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Regional Rectangular Harmonic Analysis (R-Rha) Applied to Model the Earth's Magnetic Field of Madagascar Island

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Abstract : This Article Provides A Detailed Description of The Regional Modeling Formalism Within A Rectangular Domain. This Approach Called Regional Rectangular Harmonic Analysis (R-RHA), Could Be Particularly Useful For Utilizing, For The First Time, Data From Malagasy Repeat Stations That Have Been Reoccupied Since 1983. Existing Regional Modeling Techniques Are Not Well Suited For Madagascar, As They Typically Require A Large Amount Of Data To Be Applied Correctly. However, Madagascar Has Only 25 Repeat Stations In Total, And Not All Of Them Are Reoccupied During A Magnetic Survey. The Results Obtained Using The Optimal Parameters Of The Rectangular Model Confirm Its Validity For Madagascar. Its Reliability Remains Limited To Areas Covered By Measurements. Even Though The Dataset Is Not Extensive, At Least One Measurement Is Required At Each Of The Following Stations : Antsiranana, Mahajanga, Toamasina, Toliary, And Taolagnaro. The Lack Of Data From Any Of These Regions Would Prevent The Proper Development Of Magnetic Maps Of Madagascar Using This Formalism.

Keywords : *Regional Modeling, Magnetic Field, Repeat Stations, Rectangular Domain, Madagascar.*

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I. INTRODUCTION

Regional modeling aims to describe the magnetic field within a given region. Several methods, taking into account the nature of the magnetic potential, have been proposed for regional-scale magnetic field modeling. For partial modeling, spherical harmonics (SH), commonly known as the global model, are no longer suitable because the field is no longer orthogonal when applied to specific regions. Simple techniques, such as surface polynomial modeling or rectangular harmonic modeling (Alldredge, 1981), were used before satellite data became available, but the models obtained through these methods were not successfully established (Haines, 1990).

Spherical Cap Harmonic Analysis (SCHA), proposed by Haines (1985), is an appealing modeling approach. Its formalism resembles a natural extension of spherical harmonic analysis. This method is claimed to be valid across the entire spherical cap at all altitudes above the Earth's surface. Based on these claims, SCHA has been used to model crustal anomalies (De Santis et al., 1989), the main field (Hwang & Chen, 1997), and even secular variation (Korte & Haak, 2000).

However, practitioners of SCHA encounter two main challenges. First, the convergence of SCHA is extremely slow for small caps, and an insufficient expansion leads to unrealistic oscillations when interpolating over a dense grid. A larger expansion, however, requires a higher number of data points. Second, Haines' formalism is partially incorrect because it only imposes boundary conditions on the lateral surface of the cap. This results in inconsistencies in the expressions of Gauss coefficients with altitude since the quantitative results are highly dependent on the opening angle of the spherical cap. Consequently, the horizontal and vertical components of the field cannot be expressed in terms of the same Gauss coefficients.

Thébault (2003) corrected Haines' formalism by adding boundary conditions on both the lower surface (Earth's surface) and the upper surface (high enough to include all available data). This formalism, known as Regional Spherical Cap Harmonic Analysis (R-SCHA), produces satisfactory results when both ground-based and satellite data are available. However, numerical issues persist when only ground-based data is available, as is the case in Madagascar.

Malagasy repeat station data is very limited. To reasonably utilize this data, a technique capable of

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incorporating only a small number of ground-based measurements is necessary. To achieve this, we propose reformulating rectangular harmonic modeling (RHA) (Alldredge,1981) by adding boundary conditions on all six surfaces that define the rectangular domain.

II. METHOD AND DATA USED

- *Geometry and Coordinate System*

The study domain Ω is a rectangular parallelepiped defined by the volume $-x_0 \le x \le x_0$, $-y_0 \le y \le y_0$, and $-z_0 \le z \le z_0$, with the key property that it contains no sources of the magnetic field. The Ox axis forms an angle μ (expressed in degrees) with the East-West direction. This rotation angle μ is adjusted to maximize the density of data within the domain Ω.

Consider a point P with coordinates (λ, φ, h) in the geographic reference frame, where λ represents latitude (in degrees), φ longitude (in degrees), and h altitude (in kilometers). Let (λ_0, φ_0, h_0) be the coordinates of the origin O in the rectangular reference frame, and (x, y, z) the coordinates of point P (expressed in kilometers) in the rectangular system. These coordinates are related by the following equation (1a) :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\mu & \sin\mu & 0 \\ -\sin\mu & \cos\mu & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_x(\varphi-\varphi_0) \\ C_y(\lambda-\lambda_0) \\ h-h_0 \end{pmatrix} \tag{1a}$$

The coefficients C_x and C_y convert degrees into kilometers and are expressed in km/degree. C_x corresponds to a one-degree variation in longitude for a given latitude. Using classical spherical geometry and assuming Earth is a sphere with radius R = 6371.2 km, C_x is given by :

$$C_x = R\cos^{-1}\left[\sin^2\lambda + \cos^2\lambda\cos(\pi/180)\right] \tag{1b}$$

C_y corresponds to a one-degree variation in latitude. Its value is constant and given by :

$$C_y = \pi R/180 \tag{1c}$$

For the portion of the Earth's surface to be approximated as a plane, the boundary values x_0 and y_0 in degrees must be less than 8°. In this case, the coordinates x and y are determined with an accuracy better than 1 km, which is acceptable considering that the distance between two neighboring stations in Madagascar can exceed 100 km.

