ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/25dec1100

Bianchi Type-V Model with Bulk Viscous Fluid in Presence of Electromagnetic Field in Modified Theory of Gravity

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Publication Date: 2026/01/06

Abstract: This paper, devoted to a Bianchi Type-V anisotropic model of the universe within framework of f (R, T) gravity, considering a bulk viscous fluid as the matter in the presence of an electromagnetic field. Field equations corresponding to the chosen metric are derived and solved under suitable physical assumptions relating the expansion scalar to the shear scalar. By evaluating the physical parameter, we examine the effect of matter and electromagnetic field on universe expanding nature.

Keywords: *Bianchi Type V Model, Bulk Viscous Fluid, Electromagnetic Field, Modified Theory of Gravity.*

How to Cite: L. S. Ladke; S. D. Mhashakhetri (2025) Bianchi Type-V Model with Bulk Viscous Fluid in Presence of Electromagnetic Field in Modified Theory of Gravity. *International Journal of Innovative Science and Research Technology*,10(12), 2445-2452. https://doi.org/10.38124/ijisrt/25dec1100

I. INTRODUCTION

Modern cosmology faces several fundamental challenges, including explaining expansion of the Universe, uncovering the nature of dark matter and dark energy, and the observed anisotropies in evolution of the universe. While General Relativity (GR) has been remarkably successful in describing a wide range of gravitational phenomena, it encounters limitations in addressing these issues without introducing exotic matter components. To overcome these shortcomings, modified theories of gravity such as *f(R), f(T), f(R,T),* scalar–tensor theories, and others have been proposed, these theories provide a more flexible and comprehensive framework for understanding the nature of the Universe. Offering potential explanations for cosmic acceleration and other cosmological observations without solely relying on unknown forms of matter or energy.“Recent high-precision observational data indicate that our universe is undergoing an accelerated expansion (Riess et al. [1]; Perlmutter et al. [2]). In addition, CMB radiation (Spergel et al. [3]) and large-scale structure surveys (Tegmark et al. [4]) also provide indirect evidence for this late-time cosmic acceleration. The phenomenon is generally attributed to the front of a mysterious component, termed *dark energy*. In this context, Nojiri and Odintsov [5] proposed a general framework for unifying the matter-dominated era with accelerated epoch within scalar–tensor theories and dark fluid models. Furthermore, Nojiri and Odintsov [6] discussed a comprehensive review of modified gravity theories, which have emerged as a viable gravitational alternative to explain dark energy.”

In an effort to explain the late-time cosmic acceleration without invoking exotic dark energy, modified gravity theory has gained considerable attention. Among these, the f (R, T) theory of gravity, introduced by Harko et al. [7], has emerged as a natural generalization of f (R) gravity. In this framework, the gravitational Lagrangian is taken as an arbitrary function of the R and T, allowing for a richer interaction between matter and geometry. The dependence on T arises from the consideration of quantum effects or imperfect fluid descriptions, thereby providing a new perspective on the matter–geometry coupling. This framework has been extensively explored for cosmological modelling, including anisotropic universes, bulk viscous fluids, and in the sight of electromagnetic fields.

In the explain of cosmological models, anisotropic Bianchi-type universes provide an important extension beyond the highly symmetric Friedmann–Lemaître–Robertson–Walker (FLRW) model. Among them, the Bianchi Type-V space–time is significantly as it represents a spatial homogeneous but anisotropic model with negative curvature often considered a generalization of open FLRW universe. Such models allow for a more detailed description of the early Universe, where anisotropies and nonequilibrium processes may have played a significant role before universe evolved toward the nearly isotropic state observed today.

