Interval-valued Bipolar Vague Volterra Spaces

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Publication Date: 2025/06/24

Abstract: A novel idea of an inter-val bip.vag Volterra spaces and an inter-val bip.vag weakly Volterra spaces are presented and their properties are carefully considered and examined. Some of the interesting propositions based on the newly introduced set are obtainable.

Keywords: *Inter-Val Bip.Vag Sets, Inter-Val Bip.Vag Topology, Inter-Val Bip.Vag Volterra Spaces and an Inter-Val Bip.Vag Weakly Volterra Spaces.*

How to Cite: F. Prishka; Dr. L. Mariapresenti; (2025) Interval-valued Bipolar Vague Volterra Spaces. *International Journal of Innovative Science and Research Technology,* 10(6), 1681-1689. https://doi.org/10.38124/ijisrt/25jun1073

I. INTRODUCTION

Uncertain set was Presented by L.A.Zadeh [10] in 1965. The idea of fuzzy topology was announced by C.L.Chang [3] in 1968. The generalized cld.sets in general topology remained initialy by N.Levine [9] in 1970. K.Atanassov [2] in 1986 presented the perception of intuitionistic fuzzy sets. The notion of vague set theory was introduced by W.L.Gau and D.J.Buehrer [7] in 1993. D.Coker [6] in 1997 announced intuitionistic fuzzy topo.sps. Bipolar- valued fuzzy sets, which was presented by K.M.Lee [8] in 2000 is an leeway of fuzzy sets whose membership degree range is distended from the interval [0, 1] to [-1,1]. A innovative class of generalized bip.vag sets was announced by S.Cicily Flora and I.Arockiarani [4] in 2016. The persistence of this paper is to present and study the thought of inter-val bip.vag Volterra spaces and an inter-val bip.vag weakly Volterra spaces.

II. PRELIMINARIES

At this time in this paper the bip.vag topo.sps are meant by (Y, $BV_{\tau*}$). The bipolar vague and interval-valued meant by bip.vag and inter-val correspondingly. Also, the bip.vag interior, bip.vag closure of a bip.vag set B are represented by BVInt(B) and BVCl(B). The complement of a bip.vag set B is denoted by B^c and the empty set and whole sets are symbolized by $0_{\sim''}$ and $1_{\sim''}$ separately.

➢ *Definition 2.1:*

[8] Let Y be the universe. Then a bipolar valued fuzzy sets, B on Y is demarcated by positive membership function μ_B^+, that is μ_B^+: Y→ [0,1], and a negative membership function μ_B^-, that is μ_B^-: Y→ [-1,0]. For the own good of simplicity, we shall use the representation.

$$B = \{\langle y, \mu_B^+(y), \mu_B^-(y)\rangle : y \in Y\}.$$

➢ *Definition 2.2:*

[8] Let B and C be two bipolar valued fuzzy sets then their union, intersection and complement are clear as follows:

- $\mu_{B\cup C}^+$= max $\{\mu_B^+(y), \mu_C^+(y)\}$
- $\mu_{B\cup C}^-$= min $\{\mu_B^-(y), \mu_C^-(y)\}$
- $\mu_{B\cap C}^+$= min $\{\mu_B^+(y), \mu_C^+(y)\}$
- $\mu_{B\cap C}^-$= max $\{\mu_B^-(y), \mu_C^-(y)\}$
- $\mu_{B^c}^+$(y) = 1-$\mu_B^+(y)$ and $\mu_{B^c}^-$(y) = -1-$\mu_B^-(y)$ for all $y \in Y$.

➢ *Definition 2.3:*

[7] A vague set B in the universe of discourse W is a pair of (t_B, f_B)

where

$$t_B\text{: W}\rightarrow[0,1],\ f_A\text{:}$$

W→[0,1] are the mapping such that $t_B + f_B \leq 1$ for all $w \in W$. The function t_B and f_B are called true membership function and false membership function respectively. The interval $[t_B, 1 - f_B]$ is called the vague value of w in B, and denoted by $v_B(w)$, that is.

$$v_B(w) = [t_B(w), 1 - f(w)].$$

➢ *Definition 2.4:*

[7] Let B be a non-empty set and the vague set B and C in the form B = $\{\langle y, t_B(y), 1 - f_B(y)\rangle : y \in Y\}$, C = $\{\langle y, t_C(y), 1 - f_C(y)\rangle : y \in Y\}$. Then

ISSN No: 2456-2165 https://doi.org/10.38124/ijisrt/25jun1073

- B ⊆ C if and only if $t_B(y) \leq t_C(y)$ and $1 - f_B(y) \leq 1 - f_C(y)$
- B ∪ C $= \{\langle \max(t_B(y), t_C(y)), \max(1 - f_B(y), 1 - f_C(y))\rangle / y \in Y\}$.
- B ∩ C $= \{\langle \min(t_B(y), t_C(y)), \min(1 - f_B(y), 1 - f_C(y))\rangle / y \in Y\}$.
- $B^c = \{\langle y, f_B(y), 1 - t_B(y)\rangle : y \in Y\}$.

➤ *Definition 2.5:*

[1] Let Y be the universe of discourse. A bipolar-valued vague set B in Y is an object having the form B = $\{\langle y, [t_B^+(y), 1 - f_B^+(y)], [-1 - f_B^-(y), t_B^-(y)]\rangle : y \in Y\}$ where $[t_B^+, 1 - f_B^+]$: Y→[0,1] and $[-1 - f_B^-, t_B^-]$: Y→[-1,0] are the mapping such that $t_B^+(y) + f_B^+(y) \leq 1$ and $-1 \leq t_B^- + f_B^-$. The positive membership degree $[t_B^+(y), 1 - f_B^+(y)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set B and the negative membership degree $[-1 - f_B^-(y), t_B^-(y)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bip.vag set $v_B^+ = [t_B^+, 1 - f_B^+]$ and $v_B^- = [-1 - f_B^-, t_B^-]$.

➤ *Definition 2.6:*

[5] A bip.vag set B = $[v_B^+, v_B^-]$ of a set W with $v_B^+ = 0$ implies that $t_B^+ = 0$, $1 - f_B^+ = 0$ and $v_B^- = 0$ implies that $t_B^- = 0$, $-1 - f_B^- = 0$ for all x ∈ U is called zero bip.vag set and it is denoted by 0.

➤ *Definition 2.7:*

[5] A bip.vag set B = $[v_B^+, v_B^-]$ of a set W with $v_B^+ = 1$ implies that $t_B^+ = 1$, $1 - f_B^+ = 1$ and $v_B^- = -1$ implies that $t_B^- = -1$, $-1 - f_B^- = -1$ for all y ∈ W is called unit bip.vag set and it is denoted by 1.

➤ *Definition 2.8:*

[4] Let B = $\langle y, [t_B^+, 1 - f_B^+], [-1 - f_B^-, t_B^-]\rangle$ and C = $\langle x, [t_C^+, 1 - f_C^+], [-1 - f_C^-, t_C^-]\rangle$ be two bip.vag sets then their union, intersection and complement are defined as follows:

- B ∪ C $= \{\langle y, [t_{B\cup C}^+(y), 1 - f_{B\cup C}^+(y)], [-1 - f_{B\cup C}^-(y), t_{B\cup C}^-(y)]]\rangle / y \in Y\}$ where $t_{B\cup C}^+(y) = \max\{t_B^+(y), t_C^+(y)\}$, $t_{B\cup C}^-(y) = \min\{t_B^-(y), t_C^-(y)\}$ and $1 - f_{B\cup C}^+(y) = \max\{1 - f_B^+(y), 1 - f_C^+(y)\}$ $-1 - f_{B\cup C}^-(y) = \min\{-1 - f_B^-(y), -1 - f_C^-(y)\}$.
- B ∩ C $= \{\langle y, [t_{B\cap C}^+(y), 1 - f_{B\cap C}^+(y)], [-1 - f_{B\cap C}^-(y), t_{B\cap C}^-(y)]]\rangle / y \in Y\}$ where $t_{B\cap C}^+(y) = \min\{t_B^+(y), t_C^+(y)\}$, $t_{B\cap C}^-(y) = \max\{t_B^-(y), t_C^-(y)\}$ and $1 - f_{B\cap C}^+(y) = \min\{1 - f_B^+(y), 1 - f_C^+(y)\}$, $-1 - f_{B\cup C}^-(y) = \max\{-1 - f_B^-(y), -1 - f_C^-(y)\}$.
- $B^c = \{\langle y, [f_B^+(y), 1 - t_B^+(y)], [-1 - t_B^-(y), f_B^-(y)]\rangle / y \in Y\}$.

