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A Robust Leak Localization Method Based on Energy-Weight TDOA Using Accoustic Sensor Networks

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Abstract: This paper introduces an improved Time Difference of Arrival (TDOA) method for pipeline leak localization using acoustic signals. Unlike the classical TDOA, which assumes uniform sensor reliability, the proposed method incorporates energy-based weights derived from acoustic signal strength. The resulting weighted cost function enhances robustness against noisy or poorly positioned sensors. Monte Carlo simulations in both 2D and 3D settings show that the weighted TDOA reduces the root-mean-square error (RMSE) by more than 50% compared to the classical approach. This demonstrates its effectiveness in accurately locating leaks under realistic noisy conditions.

Keywords: *Leak, Accoustic Emission, Pipeline, Sensor Networks, Fast Fourier Transform, Sound, Time Difference of Arrival.*

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I. INTRODUCTION

The increasing demand for reliable and sustainable energy distribution has placed growing pressure on fluid transport infrastructures, especially pipelines. These networks, which extend across thousands of kilometers, are essential for supplying water, oil, and gas. Yet, even minor leaks can result in substantial economic losses, environmental contamination, and risks to public safety. Consequently, early detection and accurate localization of leaks have become a priority for both researchers and industry stakeholders.

Among the various techniques proposed, acoustic monitoring has emerged as one of the most promising approaches. A leak generates acoustic emissions due to fluid turbulence and structural vibrations, which can propagate along the pipeline and be captured by sensors. Figure 1 illustrates a leaking pipeline equipped with acoustic sensors, showing how acoustic waves propagate from the leakage point to the monitoring devices. Over the years, Time Difference of Arrival (TDOA) methods have been widely adopted to estimate leak positions, since they exploit the difference in arrival times of the acoustic signal at multiple sensors. TDOA has the advantage of being non-intrusive and relatively simple to implement. Figure 2 schematically presents the principle of the classical TDOA method, where the acoustic wavefront reaches sensors at different times, producing measurable delays Δt.

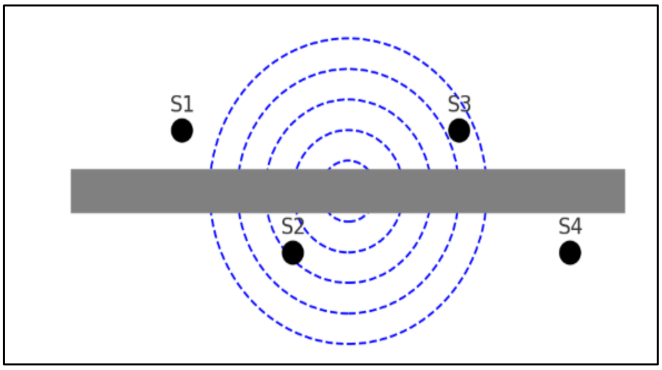

Fig 1 A Horizontal Pipeline, a Leak, Propagating Acoustic Waves and Four Sensors Around it.

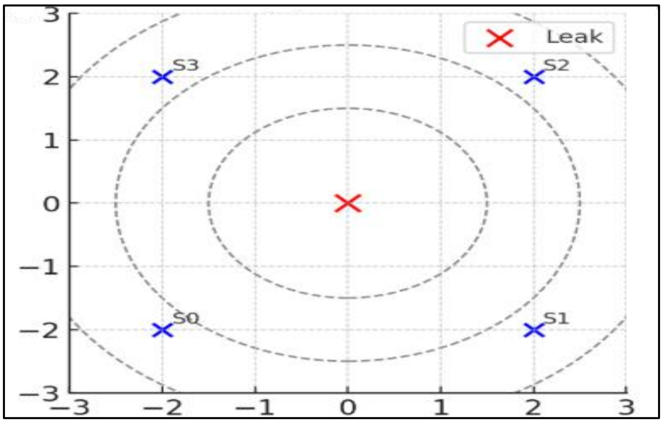

Fig 2 The Principe of the Classical TDOA

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However, classical TDOA assumes that all sensors contribute equally to the localization process. In practice, this is rarely the case: sensors may be positioned at different distances from the leak, they may be affected by environmental noise, or they may have unequal sensitivity. As a result, a single unreliable sensor can introduce significant errors into the estimation process. This limitation has motivated researchers to explore improved methods that can account for the quality of the signal received at each sensor [1], [2].