- *Problem Formulation*

Within the domain Ω, the magnetic potential V satisfies various boundary condition problems. By imposing mixed boundary conditions on the potential and its derivatives, we obtain the example of following problems:

$$\{\Delta V=0\}\{(V)_{\partial_x\Omega}=F(y,z)\}\{(V)_{\partial_y\Omega}=G(x,z)\} \tag{2a}$$

$$\{\Delta V=0\}\left\{\left(\frac{\partial V}{\partial n_x}\right)_{\partial_x\Omega}=F(y,z)\right\}\left\{\left(\frac{\partial V}{\partial n_y}\right)_{\partial_y\Omega}=G(x,z)\right\} \tag{2b}$$

The solution to such a mixed problem is not unique. The boundary conditions F, G, and H must also satisfy the divergence-free condition or flux condition. Applying Ostrogradsky-Gauss theorem to the domain Ω, we obtain:

$$\varphi = \iint_{\partial_x\Omega}\frac{\partial V}{\partial n_x}ds_x + \iint_{\partial_y\Omega}\frac{\partial V}{\partial n_y}ds_y + \iint_{\partial_z\Omega}\frac{\partial V}{\partial n_z}ds_z = 0 \tag{2c}$$

After extensive studies, the gradients of the potential were found to be orthogonal for these two problems, and their solutions converge slowly. The corresponding edge effects are relatively weak. Since these two mixed problems are equivalent, we will consider the first problem. The representation of the magnetic field using this decomposition also demonstrates strong consistency with the convergence analysis results. Even though the direct problem appears to converge slowly, there is no indication that the inverse problem cannot find coefficients that accurately fit the magnetic field.

- *Expression of the Potential V*

The main objective in solving a boundary value problem is to reconstruct the basis functions that generate a space in which the solution can be expressed. However, each problem, when considered individually, can only be easily solved if homogeneous conditions are introduced. To achieve this, the previous problem 2a is broken down into three subproblems, whose sum of solutions forms the general solution :

$$V = V1+V2+V3 \tag{3}$$

avec

$$\begin{cases} \Delta V_1 = 0 \\ (V_1)_{\partial_x\Omega} = F(y,z) \\ (V_1)_{\partial_y\Omega} = 0 \\ \left(\dfrac{\partial V_1}{\partial n_z}\right)_{\partial_z\Omega} = 0 \end{cases}$$

$$\begin{cases} \Delta V_2 = 0 \\ (V_2)_{\partial_x\Omega} = 0 \\ (V_2)_{\partial_y\Omega} = G(x,z) \\ \left(\dfrac{\partial V_2}{\partial n_z}\right)_{\partial_z\Omega} = 0 \end{cases}$$

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$$\begin{cases} \Delta V_3 = 0 \\ \left(V_3\right)_{\partial_x \Omega} = 0 \\ \left(V_3\right)_{\partial_y \Omega} = 0 \\ \left(\dfrac{\partial V_3}{\partial n_z}\right)_{\partial_z \Omega} = H(x,y) \end{cases}$$

Each potential is determined using the method of separation of variables, where the solutions are sought independently in and :

Vi(x,y,z) = Vix(x) Viy(y) Viz(z) (4)

For i=1, 2, 3

By substituting expression (4) into Vi=0 , we obtain :

$$\frac{1}{V_{ix}}\frac{d^2V_{ix}}{dx^2} + \frac{1}{V_{iy}}\frac{d^2V_{iy}}{dy^2} + \frac{1}{V_{iz}}\frac{d^2V_{iz}}{dz^2} = 0 \tag{5a}$$

The three terms in the first member of equation (5a) depend only on , , and , respectively. Therefore, each of themust be equal to a constant, leading to the following system of three ordinary differential equations :

$$\begin{cases} \dfrac{d^2V_{ix}}{dx^2} = k_x V_{ix} \\ \dfrac{d^2V_{iy}}{dy^2} = k_y V_{iy} \\ \dfrac{d^2V_{iz}}{dz^2} = k_z V_{iz} \end{cases} \tag{5b}$$

with $k_x + k_y + k_z = 0$

In other words, the potentials Vix, Viy and Viz are eigenfunctions of the « second derivative » operator, associated respectively with the eigenvalues kx, ky and kz. . The corresponding general solution is given by :

$$\begin{array}{ll} V_{ix}(x) = A_i \cos(\sqrt{-k_x}\,x) + B_i \sin(\sqrt{-k_x}\,x) & \text{si } k_x < 0 \\ V_{ix}(x) = A_i x + B_i & \text{si } k_x = 0 \\ V_{ix}(x) = A_i e^{\sqrt{k_x}\,x} + B_i e^{-\sqrt{k_x}\,x} & \text{si } k_x > 0 \end{array} \tag{5c}$$

Where Ai and Bi are constants of integration. A similar reasoning applies to and.