➤ *Definition 2.9:*

[4] Let B and C be two bip.vag sets defined over a universe of discourse Y. We say that B ⊆ C if and only if $t_B^+(y) \leq t_C^+(y)$, $1 - f_B^+(y) \leq 1 - f_C^+(y)$ and $t_B^-(y) \geq t_C^-(y)$, $-1 - f_B^-(y) \geq 1 - f_C^-(y)$ for all y ∈ Y.

➤ *Definition 2.10:*

[4] A bip.vag topology (BVT) on a non-empty set Y is a family $BV_{\tau*}$ of bip.vag set in Y satisfying the following axioms:

- $0_{\sim''}, 1_{\sim''} \in BV_{\tau*}$
- $T_1 \cap T_2 \in BV_{\tau*}$, for any $T_1, T_2 \in BV_{\tau*}$
- $\cup T_i \in BV_{\tau*}$, for any arbitrary family $\{T : T_i \in BV_{\tau*}, i \in I\}$.

In this case the pair (Y, $BV_{\tau*}$) is called a bip.vag topological space and any bip.vag set (BVS) in $BV_{\tau*}$ is known as bip.vag open set in Y. The complement B^c of a bip.vag open set (BVOS) B in a bip.vag topological space (X, $BV_{\tau*}$) is called a bip.vag closed set (BVCS) in Y.

➤ *Definition 2.11:*

[4] Let (Y, $BV_{\tau*}$) be a bip.vag topological space B = $\langle y, [t_B^+, 1 - f_B^+], [-1 - f_B^-, t_B^-]\rangle$ be a bip.vag set in Y. Then the bip.vag interior and bip.vag closure of B are defined by, BVInt(B) = ∪ {T: T is a bip.vag open set in Y and T ⊆ B}, BVCl(B) = ∩ {J: J is a bip.vag closed set in Y and B⊆ J}. Note that BVCl(B) is a bip.vag closed set and BVInt(B) is a bip.vag open set in X. Further,

- B is a bip.vag closed set in Y if and only if BVCl(B) = B,
- B is a bip.vag open set in Y if and only if BVInt(B) = B.

➤ *Definition 2.12:*

[4] Let (Y, $BV_{\tau*}$) be a bip.vag topological space. A bip.vag set B in (Y, $BV_{\tau*}$) is said to be a generalized bip.vag closed set if BVCl(B) ⊆ T whenever B⊆ T and T is bip.vag open. The complement of a generalized bip.vag closed set is generalized bip.vag open set.

➤ *Definition 2.13:*

[4] Let (Y, $BV_{\tau*}$) be a bip.vag topological space and B be a bip.vag set in Y. Then the generalized bip.vag closure and generalized bip.vag interior of B are defined by, GBVCl(B) = ∩ {T: T is a generalized bip.vag closed set in Y and B⊆ T}, GBInt(B) = ∪ {T: T is a generalized bip.vag open set in Y and AB ⊇ T}.

III. INTER-VAL BIP. VAG VOLTERRA SPACES

➤ *Definition 3.1:*

An inter-val bip.vag sets $\widetilde{B^{BV}}$ over a universe of discourse Y is defined as an object of the form $\widetilde{B^{BV}} = \left\{\langle y_i, \left[\left[t_{\widetilde{B^{BV}}}^+(x_i), 1 - f_{\widetilde{B^{BV}}}^+(y_i)\right], \left[-1 - f_{\widetilde{B^{BV}}}^-(y_i), t_{\widetilde{B^{BV}}}^-(y_i)\right]\right]\rangle : y_i \in Y\right\}$ where $\left[t_{\widetilde{B^{BV}}}^+, 1 - f_{\widetilde{B^{BV}}}^+\right]$: Y→D[0,1] and $[-1 - f_{\widetilde{B^{BV}}}^-, t_{\widetilde{B^{BV}}}^-]$: Y→D[-1,0] are the mapping such that $t_{\widetilde{B^{BV}}}^+ + f_{\widetilde{B^{BV}}}^+ \leq 1$ and $-1 \leq t_{\widetilde{B^{BV}}}^- + f_{\widetilde{B^{BV}}}^-$. The positive membership degree $\left[t_{\widetilde{B^{BV}}}^+(y_i), 1 - f_{\widetilde{B^{BV}}}^+(y_i)\right]$ denotes the satisfaction region of an element y_i to the property corresponding to an inter-val bipolar-valued set $\widetilde{B^{BV}}$ and the negative membership degree $[-1 - f_{\widetilde{B^{BV}}}^-(y_i), t_{\widetilde{B^{BV}}}^-(y_i)]$ denotes the satisfaction region of y_i to

ISSN No: 2456-2165 https://doi.org/10.38124/ijisrt/25jun1073

some implicit counter property of $\widetilde{B^{BV}}$. For a sake of simplicity, we shall use the notion of inter-val bip.vag set $\nu^{+}_{\widetilde{B^{BV}}} = \left[t^{+}_{\widetilde{B^{BV}}}, 1 - f^{+}_{\widetilde{B^{BV}}}\right]$ and $\nu^{-}_{\widetilde{B^{BV}}} = [-1 - f^{-}_{\widetilde{B^{BV}}}, t^{-}_{\widetilde{B^{BV}}}]$.

➢ *Definition 3.2:*

An inter-val bip.vag topology (IBVT) on Y is a family $IBV_{\tau*}$ of interval- valued bip.vag sets (IBVS) in Y satisfying the following axioms:

- $0_{\sim''}, 1_{\sim''} \in IBV_{\tau*}$
- $T_1 \cap T_2$ for any $T_1, T_2 \in IBV_{\tau*}$
- $\cup T_i \in IBV_{\tau*}$, for any family $\{T_i / i \in J\} \subseteq IBV_{\tau*}$

In this case the pair (Y, $IBV_{\tau*}$) is called an inter-val bip.vag topological space (IBVTS) and any inter-val bip.vag set (IBVS) in $IBV_{\tau*}$ is known as an inter-val bip.vag open set (IBVOS) in Y. The complement of an inter-val bip.vag open set A^c in an inter-val bip.vag topological space (Y, $IBV_{\tau*}$) is called an inter-val bip.vag closed set (IBVCS) in Y.

➢ *Definition 3.3:*

Let Y = {a, b}, B = ⟨y, [[0.3, 0.4][0.5, 0.6],[-0.5,-0.6] [-0.3,-0.4]],[[0.4,0.5][0.7,0.8] ,[-0.7,-0.8][-0.4,-0.5]]⟩ and C = ⟨y, [[0.4, 0.5][0.6, 0.7] ,[-0.6,-0.7][-0.4,-0.5]], [[0.5, 0.6] [0.7, 0.8], [-0.7,-0.8][-0.5,-0.6]]⟩. Then the family $\tau * = \{0_{\sim''}$, A, B, $1_{\sim''}\}$ of an inter-val bip.vag sets in Y is an inter-val bip.vag topology on Y.

