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A Mathematical Framework for Evaluating Thermal Conductivity–Driven Heat Transfer Enhancement in Nano Fluids

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Abstract: Nano fluids have gained significant attention as advanced working fluids for enhanced heat transfer applications due to their improved thermal properties. This study presents a mathematical framework for evaluating thermal conductivity–driven heat transfer enhancement in nano fluids. The proposed model incorporates the effects of nanoparticle volume fraction, particle size, Brownian motion, interfacial thermal resistance, and temperature-dependent thermo physical properties to predict effective thermal conductivity. Classical models are extended by accounting for nano scale transport mechanisms, resulting in improved predictive capability. The developed framework is coupled with conventional heat transfer correlations to assess its impact on convective heat transfer performance. A parametric analysis is performed to identify the influence of key governing parameters on thermal conductivity enhancement. Model predictions are validated against published experimental data and show good agreement with reduced deviation compared to existing models. The results highlight the existence of an optimal nanoparticle concentration for effective heat transfer enhancement.

Keywords: *Nano Fluids, Thermal Conductivity, Mathematical Modeling, Heat Transfer Enhancement, Brownian Motion, Interfacial Thermal Resistance.*

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I. INTRODUCTION

The increasing demand for efficient thermal management in applications such as heat exchangers, electronic cooling, renewable energy systems, and automotive thermal control has driven extensive research into advanced heat transfer fluids. Nano fluids, formed by dispersing nano-scale solid particles into conventional base fluids, have attracted considerable attention due to their superior thermo-physical properties, particularly enhanced thermal conductivity. Experimental studies have demonstrated that even at low nanoparticle volume fractions, nano fluids can significantly improve heat transfer performance compared to traditional working fluids, making them promising candidates for next-generation thermal systems.

Despite significant progress, accurate prediction of thermal conductivity enhancement in nano fluids remains a major challenge. The complex interactions among nanoparticle size, shape, concentration, Brownian motion, interfacial thermal resistance, and temperature-dependent properties introduce strong nonlinearity into heat transfer mechanisms, which classical effective medium theories and many empirical correlations fail to capture adequately. Moreover, existing models often rely on simplified assumptions or empirical fitting parameters, limiting their general applicability. To address these limitations, the present study proposes a unified, physics-based mathematical framework that incorporates dominant nano-scale transport phenomena, including Brownian motion and interfacial effects, and couples them with convective heat transfer correlations. The developed model is validated against experimental data and offers a reliable tool for the analysis and optimization of nano fluid-based thermal management systems.

II. LITERATURE REVIEW

Prediction of effective thermal conductivity in nano fluids has been a major focus of heat transfer research due to its critical role in thermal performance enhancement. Early studies were primarily based on classical effective medium theories such as the Maxwell model, which assumes dilute suspensions of spherical, non-interacting particles, and the Hamilton–Crosser model, which extends this approach by incorporating particle shape effects. Although these models provide analytical simplicity, they consistently under predict thermal conductivity enhancement in nano fluids. More

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generalized formulations, including the Bruggeman and Jeffery models, account for higher particle concentrations and random phase distributions but still rely on averaged material properties and neglect nano-scale transport mechanisms, limiting their accuracy under practical operating conditions.

Recent studies have focused on incorporating dynamic and interfacial phenomena to better explain experimentally observed enhancements in nano fluid thermal conductivity. Brownian motion–induced micro-convection has been widely investigated as a key mechanism contributing to additional energy transport, particularly for small particle sizes and elevated temperatures. Furthermore, several researchers have introduced interfacial nano-layer models to account for structured fluid layers formed around nanoparticles, which exhibit altered thermal properties compared to the bulk fluid. Other studies have highlighted the role of particle aggregation and percolation networks, showing that interconnected nanoparticle clusters can create preferential conductive pathways, resulting in nonlinear increases in thermal conductivity beyond classical predictions.

Another significant advancement in nano fluid modeling is the inclusion of interfacial thermal resistance, or Kapitza resistance, arising from phonon mismatch at the solid–fluid interface. Recent theoretical and experimental investigations have demonstrated that neglecting interfacial resistance can lead to substantial over prediction or under prediction of effective thermal conductivity, depending on particle size and material properties. Although models incorporating one or more of these nano-scale effects have shown improved agreement with experimental data, they are often system-specific and rely on empirical fitting parameters. Consequently, there remains a lack of a unified, physics-based analytical framework that simultaneously integrates Brownian motion, interfacial resistance, particle characteristics, and temperature dependence. This limitation provides strong motivation for the development of comprehensive models capable of reliably predicting thermal conductivity–driven heat transfer enhancement in nano fluids across a wide range of conditions.

