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A Fuzzy EOQ Model with Investment in Carbon Emission Reduction Using Kuhn – Tucker Method

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Abstract: Lessening the amount of greenhouse gas (GHG) emissions that a person, group, or nation produces refers to carbon emission reduction. In order to reduce such emissions, investment in carbon emission reduction is mandatory, and at present many researchers focus on these criteria in Economic Order Quantity(EOQ) Models and find new ideas and techniques. Concentrating on the emission of carbon, its reduction, along with the analysation of uncertain situations in the EOQ models, is worthwhile. On examining the drawbacks of vagueness and the requirement to remove it, in this present work, we implement a fuzzy approach for heptagonal fuzzy numbers. We use the sub – interval average method for defuzzification and the Kuhn – tucker method for finding the optimal solution. Optimal order quantity and total cost for both crisp and fuzzy senses are determined and compared to justify the results.

Keywords: *Emission Reduction, Heptagonal Fuzzy Number, Kuhn – Tucker Method, Sub - Interval Average Method, Uncertainty.*

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I. INTRODUCTION

Minimizing greenhouse gas (GHG) emissions produced by an individual, community, or country is known as emission reduction. Also reducing emissions is the process of lowering the amount of dangerous gases and particles released into the environment, usually by the use of technology advancements, adjustments to economic development patterns, and lifestyle changes supported by robust legislation. Lowering emissions is necessary to reach net zero, mitigate the effects of climate change, and satisfy regional and global climate targets. Financial resources and wise investments are needed to fight climate change, lower CO2, encourage adaptation to its impacts, and promote resilience-building. Whether or not we acquire money and potential avenues to riches will depend on the investment choices we make today. It is becoming increasingly clear that the world cannot afford to burn all of its fossil fuel supplies if we are to succeed in reducing climate change to levels that are sustainable and habitable. But the investment has an impact on the carbon emissions per unit generated and per replenishment. So to examine this problem carbon policy can be implemented in EOQ models. In this work, carbon cap policy is used among several carbon policies.

According to this carbon cap policy, it allows a government to "cap" or restrict the overall quantity of greenhouse gases that can be released. Companies that emit greenhouse gases excessively are required to pay for each tonne of carbon dioxide that they release under such a plan. Also a carbon cap firmly limits a company's carbon impact to be below or equivalent to a cap. In general, the cap is the maximum amount of GHG emissions permitted under a plan, or, to put it another way, the total amount of allowances (emissions budget) that covered organizations are qualified to receive. Regulators attempt to balance environmental goals with their viability from an economic standpoint when establishing a cap.

In 2013, Chen et al considered a EOQ model in which carbon is emitted per restocking and per unit items grasped in stock per unit time period. They proved that emission of carbon can be reduced importantly without importantly increasing cost. They analysed this approach under carbon cap policy which is one among several carbon emission policies. Under various regulation policies like carbon cap, carbon tax, carbon cap – and – trade and cap – and – offset along with the consideration of carbon footprint, a EOQ model is studied by Arslan and Turkay in 2013 with the assumption that carbons are emitted per order, per unit grasped in stock per unit time. In 2014, Toptal et al analysed a EOQ model with carbon emission reduction under carbon cap, cap - and – trade and carbon tax policies. However,

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according to their model, the investment's ability to reduce carbon emissions is unrelated to the carbons emitted per unit generated, per unit stored, or each replenishment, which could suggest that the expenditure is made to lower carbon emissions from facility servicing rather than from production. Yuyao Fan et al in 2018 examined a production inventory and emission reduction investment decision model under carbon cap and trade policy. They examined choices for carbon trading, production-inventory, and investment in emission reduction in both centralized and decentralized scenarios. An environmentally sustainable EOQ model with partial backordering and investment in transportation emission cost reduction is studied by hsien – Jen Lin et al in 2018. They looked into the results of enhancing investment to lower transportation's emission costs. In 2020, Jialiang pan et al developed an inventory based sustainable production model for technical collaboration on investment to lower carbon emissions under carbon cap-and-trade and carbon tax policies. In their work, they put out a production-inventory model wherein the vendor and buyer in an integrated supply chain consent to share financial resources in order to lower carbon emissions. Jun-Yeon Lee examined a EOQ model with investment in carbon emission reduction under several carbon emission policies in 2019. Hence researchers have discussed all the above cases in different criteria. But in all the reviewed models, vague situation may exist at some circumstances. And one of the most significant formal revolutions in science and mathematics this century is the idea of uncertainty.