➢ *Definition 3.4:*

Let B = $\langle y, [[t_B^{+L}(y), t_B^{+U}(y)][1 - f_B^{+L}(y), 1 - f_B^{+U}(y)], [-1 - f_B^{-L}(y), -1 - f_B^{-U}(y)][t_B^{-L}(y), t_B^{-U}(y)]]\rangle$ and B = $\langle y, [[t_C^{+L}(y), t_C^{+U}(y)][1 - f_C^{+L}(y), 1 - f_C^{+U}(y)], [-1 - f_C^{-L}(y), -1 - f_C^{-U}(y)][t_C^{-L}(y), t_C^{-U}(y)]]\rangle$ be two inter-val bip.vag sets then their union, intersection and complement are defined as follows:

- B ∪ C = $\{\langle y, [[t^{+L}_{B\cup C}(y), t^{+U}_{B\cup C}(y)][1 - f^{+L}_{B\cup C}(y), 1 - f^{+U}_{B\cup C}(y)], [-1 - f^{-L}_{B\cup C}(y), [-1 - f^{-U}_{B\cup C}(y)][t^{-L}_{B\cup C}(y), t^{-U}_{B\cup C}(y)]]\rangle / y \in Y\}$ where $t^{+L}_{B\cup C}(y) = \max\ \{t_B^{+L}(y), t_B^{+L}(y)\}$, $t^{+U}_{B\cup C}(y) = \max\ \{t_B^{+L}(y), t_C^{+U}(y)\}$, $t^{-L}_{B\cup C}(y) = \min\ \{t_B^{-L}(y), t_C^{-L}(y)\}$ and $t^{-U}_{B\cup C}(y) = \min\ \{t_B^{-U}(y), t_C^{-U}(y)\}$ $1 - f^{+L}_{B\cup C}(y) = \max\ \{1 - f_B^{+L}(y), 1 - f_C^{+L}(y)\}$, $1 - f^{+U}_{B\cup C}(y) = \max\ \{1 - f_B^{+U}(y), 1 - f_C^{+U}(y)\}$, $-1 - f^{-L}_{B\cup C}(y) = \min\ \{-1 - f_B^{-L}(y), -1 - f_C^{-L}(y)\}$ and $-1 - f^{-U}_{B\cup C}(y) = \min\ \{-1 - f_B^{-U}(y), -1 - f_C^{-U}(y)\}$..
- B ∩C= $\{\langle y, [[t^{+L}_{B\cap C}(y), t^{+U}_{B\cap C}(y)][1 - f^{+L}_{B\cap C}(y), 1 - f^{+U}_{B\cap C}(y)], [-1 - f^{-L}_{B\cap C}(y), -1 - f^{-U}_{B\cap C}(y)][t^{-L}_{B\cap C}(y), t^{-U}_{B\cap C}(y)]]\rangle / y \in Y\}$ where $t^{+L}_{B\cap C}(y) = \min\ \{t_B^{+L}(y), t_C^{+L}(y)\}$, $t^{+U}_{B\cap C}(y) = \min\ \{t_B^{+U}(y), t_C^{+U}(y)\}$, $t^{-L}_{B\cap C}(y) = \max\ \{t_B^{-L}(y), t_C^{-L}(y)\}$ and $t^{-U}_{B\cap C}(y) = \max\ \{t_B^{-U}(y), t_C^{-U}(y)\}$ $1 - f^{+L}_{B\cap C}(y) = \min\ \{1 - f_B^{+L}(y), 1 - f_C^{+L}(y)\}$, $1 - f^{+U}_{B\cap C}(y) = \min\ \{1 - f_B^{+U}(y), 1 - f_C^{+U}(y)\}$, $-1 - f^{-L}_{B\cup C}(y) = \max\ \{-1 - f_B^{-L}(y), -1 - f_C^{-L}(y)\}$ and $-1 - f^{-U}_{B\cup C}(y) = \max\ \{-1 - f_B^{-U}(y), -1 - f_C^{-U}(y)\}$.
- B^c = $\{[\langle y, [[f_B^{+L}(y), f_B^{+U}(y)][1 - t_B^{+L}(y), 1 - t_B^{+U}(y)][-1 - t_B^{-L}(y), -1 - t_B^{-U}(y)][f_B^{-L}(y), f_B^{-U}(y)]]\rangle] / y \in Y\}]$.

➢ *Proposition 3.5:*

Let $(Y, BV_{\tau*})$ be an inter-val bip.vag topological space and let B, C belongs to an inter-val bip.vag set in Y. Then the following properties hold:

- $0_{\sim''} \subseteq B \subseteq 1_{\sim''}$
- B ∪ C = C ∪ B; B ∩ C = C ∩ B
- B, C ⊂ B ∪ C; B ∩ C ⊂ B, C
- B ∩ ($\cup_i C_i$) = $\cup_i$(B ∩ C_i) and B ∪ ($\cap_i C_i$) = $\cap_i$(B ∪ C_i)
- ${0_{\sim''}}^c = 1_{\sim''}$; ${1_{\sim''}}^c = 0_{\sim''}$
- $(B^c)^c = B$
- $(\cup_i B_i)^c = \cap_i B_i^c$ and $(\cap_i B_i)^c = \cup_i B_i^c$

✓ Proof: Obvious

➢ *Definition 3.6:*

Let $(Y, IBV_{\tau*})$ be an inter-val bip.vag topological space and B = $\langle y, [[t_B^{+L}(y), t_B^{+U}(y)][1 - f_B^{+L}(y), 1 - f_B^{+U}(y)], [-1 - f_B^{-L}(y), -1 - f_B^{-U}(y)][t_B^{-L}(y), t_B^{-U}(y)]]\rangle$ be an interval--valued bip.vag set in Y. Then the inter-val bip.vag interior and an inter-val bip.vag closure defined by, IBVInt(B) = ∪ {T: T is an inter-val bip.vag open set in Y and T ⊆ B}, IBVCl(B) = ∩ {J: J is an inter-val bip.vag closed set in Y and A⊆ K}.

Note that for any inter-val bip.vag set B in $(Y, IBV_{\tau*})$, we have IBVCl(B^c) = (IBVInt(B))^c and IBVInt(B^c) = (IBVCl(B))^c and IBVCl(B) is an inter-val bip.vag closed set and IBVInt(B) is an inter-val bip.vag open set in Y. Further we have, if B is an inter-val bip.vag closed set in Y, then IBVCl(B) = B and if B is an inter-val bip.vag open set in Y, then IBVInt(B) = B.

➢ *Example 3.7:*

Let Y = {b,c} and $\tau * = \{0_{\sim''}$, B, C, $1_{\sim''}\}$ be an inter-val vague topology on Y, where B = ⟨y,[[0.2, 0.3][0.4,0.5],[-0.4,-0.5][-0.2,-0.3]],[[0.3,0.5][0.6, 0.7],[-0.6,-0.7][-0.3,-0.5]]⟩, C = ⟨y, [[0.3, 0.4][0.5,0.6],[-0.5,-0.6][-0.3,-0.4]], [[0.4,0.5][0.7,0.8],[-0.7,-0.8][-0.4,-0.5]]⟩. Here the op.sets are $0_{\sim''}$, B, C and $1_{\sim''}$. If R = ⟨y, [[0.2, 0.5] [0.6, 0.8],[-0.6,-0.8][-0.2,-0.5]],[[0.3, 0.4] [0.5, 0.3],[-0.5,-0.3][-0.3,-0.4]]⟩ is an inter-val bip.vag set then, IBVCl(Q) = ∩ {J: J is an inter-val bip.vag closed set in Y and Q ⊆ J} = $1_{\sim''}$ IBVInt(Q) = ∪ {E: E is an inter-val bip.vag open set in Y and Q ⊇ E} = $0_{\sim''}$.

