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Rotating Spacetime: Theory or Reality? A Concise Journey of General Relativity

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Abstract:- The paper reviews the theoretical formulae of different astrophysical conditions to describe spacetime and connects theory with observational evidence. The spacetime is governed by gravity, which is well-explained by the theory of General Relativity. The paper starts from the simplest version of spacetime, that is, flat spacetime which has no gravitational influence. This spacetime is described by the Minkowski metric. Then the paper goes to the properties of spacetime in the presence of gravity, which creates curved spacetime. The Schwarzschild metric defines this spacetime. Although these phenomena are well-established by experimental proof, the most intricate characteristic of spacetime has not been discovered until very recently. That is the spacetime around a rotating massive body. The paper will present the mathematical expressions for describing such spacetime, the Kerr metric, and finally will end with the observational evidence of the effect of a spinning heavy body around it. Some particular exotic effects such as "frame-dragging" and "ergosphere" will be presented in brief.

Keywords:- *General Relativity, Minkowski Space, Schwarzschild Space, Kerr Metric, Ergosphere.*

I. INTRODUCTION

General relativity is a geometric theory of gravitation which was developed by Albert Einstein in 1915. It is a generalization of special relativity, which was also developed by Albert Einstein. Together they are called the theory of relativity. It redefines the universal law of gravitation devised by Isaac Newton. General relativity provides a unified description of gravity as a geometric property of space and time. It considers space and time together as four-dimensional spacetime. The curvature of spacetime is directly related to the energy and momentum of matter and radiation, as specified by the Einstein field equations. In short, "Mass tells spacetime how to curve, and spacetime tells mass how to move" (Ciufolini and Wheeler 1995). Where there is no mass, the spacetime is flat. Newtonian mechanics can be considered as an approximation of the general relativity in flat space.

The Metric Tensor is a mathematical tool used to describe the geometry of space-time (Tong 2012). Spacetime curvature is described by these metrics, which are the solutions to Einstein's field equations. Minkowski is the metric that describes spacetime far away from any massive object, that is, where no gravitational attraction occurs (H. Minkowski and Petkov 2013). The Schwarzschild metric describes the spacetime around a non-rotating spherical mass, like a planet or a typical star like our Sun (whose rotation is slow enough to be considered static) (K. Schwarzschild 1999). Nevertheless, an alternative metric is necessary to explain the spacetime around rapidly revolving astrophysical bodies like pulsars or black holes. This is referred to as the Kerr metric (Kerr 1963). The following sections explore the theoretical formulation and mathematical expression of each of these metrics. Later some of their observational evidence and their future application will be presented.

II. FLAT SPACETIME: NO MASS

Space is flat and so is time; as we perceive through our senses. It is real indeed, only when there is no big mass. However, we cannot perceive that space and time are not separate entities; they are intertwined. This idea was brought to light by famous mathematician Hermann Minkowski. In 1908, Minkowski combined space and time into a four-dimensional model. According to this model, space is not independently three-dimensional; it has time as the fourth dimension, creating a four-dimensional space. It is called Minkowski space or Minkowski spacetime. The model explains Einstein's Special Theory of Relativity (Hermann Minkowski 1908). The Minkowski space is the spacetime in the absence of gravity, i.e. a flat, uncurved space with no gravitational source. Minkowski space is a four-dimensional space where each point is an event in spacetime. A point in this space is expressed by a four-vector (t, x, y, z). The four-vector corresponds to the event represented by the point. In Special Relativity, Lorentz transformations (Lorentz 1904) are used to describe the transformation of the coordinates of an event between two inertial frames which are moving at a constant velocity relative to each other. Rotations are equivalent to the Lorentz transformations in four-dimensional Minkowski space. In Minkowski space, the rotation angle corresponds to the relative velocity of the two observers and the rotation axis corresponds to the direction of their relative motion. According to Minkowski, time and space are equivalent. An event occurs in a unified four-dimensional spacetime continuum in the Minkowski model.