Taking into account the boundary conditions for the potentials, the eigenvalues become functions of the limits and two natural integers and. In order to obtain sufficiently complete basis functions, the final solution must be the sum of all possible elementary solutions, yielding :

$$V_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_1^{m,n} F_{a1}^{m,n} + B_1^{m,n} F_{b1}^{m,n} \tag{6a}$$

$$V_2 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_2^{m,n} F_{a2}^{m,n} + B_2^{m,n} F_{b2}^{m,n} \tag{6c}$$

The basis functions used in expressions (6a) to (6c) are such that :

$$F_{a1}^{m,n} = \left[\cos\frac{\pi m y}{2y_0}\cos\frac{\pi n z}{2z_0} + (-1)^{n+1}\cos\frac{\pi m y}{2y_0}\sin\frac{\pi n z}{2z_0} + (-1)^{m+1}\sin\frac{\pi m y}{2y_0}\cos\frac{\pi n z}{2z_0} + (-1)^{m+n}\sin\frac{\pi m y}{2y_0}\sin\frac{\pi n z}{2z_0}\right] e^{\frac{\pi}{2}\sqrt{\frac{m^2}{y_0^2}+\frac{n^2}{z_0^2}}\,x} \tag{7a}$$

$$F_{b1}^{m,n} = \left[\cos\frac{\pi m y}{2y_0}\cos\frac{\pi n z}{2z_0} + (-1)^{n+1}\cos\frac{\pi m y}{2y_0}\sin\frac{\pi n z}{2z_0} + (-1)^{m+1}\sin\frac{\pi m y}{2y_0}\cos\frac{\pi n z}{2z_0} + (-1)^{m+n}\sin\frac{\pi m y}{2y_0}\sin\frac{\pi n z}{2z_0}\right] e^{-\frac{\pi}{2}\sqrt{\frac{m^2}{y_0^2}+\frac{n^2}{z_0^2}}\,x} \tag{7b}$$

$$F_{a2}^{m,n} = \left[\cos\frac{\pi nz}{2z_0}\cos\frac{\pi nx}{2x_0} + (-1)^{n+1}\cos\frac{\pi nz}{2z_0}\sin\frac{\pi nx}{2x_0} + (-1)^{m+1}\sin\frac{\pi nz}{2z_0}\cos\frac{\pi nx}{2x_0} + (-1)^{m+n}\sin\frac{\pi nz}{2z_0}\sin\frac{\pi nx}{2x_0}\right] e^{\frac{\pi}{2}\sqrt{\frac{m^2}{z_0^2}+\frac{n^2}{x_0^2}}\,y} \tag{7c}$$

$$F_{b2}^{m,n} = \left[\cos\frac{\pi nz}{2z_0}\cos\frac{\pi nx}{2x_0} + (-1)^{n+1}\cos\frac{\pi nz}{2z_0}\sin\frac{\pi nx}{2x_0} + (-1)^{m+1}\sin\frac{\pi nz}{2z_0}\cos\frac{\pi nx}{2x_0} + (-1)^{m+n}\sin\frac{\pi nz}{2z_0}\sin\frac{\pi nx}{2x_0}\right] e^{-\frac{\pi}{2}\sqrt{\frac{m^2}{z_0^2}+\frac{n^2}{x_0^2}}\,y} \tag{7d}$$

$$G_{a3}^{m,n} = \left[\cos\frac{\pi mx}{2x_0}\cos\frac{\pi ny}{2y_0} + (-1)^{n}\cos\frac{\pi mx}{2x_0}\sin\frac{\pi ny}{2y_0} + (-1)^{m}\sin\frac{\pi mx}{2x_0}\cos\frac{\pi ny}{2y_0} + (-1)^{m+n}\sin\frac{\pi mx}{2x_0}\sin\frac{\pi ny}{2y_0}\right] e^{\frac{\pi}{2}\sqrt{\frac{m^2}{x_0^2}+\frac{n^2}{y_0^2}}\,z} \tag{7e}$$

$$G_{b3}^{m,n} = \left[\cos\frac{\pi mx}{2x_0}\cos\frac{\pi ny}{2y_0} + (-1)^{n}\cos\frac{\pi mx}{2x_0}\sin\frac{\pi ny}{2y_0} + (-1)^{m}\sin\frac{\pi mx}{2x_0}\cos\frac{\pi ny}{2y_0} + (-1)^{m+n}\sin\frac{\pi mx}{2x_0}\sin\frac{\pi ny}{2y_0}\right] e^{-\frac{\pi}{2}\sqrt{\frac{m^2}{x_0^2}+\frac{n^2}{y_0^2}}\,z} \tag{7f}$$