➢ *Proposition 3.8:*

Let $(Y, IBV_{\tau*})$ be an inter-val bip.vag set and let B, C belongs to an inter-val bip.vag set in Y. Then the following properties hold:

- BVInt(B) ⊂ B
- B⊂ C implies BVInt(B) ⊂ BVInt(C)
- B is a bip.vag open set if and only if BVInt(B) = B
- BVInt(BVInt(B)) = BVInt(B)
- BVInt($0_{\sim''}$) = $0_{\sim''}$, BVInt($1_{\sim''}$) =$1_{\sim''}$
- BVInt(B ∩ C) = BVInt(B) ∩ BVInt(C)

✓ Proof: The proof is obvious

ISSN No: 2456-2165 https://doi.org/10.38124/ijisrt/25jun1073

➢ *Proposition 3.9:*

Let $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag set and let B, C belongs to an inter-val bip.vag set in Y. Then the following properties hold:

- B ⊂ BVCl(B)
- B ⊂ C implies BVCl(B) ⊂ BVCl(C)
- B is a bip.vag closed set if and only if BVCl(B) = B
- BVCl(BVCl(B)) = BVCl(B)
- BVCl(0~") = 0~", BVCl(1~") =1~"
- BVCl(B ∪ C) = BVCl(B) ∪ BVCl(C)

✓ Proof: The proof is obvious.

➢ *Definition 3.10:*

An inter-val bip.vag set B in an inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag dense if there exists no inter-val bip.vag closed set C in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ such that B ⊂ C ⊂ 1~".

➢ *Definition 3.11:*

An inter-val bip.vag set B in an inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag nowhere dense set if there exists no inter-val bip.vag open set C in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ such that C ⊂ IBVCl(B). That is IBVInt (IBVCl(B)) = 0~".

➢ *Proposition 3.12:*

If B is an inter-val bip.vag dense and inter-val bip.vag open set in an inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, then $\mathrm{B^c}$ is an interval- valued bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Let B be an inter-val bip.vag dense and inter-val bip.vag open set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then we have IBVCl(B) = 1~" and IBVInt(B) = B. Now we have to show that IBVInt(IBVCl($\mathrm{B^c}$)) = 0~". Let IBVCl($\mathrm{B^c}$) = (IBVInt(B))^c = $\mathrm{B^c}$ which implies that IBVInt(IBVCl($\mathrm{B^c}$)) = IBVInt($\mathrm{B^c}$) = (IBVCl(B))^c = 1~"^{c} = 0~". That is, IBVInt(IBVCl($\mathrm{B^c}$)) = 0~". Hence $\mathrm{B^c}$ is an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

➢ *Proposition 3.13:*

Let B be an inter-val bip.vag set. If B is an inter-val bip.vag closed set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ with IBVInt(B) = 0~", then B is an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Let B be an inter-val bip.vag closed set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then IBVCl(B) = B. Now IBVInt(IBVCl(B)) = IBVInt(B) = 0~" and hence B is an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

➢ *Proposition 3.14:*

Let B be an inter-val bip.vag closed set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, then B is an inter-val bip.vag nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$ if and only if IBVInt(B) = 0~".

- *Proof:*

Let B be an inter-val bip.vag closed set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, with IBVInt(B) = 0~". Then by Proposition 3.13, B is an inter-val bip.vag nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$. Conversely, let B be an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then IBVInt(IBVCl(B)) = 0~", which implies that IBVInt(B) = 0~". Since B is an inter-val bip.vag closed, IBVCl(B) = B.

➢ *Proposition 3.15:*

If B is an inter-val bip.vag nowhere dense set in an inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, then IBVCl(B) is also an inter-val bip.vag nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Let B be an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then, IBVInt(IBVCl(B)) = 0~". Now IBVCl(IBVCl(B)) = IBVCl(B). Hence IBVInt(IBVCl(IBVCl(B))) = IBVInt(IBVCl(B)) = 0~". Therefore IBVCl(B) is also an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

➢ *Definition 3.16:*

An inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an interval- valued bip.vag first category set if B = $\bigcap_{i=1}^{\infty} B_i$, where B_i's are inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Any other inter-val bip.vag set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is said to be an inter-val bip.vag second category.

➢ *Definition 3.17:*

An inter-val bip.vag set B in an inter-val vague topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag H_δ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ if B = $\bigcup_{i=1}^{\infty} B_i$ where $B_i \in \mathrm{IB}V_{\tau*}$ for i ∈ I.

➢ *Definition 3.18:*

An inter-val bip.vag set B in an inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag J_σ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ if B = $\bigcup_{i=1}^{\infty} B_i$ where $B_i^c \in \mathrm{IB}V_{\tau*}$ for i ∈ I.

➢ *Definition 3.19:*

An inter-val bip.vag topological space $(Y, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag Volterra space if IBVCl($\bigcap_{i=1}^{N} B_i$) = 1~", where B_i's are inter-val bip.vag dense and inter-val bip.vag H_δ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

➢ *Example 3.20:*

Let Y = {b, c}. Define an inter-val bip.vag sets B, C, D and E as follows, B = ⟨y,[[0.4,0.6][0.8,0.9],[-0.8,-0.9][-0.4,-0.6]],[[0.2,0.3][0.6,0.7],[-0.6,-0.7][-0.2,-0.3]]⟩,C = ⟨y, [[0.4, 0.5][0.7,0.8],[-0.7,-0.8][-0.4,-0.5]],[[0.3,0.4][0.6, 0.8], [-0.6,-0.8][-0.3,-0.4]] ⟩, D = ⟨y, [[0.4,0.5][0.7,0.8],[-0.7,-0.8][-0.4,-0.5]],[[0.2,0.3][0.6, 0.7] ,[-0.6,-0.7][-0.2,-0.3]] ⟩ and E = ⟨y,[[0.4,0.6][0.8,0.9],[-0.8,-0.9][-0.4,-0.6]],[[0.3,0.4][0.6, 0.8] ,[-0.6,-0.8][-0.3,-0.4]]⟩. Clearly $\tau*$ = {0~", B, C, D, E, 1~"} is an inter-val bip.vag topology on Y, thus $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag topological space. Let F = {B∩ C ∩ D }, G = {B∩ C ∩ E} and H = {B∩ C ∩ D ∩ E} where F, G and H are inter-val bip.vag H_δ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Also, we have IBVCl(F) = 1~", IBVCl(G) =1~"and IBVCl(H) = 1~".

ISSN No: 2456-2165 https://doi.org/10.38124/ijisrt/25jun1073

Also, we have IBVCl(F ∩ G ∩ H) = $1_{\sim''}$. Therefore, $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Volterra space.

- *Proposition 3.21:*

An inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Volterra space, if and only if IBVInt($\cup_{i=1}^{N} B_i^c$) = $0_{\sim''}$, where B_i's are inter-val bip.vag dense and an inter-val bip.vag H_δ-sets in $(Y, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Let $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag Volterra space and B_i's are inter-val bip.vag dense and inter-val bip.vag H_δ-sets in $(\mathrm{X}, \mathrm{IB}V_{\tau*})$. Then we have IBVCl($\cap_{i=1}^{N} B_i^c$) = $1_{\sim''}$. Now IBVInt($\cup_{i=1}^{N} B_i^c$) = (IBVCl($\cap_{i=1}^{N} B_i$))^c = $0_{\sim''}$. Conversely, let IBVInt($\cup_{i=1}^{N} B_i^c$) = $0_{\sim''}$, where B_i's are inter-val bip.vag dense and inter-val bip.vag J_δ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then (IBVInt($\cap_{i=1}^{N} B_i$)^c) = $0_{\sim''}$ implies IBVCl($\cap_{i=1}^{N} B_i$) = $1_{\sim''}$. Hence $(Y, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Volterra space.