A shortcoming of existing inventory models is the irrational presumption that every item produced is of high quality. The entire inventory cost in a crisp environment includes all known and unambiguously defined factors, including holding costs, ordering costs, setup costs, production costs, reworking costs, backorder costs, production rate, deterioration rate, and demand rate. Certain business scenarios fulfill these criteria, however in the majority of cases and in the constantly shifting market environment, the dimensions and variables are incredibly ambiguous or inaccurate. Getting exact information regarding inventory parameters isn't always feasible in real life. Occasionally, random variables chosen from a probability distribution do not adequately reflect this kind of imprecise data. By incorporating innovative methods into supply chain models, uncertainties can be addressed. Fuzzy technique is one such innovative tool to deal with this issue. Some researchers have included fuzzy idea in their study.

Analysation of EOQ model under fuzzy methodology is proposed by cheng wang et al in 2010 with an assumption that the decision maker's knowledge of market demand, ordering costs, and holding costs is imprecise. In 2014, Xiaolong et al analysed a fuzzy retailers inventory model under carbon cap and trade mechanism where they apply trapezoidal fuzzy numbers for certain ambiguous parameters. Under deterministic and trapezoidal fuzzy demand a multi-item sustainable manufacturing model is designed by karthick and Uthayakumar with Discrete Setup Cost and Carbon Emission Reduction in 2021. A refurbishing inventory model that uses the hexagonal fuzzy number and cap-and-trade laws to reduce carbon emissions is studied by Ritu Arora et al in 2021. They concentrated more on the uncertain parameters and solved their model under graded mean integration defuzzification method to provide confident result. In 2023, Srabani Shee and Tripti Chakrabarti developed a unreliable EOQ model with effects of carbon emission regulations in a cloud fuzzy setting. In their work, they discussed about emission of carbon under Carbon tax and cap-trade regulation where they have considered demand as cloud fuzzy and general fuzzy number and solved under yager's index method in a way to remove vagueness in it. Abhishek Kumar et al investigated a fuzzy production inventory model with carbon emission under signed distance methodology for pentagonal fuzzy numbers in 2024. Narendra Kumar et al in 2024, examined a lot size model with cap-and-trade and carbon tax rules, for several products over a limited planning horizon, under the influence of learning and time-value of money by implementing fuzzy technique. Hence fuzzy set theory is used here to deal with an imprecise cost. In our work, we proposed a fuzzy EOQ model with investment in carbon emission in which we have considered three uncertain parameters as heptagonal fuzzy numbers and found the optimal solution using Kuhn – tucker method after the defuzzification process using sub – interval average methodology.

II. DEFINITIONS AND METHODOLOGIES:

- *Fuzzy Set:*

A fuzzy set $\tilde{A}$ defined on a Universe of discourse X may be written as a collection of ordered pairs, $\tilde{A} = \left\{(x,\mu_{\tilde{A}}(x)) \text{: } x \in X, \mu_{\tilde{A}} \in [0,1]\right\}$, where each pair $(x,\mu_{\tilde{A}}(x))$ is called a singleton and the element $\mu_{\tilde{A}}(x)$ belongs to the interval $[0,1]$. The function $\mu_{\tilde{A}}(x)$ is called as membership function.

- *Heptagonal Fuzzy Number:*

A fuzzy number $\tilde{A} = (a,b,c,d,e,f,g)$ where $a < b < c < d < e < f < g$ are defined on R is called heptagonal fuzzy number if its membership function is

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$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{(x-a)}{2(b-a)}, & a\le x\le b\\ \dfrac{1}{2} & b\le x\le c\\ \dfrac{(x-d)}{2(d-c)}+1, & \mathrm{c}\le x\le d\\ \dfrac{(d-x)}{2(e-d)}+1, & d\le x\le e\\ \dfrac{1}{2}\ , & e\le x\le f\\ \dfrac{(g-x)}{2(g-f)}, & f\le x\le g\\ 0, & Otherwise\end{cases}$$

- *Fuzzy Arithmetical Operations:*

Some of the fuzzy arithmetical operations for heptagonal fuzzy numbers under function principle are as follows,

Let us assume $\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7)$ and $\tilde{B}=(b_1,b_2,b_3,b_4,b_5,b_6,b_7)$ as two heptagonal fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}+\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,a_5+b_5,a_6+b_6,a_7+b_7)$, where $a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,a_7,\ b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6\ and\ b_7$ are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}\times\tilde{B}=(a_1b_1,\ a_2b_2,\ a_3b_3,\ a_4b_4,\ a_5b_5,\ a_6b_6,\ a_7b_7)$, where $a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6\ and\ b_7$ are any real numbers.