In this context, we propose a weighted TDOA model, in which the contribution of each sensor is adjusted according to the acoustic energy it records. Intuitively, sensors closer to the leak or less affected by noise will register stronger acoustic energy, and thus should exert a greater influence in the localization model. By introducing energy-based weights into the TDOA cost function, we aim to increase robustness and accuracy, particularly in noisy or complex environments.

This paper provides both the mathematical framework and simulation results of the proposed method. We compare the classical and weighted TDOA approaches in 2D and 3D settings, highlighting the gain in precision and the reduction of localization errors. The weighted approach, grounded in physical reality, offers a promising direction for more reliable leak detection systems.

II. MATERIALS AND METHODS

➢ *Classical TDOA Method*

Let the leak be located at position $p = (x, y, z)$, and i-th sensor be at $s_i = (x_i, y_i, z_i)$. The time of arrival of the acoustic wave is given by:

$$t_i = \frac{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}{v} \quad (1)$$

where v is the speed of sound in the medium. Considering sensor 0 as the reference, the time difference of arrival is:

$$\Delta t_i = t_i - t_0 \quad i = 1,2, \dots M-1 \quad (2)$$

The localization problem involves minimizing the cost function:

$$J(x,y,z) = \sum_{i=1}^{M-1} (\Delta t_i^{mes} - \Delta t_i(x,y,z))^2 \quad (3)$$

This equation is nonlinear due to the square root terms in the distance calculations, making analytical solutions impractical. Therefore, numerical optimization techniques such as Levenberg–Marquardt or gradient descent are used.

➢ *Weight TDOA Method*

In the proposed approach, the function is modified to include energy weights:

$$J(x,y,z) = \sum_{i=1}^{M-1} w_i (\Delta t_i^{mes} - \Delta t_i(x,y,z)^2 \quad (4)$$

where the weight w_i is calculated as:

$$w_i = \frac{E_i}{\sum_{j=1}^{M} E_j} \quad (5)$$

And the energy E_i is obtained from the signal recorded by each sensor:

$$E_i = \int_{f1}^{f2} |S_i(f)|^2 df \quad (6)$$

Here, $S_i(f)$ is the signal's frequency spectrum computed via Fast Fourier Transform (FFT), and f_1 , f_2 define the frequency band of interest (typically 100 – 500 Hz for leak noise). This weighting prioritizes sensors capturing higher acoustic power and mitigates errors from low-energy or noisy measurements [4].

➢ *Numerical Resolution*

To minimize $J(p)$ or $J_w(p)$, iterative algorithms are applied. The Levenberg–Marquardt algorithm balances the stability of gradient descent with the efficiency of Gauss–Newton [2] [3]. The update rule is:

$$p_{k+1} = p_k - (J^T J + \lambda I)^{-1} J^T (r) \quad (7)$$

where J is the Jacobian matrix of partial derivatives, r the residual vector, and λ a damping factor.

Figure 3 presents the geometric configuration of the sensor array and the propagation wavefronts originating from the leak position. Each circle represents the acoustic front detected by a sensor, corresponding to equal time-of-arrival contours. The intersection of these isochronous fronts defines the possible location of the leak.