A. Theoretical Formulation

Minkowski metric describes a line element in Minkowski space. It is the foundation which leads to curved spacetime. It also lays the foundation of QFT (Quantum Field Theory) in flat spacetime. It allows us to describe the behaviour of particles and EM fields in the absence of gravity. The idea of the metric can be understood by considering a rotation matrix in four-dimensional space. Since the coordinates of an event in spacetime can be expressed by a four-vector (t, x, y, z), the rotation matrix rotates the four-vector around a particular axis (Wikisource 2024). Mathematically this can be expressed as the following equation:

$$x^2 + y^2 + z^2 + (ict)^2 = \text{constant} \quad \text{......................} \quad (1)$$

Or

$$x^2 + y^2 + z^2 - c^2t^2 = \text{constant} \quad \text{....................} \quad (2)$$

The graphical representation of this expression is called the Minkowski diagram. It is a two-dimensional graph depicting events in the Minkowski spacetime (Fig. 1).

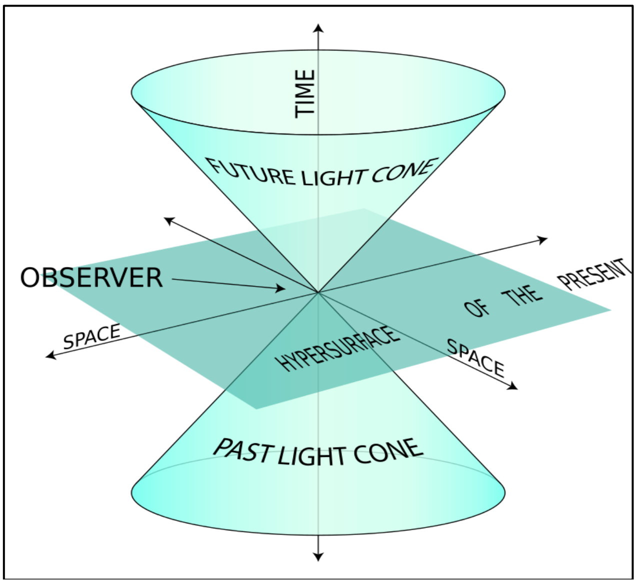

Fig 1: The Minkowski Diagram for an Event in four conditions: The Light Cone, The Absolute Future, The Absolute Past, and Elsewhere. (Sard 1970).

The Minkowski diagram consists of one space dimension and one time dimension. In Minkowski's words: "The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth, space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." (Hermann Minkowski 1908). To match with this idea, Minkowski also reformulated the Maxwell equations of electromagnetism.

➢ *The Minkowski Metric is Given by (Schmitz 2022):*

$$ds^2 = -c^2\,dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\nu\mu} dx^\mu dx^\nu \ldots\ldots(3)$$

Where ds^2 is the spacetime interval between two events. c is the speed of light. It is the universal speed limit. The Greek alphabet η stands for the flat space metric. η is expressed as the following matrix:

$$\eta = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \ldots\ldots\ldots\ldots\text{..................}(4)$$

B. Physical Meaning

Our familiar world is dominated by the Minkowski metric because most of the space in the universe is free from the gravitational influence of gigantic massive objects unless an observer is close enough to that object. The coordinate transformations between the inertial frames are linear in the Minkowski spacetime. It means the Minkowski metric represents a straight line. Being a straight line means having no curvature. Its physical interpretation is the absence of gravity. In the astrophysical scenarios, regions far from the heavy objects obey the Minkowski metric. The metric is valid where the gravitational effect is negligible, i.e. far from gravitational sources. The Minkowski metric estimates spacetime in places like our solar system, interstellar medium or weak gravitational fields around low-mass stars. It helps us to predict other conditions such as spacetime around high-mass stars.