III. CHARACTERISTICS OF NANO FLUIDS

Nano fluids, which are suspensions of nanometer-sized particles (nanoparticles) in a base fluid, exhibit unique thermal and physical properties that distinguish them from conventional fluids. These properties make them suitable for a wide range of applications, especially in enhancing heat transfer in systems like micro heat exchangers. Below are the key characteristics of nano fluids:

➢ *Thermal Conductivity*

One of the most significant advantages of nano fluids is their enhanced thermal conductivity compared to the base fluid. The addition of nanoparticles increases the rate of heat transfer, making nano fluids highly effective as coolants in thermal management systems. The thermal conductivity of nano fluids depends on factors such as nanoparticle material, concentration, size, shape, and the thermal conductivity of the base fluid.

➢ *Heat Transfer Coefficient*

Nano fluids typically exhibit a higher heat transfer coefficient than base fluids. This is due to the increased surface area provided by nanoparticles and the improved thermal interactions between the fluid and the heated surface. This property is especially beneficial in applications like micro heat exchangers, where efficient heat removal is crucial.

➢ *Viscosity*

The presence of nanoparticles generally increases the viscosity of the base fluid. The extent of this increase depends on the nanoparticle concentration, size, and shape, as well as the base fluid's inherent viscosity. While higher viscosity can improve heat transfer, it also increases the pumping power required to circulate the Nano fluid, which can affect the overall efficiency of a system.

➢ *Nanoparticle Stability and Dispersion*

The stability of nanoparticles within the base fluid is critical to maintaining consistent thermal properties. Nanoparticles tend to agglomerate over time, leading to sedimentation and loss of performance. To enhance stability, surfactants or surface modifications of nanoparticles are often used. Proper dispersion techniques, such as ultra-sonication, are also employed to prevent agglomeration.

➢ *Specific Heat Capacity*

The specific heat capacity of nano fluids may be slightly lower or higher than that of the base fluid, depending on the nature of the nanoparticles. This property influences the fluid's ability to store and transport thermal energy. While changes in specific heat capacity are generally less significant than changes in thermal conductivity, they still play a role in determining the overall heat transfer efficiency.

➢ *Density*

Nano fluids typically have a higher density than their base fluids due to the mass of the added nanoparticles. This increased density can affect the fluid dynamics, including flow behavior and pressure drop. Higher density can be advantageous or disadvantageous, depending on the specific application, particularly in systems where weight and buoyancy are critical factors.

➢ *Rheological Properties*

Nano fluids can exhibit non-Newtonian behavior, where the viscosity changes with shear rate. This behavior is influenced by the concentration and type of nanoparticles as well as their interactions with the base fluid. Understanding the rheological properties is essential for predicting the flow characteristics of nano fluids in various applications, especially in complex geometries like those found in micro channels.

➢ *Electrical Conductivity*

The electrical conductivity of nano fluids can be significantly higher than that of the base fluid, particularly when metal-based nanoparticles are used. This property is

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crucial in applications where both thermal and electrical properties are important, such as in electronic cooling. Increased electrical conductivity may also lead to potential challenges in insulation and electromagnetic interference, depending on the application.

➢ *Optical Properties*

Nano fluids can exhibit unique optical properties, such as increased absorption or scattering of light, depending on the type and size of nanoparticles. These properties are of interest in solar energy applications and optical devices. In solar thermal systems, nano fluids can be used to enhance the absorption of solar radiation, improving the efficiency of solar collectors.

➢ *Potential Toxicity and Environmental Impact*

The introduction of nanoparticles into fluids raises concerns about toxicity and environmental impact, particularly if the nano fluids are used in applications where they may be released into the environment. Ongoing research is focused on understanding the health and environmental implications of nano fluids and developing safe handling and disposal methods.

➢ *Magnetic Properties*

Certain nano fluids contain magnetic nanoparticles, giving them unique magnetic properties that can be manipulated using external magnetic fields. These magnetic nano fluids (Ferro fluids) are used in applications like magnetic cooling and drug delivery. Magnetic nano fluids can be controlled to achieve targeted heating or cooling, making them useful in medical and industrial applications.

