial x}+p\delta(2)\right]=0,\ for\ k=2$$

Therefore

$$\varphi(x,t)=\sum_{k=0}^{\infty}\Phi_k x^{k\alpha}=1-x^2+(p-4)\frac{t^\alpha}{\Gamma(\alpha+1)}, \tag{6.3}$$

The solution obtained by the residual power series method is not significantly different from the solution obtained by the method in Example 5.

Table 1 Comparison of Numerical Relative Errors for Example 1 When α = 1.5

t	x	HPM	ADM	RPSM	L-RPSM	RDTM
0.2	-10	5.51277	4.97102×10^{-5}	4.97102×10^{-5}	4.97102×10^{-5}	4.97102×10^{-5}
0.2	1	5.51277×10^{-2}	4.97102×10^{-7}	4.97102×10^{-7}	4.97102×10^{-7}	4.97102×10^{-7}
0.2	10	5.51277	4.97102×10^{-5}	4.97102×10^{-5}	4.97102×10^{-5}	4.97102×10^{-5}
0.4	-10	5.31407	2.24963×10^{-3}	2.24963×10^{-3}	2.24963×10^{-3}	2.24963×10^{-3}
0.4	1	5.31407×10^{-2}	2.24963×10^{-5}	2.24963×10^{-5}	2.24963×10^{-5}	2.24963×10^{-5}
0.4	10	5.31407	2.24963×10^{-3}	2.24963×10^{-3}	2.24963×10^{-3}	2.24963×10^{-3}
0.6	-10	4.64626	2.09224×10^{-2}	2.09224×10^{-2}	2.09224×10^{-2}	2.09224×10^{-2}
0.6	1	4.64626×10^{-2}	2.09224×10^{-4}	2.09224×10^{-4}	2.09224×10^{-4}	2.09224×10^{-4}
0.6	10	4.64626	2.09224×10^{-2}	2.09224×10^{-2}	2.09224×10^{-2}	2.09224×10^{-2}
0.8	-10	4.91059	1.01806×10^{-1}	1.01806×10^{-1}	1.01806×10^{-1}	1.01806×10^{-1}
0.8	1	4.91059×10^{-2}	1.01806×10^{-3}	1.01806×10^{-3}	1.01806×10^{-3}	1.01806×10^{-3}
0.8	10	4.91059	1.01806×10^{-1}	1.01806×10^{-1}	1.01806×10^{-1}	1.01806×10^{-1}

Table 2 Comparison of Numerical Relative Errors for Example 1 When α = 1.75

t	x	HPM	ADM	RPSM	L-RPSM	RDTM
0.2	-10	1.15719	3.70435×10^{-6}	3.70435×10^{-6}	3.70435×10^{-6}	3.70435×10^{-6}
0.2	1	1.15719×10^{-2}	3.70435×10^{-8}	3.70435×10^{-8}	3.70435×10^{-8}	3.70435×10^{-8}
0.2	10	1.15719	3.70435×10^{-6}	3.70435×10^{-6}	3.70435×10^{-6}	3.70435×10^{-6}
0.4	-10	0.92375	2.81935×10^{-4}	2.81935×10^{-4}	2.81935×10^{-4}	2.81935×10^{-4}
0.4	1	9.23749×10^{-3}	2.81935×10^{-6}	2.81935×10^{-6}	2.81935×10^{-6}	2.81935×10^{-6}
0.4	10	0.92375	2.81935×10^{-4}	2.81935×10^{-4}	2.81935×10^{-4}	2.81935×10^{-4}
0.6	-10	1.06133	3.55402×10^{-3}	3.55402×10^{-3}	3.55402×10^{-3}	3.55402×10^{-3}
0.6	1	1.06133×10^{-2}	3.55402×10^{-5}	3.55402×10^{-5}	3.55402×10^{-5}	3.55402×10^{-5}
0.6	10	1.06133	3.55402×10^{-3}	3.55402×10^{-3}	3.55402×10^{-3}	3.55402×10^{-3}
0.8	-10	1.73713	2.14579×10^{-2}	2.14579×10^{-2}	2.14579×10^{-2}	2.14579×10^{-2}
0.8	1	1.73713×10^{-2}	2.14579×10^{-4}	2.14579×10^{-4}	2.14579×10^{-4}	2.14579×10^{-4}
0.8	10	1.73713	2.14579×10^{-2}	2.14579×10^{-2}	2.14579×10^{-2}	2.14579×10^{-2}

Table 3 Comparison of numerical relative errors for Example 1 when α = 2

t	x	HPM	ADM	RPSM	L-RPSM	RDTM
0.2	-10	2.53968×10^{-7}	2.53968×10^{-7}	2.53968×10^{-7}	2.53968×10^{-7}	2.53968×10^{-7}
0.2	1	2.53968×10^{-9}	2.53968×10^{-9}	2.53968×10^{-9}	2.53968×10^{-9}	2.53968×10^{-9}
0.2	10	2.53968×10^{-7}	2.53968×10^{-7}	2.53968×10^{-7}	2.53968×10^{-7}	2.53968×10^{-7}
0.4	-10	3.25079×10^{-5}	3.25079×10^{-5}	3.25079×10^{-5}	3.25079×10^{-5}	3.25079×10^{-5}
0.4	1	3.25079×10^{-7}	3.25079×10^{-7}	3.25079×10^{-7}	3.25079×10^{-7}	3.25079×10^{-7}
0.4	10	3.25079×10^{-5}	3.25079×10^{-5}	3.25079×10^{-5}	3.25079×10^{-5}	3.25079×10^{-5}
0.6	-10	5.55429×10^{-4}	5.55429×10^{-4}	5.55429×10^{-4}	5.55429×10^{-4}	5.55429×10^{-4}

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0.6	1	5.55429×10^{-6}	5.55429×10^{-6}	5.55429×10^{-6}	5.55429×10^{-6}	5.55429×10^{-6}
0.6	10	5.55429×10^{-4}	5.55429×10^{-4}	5.55429×10^{-4}	5.55429×10^{-4}	5.55429×10^{-4}
0.8	-10	4.16102×10^{-3}	4.16102×10^{-3}	4.16102×10^{-3}	4.16102×10^{-3}	4.16102×10^{-3}
0.8	1	4.16102×10^{-5}	4.16102×10^{-5}	4.16102×10^{-5}	4.16102×10^{-5}	4.16102×10^{-5}
0.8	10	4.16102×10^{-3}	4.16102×10^{-3}	4.16102×10^{-3}	4.16102×10^{-3}	4.16102×10^{-3}

IV. CONCLUSION

The Reduced Differential Transform Method (RDTM) is offered as an efficient method for the analytical–numerical solution of linear fractional partial differential equations based on the findings of this work. Without the need for discretization, linearization, or intricate integral computations, RDTM can generate extremely accurate and quickly convergent solutions using a straightforward recursive relation. RDTM provides better convergence speed and numerical stability when compared to well-known techniques like the Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), Residual Power Series (RPS), and Laplace Residual Power Series (L-RPS).

The method's superior accuracy in solving equations such as the fractional wave equation, fractional telegraph equation, and fractional Poisson equation is amply demonstrated by the numerical data included in the error comparison tables. In particular, the RDTM solutions exactly match the analytical solutions and outcomes from other exact approaches in the classical case (\alpha=2). RDTM's flexibility in applying beginning and boundary conditions, ease of implementation, and capacity to extend to nonhomogeneous equations without requiring more computing work are some of its main features.Overall, RDTM is suggested as a dependable and potent method for resolving fractional problems in mathematical physics and engineering due to its shown effectiveness and computing advantages across numerous situations.

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