Lorenz D. [8] introduced a cosmological solution of the source free Einstein-Maxwell equations with stiff matter and an electromagnetic null field, describing a locally rotationally symmetric (LRS) Bianchi Type-V universe. Atul Tyagi, Dhirendra Chhajed. [9] Studied Bianchi type IX model in presence of electromagnetic field. Chaubey, R. [10] studied Bianchi type-V bulk viscous cosmological models within framework of Brans-Dicke theory. Baghel, Prashant Singh et. al. [11] analyzed Bianchi type V model with bulk viscous fluid, considering a time varying gravitational constant G and cosmological term Λ. Ahmed and Pradhan [12] further investigated Bianchi type-V models in f (R, T) gravity, incorporating a time-dependent cosmological constant Λ(T) and analyzing the resulting dynamical behavior. Shamir and Ali [13] analyzed Bianchi Type-V models in the vicinity of magnetized anisotropic dark energy, focusing on their impact on the anisotropic expansion of the universe. Tiwari and Mishra [14] investigated Bianchi type-V cosmological models within the framework of f (R, T) gravity, analyzing their physical properties to understand the role of modified gravity in anisotropic cosmology. Hegazy [15] studied bulk viscous Bianchi type VI_0 cosmological models in both the self-creation theory of gravitation and general relativity, focusing on the influence of viscosity on the dynamics and evolution of anisotropic universes. Al-Haysah and Hasmani [16] derived exact solutions for various Bianchi-type models in f (R, T) theory, examined their physical and dynamical properties, to gain insights into anisotropic evolution in modified gravity theories. Bhardwaj, Rana, and Yadav [17] explored bulk viscous Bianchi type-V models under the formalism of f (R, T) $=f_1(R) + f_2(R)f_3(T)$ gravity, analyzing the combined impact of viscosity and modified gravity terms on the evolution of anisotropic universes. Vinutha and Sri Kavya [18] studied Bianchi-type models in f(R,T) gravity using a quadratic functional form, examining how modified gravity terms affect anisotropic expansion. Tiwari, Sofuoğlu, and Dubey [19] investigated phase transition behavior in locally rotational symmetric Bianchi-I models highlighting the influence of f(R, T) gravity on the dynamics of anisotropic universes. Yadav [20] analyzed models in f(R, T) gravity, emphasizing the influence of viscosity in progression of anisotropic universes, while Jokweni, Singh, and Beesham [21] focused on LRS Bianchi type-I models with bulk viscosity, in f(R,T) gravity, assessing its impact on anisotropic expansion dynamics. Brahma and Dewri [22] analyzed models of Bianchi-V within f(R,T) gravity, focusing on advancement of anisotropic universes under the influence of viscosity. Pawar and Katre [23] studied perfect fluid model of Bianchi-V models in f(R, T) gravity, emphasizing the effects of anisotropy on cosmic evolution. Mete, Ingle, and Valkunde [24] investigated locally rotationally symmetric Bianchi-I space-time in modified f(R, T) gravity, focusing on the dynamical evolution of anisotropic universes. Gaikwad and Mule [25] examined Bianchi type-V models in f(R,T) gravity, analyzing contribution of anisotropy in driving development of universe. Mete and Ingle [26] studied locally rotationally symmetric perfect fluid models in non-minimally coupled f(R,T) gravity, exploring the interplay between anisotropy, matter content, and modified gravity on cosmic evolution. Sharma, Singh, and Tiwari [27] conducted a detailed mathematical study of Bianchi type-V models in modified f(R, T) gravity, incorporating a varying deceleration parameter to analyze the expansion dynamics and anisotropic behavior of the universe.

In the present work, we focus on the construction and analysis of a Bianchi Type-V model in the region of electromagnetic field and a bulk viscous fluid in modified gravity theory. The study emphasizes the effect of matter and fields on evolution of cosmological model. Insights from this work may contribute to a better understanding both the early anisotropic universe and the mechanisms driving the present cosmic acceleration.

The paper organized as follows: In the section 2, we introduce the Bianchi Type-V space time and develop the corresponding field equations in modified theory. In section 3, we present, field equations solution, Section 4, discusses the physical parameters of the model and section 5 provides conclusions.

II. MODIfiED EINSTEIN'S FIELD EQUATIONS

➢ *The Action of Modified Theory is Given by*

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\ d^4x + \int L_m\sqrt{-g}\ d^4x, \quad (1)$$

Its trace is expressed as $T = g^{ij}T_{ij}$

The matter's energy momentum tensor is

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}} \quad (2)$$

Now the action S with respect to metric tensor g_{ij}, the field equation of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \quad (3)$$

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/25dec1100

Where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2}{\partial g^{ij}\partial g^{lm}} \quad (4)$$

Here $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$,

$\Box \equiv \nabla^i \nabla_i$ where ∇_i denotes the covariant derivatives.

If the matter is treated as perfect fluid, then T_{ij} for bulk viscous fluid in connection with an electromagnetic field is

$$T_{ij} = (\bar{p} + \rho)u_i u_j - \bar{p}g_{ij} - E_{ij} \quad (5)$$

Where $\bar{p} = p - 3\xi H = \omega\rho$

We consider ρ , $\bar{p}$ and $\xi(t)$ are function of t.