III. CURVED SPACETIME: NON-ROTATING MASS

Space is curved even though we perceive it as flat in our regular life. Flat spacetime is an approximation of curved spacetime like Earth's surface appears flat to us even though it is indeed curved as it is the surface of the spherical-shaped Earth. In 1916, the first exact solution to the Einstein field equations of the theory of general relativity was found by German physicist and astronomer Karl Schwarzschild (Karl Schwarzschild 1916). The solution assumes that the central mass is charge neutral (no electric charge) and non-rotating (no angular momentum). This solution is called Schwarzschild metric (K. Schwarzschild 1999). The geometry of the spacetime around a non-rotating as well as non-charged object can be described by the Schwarzschild metric. Therefore, the solution allows us to understand the geometry of our familiar territory because the solar system contains mainly slowly rotating astronomical objects like many planets and asteroids, including Earth and the Sun.

A. Theoretical Formulation

The Schwarzschild metric assumes the central body is spherical. As such, the metric is expressed in terms of spherical coordinates, together with the time dimension which is called Schwarzschild coordinates (t, r, θ, ϕ). The line element for the proper time in Schwarzschild space, i.e.,

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the Schwarzschild metric is given by (Fromholz, Poisson, and Will 2014):

$$ds^2 = c^2 d\tau^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Where τ is the proper time defined as the time measured by a clock which is moving along the same world line as a test particle. Here G is the universal gravitational constant and M is the mass of the central body. The gravitational potential created by the mass M is given by the term $2GM/r$. It allows us to define a surface around the central object beyond which nothing can escape; even light is trapped by the gravitational potential of the central mass inside this region. This hypothetical surface is called the event horizon. It is not a physical surface; it determines the radius of the central body at which point the escape velocity of the central mass becomes equal to the speed of light. This distance is called the Schwarzschild radius, r_s. It is expressed as

$$R_s = \frac{2GM}{c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

If a mass which is non-rotating and non-charged becomes smaller than its Schwarzschild radius, it will turn into a black hole. Therefore, any object can form a black hole in principle.

Schwarzschild black hole has a photon capture radius, which is different from the Schwarzschild radius. The photon capture radius is given by

$$R_c = \sqrt{27}\frac{GM}{c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

The significance of this radius is that if the impact parameter b of the photons approaching the black hole is less than the photon capture radius, i.e. $b < R_c$, then the photons are captured and plunge into the black hole (Renn and Stachel 2007). If the impact parameter is larger than the photon capture radius, i.e. $b > R_c$, then the photons escape to infinity. The photons with $b = R_c$ are captured on an unstable circular orbit around the black hole. Thus the "photon ring", the ring of light around the black hole is produced.

From Eq. (6) and (7) we can see that the photon capture radius, R_c , is larger than the Schwarzschild radius R_s . That means the photon capture radius resides outside the event horizon of a Schwarzschild black hole.

B. *Physical Meaning*

The Schwarzschild metric is valid outside of the gravitating body only since it is a solution of Einstein's field equations in empty space. In other words, the solution is true for $r > R$ for a spherical body of radius R. The Schwarzschild metric is valid for our solar system because it describes slowly rotating astrophysical bodies such as general stars, like the Sun and planets, like Earth. It also allows us to model other types of stellar objects such as giant planets and white dwarfs. Major astronomical incidents like gravitational time dilation, Redshift due to time dilation, light bending around stars, and the trajectories of objects moving in the gravitational field of a star, can be calculated using the Schwarzschild metric. The existence of black holes was anticipated by the Schwarzschild metric. Astrophysicists use the metric to explain non-rotating black holes, planetary orbits, and gravitational lensing.

The biggest contribution of the Schwarzschild metric is the idea of the existence of black holes. The black holes described by this metric are called Schwarzschild black holes. These are the static, i.e. non-rotating black hole. Their properties, e.g. Schwarzschild radius, are determined only by their mass. Space curves so much around a Schwarzschild black hole that even light beams are strongly deflected. Light near the black hole may be deflected so strongly that it circles the hole multiple times and creates an accretion disk around it (Sneppen 2021).