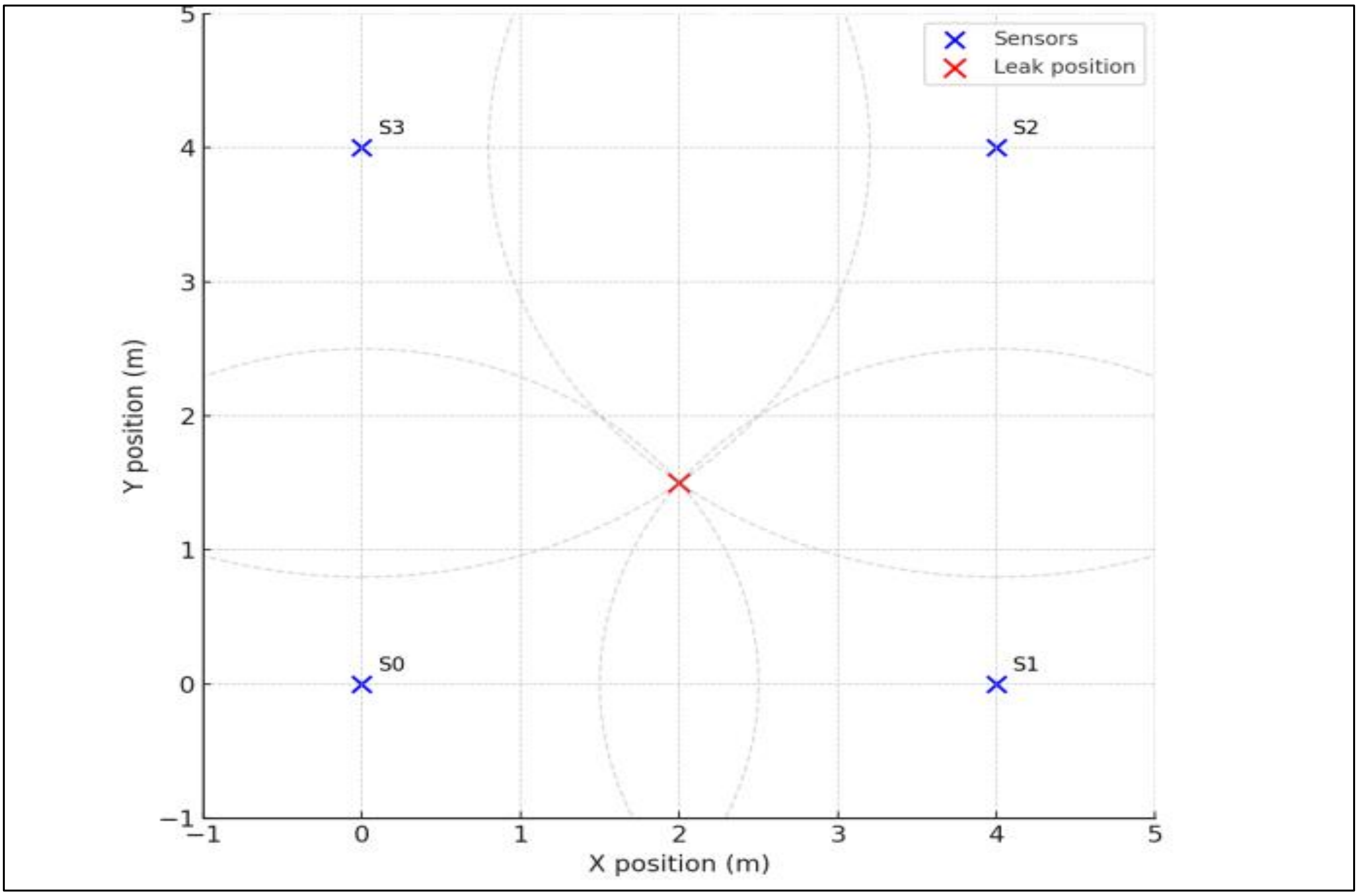

Fig 3 Geometric Configuration and Propagation Fronts (TDOA Illustration)

III. SIMULATION

The simulation setup is designed to reproduce realistic conditions of acoustic leak detection along a fluid pipeline. In this configuration, four sensors are strategically placed around a section of the pipeline to record the acoustic emissions produced by a leak. The goal is to evaluate how both classical and energy-weighted TDOA models perform under controlled yet realistic scenarios.

The simulated environment considers a leak located at position (2.0,1.5) meters and four sensors forming a square geometry at positions (0,0), (4,0), (4,4), (0,4). The propagation velocity of acoustic waves is set to v=1500 m/s, corresponding to typical sound speed in water. The frequency range of the leak-generated acoustic signal is assumed between 100 and 500 Hz, consistent with experimental studies reporting leak emission frequencies within low-frequency bands. To emulate real operating conditions, Gaussian noise is introduced into the sensor measurements with a standard deviation of 5×10^{-4}. These simulation parameters are summarized in Table 1.