(iii) The subtraction of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A}-\tilde{B}=(a_1-b_7,\ a_2-b_6,\ a_3-b_5,\ a_4-b_4,\ a_5-b_3,\ a_6\text{-}b_2,\ a_7\text{-}b_1)$,where $-\tilde{B}=(-b_7,-b_6,-b_5,-b_4,-b_3,-b_2,-b_1)$,also $a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6\ and\ b_7$ are any real numbers.

(iv) The division of $\tilde{A}$ and $\tilde{B}$ is $\dfrac{\tilde{A}}{\tilde{B}}=\left(\dfrac{a_1}{b_7},\ \dfrac{a_2}{b_6},\ \dfrac{a_3}{b_5},\ \dfrac{a_4}{b_4},\ \dfrac{a_5}{b_3},\ \dfrac{a_6}{b_2},\ \dfrac{a_7}{b_1}\right)$, where $\dfrac{1}{\tilde{B}}=\tilde{B}^{-1}=\left(\dfrac{1}{b_7},\ \dfrac{1}{b_6},\ \dfrac{1}{b_5},\ \dfrac{1}{b_4},\ \dfrac{1}{b_3},\ \dfrac{1}{b_2},\ \dfrac{1}{b_1}\right)$, $b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6\ and\ b_7$ are positive real numbers. Also $a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6\ and\ b_7$ are any real numbers.

(v) For any $\alpha\in R$,

a) If $\alpha\ge 0$, then $\alpha\times\tilde{A}=(\alpha a_1,\alpha a_2,\alpha a_3,\alpha a_4,\alpha a_5,\alpha a_6,\alpha a_7)$.

b) If $\alpha<0$, then $\alpha\times\tilde{A}=(\alpha a_7,\alpha a_6,\alpha a_5,\alpha a_4,\alpha a_3,\alpha a_2,\alpha a_1)$.

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- *Sub interval Average method:*

If $\tilde{B} = (b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6,\ b_7)$ is a heptagonal fuzzy number, then sub interval average method formula for $\tilde{B}$ is given by,

$$R(\tilde{B}) = \frac{8\left(b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7\right)}{56}$$

- *Kuhn- Tucker Method:*

The Kuhn-Tucker method is a method for finding optimal solutions for non-linear programming problems containing differentiable functions. The Kuhn-tucker conditions are based on the extension of lagrangian method.

Suppose we consider an optimization problem,
Minimize $Y = f(x)$ subject to the constraints $g_i\left(x\right) \geq 0$, i=1,2,………..m.

The non-negativity constraints may be converted into equations by using non negative surplus variables.

Let $\lambda = \left(\lambda_1, \lambda_2, \lambda_3, ..., \lambda_m\right)$

$g(x) = (g_1(x), g_2(x), g_3(x), ..., g_m(x))$ and

$S^2 = (S_1^{\ 2}, S_2^{\ 2}, S_3^{\ 2}, ..., S_m^{\ 2})$.

The Kuhn-Tucker conditions need X and λ to be a stationary point of this problem of minimization, which can be expressed as follows,

$$\lambda_i \leq 0,$$
$$\nabla f(x) - \lambda \nabla g(x) = 0,$$
$$\lambda_i g_i(x) = 0, i = 1, 2, ..., m,$$
$$g_i(x) \geq 0, i = 1, 2, ..., m.$$

III. ASSUMPTIONS AND NOTATIONS

- *ASSUMPTIONS:*

- Investment in carbon emission reduction under carbon cap policy is considered here.
- A fuzzy technique is used to solve the uncertainty of specific parameters.
- Sub interval average method is involved here for defuzzification.
- Cost of ordering items per replenishment, cost of holding inventory per unit per year, penalty of carbon per ton are taken as heptagonal fuzzy numbers.