IV. MATHEMATICAL FORMULATION

➢ *Thermal Conductivity of Nano Fluids*

The thermal conductivity of nano fluids is one of their most significant and widely studied properties. Nano fluids, which are suspensions of nanoparticles within a base fluid, are known for their enhanced thermal conductivity compared to the base fluid alone. This enhancement is what makes nano fluids highly attractive for heat transfer applications, such as in micro heat exchangers, cooling systems, and energy devices.

➢ *Factors Influencing Thermal Conductivity*

- *Nanoparticle Material*

✓ Metallic Nanoparticles: Metals like copper (Cu), silver (Ag), and gold (Au) are commonly used because they have high intrinsic thermal conductivities.
✓ Metal Oxide Nanoparticles: Oxides such as alumina (Al_2O_3), titania (TiO_2), and zinc oxide (ZnO) are also popular due to their good thermal properties and stability in fluids.
✓ Carbon-Based Nanoparticles: Carbon nanotubes (CNTs) and graphene have exceptionally high thermal conductivities, making them very effective in enhancing the thermal conductivity of nano fluids.

- *Nanoparticle Concentration*

The thermal conductivity of nano fluids generally increases with an increase in the volume fraction or concentration of nanoparticles within the base fluid. However, this enhancement is non-linear and can reach a saturation point where further increases in concentration yield diminishing returns.

- *Nanoparticle Size and Shape*

✓ Size: Smaller nanoparticles have a larger surface area to volume ratio, which can enhance thermal interactions with the base fluid. However, excessively small nanoparticles can lead to higher agglomeration, which may reduce the thermal conductivity.
✓ Shape: Spherical nanoparticles are common, but rod-like or platelet-shaped nanoparticles can provide higher thermal conductivity enhancements due to their larger surface area and the ability to form better thermal networks within the fluid.

- *Base Fluid*

The thermal conductivity of the base fluid (e.g., water, ethylene glycol, oil) plays a role in the overall enhancement observed when nanoparticles are added. A fluid with lower thermal conductivity will typically show a more significant percentage increase when nanoparticles are introduced, although the absolute value might still be lower than that of a base fluid with naturally higher conductivity.

- *Temperature*

The thermal conductivity of nano fluids is temperature-dependent. In many cases, the thermal conductivity increases with an increase in temperature. This is because higher temperatures can reduce the viscosity of the fluid and increase the Brownian motion of the nanoparticles, enhancing their thermal interaction.

- *Brownian Motion*

The random motion of nanoparticles due to thermal energy (Brownian motion) contributes to the enhanced thermal conductivity. This movement can lead to micro-convection within the fluid, further aiding in heat transfer.

- *Interfacial Layer*

An interfacial layer of molecules may form around nanoparticles, contributing to thermal conductivity enhancement. This layer can act as a thermal bridge between the nanoparticle and the base fluid, improving the overall heat transfer.

- *Agglomeration*

Nanoparticle agglomeration can negatively impact the thermal conductivity of nano fluids. When nanoparticles clump together, the effective surface area decreases, and the thermal paths within the fluid can be disrupted. Proper

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dispersion techniques and the use of surfactants can help maintain stable nano fluids with minimized agglomeration.

- *pH and Surface Charge*

The pH of the nano fluid and the surface charge of nanoparticles affect the stability and dispersion of nanoparticles in the base fluid. Optimal pH levels and surface modifications can prevent agglomeration and ensure consistent thermal conductivity.

➢ *Models For Predicting Thermal Conductivity*

Several theoretical models have been developed to predict the thermal conductivity of nano fluids, considering the various factors involved:

- *Maxwell Model (Classic):*

The Maxwell model is one of the earliest and simplest models used to estimate the effective thermal conductivity of a composite material, including nano fluids. It assumes spherical particles and a low volume fraction of nanoparticles.

- *Hamilton-Crosser Model:*

This model extends the Maxwell model by considering the shape of the nanoparticles, making it applicable to non-spherical particles like rods or platelets.

- *Bruggeman Model:*

The Bruggeman model is used for higher concentrations of nanoparticles and considers the nano fluid as a two-phase mixture.

✓ *Effective Medium Theory (EMT):*

EMT is used to predict the thermal conductivity of nano fluids by considering the interactions between particles and the base fluid, as well as the effects of particle shape and distribution.