Table 1 Parameters Used for the Basic Simulation

Parameters	**Value**	**Descriptions**
Sensors positions	(0,0) (4,0) (4,4) (0,4)	Positions of each sensor
Leak position	(2, 1.5)	Proposed leak position
Frequence range	100-500Hz	proposed water leak frequency interval [5] [6]
Engergy leves	0.27, 0.27, 0.46, 0	signal energy received by each sensor

The numerical simulation computes the theoretical Time Differences of Arrival (TDOA) for each sensor relative to a reference. These time delays are then perturbed with random noise and used as input for both classical and weighted TDOA estimations. The weighting coefficients are calculated from the acoustic energy captured by each sensor, representing their signal reliability. This configuration enables a direct comparison between unweighted and energy-weighted localization accuracy.

The results of the simulation are visualized in both 2D and 3D. The 3D view provides an intuitive understanding of how acoustic waves propagate and interact with sensor positions around the leak source. The figure 4shows the geometric setup and simulated acoustic wave propagation.

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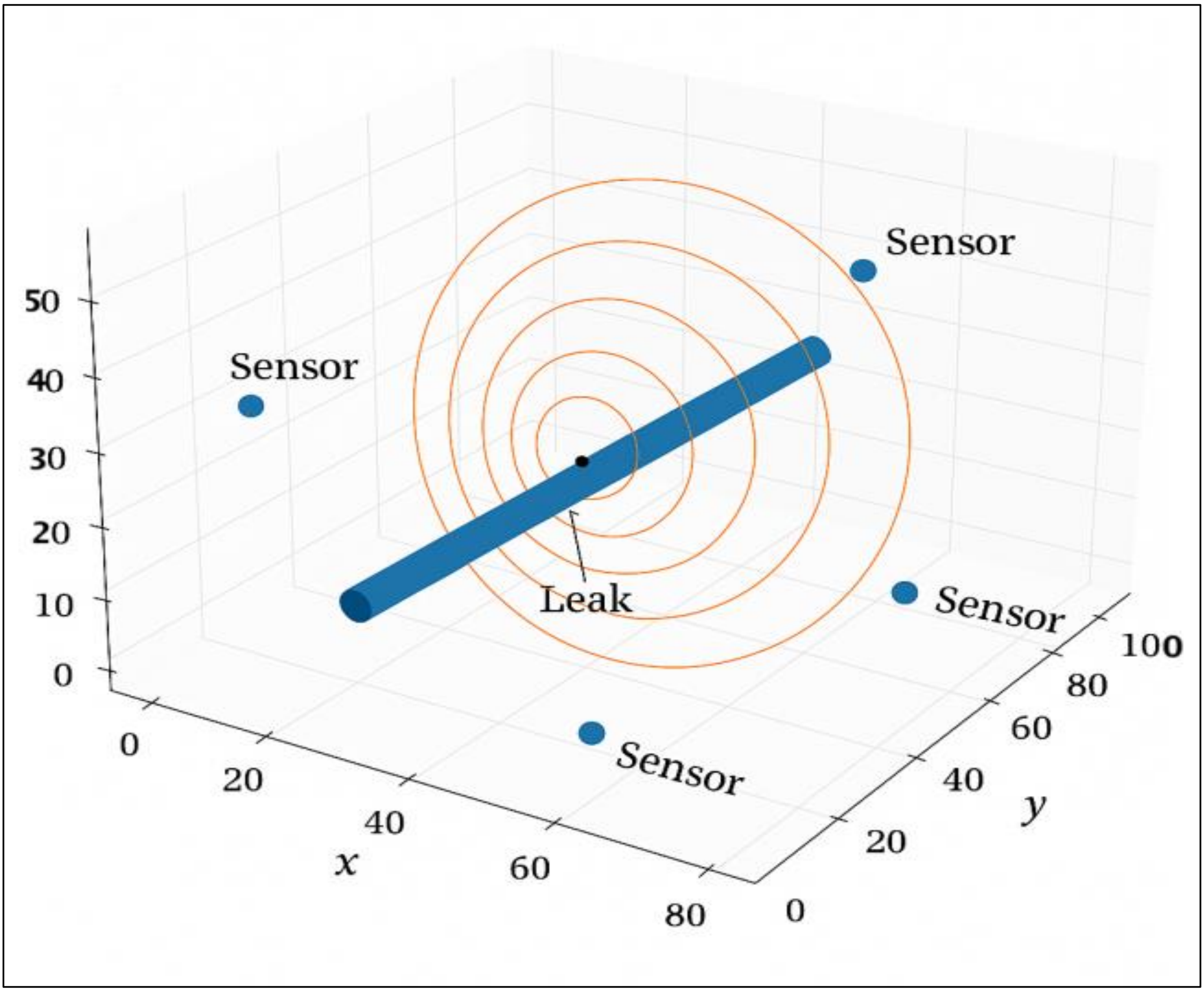