➢ *Expression Of The Field And Inverse Problem*

We start with the components X (North), Y (East), and Z (vertical, directed positively downwards) of the magnetic field in the local geographic reference frame. The rectangular components Bx, By, and Bz of the field are given by :

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} \cos\mu & \sin\mu & 0 \\ -\sin\mu & \cos\mu & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} y \\ X \\ -Z \end{pmatrix} \tag{8}$$

Fig 1where M Is The Rotation Angle Illustrated In

However, the magnetic field in the domain Ω is expressed as the gradient of the potential V, defined by relation (3) and expressions (6a) to (6c). In practice, each development is limited by maximum truncation indices Mmax and Nmax, and the field components are expressed as :

$$B_x = \sum_{m=1}^{M_{max}}\sum_{n=1}^{N_{max}}\left[A_1^{m.n}\frac{\partial F_{a1}^{m.n}}{\partial x} + B_1^{m.n}\frac{\partial F_{b1}^{m.n}}{\partial x} + A_2^{m.n}\frac{\partial F_{a2}^{m.n}}{\partial x} + B_2^{m.n}\frac{\partial F_{b2}^{m.n}}{\partial x} + A_3^{m.n}\frac{\partial G_{a3}^{m.n}}{\partial x} + B_3^{m.n}\frac{\partial G_{b3}^{m.n}}{\partial x}\right] \tag{9a}$$

$$B_y = \sum_{m=1}^{M_{max}}\sum_{n=1}^{N_{max}}\left[A_1^{m.n}\frac{\partial F_{a1}^{m.n}}{\partial y} + B_1^{m.n}\frac{\partial F_{b1}^{m.n}}{\partial y} + A_2^{m.n}\frac{\partial F_{a2}^{m.n}}{\partial y} + B_2^{m.n}\frac{\partial F_{b2}^{m.n}}{\partial y} + A_3^{m.n}\frac{\partial G_{a3}^{m.n}}{\partial y} + B_3^{m.n}\frac{\partial G_{b3}^{m.n}}{\partial y}\right] \tag{9b}$$

$$B_z = \sum_{m=1}^{M_{max}}\sum_{n=1}^{N_{max}}\left[A_1^{m.n}\frac{\partial F_{a1}^{m.n}}{\partial z} + B_1^{m.n}\frac{\partial F_{b1}^{m.n}}{\partial z} + A_2^{m.n}\frac{\partial F_{a2}^{m.n}}{\partial z} + B_2^{m.n}\frac{\partial F_{b2}^{m.n}}{\partial z} + A_3^{m.n}\frac{\partial G_{a3}^{m.n}}{\partial z} + B_3^{m.n}\frac{\partial G_{b3}^{m.n}}{\partial z}\right] \tag{9c}$$

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Equations (9a) to (9c) are linear in the Gauss coefficients $A_i^{m,n}$ and $B_i^{m,n}$ (for i = 1, 2, 3) and can be written as:

$$D = FP \tag{10a}$$

Where D is the data vector. Let N_D be the number of data points to be introduced into the model. D is a column vector consisting of $3N_D$ rows (since there are three components per data point).

P is the parameter vector formed by the Gauss coefficients, arranged starting with the letters A and B in a one-dimensional order. For given values of Mmax and Nmax, there are :

$$N_P = 6M_{max}N_{max} \tag{10b}$$

parameters to be determined, and P is a column vector with NP rows.

F is the function matrix formed of 3ND rows and NP columns. Its elements are the basis functions defined by formulas (7a) to (7f).

Since the data are assumed to be noisy, the classical statistical model is used:

$$D = FP + \varepsilon \tag{10c}$$

where ε is a Gaussian random variable with zero mean and variance σ^2.

The solution to equation (10c) can be obtained by the least squares method, which is fast in terms of convergence, and the vector P is obtained by :

$$P = (FtF)^{-1} FtD \tag{10d}$$

Where F^t denotes the transpose of F.

Here, we consider equal weight for all data, and we impose no constraints in our inversion. We also limit ourselves to the case where the Gauss coefficients are constant, i.e., the data used are assumed to be taken at the same time. In practice, this assumption holds in the data were acquired over a period of about one month.

- *Data Used and Error Estimation*

Since this is a first attempt at inversion using the previous formalism, we first need to consider synthetic data calculated by a global model to test its validity. This is the normal procedure in such problems. We will then verify whether our formalism is capable of representing a magnetic field while determining the best possible developments and the reconstruction error resulting from the choice of truncation indices.