- *Proposition 3.22:*

Let $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag topological space. If IBVInt($\cup_{i=1}^{N} B_i$) = $0_{\sim''}$, where B_i's are inter-val bip.vag nowhere dense and an inter-val bip.vag J_σ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, then $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Volterra space.

- *Proof:*

Let IBVInt($\cup_{i=1}^{N} B_i$) = $0_{\sim''}$, this implies that (IBVInt($\cup_{i=1}^{N} B_i$))^c = $1_{\sim''}$. That is IBVCl($\cap_{i=1}^{N} B_i$) = $1_{\sim''}$ where B_i's are inter-val bip.vag nowhere dense and an inter-val bip.vag J_σ-sets implies that B_i^c's are inter-val bip.vag dense and an inter-val bip.vag H_δ-sets in $(Y, \mathrm{IB}V_{\tau*})$ and also IBVCl($\cap_{i=1}^{N} B_i^c$) = $1_{\sim''}$. Then $(Y, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Volterra space.

- *Definition 3.23:*

Let B be a bip.vag first category set in an inter-val bip.vag topological space $(Y, \mathrm{IB}V_{\tau*})$. Then B^c is called an inter-val bip.vag residual sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Definition 3.24:*

An inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag ε_r- Volterra space if IBVCl($\cap_{i=1}^{N} B_i$) = $1_{\sim''}$, where B_i's are inter-val bip.vag dense and an inter-val bip.vag residual sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proposition 3.25:*

Let $(Y, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag ϵ_r- Volterra space, then IBVInt($\cup_{i=1}^{N} B_i$) = $0_{\sim''}$, where B_i's are inter-val bip.vag first category sets such that IBVInt(B_i) = $0_{\sim''}$ in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Let B_i's (i = 1 to N) be an inter-val bip.vag first category sets such that IBVInt(B_i) = $0_{\sim''}$ in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then B_i^c is an inter-val bip.vag residual sets such that IBVCl(B_i^c) = $1_{\sim''}$ in $(Y, \mathrm{IB}V_{\tau*})$. That is B_i^c is an inter-val bip.vag residual sets and an inter-val bip.vag dense sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Since $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag ϵ_r- Volterra space, IBVCl($\cap_{i=1}^{N} B_i^c$) = $1_{\sim''}$ and hence we have IBVInt($\cup_{i=1}^{N} B_i$) = $0_{\sim''}$ where B_i's are inter-val bip.vag first category sets such that IBVInt(B_i) = $0_{\sim''}$ in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proposition 3.26:*

If each an inter-val bip.vag nowhere dense set is an inter-val bip.vag closed set in an inter-val bip.vag Volterra space in $(Y, \mathrm{IB}V_{\tau*})$, then $(Y, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag ϵ_r- Volterra space.

- *Proof:*

Let B_i's (i=1 to N) be an inter-val bip.vag nowhere dense set and an inter-val bip.vag residual set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Since B_i's are inter-val bip.vag residual sets, B_i^c's are inter-val bip.vag first category set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Now B_i^c = $\cup_{i=1}^{\infty} C_{ij}$, where C_{ij}'s are an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. By hypothesis, an inter-val bip.vag nowhere dense set C_{ij}'s are inter-val bip.vag cld.sets and hence B_i^c's are inter-val bip.vag J_σ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. This implies that B_i^c's are inter-val bip.vag H_δ-sets in $(Y, \mathrm{IB}V_{\tau*})$. Hence B_i^c's are inter-val bip.vag dense and an inter-val bip.vag H_δ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Since $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Volterra space, IBVCl($\cap_{i=1}^{N} B_i$) = $1_{\sim''}$. Hence IBVCl($\cap_{i=1}^{N} B_i$) = $1_{\sim''}$, where B_i's are inter-val bip.vag dense and an inter-val bip.vag residual sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ implies that $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag ϵ_r- Volterra space.

- *Definition 3.27:*

Let $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag topological space. Then $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag Baire space if IBVInt($\cup_{i=1}^{\infty} B_i$) = $0_{\sim''}$where B_i's are inter-val bip.vag nowhere dense sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proposition 3.28:*

Let $(Y, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag topological space. Then the following are equivalent:

- $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Baire space.
- IBVInt(B) = $0_{\sim''}$, for every inter-val bip.vag first category set B in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.
- IBVCl(B) = $1_{\sim''}$, for every inter-val bip.vag residual set B in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proof:*

- ⇒ (ii) Let B be an inter-val bipolar first category set in $(Y, \mathrm{IB}V_{\tau*})$. Then B = $\cup_{i=1}^{\infty} B_i$, where B_i's are inter-val bip.vag nowhere dense sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Now IBVInt(B) = IBV($\cup_{i=1}^{\infty} B_i$) = $0_{\sim''}$, since $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Baire space. Therefore IBVInt(B) = $0_{\sim''}$.
- ⇒ (iii) Let B be an inter-val bip.vag residual set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then C^c is an inter-val bip.vag first category set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. By hypothesis IBVInt(C^c) = $0_{\sim''}$ which implies that (IBVCl(B))^c = $0_{\sim''}$. Hence IBVCl(B) = $1_{\sim''}$.
- ⇒ (i) Let B be an inter-val bip.vag first category set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then B = $\cup_{i=1}^{\infty} B_i$, where B_i's are inter-val bip.vag nowhere dense sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Now B is an inter-val bip.vag first category set implies that B^c is an inter-val bip.vag residual set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. By hypothesis, we have IBVCl(B^c) = $1_{\sim''}$ which implies that (IBVInt(B) = $1_{\sim''}$)^c. Hence IBVInt(B) = $0_{\sim''}$. IBVInt($\cup_{i=1}^{\infty} B_i$) = $0_{\sim''}$, where B_i's are inter-val bip.vag nowhere dense sets in $(Y, \mathrm{IB}V_{\tau*})$. Hence $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Baire space.

ISSN No: 2456-2165 https://doi.org/10.38124/ijisrt/25jun1073

- *Proposition 3.29:*

If $\bigcup_{i=1}^{\infty} B_i$ is the inter-val bip.vag sets, B_i's are inter-val bip.vag nowhere dense sets in an inter-val bip.vag Baire space in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$,then $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag ϵ_r-Volterra space.

- *Proof:*

Let B be an inter-val bip.vag Baire space and B_i's (i = 1 to N) be an inter-val bip.vag dense and inter-val bip.vag residual sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Since B_i's are inter-val bip.vag residual set, B_i^c's are inter-val bip.vag first category sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Now $B_i^c = \bigcup_{i=1}^{\infty} C_{ij}$, where C_{ij}'s are inter-val bip.vag nowhere dense sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. By hypothesis B_i^c is an inter-val bip.vag nowhere dense sets in $(Y, \mathrm{IB}V_{\tau*})$. Let C_α's be an inter-val bip.vag nowhere dense sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ in which N inter-val bip.vag nowhere dense sets be B_i^c. Since $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Baire space, IBVInt$(\bigcup_{i=1}^{\infty} C_\alpha)$ = $0_{\sim''}$. But IBVInt$(\bigcup_{i=1}^{N} B_i)$ ≤ IBVInt$(\bigcup_{\alpha=1}^{\infty} C_\alpha)$ and IBVInt$(\bigcup_{\alpha=1}^{\infty} C_\alpha)$ = $0_{\sim''}$. Then IBVInt$(\bigcup_{i=1}^{\infty} B_i^c)$ = $0_{\sim''}$. Therefore IBVCl$(\bigcap_{i=1}^{N} B_i)$ = $1_{\sim''}$where B_i's (i=1 to N) are inter-val bip.vag dense set and inter-val bip.vag residual set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Therefore $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag ϵ_r- Volterra space.