Fig 4 Simulation Geometry in 3D Illustrating the Acoustic Wave Propagation Between the Leak and the Four Sensors Surrounding the Pipeline. Spherical Wavefronts Represent Equal Time-of-Arrival Contours, Forming the Basis of the TDOA Estimation Model.

The model was implemented using Python with NumPy and Matplotlib libraries.

IV. RESULTATS AND DISCUSSIONS

➢ *Localization Accuracy*

Monte Carlo simulations (200 runs) were performed to compare classical and weighted TDOA. The results demonstrate a significant improvement (Table 3).

Table 2 Localization Accuracy

Metric	**Classical TDOA**	**Weight TDOA**
RMSE	0.42	0.18
MAE	0.36	0.15
Median Error	0.33	0.14
%Error < 0.5m	72%	94%

To provide a visual comparison between the two localization methods, Figure 4 illustrates the estimated leak positions obtained using both classical and energy-weighted TDOA models. The classical TDOA estimate appears slightly offset from the true leak location, whereas the weighted model converges much closer to the actual point. This improvement confirms that integrating energy-based weighting enhances spatial accuracy and robustness, especially when certain sensors are affected by noise or attenuation.

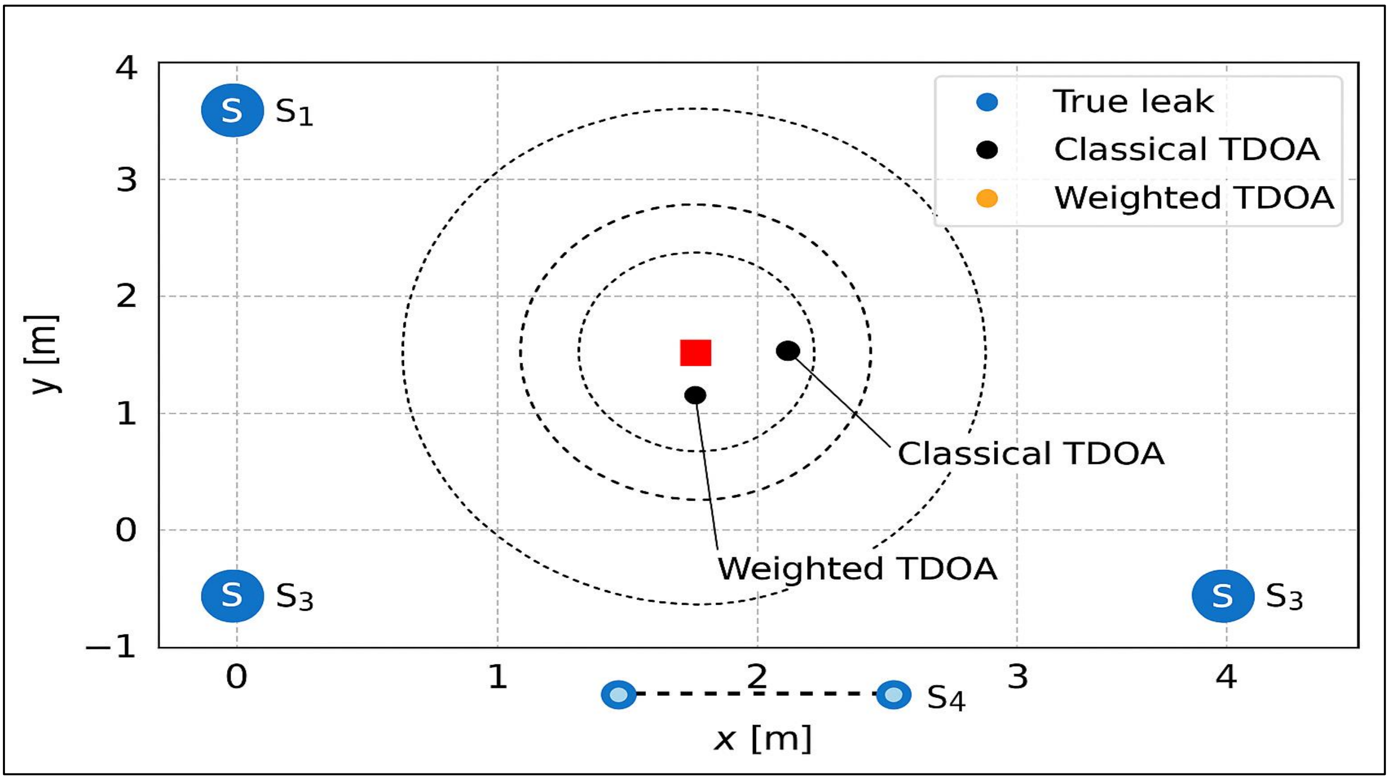

Fig 5 Comparison of Estimated Leak Positions

➢ *Error Computation*

To statistically validate the robustness of the proposed localization model, a series of 1000 Monte Carlo simulations was performed. In each simulation run, random variations were applied to the measurement noise and sensor readings to emulate real-world uncertainties. By repeating the estimation process over numerous trials, a realistic distribution of localization errors was obtained, allowing for the computation of statistical indicators such as the RMSE, MAE, and median error.

Figure 5 displays the resulting histogram of localization errors for both classical and energy-weighted TDOA methods. The weighted model exhibits a sharper and more concentrated distribution near zero, confirming its superior stability and precision even under noisy conditions

This confirms that the proposed energy-based formulation enhances both the stability and the reliability of leak localization, particularly under asymmetric or noisy sensor configurations.

