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An EOQ Model for Deteriorating Item with Preservation Technology, Linear Holding Cost, and Multi-Variate Demand

Nargis Jahan[1]

[1]Department of Mathematics College of Science, Jazan University, Jazan Saudi Arabia

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Abstract: Nowadays, customer's purchasing behaviour gradually changes. They are more shifting towards green products keeping the selling price of the products in mind. Further, sales team efforts attract the customers also. In the current study, an EOQ model is developed considering selling price, green level, and sales team effort sensitive demand. The holding cost is considered as linear function of time. Ordering cost and purchasing cost depends on the greenness level of the products. Preservation technology is adopted to reduce the economic loss due to deterioration. Optimal values of greenness level and ordering quantity are obtained so that total profit of the system is maximum. Numerical analysis is carried out and sensitivity is performed with respect to key parameters. Numerically, it is observed that greenness level has positive impact on the environment as well as on the profit of the system.

Keywords: *EOQ Model, Deterioration, Preservation Technology, Multi-Variate Demand.*

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I. INTRODUCTION

Nowadays, countries across the globe are committed to foster a sustainable and environmentally friendly economy. Different countries started green credit programme in which green credits awarded to various sectors who initiated tree plantation, water conservation, and sustainable agriculture. In this direction, Indian government adopted Eco-Mark certification rules by replacing 1991 Eco-Mark Scheme to encourage the consumption and production of eco-friendly products. Initiatives taken by different countries change the purchasing attitude of customers and inclined them towards eco-friendly products. According to Accenture report, 45% of consumers consider environmental consequences while taking purchasing decisions. Purchasing behaviour of customers especially from developing countries also influenced by the selling price of the products. Customers try to balance between level of greenness of products and its price. Sales team is the backbone of any business organization. It is observed that active engagement of sales team with customers enhance the demand of the products. It helps to create the brand image of products in front of the customers. From above discussion, it can be summarized that demand of the customers depend on selling price, greenness level, and efforts of sales team.

Deterioration, a natural phenomenon, linked with inventory system. It refers to degradation in quality or decline in utility of the inventory over the time. Some economic value is linked with the deterioration which results substantial losses for the system. As per report[1], because of deterioration, annual cost of food waste in the US is around 100 billion. It is observed that around 10% to 30% of inventory deteriorated due to the deterioration process from different industries[2]. Therefore, it is important for the decision-maker to explore the options to mitigate the deterioration rate and save the capital for the system for further business. In this direction, preservation technology is one of the options. Various physical preservation mechanisms such as vacuum packaging, modified atmosphere packaging, dehydration, freezing, refrigeration, etc., are available based on the properties of products. Further, chemical preservation mechanism (such as antioxidants, preservative, etc.) and chemical preservation mechanism (such as pasteurization, sterilization, biological control, etc.) are also available to mitigate the deterioration rate and save economic losses to the retailer.

- *Structure of Study*

There are seven sections in the current study. Section-1 consists introduction of study which is followed by the literature review section-2. Basic assumptions and notations

[1] https://www.rts.com/resources/guides/food-waste-america/ visited on 04.04.2025

[2] https://www.iasplus.com/en/standards/ias/ias2 visited on 04.04.2025

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to frame the mathematical framework is provided in Section-3. This section is followed by the section devoted to the development of mathematical framework of current study. Section-5 provides the solution methodology to obtain optimal solution. Numerical experiments and sensitivity analysis are provided in Section-6 and Section-7 respectively. Finally, concluding remarks and future extension is provided in Section-8.

II. LITERATURE REVIEW

This section is devoted to identify the research gaps and motivation of the current study. In this section, literature review is carried out under different keywords.