IV. SPACETIME AROUND ROTATING MASS

Now here comes the twist. Curved spacetime is already bizarre to perceive by our senses, yet that is not the end! The space gets more twisted in the presence of extremely conditioned objects! This idea comes from another exact solution of the field equations of general relativity. Nearly a half-century after the discovery of Einstein's theory of relativity, mathematician Roy Patrick Kerr discovered an exact solution to the Einstein field equation of general relativity in 1963 (Kerr 1963). This solution is called the Kerr geometry. Like the Schwarzschild metric, this solution is also valid in the gravitational field outside a gigantic object. But the difference is that it allows the object to be rotating. The Kerr metric also assumes the mass to be uncharged. Therefore, the Kerr metric is a generalization of the Schwarzschild metric in the case of a rotating body. The Kerr geometry explains the geometry of empty spacetime in the vicinity of a charge-neutral spinning body which is spherically symmetric.

A. *Theoretical Formulation*

The line element in the Kerr spacetime, i.e. the Kerr metric in the vicinity of a spinning mass M with angular momentum J is given by: (Hirata 2012).

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4aMr}{\Sigma}\sin^2\theta \, dt \, d\phi + \frac{(r^2+a^2)^2 - a^2 \Delta \sin^2\theta}{\Sigma}\sin^2\theta \, d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma \, d\theta^2, \ldots\ldots\ldots \quad (8)$$

Where M is the mass of the spinning object, a is its spin parameter given by

$$a = \frac{J}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

The other symbols' meanings are as follows:

$$\Sigma = r^2 + a^2\cos^2\theta \quad \text{and} \quad \Delta = r^2 - 2Mr + a^2 \ \ldots.. \quad (10)$$

The Kerr metric implies coupling between time and motion in the plane of rotation which is evident from the cross-term $dtd\phi$. Consequently, spacetime is “twisted" due to the rotation of the mass, creating a unique effect called frame-dragging. Simply speaking, the effect predicts that a body close enough to the rotating body will be bound to participate in the rotation, not because of any applied force or torque, but because the space itself is part of the rotation of the central body.

In the case of the Kerr black hole, the photon capture radius R_c (Eq. 7) depends on the rotation of the black hole. R_c changes with the angular-momentum vector of the Kerr black hole. If the black hole rotates fast enough, the photon ring might appear as a “crescent” (Lu et al. 2014). The observed ring appears so due to the gravitational lensing.

B. Physical Meaning

Kerr metric allows us to study several extreme astrophysical events such as quasars, active galactic nuclei, relativistic jets, and accretion disks (Teukolsky 2015). The discovery of spinning black holes was made possible by the Kerr metric, which predicted the likely existence of rotating gigantic objects. According to Kerr metric, a spinning body has a unique impact on spacetime known as frame-dragging (Costa and Natário 2021). A spinning body bends spacetime around it and makes it move along its rotation, causing objects near it to participate in its rotation. This phenomenon, not only a theory but also a reality, has been proven by the Gravity Probe B experiment (Everitt et al. 2011). Light can travel around the rotating black hole multiple times, creating multiple images of the same object (NASA 2021). The swirling curvature of spacetime associated with the rotating body is responsible for this rotation.

Another distinguished prediction of the Kerr metric is the “ergosphere", a unique region around a Kerr black hole. The area outside the outer event horizon of a revolving black hole is known as the ergosphere. In this region, no object can remain at rest relative to distant observers. The ergosphere is a region that spreads to a larger radius at the equator of a revolving black hole and reaches the event horizon at its poles. In a black hole with moderate angular momentum, the ergosphere has an oblate spheroid shape; with higher spins, the ergosphere takes on a more pumpkin-like form. The polar radius is the smallest radius of the event horizon, and the equatorial radius is the Schwarzschild radius (Visser 2008).

V. OBSERVATIONAL EVIDENCE

The Minkowski metric, Schwarzschild metric and Kerr metric represent a profound physical interpretation of spacetime around various types of astrophysical objects. The metrics provide not only theoretical advancements but also observational proofs of general relativity. Here we shall present some of the observational evidence of the metric representation of spacetime.