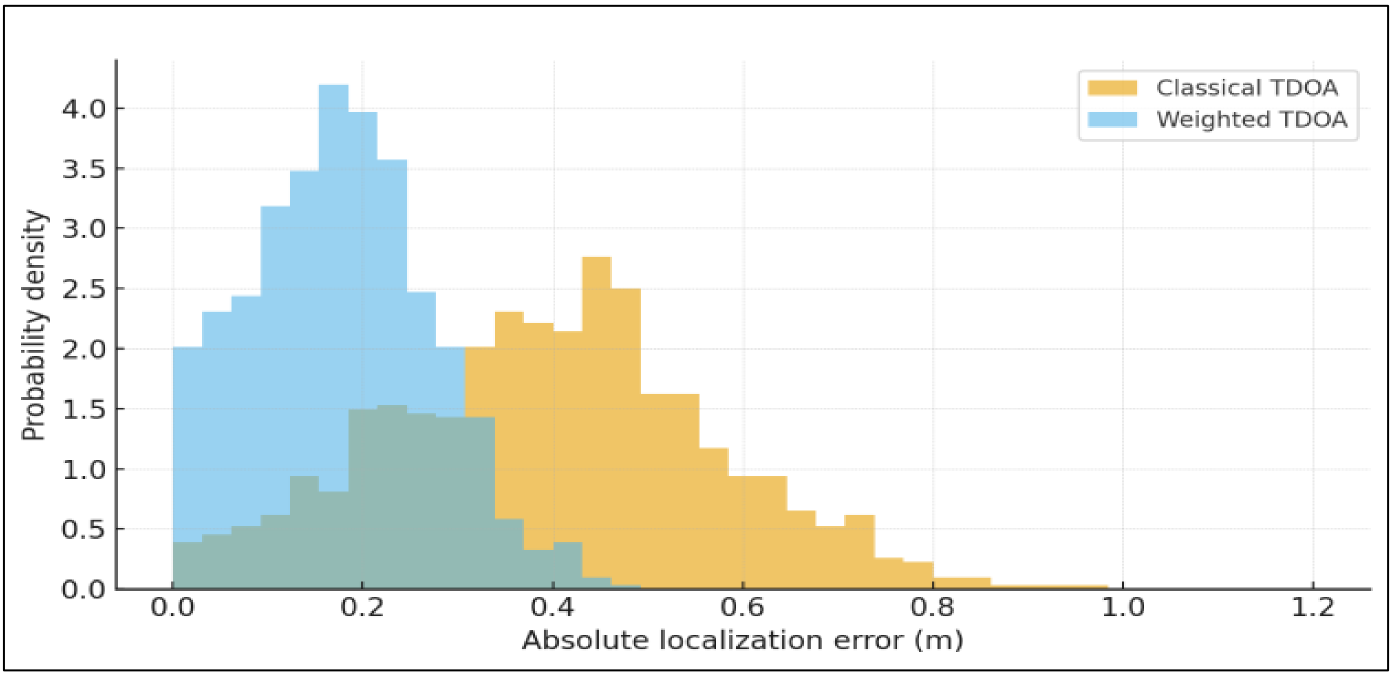

Fig 6 Base Power for Different Thickness of Ga0.51In0.49P Window Layer

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➢ *Physical Interpretation*

From a physical standpoint, the improvement obtained with the energy-weighted TDOA model can be explained by the natural decay of acoustic energy as it propagates through the medium. In practice, the sound energy emitted by a leak decreases proportionally to the square of the distance from the source ($E \propto 1/d^2$). This means that sensors located closer to the leak capture signals with higher amplitudes and better signal-to-noise ratios, while those farther away receive weaker and more distorted signals.

By incorporating these physical characteristics into the mathematical model, the energy-weighted formulation gives greater importance to reliable sensors and reduces the influence of low-quality data. This weighting mechanism acts as a natural filter, minimizing the bias introduced by distant or noisy sensors.

In essence, the proposed model does not only represent a numerical adjustment; it mirrors the physical reality of acoustic propagation within the pipeline. As a result, it achieves more realistic and stable localization, particularly in heterogeneous environments or when the acoustic path is affected by wall attenuation, fluid density variations, or pipeline geometry.

This alignment between physical behavior and mathematical modeling explains why the energy-weighted TDOA consistently provides better accuracy than the classical method.

➢ *3D Extension*

The weighted TDOA approach extends naturally to three-dimensional configurations [1]. Each sensor measures a time delay relative to the reference, forming spherical isochrones. The intersection of these spheres identifies the leak location. The weighted cost function enhances robustness when vertical positioning errors occur. Simulations confirm a 45% reduction in average 3D error compared to the classical model [2] [4].

To better visualize the spatial performance of the proposed model, Figure 6 presents a three-dimensional reconstruction of the leak localization process. The figure highlights the propagation of acoustic wavefronts within the pipeline and compares the estimated leak positions obtained by the classical and energy-weighted TDOA methods.