➢ *Literature Review for Multi-Variate Demand*

Recently, inventory practitioners are exploring different factors that are producing the impact on the customers purchasing behaviour. They observed that selling price, greenness level, stock level, advertisement policy, etc., are the various factors that produced impact on the demand of customers. Many researchers (such as Yadav et al., 2015; Dey et al., 2019; Kumar et al., 2025) studied the impact of selling price on the demand of the customers. They observed that rise in selling price have negative impact on demand and hence on the profit of the system (Rastogi and Singh, 2018). Authors (Bhatnagar et al., 2022; Shekhar et al., 2024) also explored the impact of sales team on the demand. They suggested that efforts of sales team boost the demand and hence the profit of the system (Kumar et al., 2022). Many authors (Alharbi, 2022) also explored the impact of greenness level on the demand of the customers. Many authors explored the combined effect of selling price and advertisement (Khan et al., 2023), selling price and greenness dependent demand (Das et al., 2024) and advertisement and greenness level dependent demand (Dolai et al., 2023).

➢ *Literature Review on Preservation Technology*

How the negative effect of deterioration can be minimized is the big challenge for every business dealing in inventory system. As because of deterioration, economic loss faces by the organization. Extension review work was carried out by the Bakkar et al. (2012) for the deterioration process and its effect on the decision-making process. On observing the negative impact of deterioration, Hsu et al. (2010) suggested an investment in preservation technology to control the deterioration rate. Work of Hsue et al. (2010) was extended by many authors such as Yadav et al. (2021), Mahapatra et al. (2022), Kaushik (2024) focussing investment in preservation technology. Yadav et al. (2024) observed that investment in preservation technology based on selling price is more prudent to control the deterioration rate. Further, they suggested that for the deteriorated units, salvage trading is one of good options. Luo et al. (2025) adopted the preservation technology for food industries. They suggested that government intervention is necessary to support the preservation technology investment and for framing the effective policy strategies for food waste. Yadav et al. (2025) presented retailer-based inventory model incorporating preservation technology and green technology to control deterioration and carbon emissions respectively. They observed that preservation technology increases cycle length and significant decline is also observed in total cost.

➢ *Literature Review on Variable Holding Cost*

Holding cost is bear by the retailer to hold the inventory in stock which is needed to satisfy the demand of the customers. Holding cost consist two components: (i) constant component and (ii) variable component. Most of the inventory practitioners taken holding cost as constant parameters (Yadav et al., 2021; Yadav et al., 2022; Kumar et al., 2024). Practically, this assumption is.

III. NOTATIONS AND ASSUMPTIONS

This section is devoted to represent notations and assumptions required to developed mathematical framework of the proposed model.

➢ *Notations*

A : $(= A_f + A_g g)$Ordering cost
A_f : fixed part of the ordering cost
A_g : ordering cost depends on the greenness dependent
D_0 : Base demand
D_1 : scale parameters of selling price
D_2 : scale parameter for sales team effort
D_3 : sensitivity parameter linked with greenness.
s : Selling Price
ρ : Sales team effort
g : Greenness level
θ_0 : Deterioration rate in absence of preservation technology
ϵ : Investment in preservation technology
$\theta(\epsilon)$: $(= \theta_0 e^{-\alpha\epsilon})$Effect deterioration rate under the effect of preservation technology
α : Efficiency parameter of preservation technology
p : $(= p_0 + p_g g)$Purchasing cost
p_0 : Fixed component of purchasing price
p_g : Greenness dependent purchasing price
$h(t)$: $(= h_0 + h_1 t)$Holding cost
h_0, h_1 : Constant parameter
Q : Ordered quantity
T : Cycle time
C_θ : Deterioration cost
m : Marked price

➢ *Assumptions*

- Current study considered single item and the rate of replenishment is instantaneous.
- Various factors such as selling price, sales team efforts, and greenness level of product affects the demand of the customer. So, following demand pattern is considered in the model:

$$D(s, \rho, g) = D_0 - D_1 s + D_2 \rho + D_3 g$$

Where D_0 is the base demand, D_1 scale parameters of selling price, D_2 scale parameter for sales team effort, and D_3 sensitivity parameter linked with greenness.