The creation of synthetic data is done according to the following steps: first, we create fictitious measurement points, either regularly or randomly distributed, in sufficient numbers such that the grid spacing is approximately 100 km. For the case of Madagascar, we will consider N_D = 7x17 = 119 points, corresponding to a spacing of approximately 102 km. Then, we calculate the field at each point using the CM4 global model (Sabaka et al., 2004). We chose the CM4 model because it better calculates the internal field (with continuous secular variation) compared to the international reference model IGRF (which represents secular variation jumps every five years). After, we add Gaussian white noise with zero mean and standard deviation σ = 5nT (absolute uncertainty in determining the components of the internal field).

Once the data are prepared, we can determine the parameter vector P. Knowing this, we can determine the field calculated by the rectangular model. In accordance with statistical analysis practices, we focus on the residuals, whose means are defined by :

$$\rho_X = \frac{1}{N_D}\sum_{i=1}^{N_D}(X_i - X_{Ci}) \tag{11a}$$

$$\rho_y = \frac{1}{N_D}\sum_{i=1}^{N_D}(y_i - y_{Ci}) \tag{11b}$$

$$\rho_Z = \frac{1}{N_D}\sum_{i=1}^{N_D}(Z_i - Z_{Ci}) \tag{11c}$$

where XCi, YCi, and ZCi are the components estimated by the rectangular model.

In ordinary least squares inversion, the mean residuals should be close to zero if the model is in good agreement with the data. In this case, the model error is classically evaluated by :

$$\sigma_X = \sqrt{\frac{1}{N_D}\sum_{i=1}^{N_D}(X - X_{Ci} - \rho_X)^2} \tag{12a}$$

$$\sigma_y = \sqrt{\frac{1}{N_D}\sum_{i=1}^{N_D}(y - y_{Ci} - \rho_y)^2} \tag{12b}$$

$$\sigma_Z = \sqrt{\frac{1}{N_D}\sum_{i=1}^{N_D}(Z - Z_{Ci} - \rho_Z)^2} \tag{12c}$$

Once the validity of the rectangular model is verified, we can then apply it to the actual data from the Malagasy reoccupied repetition stations since 1983, the numbers of which are given in Table 1.

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Table1 Number Of Available Data For Malagasy Reoccupied Repetition Stations Since 1983.

Year	**1983**	**1990**
Number of Measurements	**6**	**13**
Stations	TAN-TUL-FDF-FNV-DGS-MJG	TAN-FDF-MJG-FNV-AZK-TMV-DGS-SBV-TLH-VHM-PBG-THH-ABJ

The spatial distribution of the measurements for each magnetic campaign will be indicated on the magnetic maps of Madagascar that we will create.

Table 2 Malagasy Reoccupied Repetition Stations Since 1983.

Station	**Nom**	**Coordonnées Géographiques**		
		Latitude	**Longitude**	**Altitude**
DGS	Antsiranana	-12°21’00 ‘’	49°17’40’’	74m
ABB	Ambilobe	-13°11’24’’	48°58’54’’	121m
VHM	Vohemar	-13°22’04 ‘’	50°00’00’’	67m
ABJ	Ambanja	-13°38’24’’	48°27’06’’	83m
SBV	Sambava	-14°16’39’’	50°10’27’’	148m
TLH	Antalaha	-14°59’51 ‘’	50°19’16’’	73m
THH	Antsohihy	-14°54’00’’	47°39’00’’	269m
PBG	Port bergé	-15°34’48’’	47°37’18’’	308m
MJG	Mahajanga	-15°39’57’’	46°21’03’’	44m
MVT	Maevatanana	-16°57’11’’	46°49’57’’	1013m
KZB	Ankazobe	-18°19’49’’	47°07’35’’	1213m
FNV	Fenoarivo Est	-17°25’30’’	49°26’06’’	42m
AZK	Ambatondrazaka	-17°47’32’’	48°26’12’’	650m
TMV	Toamasina	-18°07’00’’	49°23’36’’	36m
MRG	Moramanga	-18°54’48’’	48°12’54’’	517m
TRB	Antsirabe	-19°49’50’’	47°03’04’’	1493m
ABS	Ambositra	-20°32’51’’	47°14’40’’	868m
MNJ	Mananjary	-21°12’17’’	48°21’24’’	54m
FNT	Fianarantsoa	-21°26’15’’	47°07’06’’	1127m
IHS	Ihosy	-22°24’34’’	46°10’07’’	971m
MDV	Morondava	-20°17’24’’	44°21’00’’	88m
MRB	Morombe	-21°45’18’’	43°32’18’’	94m
TUL	Toliara	-23°23’12’’	43°49’30’’	56m
FDF	Taolanaro	-25°02’00’’	46°57’36’’	114m
TAN	Antananarivo	18°55’00’’	47°33’00’’	1375m

III. RESULTS AND DISCUSSIONS

The most appropriate parameters for the rectangular domain in the case of Madagascar are as follows: $\lambda_o = -18.52°$, $\varphi_o = 46.55°$, $h_o = 0.765$ km (average of the altitudes of all the repetition stations), $x_o = 321.871$ km (or 3.06°), $y_o = 811.140$ km (or 7.31°), $z_o = 0.729$ km, and $\mu = -18.0°$.