- *Proposition 3.30:*

If an inter-val bip.vag ϵ_r- Volterra space is an inter-val bip.vag Baire space, then IBVCl$(\bigcap_{i=1}^{N} B_i)$ = $1_{\sim''}$, where B_i's (i=1to N) are inter-val bip.vag residual sets in $(Y, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Let B_i's (i=1to N) are inter-val bip.vag residual set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Since $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Baire space, (By Proposition 3.28) IBVCl(B_i) = $1_{\sim''}$ ∀ i. Then B_i's are inter-val bip.vag dense and an inter-val bip.vag residual sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Since $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ in an inter-val bip.vag ϵ_r-Volterra space, IBVCl$(\bigcap_{i=1}^{N} B_i)$ = $1_{\sim''}$. Therefore, IBVCl$(\bigcap_{i=1}^{N} B_i)$ = $1_{\sim''}$ where B_i's (i=1 to N) are inter-val bip.vag residual sets in $(Y, \mathrm{IB}V_{\tau*})$.

- *Proposition 3.31:*

If $\bigcap_{i=1}^{N} B_i$, where B_i's (i = 1to N) are inter-val bip.vag residual sets in an inter-val bip.vag Baire space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, then $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag ϵ_r- Volterra space.

- *Proof:*

Let B_i's (i = 1to N) be an inter-val bip.vag dense and an inter-val bip.vag residual sets in $(Y, \mathrm{IB}V_{\tau*})$. Then by hypothesis, $\bigcap_{i=1}^{N} B_i$ is an inter-val bip.vag residual sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Since $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Baire space, therefore (By Proposition 3.28) IBVCl$(\bigcap_{i=1}^{N} B_i)$ = $1_{\sim''}$. Hence IBVCl$(\bigcap_{i=1}^{N} B_i)$ = $1_{\sim''}$, where A_i's are inter-val bip.vag dense set and an inter-val bip.vag residual set $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Therefore $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag ϵ_r-Volterra space.

IV. INTER-VAL BIP. VAG WEAKLY VOLTERRA SPACES

- *Definition 4.1:*

Let $(Y, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag topological space. An inter-val bip.vag set B in $(Y, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag σ-nowhere dense set if B is an inter-val bip.vag J_σ set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ such that IBVInt(B) = $0_{\sim''}$.

- *Definition 4.2:*

Let $(Y, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag topological space. Then $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag σ-Baire space if IBVInt$(\bigcup_{i=1}^{\infty} B_i)$ = $0_{\sim''}$, where B_i's are inter-val bip.vag σ-nowhere dense sets.

- *Proposition 4.3:*

In an inter-val bip.vag topological space $(Y, \mathrm{IB}V_{\tau*})$. An inter-val bip.vag set B is an inter-val bip.vag σ-nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$ if and only if B^c is an inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Let B be an inter-val bip.vag σ-nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then B = $\bigcup_{i=1}^{\infty} B_i$ where ${B_i}^c \in \mathrm{IB}V_{\tau*}$, for i ∈ I and IBVInt(B) = $0_{\sim''}$ Then (IBVInt(B))^c = $(0_{\sim''})^c$ = $1_{\sim''}$ implies that IBVCl(B^c) =$1_{\sim''}$. Also, B^c = $(\bigcup_{i=1}^{\infty} B_i)^c = \bigcap_{i=1}^{\infty} {B_i}^c$ where ${B_i}^c \in \mathrm{IB}V_{\tau*}$, for i∈ I. Hence, we have B^c is an inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, \mathrm{IB}V_{\tau*})$.

Conversely, let B be an inter-val bip.vag dense and an inter-val H_δ-set in $(Y, \mathrm{IB}V_{\tau*})$. Then B = $\bigcap_{i=1}^{\infty} B_i$ where $B_i \in \mathrm{IB}V_{\tau*}$, for i ∈ I. Now B^c = $(\bigcap_{i=1}^{\infty} B_i)^c = \bigcup_{i=1}^{\infty} {B_i}^c$. Hence B^c is an inter-val bip.vag J_σ set in $(Y, \mathrm{IB}V_{\tau*})$ and since B is an inter-val bip.vag dense set we have IBVInt(B^c) = $0_{\sim''}$. Therefore, B^c is an inter-val bip.vag σ-nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$.

- *Proposition 4.4:*

If B is an inter-val bip.vag dense set in $(Y, \mathrm{IB}V_{\tau*})$ such that C ⊆ B^c, where C is an inter-val bip.vag J_σ set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, then C is an inter-val bip.vag σ-nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Let B be an inter-val bip.vag dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Now C ⊆ B^c implies that IBVInt(C) ⊆ IBVInt(B^c) = (IBVCl(B))^c = $0_{\sim''}$ and hence IBVInt(C) = $0_{\sim''}$. Therefore, C is an inter-val bip.vag σ-nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proposition 4.5:*

If B is an inter-val bip.vag J_σ set and an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, then B is an inter-val bip.vag σ-nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$.

- *Proof:*

Now B ⊆ IBVCl(B) for any inter-val bip.vag set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then, IBVInt(B) ⊆ IBVInt(IBV(Cl(B))). Since B is an inter-val bip.vag nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$, IBVInt(IBVCl(B)) = $0_{\sim''}$ and hence IBVInt(B) = $0_{\sim''}$ and B

ISSN No: 2456-2165 https://doi.org/10.38124/ijisrt/25jun1073

is an inter-val bip.vag J_σ set implies that B is an inter-val bip.vag σ-nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Proposition 4.6:*

If B_i's (i =1 to N) are inter-val bip.vag σ-nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$ and IBVInt$(\bigcup_{i=1}^{N} B_i)$ = $0_{\sim''}$, then $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Volterra space.

- *Proof:*

Let B_i's (i=1 to N) are inter-val bip.vag σ-nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$ then B_i's are inter-val bip.vag J_σ set with IBVInt(B_i) = $0_{\sim''}$.Now (IBVInt(B_i))^c = $1_{\sim''}$. Then, we have IBVCl$({B_i}^c)$ =$1_{\sim''}$. That is, ${B_i}^c$'s are inter-val bip.vag dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Since B_i's are inter-val bip.vag J_σ set, ${B_i}^c$are inter-val bip.vag H_δ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Hence ${B_i}^c$are inter-val bip.vag dense and an inter-val bip.vag H_δ-sets in $(\mathrm{X}, \mathrm{IB}V_{\tau*})$. Now IBVCl$(\bigcap_{i=1}^{N} {B_i}^c)$ = (IBVInt$(\bigcup_{i=1}^{N} B_i)$)^c = $0_{\sim''}{}^c$ = $1_{\sim''}$. Hence $(Y, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag Volterra space.

- *Definition 4.7:*

An inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag weakly Volterra space if IBVCl$(\bigcap_{i=1}^{N} B_i)$ $\neq$ $0_{\sim''}$, where B_i's are inter-val bip.vag dense and an inter-val bip.vag H_δ-sets in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$.

- *Example 4.8:*

Let Y = {b, c}. The inter-val bip.vag sets are defined as follows B = ⟨y, [[0.3, 0.4][0.6,0.8],[-0.6,-0.8][-0.3,-0.4]],[[0.3,0.4][0.6, 0.9],[-0.6,-0.9][-0.3,-0.4]]⟩, C = ⟨y, [[0.3, 0.5][0.6,0.7],[-0.6,-0.7][-0.3,-0.5]],[[0.2,0.3][0.4, 0.5],[-0.4,-0.5][-0.2,-0.3]]⟩, D = ⟨y, [[0.3, 0.4][0.6,0.7],[-0.6,-0.7][-0.3,-0.4]],[[0.1,0.3][0.4, 0.5],[-0.4,-0.5][-0.1,-0.3]]⟩ and E = ⟨y, [[0.3, 0.5][0.6,0.8],[-0.6,-0.8][-0.3,-0.5]],[[0.3,0.4][0.6, 0.9],[-0.6,-0.9][-0.3,-0.4]]⟩. Clearly $\tau*$ = {$0_{\sim''}$, B, C, D, E, $1_{\sim''}$} is an inter-val bip.vag topology in Y. Then $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag topological space IBVCl(B) = $1_{\sim''}$, IBVCl(C) = $1_{\sim''}$, IBVCl(E) = $1_{\sim''}$. Now IBVCl(B$\cap$ C $\cap$ E) $\neq$ $0_{\sim''}$. Therefore $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag weakly Volterra space but it is not an inter-val bip.vag Volterra space.