- Ordering cost has two components: (i) constant part and (ii) greenness dependent part.
- Therefore, ordering cost is $A = A_f + A_g g$ where A_f is fixed part of the ordering cost and A_g depends on the greenness dependent.
- Physical presence of inventory in stock results deterioration process of product. Therefore, constant rate of deterioration is considered in the model.
- To control the deterioration rate, preservation technology is considered in the model. Under the effect of preservation technology, effective deterioration rate is as follows:

$$\theta(\epsilon) = \theta_0 e^{-\alpha\epsilon}$$

Where θ_0 is initial deterioration rate, and ϵ is the investment in preservation technology.

- Price of green products also depends on greenness level. Therefore, following form of purchasing cost is considered in the model:

$$p = p_0 + p_g g$$

Where p_0 is the fixed component of purchasing price and p_g is the greenness dependent purchasing price.

- Holding cost of the products increase with the rise in holding period. Therefore, holding cost is as follows:

$$h(t) = h_0 + h_1 t$$

IV. MATHEMATICAL FRAMEWORK OF EOQ MODEL

Let at $t = 0$, an order of Q quantity is place by the decision-maker to meet the demand of the customer. Inventrory level decreases due to demand and deterioration. At $t = T$, inventory levels become zero. Graphically, this situation can be represented in Fig.1.

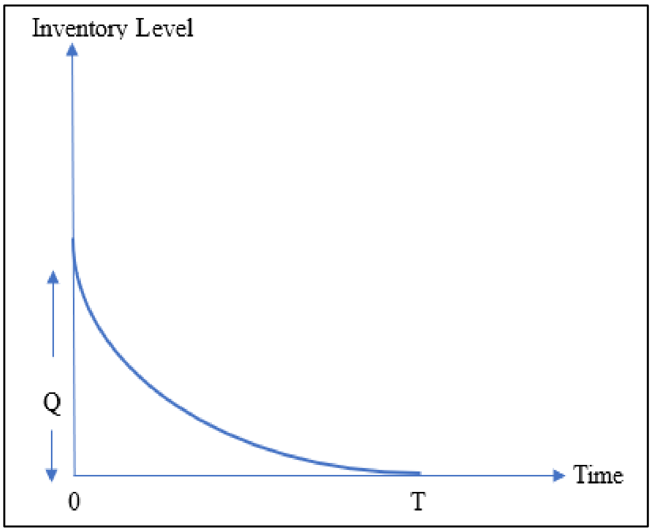

Fig.1 Graphical Representation of Inventory Level of Green

Mathematical, inventory level can be represented as follows:

$$\frac{dI(t)}{dt} = -D(s,\rho,g) - \theta(\epsilon)I(t), \quad 0 \le t \le T \qquad (1)$$

- Boundary condition: $I(0) = Q, and\ I(T) = 0$.

On solving the equation (1) applying the boundary condition $I(T) = 0$, we get

$$I(t) = \frac{D(s,\rho,g)}{\theta(\epsilon)}\left(e^{\theta(\epsilon)(T-t)} - 1\right), \quad 0 \le t \le T \qquad (2)$$

Using the condition $I(0) = Q$, we get

$$T = \frac{1}{\theta(\epsilon)} log\left(\frac{Q\theta(\epsilon)}{D(s,\rho,g)} + 1\right) \qquad (3)$$

Objective of current model is to maximize the profit of the system. For this, we evaluate different costs associated with the inventory system.

- Ordering Cost: To satisfy the demand of customers, order of green product is placed by the retailer. So, ordering cost is

$$OC = A_f + A_g g.$$

- Purchasing cost: Here, purchasing cost depends on the greenness of the purchased products. So, purchasing cost is

$$PC = \left(p_0 + p_g g\right)Q.$$

- Holding Cost: To satisfy the regular demand of customers, green items are hold in the stock. So, holding cost is

$$HC = \int_0^T (h_0 + h_1 t)I(t)dt$$

$$= \frac{D(s,\rho,g)}{\theta(\epsilon)}\left[(h_0 + h_1 t)\left\{\frac{1}{\theta(\epsilon)}\left(e^{\theta(\epsilon)T} - 1\right) - T\right\} + h_1\left\{\frac{1}{\theta^2(\epsilon)}\left(e^{\theta(\epsilon)T} - 1\right) + \frac{T^2}{2}\right\}\right].$$