The validity of our formalism is primarily checked by examining the evolution of the global modeling errors.

- *Evolution of Reconstruction Errors*

The figure 1 shows the global evolution of reconstruction errors as a function of the truncation indices Mmax and Nmax. To standardize the representations, we limited the average error between 0 and 5nT, and the standard deviation between 0 and 50nT for all components of the magnetic field. Thus, the red color corresponds to absolute values greater than or equal to 5nT for the average error and 50nT for the standard deviation. It is worth noting that the 50nT value corresponds to the uncertainty in determining the internal field using a global model. The presence of the green color, which corresponds to absolute values less than 1nT for the average error and 10nT for the standard deviation, demonstrates the ability of our formalism to model the Earth's magnetic field for well-chosen values of Mmax and Nmax.

Since we have N_D=119 total data points and the number of equations ($3N_D$=357) must be greater than or equal to the number of unknowns N_P given by relation (10b), we focus on the value of Mmax or Nmax such that one of the truncation indices Mmax or Nmax must be less than or equal to $M_{max} \approx N_{max} \approx \sqrt{357/6} \approx 7$.

This rectangular model can estimate the components of the field with a maximum uncertainty of approximately 25nT (10nT inside the domain and 15nT near the boundary). Since this value is half of that obtained with a global model (greater than 50nT), we can consider applying it to real data.

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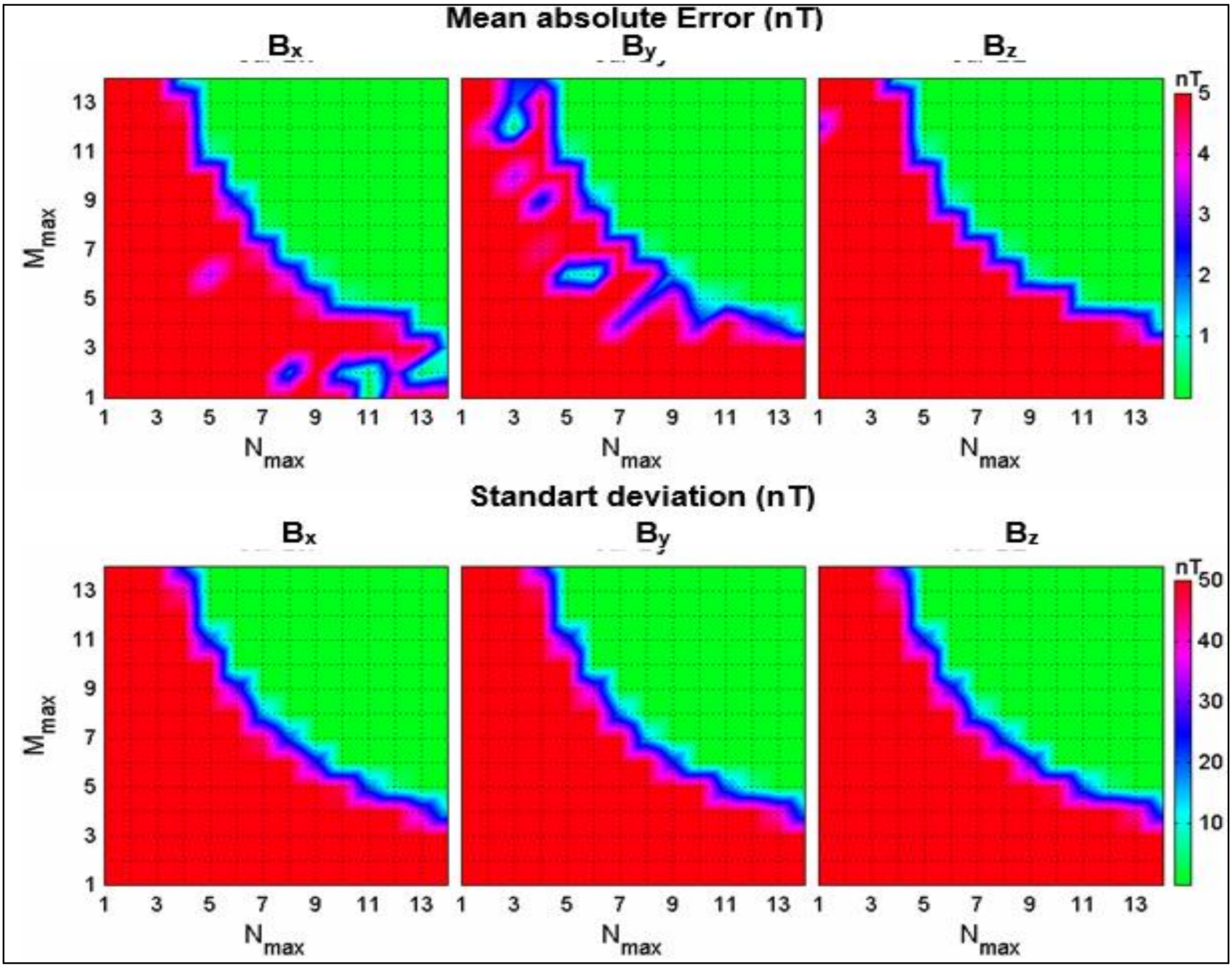