A. Active Galactic Nuclei

The curvature induced by mass and rotation plays a significant role in the transition of a flat space into a curved or twisted one. The observational evidence of the Kerr metric is the image of a supermassive black hole named M87* (Fig. 2). In 2019, the first-ever image of a black hole was published by the Event Horizon Telescope (Collaboration 2019). It is the supermassive black hole residing at the centre of the giant elliptical galaxy called M87. It is a Kerr black hole as predicted by general relativity. The Kerr black hole can be modelled using general-relativistic magnetohydrodynamics. A turbulent, hot, magnetized disk orbiting a Kerr black hole is predicted according to the general-relativistic magnetohydrodynamics model. The black hole is characterized by two parameters: Firstly, the spin parameter.

$$a = \frac{Jc}{GM^2} \qquad \text{..} \qquad (11)$$

where J is the spin angular momentum of the Kerr black hole and M is its mass. Secondly, the magnetic flux over the event horizon

$$\phi = \frac{\Phi}{(\dot{M} R_g{}^2)^{1/2}} \qquad \text{...} \qquad (12)$$

where Φ is the magnetic flux across the horizon and $\dot{M}$ is the mass flux, known as accretion rate, across the horizon of the Kerr black hole. The accretion disks are prograde for $a \geq 0$ and retrograde for $a < 0$ with respect to the spin axis of the black hole. In the picture 2 we can see the accretion flow orbiting a Kerr black hole. The ring of light is created due to the emission of photons from a plasma rotating close to the speed of light around a black hole. The ring is asymmetric. It is created due to strong gravitational lensing and relativistic beaming. Inside the ring, we can see the darkness which is the observational signature of the black hole. We also see that the ring is brighter on the southern hemisphere of the black hole. The asymmetry of the ring is due to the relative motion between the spin of the black hole and the rotation of the accretion disk. The luminous side of the ring means that matter in the bottom part of the image is moving towards Earth, i.e. the observer.

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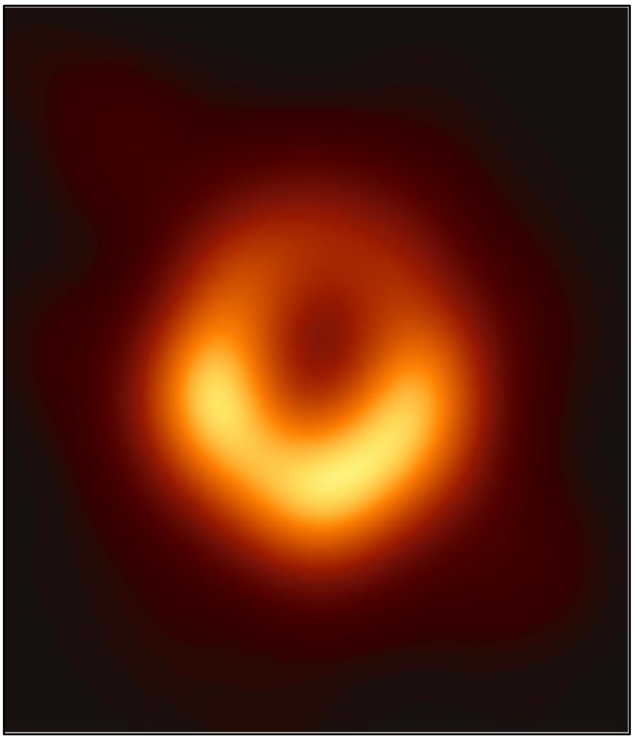

Fig 2: Event Horizon Telescope Image of M87* from Observations on 2017 April 11 (Collaboration 2019).

Apart from a Kerr black hole, other exotic entities are also possible theoretically. Few of them are: naked singularities (Shaikh et al. 2018), boson stars (Liebling and Palenzuela 2023), and gravastars (Chirenti and Rezzolla 2007). All these are possible according to General Relativity, though evidence of their existence is far from reality as per our present technology.