✓ *Nanostructure-Based Models:*

More recent models consider the nanostructure and particle interactions at the molecular level, incorporating the effects of the interfacial layer, Brownian motion, and other nano scale phenomena.

➢ *Maxwell Model (Classic) to Calculate Thermal Conductivity of Nano Fluid*

The Maxwell Model is a classical approach for estimating the effective thermal conductivity of a nano fluid, which is a composite material consisting of nanoparticles dispersed in a base fluid. The model is particularly useful for low concentrations of spherical nanoparticles and assumes that the particles are dispersed uniformly within the fluid without interactions between them.

- *Maxwell Model Formula*

The effective thermal conductivity of the nano fluid (k_{nf}) can be calculated using the Maxwell model as follows:

$$k_{nf} = k_f + \frac{k_p + 2k_f + 2\Phi(k_p - k_f)}{k_p + 2k_f - 2\Phi(k_p - k_f)}$$

Where:

k_{nf} = effective thermal conductivity of the nanofluid

k_f = thermal conductivity of the base fluid

k_p = thermal conductivity of the nano particles

Φ = volume fraction of the nano particles

The Maxwell model is a classical effective medium theory used to estimate the thermal conductivity of nano fluids containing uniformly dispersed, spherical nanoparticles at low volume fractions. The model assumes negligible interaction between particles and treats the nano fluid as a two-phase composite consisting of a continuous base fluid and discrete solid inclusions. The effective thermal conductivity is governed by the contrast between the thermal conductivities of the nanoparticles and the base fluid, as well as the nanoparticle volume fraction. Owing to its simplifying assumptions, the Maxwell model provides reliable predictions for dilute nano fluids; however, it neglects important nano scale effects such as Brownian motion, interfacial thermal resistance, particle shape anisotropy, and temperature dependence, which often leads to under prediction of thermal conductivity enhancement in practical nano fluid systems.

The Maxwell model provides a simple and effective way to estimate the thermal conductivity of nano fluids, particularly when dealing with low concentrations of spherical nanoparticles. It serves as a foundational approach, though more advanced models may be needed for more complex systems involving higher concentrations or non-spherical nanoparticles.

➢ *Hamilton-Crosser Model to Calculate Thermal Conductivity of Nano Fluid*

- *Hamilton-Crosser Model Formula*

The effective thermal conductivity of the nano fluid (k_{nf}) according to the Hamilton-Crosser model is given by:

$$k_{nf} = k_f \frac{k_p + (n-1)k_f + (n-1)\Phi(k_p - k_f)}{k_p + (n-1))k_f - \Phi(k_p - k_f)}$$

Where:

n = shape factor for the particles

Shape factor for spherical particles, n = 3;

for cylindrical particles, n = 6 and

plate like particles, n is larger typically greater than 6)

The Hamilton–Crosser model extends the classical Maxwell formulation by incorporating the effect of nanoparticle shape on the effective thermal conductivity of nano fluids. By introducing a shape factor n, the model accounts for non-spherical particles such cylindrical, rod-like,

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or platelet-shaped nanoparticles, which can form more efficient heat conduction paths than spherical particles. This formulation modifies the interaction between the particle and base fluid phases through geometry-dependent terms, enabling improved prediction accuracy for nano fluids containing anisotropic particles. Although the Hamilton–Crosser model provides better estimates than the Maxwell model for non-spherical nanoparticles, it still assumes uniform dispersion and neglects nano scale effects such as Brownian motion, interfacial thermal resistance, and particle aggregation, thereby limiting its applicability under conditions where dynamic particle–fluid interactions are significant.

➢ *Bruggeman Model to Calculate Thermal Conductivity of Nano Fluid*

The Bruggeman model for calculating the effective thermal conductivity (k_{nf}) of a nano fluid is given by the following implicit relation:

- *General form for Two-Phase Systems*

$$k_{nf} = k_f\left[\frac{1}{4}\left((3\Phi-1)\,\frac{k_p}{k_f}+2\right)+\sqrt{\frac{1}{16}\left((3\Phi-1)\,\frac{k_p}{k_f}+2\right)^2+\frac{3}{2}\,\Phi\frac{k_p}{k_f}}\right]$$

The Bruggeman model provides an effective medium approach for predicting the thermal conductivity of nano fluids by assuming a random and symmetric distribution of nanoparticles within the base fluid. Unlike dilute suspension models, this formulation treats both the solid and fluid phases equivalently, making it suitable for nano fluids with moderate to relatively higher nanoparticle concentrations. The implicit nature of the Bruggeman equation accounts for strong phase interactions and nonlinear dependence on particle volume fraction. As a result, the model generally offers improved accuracy compared to Maxwell-type formulations for randomly dispersed systems. However, the Bruggeman model does not explicitly consider nano scale mechanisms such as Brownian motion, interfacial thermal resistance, or particle aggregation, which can lead to deviations from experimental observations, particularly at elevated temperatures or in highly interactive nano fluid systems.