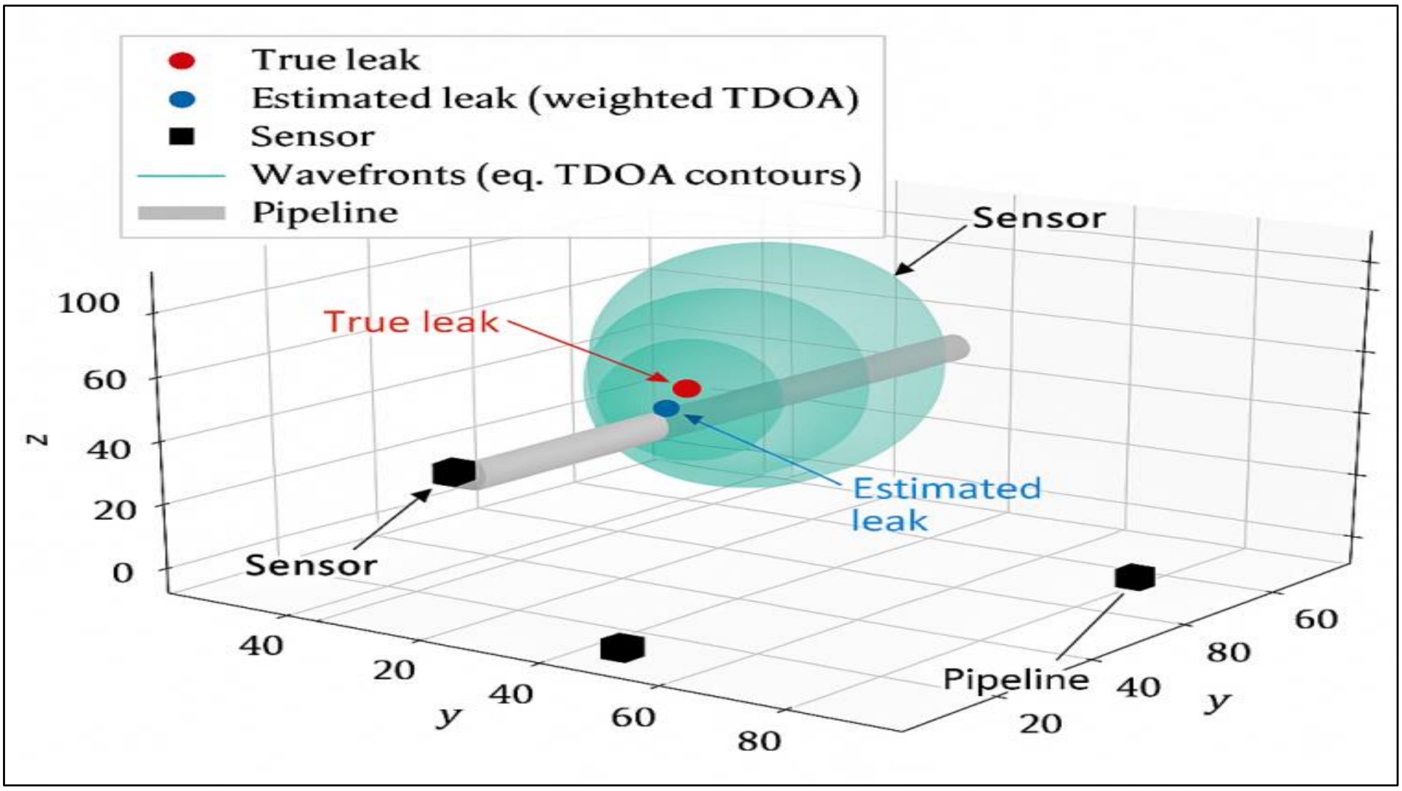

Fig 7 3D Reconstruction of Leak Localization with Four Sensors, Comparing Estimated Position to True Position

V. CONCLUSION

This research introduced an improved acoustic leak localization model based on an energy-weighted Time Difference of Arrival (TDOA) approach. The method addresses the limitations of classical TDOA models, which assume uniform sensor reliability, by integrating a physically meaningful weighting scheme derived from the acoustic energy received at each sensor.

Through mathematical modeling and numerical simulations, we demonstrated that incorporating sensor energy information significantly improves localization accuracy and robustness. The results highlight a reduction in root mean square error (RMSE) of more than 50% compared to the traditional TDOA formulation, particularly in noisy or asymmetric sensor configurations. This improvement stems from the fact that energy weighting naturally reflects acoustic

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propagation laws, where energy decays with distance and is affected by environmental attenuation.

Moreover, the proposed model preserves the simplicity of the TDOA framework while enhancing its adaptability to real-world conditions. The 3D extension further validated the model's scalability for complex industrial networks, confirming its potential for real-time pipeline monitoring applications.

In future work, we plan to perform experimental validation on a physical testbed and to incorporate frequency-dependent weighting to capture multi-band leak characteristics. Additionally, integrating the proposed algorithm into an autonomous sensor network could enable continuous, self-calibrating leak detection systems for smart infrastructures.

In summary, the energy-weighted TDOA model represents a practical step toward more accurate, resilient, and physics-informed leak localization, bridging the gap between theoretical acoustic models and their industrial applications.

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