B. Twin Quasar

The observational evidence of the Schwarzschild metric is the gravitational lensing, first observed in 1979 (Walsh, Carswell, and Weymann 1979). As general relativity describes how mass concentrations distort the space around them, an object big enough can create a gravitational field strong enough that causes a sufficient curvature of spacetime to bend the light from distant galaxies behind it, like a lens (Space. com Staff 2023a). This effect is called gravitational lensing (H. Staff 2024). In 1979, astronomers discovered two quasars which seemed identical (Walsh, Carswell, and Weymann 1979). Since their positions are close to each other, both of their distances are the same from us and have similar properties, they were named "Twin Quasar". Being so close distanced is weird enough for such big astronomical entities which led to thorough investigation. Soon it was revealed that these "twins" are indeed the same object, named as the double quasar. Its image is lensed due to spacetime curvature created by the gravitational field of the massive galaxy YGKOW G1. The galaxy acts as a gravitational lens, an object with a mass so massive that it can bend the light from objects lying behind it. This galaxy's immense gravitational force bends the light from the quasar behind it, creating a double image of the same quasar (Fig. 3).

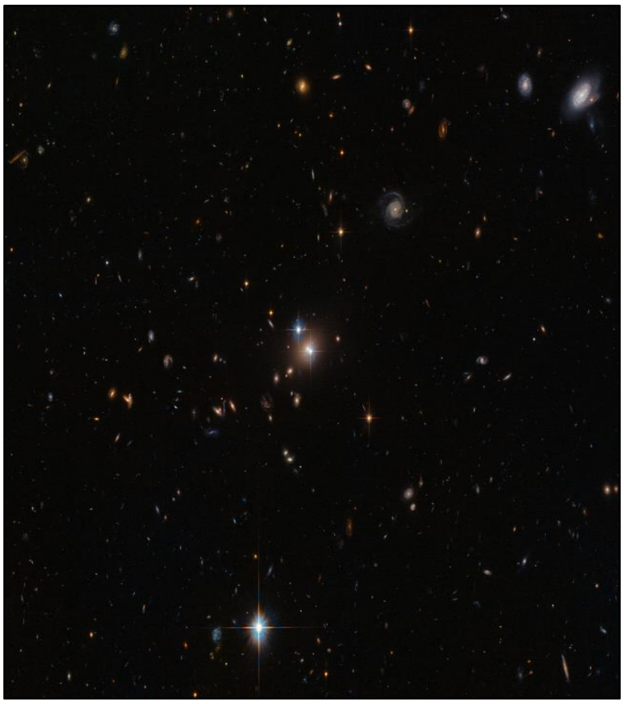

Fig 3: The Twin Quasar (named QSO 0957+561) is a Single Quasar, 8.7 Billion Light-Years away from Earth, which can be Seen as Two Adjacent Stars in the Middle of the Picture (ESA Hubble and NASA 2014).

This is not only a fascinating optical illusion but also a crucial effect on space, warping and bending the spacetime environment. This discovery is the evidence for Einstein's theory of general relativity, which predicted gravitational lensing as one of its observable effects in 1936 (Einstein 1936).

C. Gravitational Wave

The Kerr metric predicts several high-energy astrophysical phenomena, such as quasars, gamma-ray bursts, and gravitational waves. The existence of gravitational waves was discovered in 2016 by LIGO (Laser Interferometer Gravitational-Wave Observatory) (LIGO 2016).