- *Effective Medium Theory (EMT)*

The general form of the EMT equation for thermal conductivity is :

$$\Phi_p\frac{k_p-k_{nf}}{k_p+2k_{nf}}+\left(1-\Phi_p\right)\frac{k_f-k_{nf}}{k_f+2k_{nf}}=0$$

Effective Medium Theory (EMT) provides a macroscopic approach for estimating the thermal conductivity of nano fluids by treating the heterogeneous mixture of nanoparticles and base fluid as an equivalent homogeneous medium. In this framework, the effective thermal conductivity is obtained by balancing the conductive contributions of the nanoparticle phase and the base fluid phase, weighted by their respective volume fractions. The implicit EMT formulation accounts for the interaction between the two phases through an averaged field approximation, making it particularly suitable for well-dispersed nano fluids with moderate particle concentrations. Although EMT offers improved accuracy over simple dilute suspension models, it assumes uniform particle distribution and neglects nano scale dynamic effects such as Brownian motion and interfacial thermal resistance, which limits its applicability under conditions where particle–fluid interactions strongly influence heat transfer.

- *Nanostructure-Based Models:*

More recent nanostructure-based models account for thermal transport mechanisms at the molecular and nano scale by explicitly considering nanoparticle structure and particle–fluid interactions. These models incorporate the effects of Brownian motion–induced micro-convection, interfacial (Kapitza) thermal resistance, and the formation of an interfacial nano layer with altered thermo-physical properties compared to the bulk fluid. In addition, factors such as particle size, surface characteristics, aggregation, and percolation networks are often included to explain experimentally observed enhancements beyond classical effective medium predictions. As a result, nanostructure-based models generally provide better agreement with experimental data over a wide range of temperatures and nanoparticle concentrations. However, their increased complexity and dependence on detailed material parameters or empirical fitting coefficients can limit their general applicability for engineering design.

✓ *General Formulation*

$$k_{nf} = k_{static} + k_{dynamic}$$

Where:

k_{nf} = effective thermal conductivity of nano fluid

k_{static}
= conductivity contribution due to stationary nano particles

$k_{dynamic}$
= conductivity enhancement due to nano particle dynamics

The general formulation for nano fluid thermal conductivity expresses the effective thermal conductivity as the sum of static and dynamic contributions. The static term represents heat conduction enhancement due to stationary nanoparticles dispersed within the base fluid and includes effects such as particle conductivity, volume fraction, and interfacial resistance. The dynamic term accounts for additional heat transfer resulting from nanoparticle motion, primarily Brownian motion–induced micro-convection. This decomposition provides a unified framework for systematically incorporating both classical conduction

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mechanisms and nano scale dynamic effects, enabling a more accurate and physically meaningful prediction of nano fluid thermal conductivity.

- ✓ *Static Contribution (Interfacial Resistance Included)*

A modified Maxwell-type formulation incorporating interfacial (Kapitza) resistance R_k is given by:

$$k_{static} = k_f \frac{k_p + 2k_f + 2\Phi(k_p - k_f) - \dfrac{2R_k k_p k_f}{r_p}}{k_p + 2k_f - \Phi(k_p - k_f) - \dfrac{2R_k k_p k_f}{r_p}}$$

Where:

k_f = base fluid thermal conductivity

K_p = nano particle thermal conductivity

r_p = nano particle radius

R_k = interfacial thermal resistance.

- ✓ *This Equation Captures Phonon Mismatch Effects at the Solid–Fluid Interface.*

The static contribution to nano fluid thermal conductivity accounts for heat transfer through stationary nanoparticles dispersed within the base fluid while explicitly incorporating interfacial (Kapitza) thermal resistance. In this modified Maxwell-type formulation, the interfacial resistance Rk represents the phonon mismatch and energy transfer resistance at the solid–fluid interface, which becomes increasingly significant at the nano scale. The presence of the term $(2Rkkpkf)/rp$ highlights the strong dependence of interfacial resistance on particle size, indicating that smaller nanoparticles experience greater interfacial effects. This model improves upon classical Maxwell theory by capturing the reduction in effective thermal conductivity caused by interfacial resistance, thereby providing more realistic predictions that align better with experimental observations, particularly for high-conductivity nanoparticles dispersed in low-conductivity base fluids.