- *NOTATIONS:*

- *Crisp Parameters:*

$D_\lambda -$ Demand.

$F -$ Cost of ordering items per replenishment.

$H_C -$ Cost of holding inventory per unit per year.

$\alpha_F -$ Carbon footprint's cap.

$C_P -$ Penalty of carbon per ton.

$f_0, f_1, f_2 -$ Parameters related with carbon emissions per replenishment.

$g_0, g_1, g_2 -$ Parameters related with variable carbon emissions.

$Q_C^{\ *} -$ Optimum Order Quantity.

$A_G -$ Investment amount in carbon emission reduction.

$TC -$ Total cost.

$E_f\left(A_G\right) -$ Emission of carbon fixed.

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$E_V\left(A_G\right)-$varying emission of carbon.

- *Fuzzy Parameters:*

$\tilde{F}-$ Fuzzy Cost of ordering items per replenishment.

$\tilde{H}_C-$ Fuzzy Cost of holding inventory per unit per year.

$\tilde{C}_P-$ Penalty of carbon per ton under fuzzy.

$\tilde{Q}_C{}^*-$ Fuzzy Optimum Order Quantity.

$R(T\tilde{C})-$ Fuzzy Total Cost.

IV. INVENTORY MODEL WITH INVESTMENT IN CARBON EMISSION REDUCTION UNDER CARBON CAP POLICY

The annual total cost of the inventory model with investment in carbon emission reduction under carbon cap policy is given by,

$$TC=\left(F+C_PE_f\left(A_G\right)\right).\frac{D_\lambda}{Q_C}+\frac{H_cQ_C}{2}+C_PD_\lambda E_V\left(A_G\right)+A_G-C_P\alpha_F \quad \text{-------------- (4.1)}$$

Where the emission of carbon both for fixed and varying case $E_f\left(A_G\right)$ and $E_V\left(A_G\right)$ is expressed as follows,

$$E_f\left(A_G\right)=f_0-f_1A_G+f_2A_G{}^2$$
$$E_V\left(A_G\right)=g_0-g_1A_G+g_2A_G{}^2$$

Differentiating eqn (4.1) with respect to Q_C we get the required optimum order quantity and it is given by,

$$Q_C{}^*=\sqrt{\frac{2D_\lambda\left(F+C_PE_f\left(A_G\right)\right)}{H_C}}.$$

V. FUZZY INVENTORY MODEL UNDER CARBON CAP POLICY

The above green inventory model with investment in carbon emission reduction under carbon cap policy is now considered in fuzzy sense to remove uncertain, unclearness and ambiguous situations in few parameters. Hence the crisp parameters such as cost of ordering items per replenishment, cost of holding inventory per unit per year, penalty of carbon per ton are taken as fuzzy parameters.

Now let us assume

$\tilde{Q}_C=(q_{c_1},q_{c_2},q_{c_3},q_{c_4},q_{c_5},q_{c_6},q_{c_7})$,

$\tilde{F}=\left(F_1,F_2,F_3,F_4,F_5,F_6,F_7\right)$,

$\tilde{H}_C=\left(H_{C_1},H_{C_2},H_{C_3},H_{C_4},H_{C_5},H_{C_6},H_{C_7}\right)$,

$\tilde{C}_P=\left(C_{P_1},C_{P_2},C_{P_3},C_{P_4},C_{P_5},C_{P_6},C_{P_7}\right)$ as heptagonal fuzzy numbers.

Hence the total cost for the above considered green inventory model in fuzzified form is given by,

$$T\tilde{C}\left(Q_C\right)=\left(TC\left(q_{c_1}\right),TC\left(q_{c_2}\right),TC\left(q_{c_3}\right),TC\left(q_{c_4}\right),TC\left(q_{c_5}\right),TC\left(q_{c_6}\right),TC\left(q_{c_7}\right)\right)$$

Where $TC\left(q_{c_i}\right)=\left(F_i+C_{P_i}E_f\left(A_G\right)\right).\frac{D_\lambda}{q_{c_j}}+\frac{H_{c_i}q_{c_i}}{2}+C_{P_i}D_\lambda E_V\left(A_G\right)+A_G-C_{P_i}\alpha_F$

for $i=1,2,3,4,5,6,7$ and $j=7,6,5,4,3,2,1$.