Fig 1 Global Evolution of Errors As A Function of the Truncation Indices M_{max} and N_{max}.

➢ *Application with Real Data*

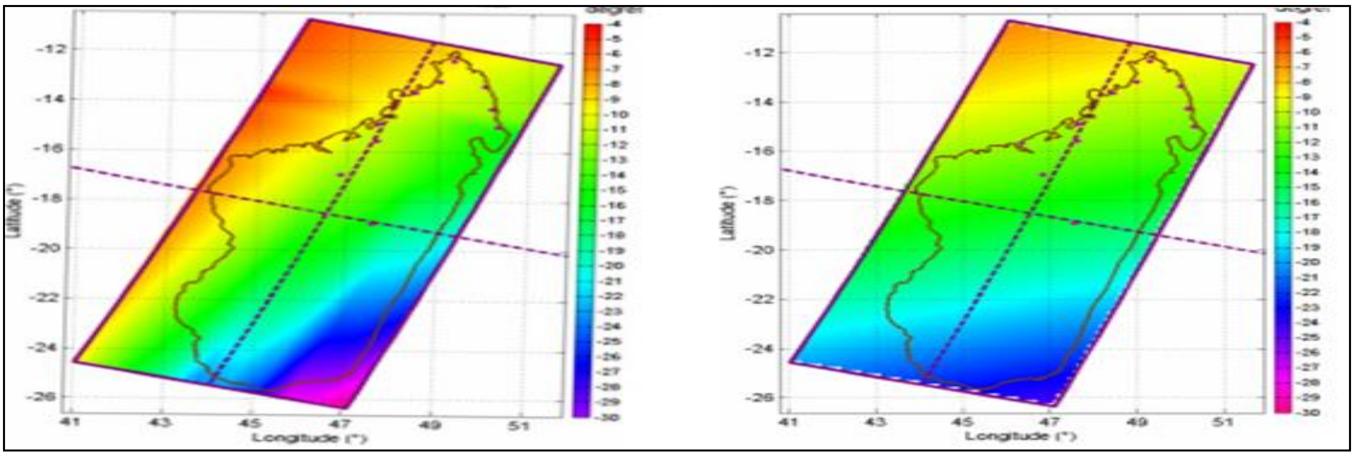

Fig 2 Magnetic Declination Maps Obtained from The Rectangular Model (Left) Compared to Those Established with the Global Model CM4 (Right) For 1983.