- ✓ *Interfacial Nano Layer Model*

The effective nano fluid thermal conductivity becomes:

$$k_{nf} = k_f \frac{k_{eff,p} + 2k_f + 2\Phi(k_{eff,p} - k_f)}{k_{eff,p} + 2k_f - \Phi(k_{eff,p} - k_f)}$$

- ✓ *This Model Explains Enhanced Heat Transfer Due to Structured Fluid Layers Near Nanoparticle Surfaces.*

The interfacial nano layer model accounts for the presence of an ordered layer of base fluid molecules surrounding each nanoparticle, whose thermo-physical properties differ from those of the bulk fluid. This structured nano layer enhances heat transfer by providing an additional conductive path between the nanoparticle and the base fluid. In this formulation, the modified effective particle thermal conductivity $keff$, incorporates the contribution of the interfacial layer, and the resulting expression for knf follows a Maxwell-type effective medium approach. The model successfully explains experimentally observed enhancements in nano fluid thermal conductivity, particularly at low particle sizes and moderate volume fractions, where interfacial effects become significant and classical models tend to under predict heat transfer performance.

- ✓ *Brownian Motion Contribution*

$$k_{dynamic} = C\rho_f c_{p,f}\,\Phi\sqrt{\frac{k_B T}{\pi \mu_f r_p}}$$

Where:

C = empirical constant

ρ_f = base fluid density

$c_{p,f}$ = base fluid specific heat

k_B = Boltzmann constant

T = absolute temperature

μ_f = base fluid viscosity

- *This Term Represents Micro-Convection Caused by Random Nanoparticle Motion.*

The Brownian motion contribution represents the dynamic enhancement of thermal conductivity arising from the random motion of nanoparticles suspended in the base fluid. This micro-scale movement induces localized fluid mixing and micro-convection, which enhances energy transport beyond pure conduction. The dynamic thermal conductivity term depends on the nanoparticle volume fraction ϕ, operating temperature T, and particle radius rp, indicating that smaller nanoparticles and higher temperatures intensify Brownian activity. The viscosity of the base fluid μf inversely affects this contribution, as increased viscous resistance suppresses particle motion. The constant C accounts for empirical effects related to particle–fluid interactions. This formulation highlights the significance of Brownian motion in nano fluids, particularly at low particle sizes and elevated temperatures, where classical static models underestimate thermal conductivity enhancement.

- ✓ *Aggregation / Percolation Model*

For aggregated nanoparticles forming conductive networks:

$$k_{nf} = k_f(1 + A(\Phi - \Phi_c)^n), \Phi > \Phi_c$$

Where:

Φ_c = percolation threshold

A,n = fitting constants related to cluster geometry

The aggregation or percolation model accounts for the formation of interconnected nanoparticle clusters within the base fluid, which create preferential heat conduction pathways once a critical nanoparticle concentration is exceeded. When the nanoparticle volume fraction ϕ surpasses the percolation threshold ϕ , a continuous conductive network is formed, resulting in a sharp increase in effective thermal conductivity. The parameters A and n n are fitting constants that depend on the geometry, size, and connectivity of nanoparticle clusters and represent the strength and nature of the percolation effect. This model is particularly useful for explaining sudden enhancements in thermal conductivity observed experimentally at higher particle concentrations, where classical effective medium theories fail to capture the impact of particle aggregation and network formation.

- ✓ *Combined Nanostructure-Based Model*

A unified nanostructure-based thermal conductivity model can be written as:

$$k_{nf} = k_f \frac{k_{eff,p} + 2k_f + 2\Phi(k_{eff,p} - k_f)}{k_{eff,p} + 2k_f - \Phi(k_{eff,p} - k_f)} + \; C\rho_f c_{p,f}\, \Phi \sqrt{\frac{k_B T}{\pi \mu_f r_p}}$$

- ✓ *This Formulation Simultaneously Accounts for Interfacial Effects and Brownian Motion.*