Here $\bar{p}$ is the total pressure which includes the proper pressure p , ρ be the rest energy density of the matter, $\xi(t)$ denoted as the coefficient of Bulk viscosity, $3\xi H$ is generally known as Bulk viscous pressure, E_{ij} is Electromagnetic energy tensor defines as

$$E_{ij} = -F_{ir}F^{kr}g_{kj} + \frac{1}{4}F_{ab}F^{ab}g_{ij} \quad (6)$$

Now we denote velocity vector is $u^i = (0,0,0,1)$ in commoving coordinates system satisfying the condition $u_i u^i = +1$, we assume that there is no unique choice for matter Lagrangian, $L_m = -\bar{p}$

We assume that F_{13} is non-vanishing component of (F_{ij}) Electromagnetic field tensor which corresponding to the pressure of magnetic field along y-direction.

➢ *The Energy Momentum Tensor is Given by*

$$T = \rho - 3\bar{p}$$

From Eqns (4), T_{ij} for perfect fluid becomes

$$\Theta_{ij} = -2T_{ij} - \bar{p}g_{ij} \quad (7)$$

In field equation, the matter field depends only on Θ_{ij}

Here apply three cases of Harko et.al.(2011). We can obtain choice of different explicit forms of $f(R,T)$ as

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} . \quad (8)$$

Here second case is used to describe the behaviour of the universe model in $f(R,T)$ as

$$f(R,T) = f_1(R) + f_2(T) \quad (9)$$

Now using Eqns (9) in Eqns (3)

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} = 8\pi T_{ij} + f_2'(T)T_{ij} + g_{ij}\left[\bar{p}f_2'(T) + \frac{1}{2}f_2(T)\right] \quad (10)$$

If we considered $f_1(R) = \lambda_1(R)$ and $f_2(R) = \lambda_2(T)$ then Eqns (9) gives

$$f(R,T) = \lambda_1(R) + \lambda_2(T)$$

Where λ_1 and λ_2 is arbitrary constant and in this condition the above Eqns (10) reduce to

$$R_{ij} - \frac{1}{2}Rg_{ij} = \frac{(8\pi + \lambda_2)}{\lambda_1}T_{ij} + g_{ij}\left[\bar{p} + \frac{1}{2}T\right]\frac{\lambda_2}{\lambda_1} \quad (11)$$

The metric in Bianchi type-V is

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 - C^2(t)dz^2] \quad (12)$$

The corresponding Ricci curvature is

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{C}\dot{B}}{CB} - \frac{3m^2}{A^2}\right] \quad (13)$$

The field Eqns (11) for Metric Eqns (12) can be founds as

$$\frac{m^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{C}\dot{B}}{CB} = \frac{(16\pi + 3\lambda_2)}{2\lambda_1}\bar{p} - \frac{\lambda_2\rho}{2\lambda_1} + \frac{(F_{13})^2 e^{-2mx}}{2A^2C^2}\left[\frac{(8\pi + 3\lambda_2)}{2\lambda_1}\right] \quad (14)$$

$$\frac{m^2}{A^2}-\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}-\frac{\dot{A}\dot{C}}{AC}=\frac{(16\pi+3\lambda_2)}{2\lambda_1}\,\overline{p}-\frac{\lambda_2\rho}{2\lambda_1}+\frac{(F_{13})^2e^{-2mx}}{2A^2C^2}\left[\frac{(8\pi+3\lambda_2)}{2\lambda_1}\right] \tag{15}$$

$$\frac{m^2}{A^2}-\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}-\frac{\dot{B}\dot{A}}{AB}=\frac{(16\pi+3\lambda_2)}{2\lambda_1}\,\overline{p}-\frac{\lambda_2\rho}{2\lambda_1}+\frac{(F_{13})^2e^{-2mx}}{2A^2C^2}\left[\frac{(8\pi+3\lambda_2)}{2\lambda_1}\right] \tag{16}$$

$$\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{A}\dot{C}}{AC}-\frac{\dot{C}\dot{B}}{CB}-\frac{3m^2}{A^2}=\frac{(16\pi+3\lambda_2)}{2\lambda_1}\rho-\frac{\lambda_2\overline{p}}{2\lambda_1}+\frac{(F_{13})^2e^{-2mx}}{2A^2C^2}\left[\frac{(8\pi+3\lambda_2)}{2\lambda_1}\right] \tag{17}$$