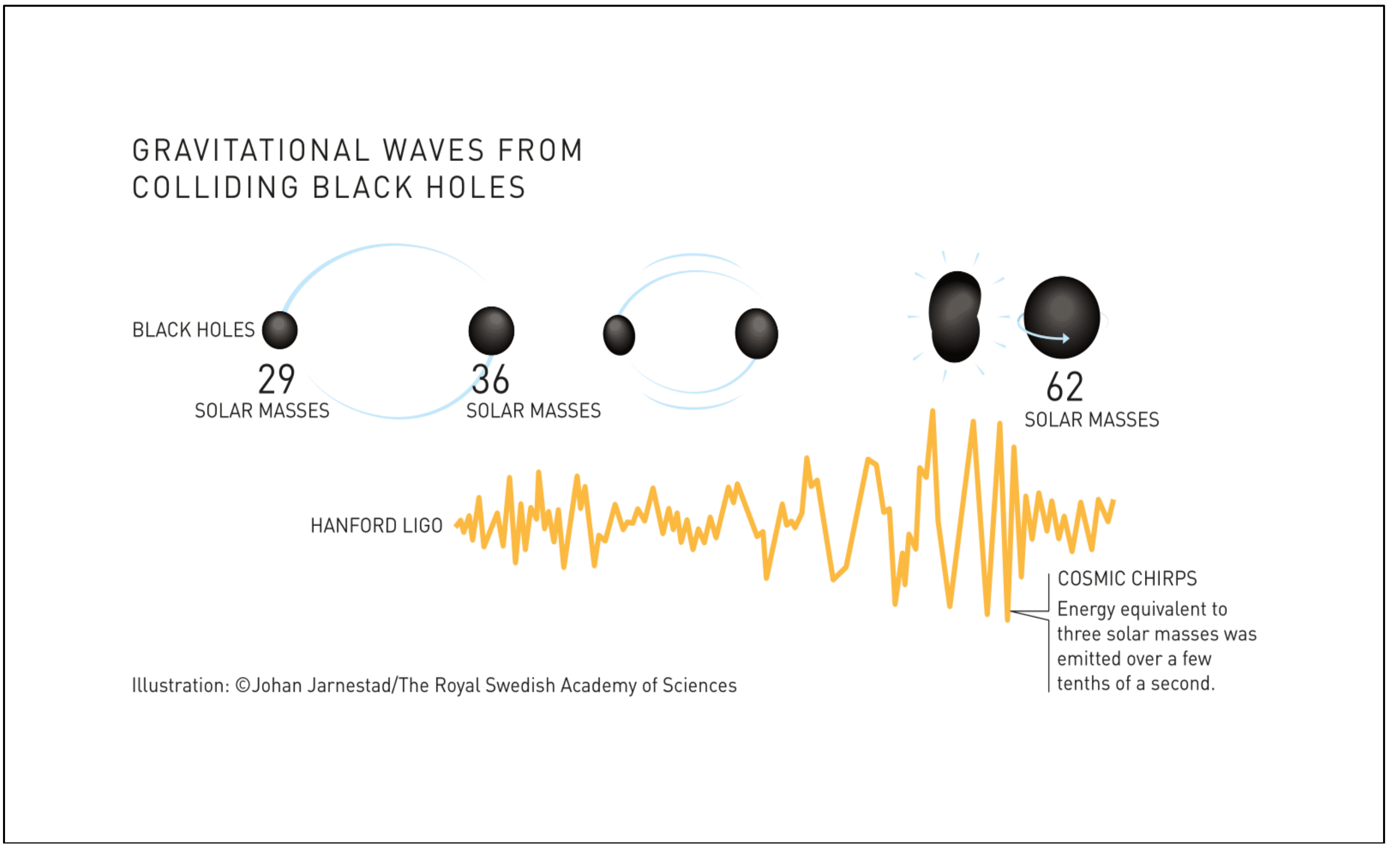

Fig 4: Schematic Diagram of How Gravitational Waves are Created from Colliding Black Holes ("Press Release. NobelPrize.org. Nobel Prize Outreach AB 2017." 2017).