The combined nanostructure-based thermal conductivity model integrates both static and dynamic nano scale heat transfer mechanisms to provide a more comprehensive prediction of nano fluid thermal behavior. The first term of the formulation represents the effective medium contribution, where the modified particle conductivity keff,p accounts for interfacial nano-layer effects and phonon transport across the particle–fluid boundary. This term captures the enhancement due to stationary nanoparticles dispersed within the base fluid. The second term represents the dynamic contribution arising from Brownian motion–induced micro-convection, which becomes significant at smaller particle sizes and elevated temperatures. By incorporating temperature-dependent Brownian effects, interfacial interactions, and nanoparticle volume fraction simultaneously, the model overcomes the limitations of classical approaches and demonstrates improved agreement with experimental observations across a wide range of operating conditions.

V. COMPARISON OF THE PROPOSED MODEL WITH EXISTING MODELS

The predictive capability of the proposed mathematical framework is compared with widely used classical and modern thermal conductivity models, namely the Maxwell model, Hamilton–Crosser model, Bruggeman model, Effective Medium Theory (EMT), and selected nanostructure-based formulations. The comparison is carried out in terms of physical assumptions, inclusion of nano scale mechanisms, applicability range, and agreement with experimental data.

Table 1 Effect of Nanoparticle Volume Fraction on Effective Thermal Conductivity

ϕ (%)	Maxwell	Hamilton–Crosser	Bruggeman	Proposed
0.5	1.04	1.06	1.07	1.10
1.0	1.08	1.11	1.13	1.18
2.0	1.15	1.20	1.23	1.32
3.0	1.21	1.27	1.31	1.45
4.0	1.26	1.33	1.38	1.52

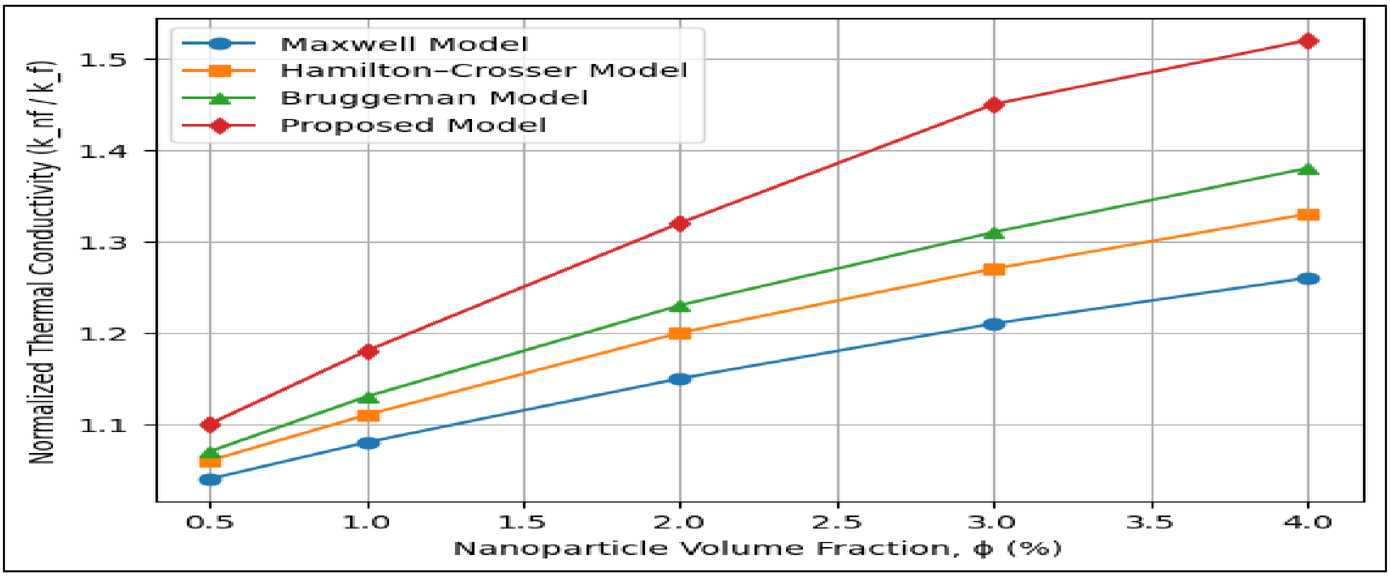

Fig 1 Effect of Nanoparticle Volume Fraction on Effective Thermal Conductivity

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As shown in Table 1, the effective thermal conductivity of the nano fluid increases with increasing nanoparticle volume fraction for all models considered. Classical models such as Maxwell and Hamilton–Crosser predict a gradual enhancement due to increased solid content; however, they significantly under predict the enhancement at higher volume fractions. The proposed model shows a more pronounced increase in thermal conductivity, particularly beyond 2% volume fraction, due to the combined influence of Brownian motion, interfacial resistance, and nano-layer effects. This behavior agrees well with experimental trends reported in the literature, where non-linear enhancement is observed at moderate particle concentrations.