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For the defuzzification of the fuzzy total cost, sub interval average method is used and it is given by,

$$R\left(T\tilde{C}\left(Q_C\right)\right)=\frac{8\left(TC\left(q_{c_1}\right)+TC\left(q_{c_2}\right)+TC\left(q_{c_3}\right)+TC\left(q_{c_4}\right)+TC\left(q_{c_5}\right)+TC\left(q_{c_6}\right)+TC\left(q_{c_7}\right)\right)}{56}.$$

with $0<q_{c_1}\le q_{c_2}\le q_{c_3}\le q_{c_4}\le q_{c_5}\le q_{c_6}\le q_{c_7}$ ------------(5.1)

We now apply Kuhn – tucker Method to the equation (5.1), to minimize the fuzzy total cost $R\left(T\tilde{C}\left(Q_C\right)\right)$ subject to the constraints

$q_{C_7}-q_{C_6}\ge 0,$

$q_{C_6}-q_{C_5}\ge 0,$

$q_{C_5}-q_{C_4}\ge 0,$

$q_{C_4}-q_{C_3}\ge 0,$

$q_{C_3}-q_{C_2}\ge 0,$

$q_{C_2}-q_{C_1}\ge 0$ and $q_{C_1}>0.$

After applying Kuhn – tucker four conditions and solving we obtain the Fuzzy Optimum Order Quantity and it is given by,

$$\tilde{Q}_C^{\ *}=\sqrt{\frac{2D_\lambda\left[8\left(F_1+F_2+F_3+F_4+F_5+F_6+F_7\right)+\left(8\left(C_{P_1}+C_{P_2}+C_{P_3}+C_{P_4}+C_{P_5}+C_{P_6}+C_{P_7}\right)\times E_f\left(A_G\right)\right)\right]}{8\left(H_{C_1}+H_{C_2}+H_{C_3}+H_{C_4}+H_{C_5}+H_{C_6}+H_{C_7}\right)}}$$

VI. NUMERICAL EXAMPLE

- *Crisp Model:*

The values for different parameters given in the green inventory model are as follows,

$D_\lambda=1000, F=100, C_P=6.022, H_C=4, A_G=396.50, f_0=200, f_1=0.1, f_2=0.0001,$

$g_0=4, g_1=0.01, g_2=0.00001, \alpha_F=2000.$

By using these values Order Quantity and Total cost for the crisp green inventory system is calculated and it is given by,

Order Quantity:

$Q_C^{\ *}=761.676085.$

Total Cost:

$TC=1077.296035.$

- *Fuzzy Model:*

Depending on the set of data taken, as in the crisp green inventory system, the values of fuzzy parameters, which are taken as heptagonal fuzzy numbers, are given below.

$\tilde{F}=\left(50,70,90,100,110,120,130\right),$

$\tilde{H}_C=\left(1,2,3,4,5,6,7\right),$

$\tilde{C}_P=\left(3.022,4.022,5.022,6.022,7.022,8.022,9.022\right)$

Using these values, the fuzzy order quantity and fuzzy total cost is calculated.

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Fuzzy Order Quantity:

$$\tilde{Q}_C^{\,*} = 760.2681115.$$

Fuzzy Total Cost:

$$R\left(T\tilde{C}\left(Q_C\right)\right) = 1071.664143.$$

VII. CONCLUSION

As a conclusion, this research has successfully developed a fuzzy Economic Order Quantity (EOQ) model that incorporates the investment in carbon emission reduction under carbon cap policy, utilizing the Kuhn-Tucker method for optimal solutions by facing uncertainty among few parameters. The model addresses the inherent uncertainties in three parameters, providing a more realistic framework for decision-making in sustainable supply chain management. By integrating fuzzy methodology with environmental concerns and traditional inventory management, this approach offers a balance between rectifying vagueness and fulfilling corporate social responsibility goals. The Kuhn-Tucker method proves to be a powerful tool for deriving fuzzy optimal order quantity and total cost, ensuring that firms can achieve a sustainable equilibrium between profitability under ambiguous nature and environmental impact. Future research could explore the extension of this model to incorporate additional uncertainties or other environmental factors, further advancing the integration of green practices into business strategies.

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