III. SOLUTIONS OF THE FIELD EQUATIONS

- *Subtract Eqns (14) from Eqns (15)*

$$\frac{\dot{B}}{B}-\frac{\dot{A}}{A}=0 \tag{18}$$

Integrate Eqns (18)

$$A=Bk \tag{19}$$

Where k = constant of integration, taking $k=1$, we have

$$A=B \tag{20}$$

Now by using this Eqns (20), the field Equations (14)-(17), reduce three non-linear independent equations with unknown $B, C, p, \rho \, and \, E_i$

$$\frac{m^2}{A^2}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}-\frac{\dot{C}\dot{B}}{CB}=\frac{(16\pi+3\lambda_2)}{2\lambda_1}\,\overline{p}-\frac{\lambda_2\rho}{2\lambda_1}+\frac{(F_{13})^2e^{-2mx}}{2A^2C^2}\left[\frac{(8\pi+3\lambda_2)}{2\lambda_1}\right] \tag{21}$$

$$\frac{m^2}{A^2}-\frac{2\ddot{B}}{B}-\frac{\dot{B}^2}{B^2}=\frac{(16\pi+3\lambda_2)}{2\lambda_1}\,\overline{p}-\frac{\lambda_2\rho}{2\lambda_1}+\frac{(F_{13})^2e^{-2mx}}{2A^2C^2}\left[\frac{(8\pi+3\lambda_2)}{2\lambda_1}\right] \tag{22}$$

$$\frac{\dot{B}^2}{B^2}-\frac{2\dot{C}\dot{B}}{CB}-\frac{3m^2}{A^2}=\frac{(16\pi+3\lambda_2)}{2\lambda_1}\rho-\frac{\lambda_2\overline{p}}{2\lambda_1}+\frac{(F_{13})^2e^{-2mx}}{2A^2C^2}\left[\frac{(8\pi+3\lambda_2)}{2\lambda_1}\right] \tag{23}$$

Adding Eqns (21) & Eqns (23), we get

$$\frac{\dot{B}^2}{B^2}-\frac{2m^2}{A^2}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{C}\dot{B}}{CB}=\frac{(16\pi+3\lambda_2)}{2\lambda_1}(\overline{p}+\rho)-\frac{\lambda_2}{2\lambda_1}(\overline{p}+\rho) \tag{24}$$

Adding Eqns (22) & Eqns (23), we obtained

$$\frac{2\dot{C}\dot{B}}{CB}-\frac{2m^2}{A^2}-\frac{2\ddot{B}}{B}=\frac{(16\pi+3\lambda_2)}{2\lambda_1}(\overline{p}+\rho)-\frac{\lambda_2}{2\lambda_1}(\overline{p}+\rho) \tag{25}$$

Subtracts Eqns (24) from Eqns (25)

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/25dec1100

$$\frac{\ddot{C}}{C}+\frac{\dot{C}\dot{B}}{CB}-\frac{\dot{B}^2}{B^2}-\frac{\ddot{B}}{B}=0 \tag{26}$$

Assuming that the expansion scalar is proportional to shear scalar, $\sigma^2 \alpha\ \theta$ gives a relation among the metric potentials, which we take as

$$B = C^n,\ n \neq 0 \tag{27}$$

Where n is arbitrary constant.

Inserting Eqns (26) with the help of Eqns (20) & Eqns (27), we obtain solution as

$$A=\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{\frac{n}{2n+1}} \tag{28}$$

$$B=\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{\frac{n}{2n+1}} \tag{29}$$

$$C=\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{\frac{1}{2n+1}} \tag{30}$$

Where $k_1 \& k_2$ are constant of integration.