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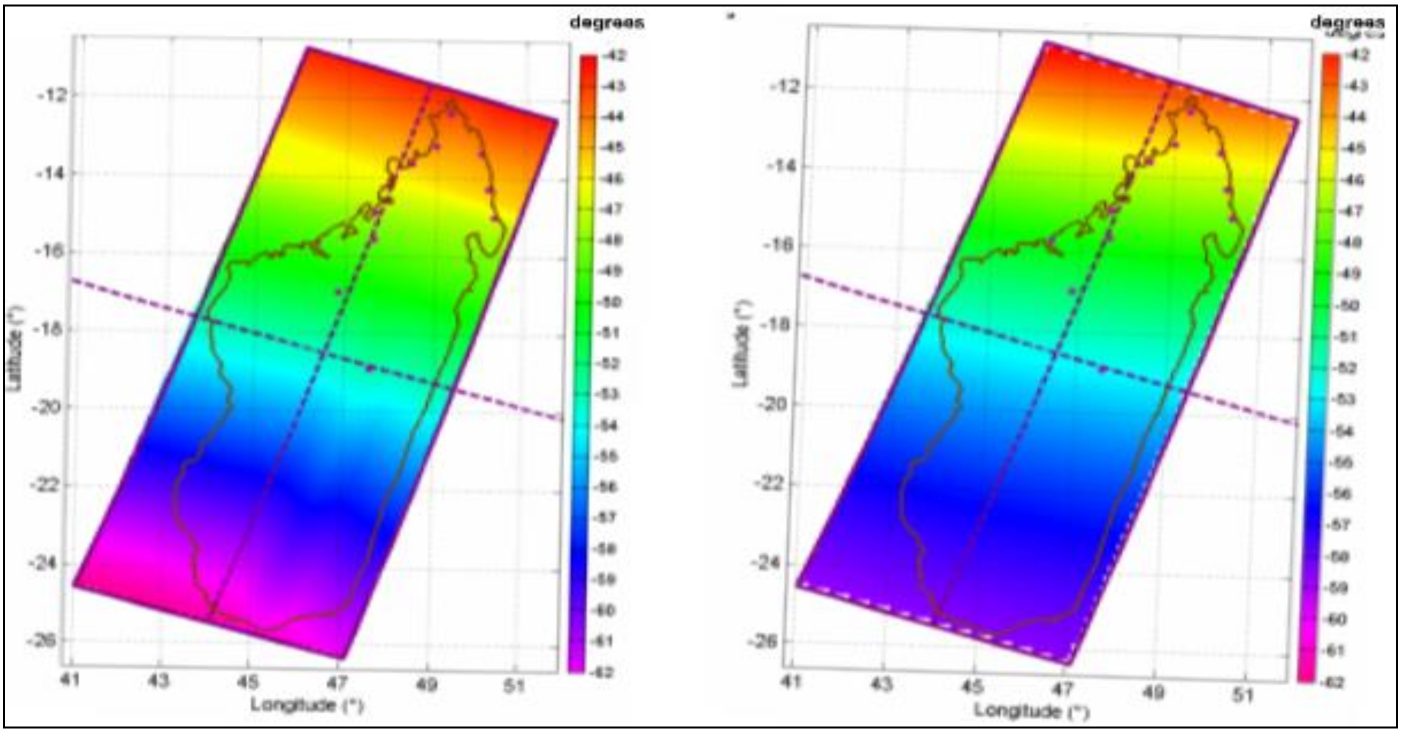

Fig 3 Magnetic Declination Maps Obtained From The Rectangular Model (Left) Compared to Those Established With the Global Model CM4 (Right) For 1990.

Let's consider the data from Malagasy repetition stations since 1983 (Table 2). It should be noted that in the case of real data, the model's residuals represent the regional magnetic anomalies of the considered region. By definition, anomalies are the differences between actual measurements and theoretical values estimated by the model. Thus, the anomaly maps will not be significant unless we have sufficient measurements, as observed in Table 1. Therefore, we limit ourselves to representing the internal field calculated by the model at every point in the domain from the available measurements for each year. We will consider the years 1983 and 1990. For simplicity, we will also limit ourselves to the magnetic declination maps of Madagascar. To validate our results, we will compare them with those obtained by the global model CM4. It is important to note that the difference between a global model and a regional model should not be too large, since for magnetic declination, regional anomalies caused by superficial crustal structures rarely exceed 5° in absolute values (De Santis et al., 1989).

1983 : Measurements exist in the North, North-West, East, South-West, and South-East regions, although they are not numerous. The declination estimated by the rectangular model with Mmax=7 and Nmax=5 and that calculated by CM4 generally show the same spatial variations. This indicates that the measurements taken in 1983 are reliable and agree with both models.

1990 : Measurements exist in the North, North-West, East, and South-East regions, but there are none in the South. The comparison between the rectangular model with Mmax=7 and Nmax=5 and the CM4 model also shows a difference, especially near x=-x0 and y=-y0. This is likely due to boundary effects accentuated by the absence of measurements in the South. Otherwise, the available data agree with both models and are therefore reliable.

This application to real data allowed us to examine the validity of the rectangular model formalism in the case of Madagascar, on the one hand, and to verify the data from Malagasy repetition stations on the other. Generally, the model's reliability remains only within the areas covered by the data, as the estimated values outside these areas are not reliable. In any case, the existing data do not allow us to accurately track the temporal evolution of the field across the entire Malagasy territory because the repetition stations change from year to year or from campaign to campaign. Nevertheless, the best case from 1983 suggests that to establish a regional model for Madagascar, data should be simultaneously available for the North, West, East, South-West, and South-East regions, particularly in Antsiranana, Mahajanga, Toamasina, Toliary, and Taolagnaro, even if these data are limited in number.

IV. CONCLUSION

In the context of regional modeling, boundary conditions are what determine the solutions. We have reformulated the formalism of rectangular harmonics, which proved to be incomplete, by introducing boundary condition issues in the rectangular domain. Preliminary formal studies allowed us to consider all conditions that could be applied to the magnetic potential in the domain in question. Then, the study of the characteristics of the magnetic field reconstructed from the eight possible problems led us to determine the best decomposition. Our initial trials with synthetic data confirm the validity of the rectangular model thus obtained. Finally, its application to the data from Malagasy repetition stations allowed us to predict which

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stations should at least be revisited if we want to use this model to create valid magnetic maps of Madagascar in the future.

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