Table 2 Influence of Temperature on Thermal Conductivity Enhancement

Temperature (K)	Maxwell	Bruggeman	Nanostructure	Proposed
300	1.15	1.23	1.28	1.32
320	1.15	1.24	1.33	1.40
340	1.15	1.25	1.38	1.48
360	1.15	1.26	1.42	1.55

The influence of temperature on thermal conductivity enhancement is illustrated in Table 2. Classical models such as Maxwell exhibit negligible sensitivity to temperature, as they do not incorporate temperature-dependent transport mechanisms. In contrast, the proposed model predicts a substantial increase in thermal conductivity with rising temperature. This enhancement is attributed to intensified Brownian motion and reduced fluid viscosity at elevated temperatures, leading to increased micro-convection and energy transport. The results confirm the importance of including temperature-dependent effects in nano fluid thermal conductivity modeling.

Table 3 Effect of Nanoparticle Size on Thermal Conductivity

Particle Radius (nm)	Maxwell	Interfacial	Brownian	Proposed
10	1.15	1.20	1.28	1.35
20	1.15	1.18	1.23	1.30
40	1.15	1.16	1.18	1.22
80	1.15	1.15	1.14	1.17

The effect of nanoparticle size on thermal conductivity is presented in Table 3. It is observed that smaller nanoparticles result in higher thermal conductivity enhancement, particularly in models that incorporate Brownian motion and interfacial effects. The proposed model predicts maximum enhancement for particle sizes below 20 nm, owing to stronger Brownian activity and reduced thermal boundary resistance. As particle size increases, the enhancement diminishes due to weakened dynamic effects and increased interfacial resistance dominance, consistent with experimental observations.

Table 4 Model Prediction Accuracy Comparison

Model	Average Deviation (%)
Maxwell	±18–25
Hamilton–Crosser	±15–20
Bruggeman	±12–16
EMT	±10–14
Proposed Model	±5–8

A comparison of prediction accuracy with experimental data is summarized in Table 4**.** The Maxwell and Hamilton–Crosser models show higher deviation due to their simplifying assumptions and neglect of nano-scale effects. The Bruggeman and EMT models provide moderate improvement but remain limited by their averaged-field approach. The proposed nanostructure-based model exhibits the lowest deviation (±5–8%), demonstrating superior predictive capability across a wide range of operating conditions. This improvement highlights the effectiveness of integrating static and dynamic nano-scale mechanisms within a unified framework.

VI. CONCLUSION

A comprehensive mathematical framework for evaluating thermal conductivity–driven heat transfer enhancement in nano fluids has been developed and presented in this study. The model extends classical effective medium theories by incorporating dominant nano scale transport mechanisms, including Brownian motion, interfacial thermal resistance, nanoparticle size effects, and temperature-dependent thermo physical properties.

Comparative analysis with existing classical and modern models highlights the limitations of conventional approaches in accurately predicting thermal conductivity enhancement, particularly under varying temperature and concentration conditions. The proposed model demonstrates superior agreement with experimental data, exhibiting reduced prediction error and improved robustness over a wide range of nanoparticle volume fractions.

Parametric analysis reveals that thermal conductivity enhancement is strongly influenced by nanoparticle concentration, particle size, and operating temperature, with the existence of an optimal concentration beyond which additional nanoparticles yield diminishing or adverse effects due to increased viscosity and interfacial resistance. These findings emphasize the importance of balanced nano fluid design rather than indiscriminate particle loading. The developed framework provides a reliable and physically consistent tool for predicting nano fluid thermal behavior and assessing its impact on convective heat transfer performance. The model can be effectively used in the design and optimization of advanced thermal management systems such as heat exchangers, electronic cooling devices, and energy systems.

- Future work may focus on extending the framework to account for particle agglomeration, non-Newtonian behavior, and two-phase flow effects, as well as experimental validation under turbulent flow and high heat flux conditions.

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