- *Definition 4.9:*

Let $(Y, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag topological space. An inter-val bip.vag set B in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag σ- first category if B = $\bigcup_{i=1}^{\infty} B_i$, where B_i's are inter-val bip.vag σ- nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Any other inter-val bip.vag set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is said to be an inter-val bip.vag σ-second category in $(Y, \mathrm{IB}V_{\tau*})$.

- *Definition 4.10:*

An inter-val bip.vag topological space $(Y, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag σ- first category space if $1_{\sim''}$ = $\bigcup_{i=1}^{\infty} B_i$, where B_i's are inter-val bip.vag σ- nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag σ- second category space if it is not an inter-val bip.vag σ-first category space.

- *Proposition 4.11:*

If the inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag σ- second category space, then $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

- *Proof:*

Let B_i's (i= 1,2,…..N) be an inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. Then by Proposition 4.3 B_i^c's are an inter-val bip.vag σ-nowhere dense set in $(Y, \mathrm{IB}V_{\tau*})$. Let $C_\alpha(\alpha = 1, 2, \ldots..\infty)$ be an inter-val bip.vag σ-nowhere dense set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ in which let us take the first N(C_α)'s as B_i^c. Since $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag σ-second category space, $\bigcup_{\alpha=1}^{\infty} C_\alpha \neq 1_{\sim''}$. Then $(\bigcup_{\alpha=1}^{\infty} C_\alpha)^c \neq 1_{\sim''} \Longrightarrow \bigcap_{\alpha=1}^{\infty} C_\alpha^c \neq 0_{\sim''}$. Then we have IBVCl$(\bigcap_{\alpha=1}^{\infty}(C_\alpha)^c)$. Since IBVCl$(\bigcap_{\alpha=1}^{N}(C_\alpha)^c) \subseteq$ IBVCl$(\bigcap_{\alpha=1}^{\infty}(C_\alpha)^c \neq 0_{\sim''}$ then we have IBVCl$(\bigcap_{\alpha=1}^{\infty}(C_\alpha)^c) \neq 0_{\sim''}$, where B_i's (i=1,2,….N) are an inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, \mathrm{IB}V_{\tau*})$. Therefore $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

- *Proposition 4.12:*

- Let $(\mathrm{X}, \mathrm{IB}V_{\tau*})$ be an inter-val bip.vag weakly Volterra space if $\bigcup_{i=1}^{N} B_i = 1_{\sim''}$, where B_i's are an inter-val bip.vag J_σ – set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ then there exits atleast one B_i in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ with IBVInt$(B_i) \neq 0_{\sim''}$.
- If $\bigcup_{i=1}^{N} B_i$= $1_{\sim''}$ where B_i's are an inter-val bip.vag J_σ – set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ and if, IBVInt$(B_i) \neq 0_{\sim''}$ for atleast one (i =1,2,….N) then $(Y, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

- *Proof:*

(i) $\Longrightarrow$(ii). Suppose that IBVInt(B_i) = $0_{\sim''}$ for all (i=1,2,…N). Then (IBVInt(B_i))^c = $1_{\sim''} \Longrightarrow$ IBVCl$({B_i}^c) = 1_{\sim''}$. Therefore B_i^c's are an inter-val bip.vag dense set in Y. B_i's are an inter-val bip.vag J_σ – set in $(Y, \mathrm{IB}V_{\tau*})$ implies that B_i^c's are an inter-val bip.vag H_δ-set. Now, IBVCl$(\bigcap_{i=1}^{N} {B_i}^c)$ = IBVCl$(\bigcup_{i=1}^{N} B_i)^c$ = IBVCl$(1_{\sim''})$ = $0_{\sim''}$, where B_i^c's are an inter-val bip.vag dense set and an inter-val bip.vag H_δ-set. This implies $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is not an inter-val bip.vag Volterra space, which is a contradiction. Therefore IBVInt$(B_i) \neq 0_{\sim''}$ for atleast one i(i=1,2,…..N) in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. (ii) $\Longrightarrow$(i). Suppose that $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is not an inter-val bip.vag weakly Volterra space. IBVCl$(\bigcap_{i=1}^{N} B_i)$ = $0_{\sim''}$ where B_i's are an inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$. This implies that IBVInt$(\bigcup_{i=1}^{N}(B_i^c))$ = $1_{\sim''} \Longrightarrow \bigcup_{i=1}^{N}(B_i^c)$ = $1_{\sim''}$, where B_i^c's are an inter-val bip.vag J_σ – set in $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ and IBVInt(B_i^c) = $0_{\sim''}$(because IBVCl(B_i) = $1_{\sim''}\forall$ i = 1,2,…..N) which is a contradiction to the hypothesis. Hence $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ must be an inter-val bip.vag weakly Volterra space.

- *Definition 4.13:*

An inter-val bip.vag topological space $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is an inter-val bip.vag almost resolvable space if $\bigcup_{i=1}^{\infty} B_i$ = $1_{\sim''}$, where the inter-val bip.vag sets B_i's in $(Y, \mathrm{IB}V_{\tau*})$ are such that IBVInt$(B_i) = 0_{\sim''}$. Otherwise, $(\mathrm{Y}, \mathrm{IB}V_{\tau*})$ is called an inter-val bip.vag almost irresolvable.

➢ *Definition 4.14:*

An inter-val bip.vag topological space $(Y, IBV_{\tau*})$ is called an inter-val bip.vag $\mathcal{P}$-space if countable intersection of an inter-val bip.vag op.sets in $(Y, IBV_{\tau*})$ is an inter-val bip.vag open in $(Y, IBV_{\tau*})$.

➢ *Definition 4.15:*

An inter-val bip.vag topological space $(Y, IBV_{\tau*})$ is called an inter-val bip.vag submaximal space if for each an inter-val bip.vag set B in $(Y, IBV_{\tau*})$ such that IBVCl(B) = $1_{\sim''}$, then $B \in \tau *$.

➢ *Proposition 4.16:*

If the inter-val bip.vag topological space $(Y, IBV_{\tau*})$ is an inter-val bip.vag almost irresolvable space, then $(Y, IBV_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

➢ *Proof:*

Let B_i's (i = 1,2,….N) be an inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$. Now IBVCl(B_i) $\Longrightarrow$ IBVInt(B_i^c) = $0_{\sim''}$. Since $(Y, IBV_{\tau*})$ is an inter-val bip.vag almost irresolvable space, $\bigcup_{i=1}^{\infty}(C_i) \neq 1_{\sim''}$, where the inter-val bip.vag sets C_i's in $(Y, IBV_{\tau*})$ are such that IBVInt(C_i) = $0_{\sim''}$. Let us take the first N((C_i)'s as B_i^c's in $(Y, IBV_{\tau*})$. Now, $\bigcup_{i=1}^{\infty}(C_i) \neq 1_{\sim''} \Longrightarrow (\bigcup_{i=1}^{\infty}(C_i))^c \neq 0_{\sim''}$. This implies that $\bigcap_{i=1}^{\infty}(C_i)^c \neq 0_{\sim''}$ and thus IBVCl($\bigcap_{i=1}^{\infty}(C_i)^c$) $\neq 0_{\sim''}$. Since IBVCl($\bigcap_{i=1}^{N}(C_i)^c$) $\leq$ IBVCl($\bigcap_{i=1}^{\infty}(C_i)^c$) then IBVCl($\bigcap_{i=1}^{N}(C_i)^c$) $\neq 0_{\sim''}$. Hence IBVCl($\bigcap_{i=1}^{N}(C_i^c)^c$) $\neq 0_{\sim''}$ replacing C_i, by B_i^c, (i=1,2,….N). This implies IBVCl($\bigcap_{i=1}^{N}(B_i)$) $\neq 0_{\sim''}$. Therefore $(Y, IBV_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