Thus, line element is now becoming

$$ds^2 = dt^2 - \left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{\frac{2n}{2n+1}} dx^2 - e^{2mx}\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{\frac{2n}{2n+1}} dy^2 - e^{2mx}\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{\frac{2}{2n+1}} dz^2 \tag{31}$$

- *The Physical Character of the Universe*

The spatial volume V and Average scale factor a for model are given by

$$V = e^{2mx}\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right] \text{and } a = e^{\frac{2mx}{3}}\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{\frac{1}{3}} \tag{32}$$

The spatial volume V increases with time t if $1>0$ model is expanding

The directional Hubble's parameters H_1, H_2 and H_3 in direction of x, y and z axis are

$$H_1 = \frac{nk_1}{(1-n)}\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{-1} H_2 = \frac{nk_1}{(1-n)}\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{-1} \& H_3 = \frac{k_1}{(1-n)}\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{-1}$$

Whereas the Average generalized Hubble's parameters H is given by

$$H = \frac{(2n+1)k_1}{3(1-n)}\left[\frac{(1+2n)k_1 t}{(1-n)}+k_2\right]^{-1} \tag{33}$$

The dynamical parameters as the expansion scalar θ, the shear scalar σ^2 and the mean anisotropy parameters A_m

$$\theta = \frac{(2n+1)k_1}{(1-n)}\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{-1} \tag{34}$$

$$\sigma^2 = \frac{k_1^2}{3}\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{-2} \tag{35}$$

$$A_m = \frac{(1+2n)^2}{2(1-n)^2} \tag{36}$$

The corresponding Ricci Curvature

$$R = 2\left[\frac{(2n+2-n^2)k_1^2}{(1-n)^2}\right]\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{-2} + 6m^2\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{\frac{-2n}{2n+1}} \tag{37}$$

From Eqn (25), Energy density of the matter are given by

$$\rho = \left[\frac{2n(n+2)k_1^2\lambda_1}{(1-n)^2(\omega-1)(8\pi+\lambda_2)}\right]\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{-2} - \frac{2m^2\lambda_1}{(\omega-1)(8\pi+\lambda_2)}\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{\frac{-2n}{2n+1}} \tag{38}$$

Total pressure and proper pressure given as

$$\bar{p} = \left[\frac{2n\omega(n+2)k_1^2\lambda_1}{(1-n)^2(\omega-1)(8\pi+\lambda_2)}\right]\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{-2} - \frac{2m^2\omega\lambda_1}{(\omega-1)(8\pi+\lambda_2)}\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{\frac{-2n}{2n+1}} \tag{39}$$

$$p = \left[\frac{2n\omega_0(n+2)k_1^2\lambda_1}{(1-n)^2(\omega-1)(8\pi+\lambda_2)}\right]\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{-2} - \frac{2m^2\omega_0\lambda_1}{(\omega-1)(8\pi+\lambda_2)}\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{\frac{-2n}{2n+1}} \tag{40}$$

Bulk viscous pressure of matter

$$3\xi H = \left[\frac{2n(\omega_0-\omega)(n+2)k_1^2\lambda_1}{(1-n)^2(\omega-1)(8\pi+\lambda_2)}\right]\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{-2} - \frac{2m^2(\omega_0-\omega)\lambda_1}{(\omega-1)(8\pi+\lambda_2)}\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{\frac{-2n}{2n+1}} \tag{41}$$

Coefficient of bulk viscosity

$$\xi = \left[\frac{2n(\omega_0-\omega)(n+2)k_1\lambda_1}{(1-n)(2n+1)(\omega-1)(8\pi+\lambda_2)}\right]\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{-1} - \frac{2m^2(\omega_0-\omega)(1-n)\lambda_1}{(\omega-1)(2n+1)(8\pi+\lambda_2)k_1}\left[\frac{(1+2n)k_1 t}{(1-n)} + k_2\right]^{\frac{-2n}{2n+1}+1} \tag{42}$$

IV. CONCLUSION

Here, we analyzed a Bianchi type-V anisotropic cosmological model in f(*R*, *T)* gravity theory, where bulk viscous fluid is taken as matter with impact of electromagnetic field. As time increases expansion of model decreases and it stop at $t=\infty$.

ISSN No: -2456-2165 https://doi.org/10.38124/ijisrt/25dec1100

The ratio $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}(\neq 0)$ is constant; it implies that this model does not approach isotropy for large value of t. When $k_2 \to 0$, then $\theta = \frac{1}{t}$ which show that the enlargement of the model tends to zero for large value of cosmic time t in the absence of field. From expression for the spatial volume, it is observed that volume increases with time, indicating model is expanding. The positive value of the Hubble parameter further signifies an accelerated expansion of the model. Energy density of the model seems to be decreasing function of time, and the bulk viscous pressure also decreases as cosmic time increases. As A_m is not depending on cosmic time, it remains constant throughout the evolution. These results show that model presented here corresponding to an expanding and accelerating universe. Therefore, the model provides a feasible framework for describing both the early and the present evolution of the universe.

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