- Deterioration Cost: Physical presences of green products results deterioration process. Means there is some economic loss to the retailer. So, deterioration cost is

$$DC = C_\theta \int_0^T \theta(\epsilon)I(t)dt = C_\theta D(s,\rho,g)\left\{\frac{1}{\theta(\epsilon)}\left(e^{\theta(\epsilon)T} - 1\right) - T\right\}.$$

- Preservation Technology Investment: To mitigate the economic loss due to deterioration, retailer applied preservation technology. Therefore, some investment is made by the retailer for this. So, investment in preservation technology is

$$PT = \epsilon T.$$

- Sales Revenue: On selling the good green products, revenue is generated the retailer. In addition to this, there

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is secondary market for the deteriorated products. Therefore, generated revenue is

$$SR = s\int_0^T D(s,\rho,g)\,dt + sm\int_0^T \theta(\epsilon)I(t)dt$$

$$= sD(s,\rho,g)T + smD(s,\rho,g)\left\{\frac{1}{\theta(\epsilon)}\left(e^{\theta(\epsilon)T}-1\right)-T\right\}.$$

Therefore, total profit for the retailer per unit time is

$$TP = \frac{1}{T}(SR - OC - PC - HC - DC - PT) \quad (4)$$

- *Objective Function:*

$$Max\ (Z) = TP(g,Q)$$

Subjected to $g > 0, Q > 0$.

Where value of $TP(g,Q)$ is given by equation (4).

V. SOLUTION METHODOLOGY

➢ *To Obtain the Optimal Solution, Following Steps are Followed:*

- Step-1: Find $\frac{\partial TP(g,Q)}{\partial g}$ and $\frac{\partial TP(g,Q)}{\partial Q}$.
- Step-2: Apply the necessary condition for optimality as follows:

$$\frac{\partial TP(g,Q)}{\partial g} = 0, \quad \frac{\partial TP(g,Q)}{\partial Q} = 0.$$

- Step-3: On solving above mentioned system of equations, we get the values of g and Q.
- Step-4: We obtain the Hessian matrix and sign of its principal minors at the derived values in step-3 as follows:

$$\text{Hessian Matrix} = \begin{bmatrix} \frac{\partial^2 TP(g,Q)}{\partial g^2} & \frac{\partial^2 TP(g,Q)}{\partial g \partial Q} \\ \frac{\partial^2 TP(g,Q)}{\partial Q \partial g} & \frac{\partial^2 TP(g,Q)}{\partial Q^2} \end{bmatrix}$$

Now, we check the sign of principle minor.

- *First Principal Minor:*

$$H_{11} = \left(\frac{\partial^2 TP(g,Q)}{\partial g^2}\right)_{(g,Q)}$$

- *Second Principal Minor:*

$$H_{22} = \left(\frac{\partial^2 TP(g,Q)}{\partial g^2}\frac{\partial^2 TP(g,Q)}{\partial Q^2} - \left(\frac{\partial^2 TP(g,Q)}{\partial g \partial Q}\right)^2\right)_{(g,Q)}$$

- Step-5: We check the optimality conditions as follows:

$$H_{11} < 0, H_{22} > 0,$$

- Step-6: If conditions of step-5 hold then (g^*, Q^*) is the optimal value of decision variables and maximum profit is $TP(g^*, Q^*)$.

VI. NUMERICAL EXPERIMENT

In this section, proposed model is illustrated with the help of numerical example. For this, data has been taken from the literature with appropriate modification as per the requirement of the model. Required data is as follows:

$\theta_0 = 0.004$ units/month, $A_f = \$550/cycle$, $A_g = \$50/cycle$, $D_0 = 80$units/month, $D_1 = 0.2$, $D_2 = 0.4$, $D_3 = 0.3$, $C_\theta = \$4/unit$, $\rho = 10$, $p_0 = \$20/unit$, $p_g = \$0.8$/unit, $h_0 = \$8unit/month$, $h_1 = \$6unit/month^2$, $s = \$80/unit$, $m = 0.4$, $\alpha = 0.1$, $\epsilon = \$89$.