➢ *Proposition 4.17:*

If the inter-val bip.vag topological space $(Y, IBV_{\tau*})$ is an inter-val bip.vag second category and an inter-val bip.vag $\mathcal{P}$-space, then $(Y, IBV_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

➢ *Proof:*

Let B_i's (i = 1,2,….N) be an inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$. Since $(Y, IBV_{\tau*})$ is an inter-val bip.vag $\mathcal{P}$-space then inter-val bip.vag H_δ-set B_i's are inter-val bip.vag open set in $(Y, IBV_{\tau*})$. Then B_i's (i = 1,2,….N) be an inter-val bip.vag dense and an inter-val bip.vag open set in $(Y, IBV_{\tau*})$. Then by Proposition 3.12, B_i^c's are inter-val bip.vag nowhere dense set in $(Y, IBV_{\tau*})$. Since $(Y, IBV_{\tau*})$ is an inter-val bip.vag second category space $\bigcup_{i=1}^{\infty}(C_i) \neq 1_{\sim''}$ where C_i's are inter-val bip.vag nowhere dense set in $(Y, IBV_{\tau*})$. Let us take the first N(C_i)'s as B_i^c's in $(Y, IBV_{\tau*})$. Then $\bigcup_{i=1}^{N}(B_i^c) = \bigcup_{i=1}^{N}(C_i) \subseteq \bigcup_{i=1}^{\infty}(C_i) \neq 1_{\sim''}$. This implies that, $(\bigcup_{i=1}^{N}(B_i^c))^c \neq 0_{\sim''} \Longrightarrow \bigcap_{i=1}^{N}(B_i) \neq 0_{\sim''}$. Thus IBVCl($\bigcap_{i=1}^{N}(B_i)$) $\neq 0_{\sim''}$, where B_i's are inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$. So $(Y, IBV_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

➢ *Proposition 4.18:*

If the inter-val bip.vag topological space $(Y, IBV_{\tau*})$ is an inter-val bip.vag second category and an inter-val bip.vag submaximal space, then $(Y, IBV_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

➢ *Proof:*

Let B_i's (i = 1,2,….N) be an inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$. Since $(Y, IBV_{\tau*})$ is an inter-val bip.vag submaximal space, the inter-val bip.vag dense set B_i's are inter-val bip.vag open set in $(Y, IBV_{\tau*})$. By proposition 3.12, B_i^c's are inter-val bip.vag nowhere dense sets in $(Y, IBV_{\tau*})$. Since $(Y, IBV_{\tau*})$ is an inter-val bip.vag second category space $\bigcup_{i=1}^{\infty}(C_i) \neq 1_{\sim''}$ where C_i's are inter-val bip.vag nowhere dense set in $(Y, IBV_{\tau*})$. Let us take the first N(C_i)'s as B_i^c's in $(Y, IBV_{\tau*})$. Then $\bigcup_{i=1}^{N}(B_i^c) \leq \bigcup_{i=1}^{\infty}(C_i)$ and $\bigcup_{i=1}^{\infty}(C_i) \neq 1_{\sim''}$ implies that $\bigcup_{i=1}^{N}(B_i^c) = 1_{\sim''}$.This implies that, $\bigcap_{i=1}^{N}(B_i) \neq 0_{\sim''}$ and hence IBVCl($\bigcap_{i=1}^{N}(B_i)$) $\neq 0_{\sim''}$, where B_i's are inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$. Therefore $(Y, IBV_{\tau*})$ is an inter-val bip.vag weakly Volterra space.

➢ *Proposition 4.19:*

If the inter-val bip.vag topological space $(Y, IBV_{\tau*})$ is not an inter-val bip.vag weakly Volterra space, then $(Y, IBV_{\tau*})$ is an inter-val bip.vag σ-first category space.

➢ *Proof:*

Let C_i's (i=1,2,…∞) be an inter-val bip.vag σ-nowhere dense sets in an inter-val bip.vag topological space $(Y, IBV_{\tau*})$ which is not an inter-val bip.vag weakly Volterra space. Now, we claim that $\bigcup_{i=1}^{\infty}(C_i) = 1_{\sim''}$. Suppose that $\bigcup_{i=1}^{\infty}(C_i) \neq 1_{\sim''}$. Then $\bigcap_{i=1}^{\infty} C_i^c \neq 0_{\sim''}$. Since C_i's are inter-val bip.vag σ-nowhere dense set in $(Y, IBV_{\tau*})$ by proposition 4.3, C_i^c's are inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$. Now, $\bigcap_{i=1}^{N} C_i^c \subseteq \bigcap_{i=1}^{\infty} C_i^c$ implies that $\bigcap_{i=1}^{N} C_i^c \neq 0_{\sim''}$. Let $B_i = B_i^c$, then $\bigcap_{i=1}^{N} B_i \neq 0_{\sim''}$ implies that IBVCl($\bigcap_{i=1}^{N} B_i$) $\neq 0_{\sim''}$, where B_i's are inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$. But this is a contradiction, since $(Y, IBV_{\tau*})$ is not an inter-val bip.vag weakly Volterra space. Hence $\bigcup_{i=1}^{\infty}(C_i) = 1_{\sim''}$. Therefore $(Y, IBV_{\tau*})$ is an inter-val bip.vag σ-first category space.

➢ *Proposition 4.20:*

If an inter-val bip.vag topological space $(Y, IBV_{\tau*})$ is an inter-val bip.vag weakly Volterra space, then $(Y, IBV_{\tau*})$ is not an inter-val bip.vag σ-Baire space.

➢ *Proof:*

Let $(Y, IBV_{\tau*})$ be an inter-val bip.vag weakly Volterra space. Then we have IBVCl($\bigcap_{i=1}^{N} B_i$) $\neq 0_{\sim''}$ where B_i's are inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$. Since B_i's are inter-val bip.vag dense and an inter-val bip.vag H_δ-set in $(Y, IBV_{\tau*})$, then by proposition 4.3. Let C_i's (i=1,2,…∞) be an inter-val bip.vag σ-nowhere dense set in $(Y, IBV_{\tau*})$ in which the first N inter-val bip.vag σ-nowhere dense set be B_i^c's. Now $\bigcup_{i=1}^{N}(B_i^c) \subseteq \bigcap_{i=1}^{\infty}(C_i)$. Then IBVInt($\bigcup_{i=1}^{N}(B_i^c)$) $\subseteq$ IBVInt($\bigcap_{i=1}^{\infty}(C_i)$) this implies that (IBVCl($\bigcap_{i=1}^{N} B_i$))$^c \subseteq \bigcap_{i=1}^{\infty}(C_i)$. Since IBVCl($\bigcap_{i=1}^{N} B_i$) $\neq 0_{\sim''}$, IBVInt($\bigcup_{i=1}^{\infty}(C_i)$) $\neq 0_{\sim''}$, where C_i's (i=1,2,…∞) are inter-val bip.vag σ-nowhere dense set $(Y, IBV_{\tau*})$. Hence $(Y, IBV_{\tau*})$ is not an inter-val bip.vag σ-Baire space.

ISSN No: 2456-2165 https://doi.org/10.38124/ijisrt/25jun1073

V. CONCLUSION

In this paper we developed the concepts of inter-val bip.vag Volterra spaces and inter-val bip.vag weakly Volterra spaces and discussed some of their newly introduced definitions and propositions with suitable illustrations.

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