On applying the solution methodology mentioned in section 4, optimal solution is as follows:

$$g^* = 0.29, Q^* = 233, TP(g^*, Q^*) = \$6296$$

➢ *Sensitivity Analysis*

In this section, sensitivity with respect to important parameters is carried out. Here, to perform the sensitivity analysis, one parameter is changed on and from -20% to +20% keeping rest parameters fixed.

Table 1 Sensitivity Analysis

Parameter	**% change**	**% change in profit**
h_0	-20%	+3.89%
	-10%	+1.29%
	+10%	-1.09%
	+20%	-4.01%
h_1	-20%	+1.09%
	-10%	+0.41%
	+10%	-0.39%
	+20%	-1.11%
p_0	-20%	_+2.34%
	-10%	+1.08%
	+10%	-0.97%
	+20%	-2.74%
	-20%	+1.01%

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p_g	-10%	+0.54
	+10%	-0.38%
	+20%	-1.14%
D_0	-20%	-9.87%
	-10%	-5.81%
	+10%	+4.01%
	+20%	+8.01%
A_f	-20%	+6.03%
	-10%	+2.99%
	+10%	-3.022%
	+20%	-5.97%
A_g	-20%	+1.99%
	-10%	+0.92%
	+10%	-0.88%
	+20%	-2.12%
ρ	-20%	-6.93%
	-10%	-2.98%
	+10%	+3.16%
	+20%	+7.29%
s	-20%	-4.03%
	-10%	-3.02%
	+10%	+2.16%
	+20%	+5.12%
α	-20%	-7.13%
	-10%	-4.02%
	+10%	+3.76%
	+20%	+8.12%

➢ *Observations:*

- Different components of holding cost have negative impact on the total profit of the system. It is observed that as different components increase, total profit of the system decreases. Total profit of the system more affected due to the change in h_0 in comparison to h_1.

- Price of the product has crucial effect on the profit of the system. From the Table-1, it is observed that total profit and different components of price are negatively correlated with each other. Fixed component of purchasing cost has more negative impact in comparison to the component of greenness dependent purchasing cost.

- From the Table-1, it is observed that as the base demand increases profit of the system also increases. Around 9% variation in profit is observed due to the change in base demand. This result suggest that decision-maker have to work out to increase the loyal customer's base.

- Efforts of sales team have positive impact on the profit of the system. More efforts result more customer for the system and hence more profit for the system. Around 7% rise in profit is observed due to the change in the sales team efforts.

- From Table-1, it is observed that different components of ordering cost have negative impact on the profit of the system. As ordering cost component rises, profit of the system decreases.

- From the Table-1, it is observed that selection of preservation mechanism is very important to earn profit for the system. Here, results indicate that as the efficiency of preservation technology rises profit of the system also rises upto 8%.

VII. CONCLUSION AND FUTURE EXTENSIONS

In this study, we have studied an EOQ model for deteriorating items considering greenness policy of products. In the model, preservation technology is considered to control the deterioration rate. Further, customers demand affected by the selling price, sales team efforts, and greenness level of the product. Optimal value of greenness degree and ordered quantity is obtained so that total profit of the system is maximum. Numerical experiment is carried out and sensitivity analysis is performed with respect to different key parameters. From the experiment, greenness degree is obtained as 0.29 and ordered quantity as 233 so that total profit is maximum. In this situation, maximum profit is \$6296. From the sensitivity analysis, it is observed that total profit is negatively correlated with different components of holding cost, purchasing cost, and ordering cost whereas positively correlated with sales team efforts, efficiency of preservation technology, and base demand. Base demand, sales team efforts, and efficiency of preservation technology are the most influence factor for decision-making process of inventory problem. For future study, researcher may considered green technology, freshness dependent demand, carbon cap-and-trade policy, and trade credit to enhance the applicability of the developed model. Comparative study can

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be done within the crisp model and fuzzy model considering different cost components as imprecise parameters.

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