The waves were created due to the collision of two rapidly rotating black holes aka Kerr black holes. It is the first direct observation of a pair of Kerr black holes (Fig. 4). The waves took 1.3 billion years to arrive at the LIGO detector. LIGO is a gravitational wave detector consisting of two gigantic identical interferometers lying 3,000 km apart (LIGO 2017). Gravitational waves are created when a mass accelerates, for example when a pair of black holes rotate around each other. Gravitational waves spread like EM waves at the speed of light. Even though general relativity directly predicts the existence of gravitational waves, Einstein himself did not believe it could ever be detected. The revolutionary gravitational detection led to the Nobel Prize in Physics in the year 2017 ("Press Release. NobelPrize.org. Nobel Prize Outreach AB 2017." 2017).

D. Pulsars

The existence of rapidly rotating stars, predicted as per the Kerr metric, was validated by the discovery of a pulsar in 1968 (Burnell et al. 1968). The Mullard Radio Astronomy Observatory, USA, received unknown signals from a pulsating radio source, that seemed producing due to the oscillations of white dwarf or neutron stars residing within our galaxy. Since the signal maintains its period very strictly like a pulse, the source of the signal is called pulsar. A pulsar is basically a highly magnetized neutron star, which rotates rapidly. As a result, the star emits radiation pulses at regular intervals ranging from seconds to milliseconds. The poles of the magnetic field aren't aligned with the axis of spin of the pulsar, hence the light beams are swept around the space as the pulsar rotates like a lighthouse (Space. com Staff 2023b).

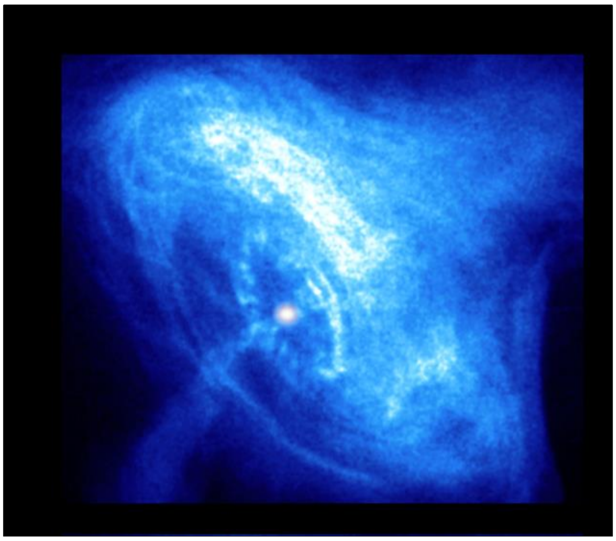

Fig 5: Crab Nebula Pulsar: A Rapidly Rotating Neutron Star Seen as a Bright White Dot Near the Centre of the Image, with Jets of Matter Expelling Away. The Image was Taken by the Chandra X-ray Observatory (Observatory 2002).

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We can see the pulsar inside the Crab Nebula (Fig. 5). The image was taken by the coordination between Hubble Space Telescope and Chandra X-ray Observatory (Hester et al. 2002).

E. Ergosphere (Future Prospect)

A rotating black hole has a region called ergosphere outside its event horizon. This area is a potential source of nearly endless energy (Misner et al. 2017). In theory, it is possible to extract mass and energy from the ergosphere of a Kerr black hole. Objects that enter the ergosphere with enough velocity can still escape the black hole's gravitational attraction since the ergosphere is outside the event horizon. By entering the black hole's rotation and then leaving, an object could gather energy and take some of the black hole's energy with it (Bhat, Dhurandhar, and Dadhich 1985). As energy is being removed from the Kerr black hole, its angular momentum starts decreasing. As a result, spacetime dragging also decreases. Eventually, the rotation stops and the ergosphere disappears. Although harnessing energy from a black hole is an experimental fantasy at the moment, the phenomenon occurs naturally as gamma-ray bursts. As Rojer Penrose said: "By injecting matter into a black hole in a carefully chosen way, one can decrease the total mass-energy of the black hole-i.e., one can extract energy from the hole." (Penrose 1969). A schematic diagram of the idea is shown in Fig. 6.

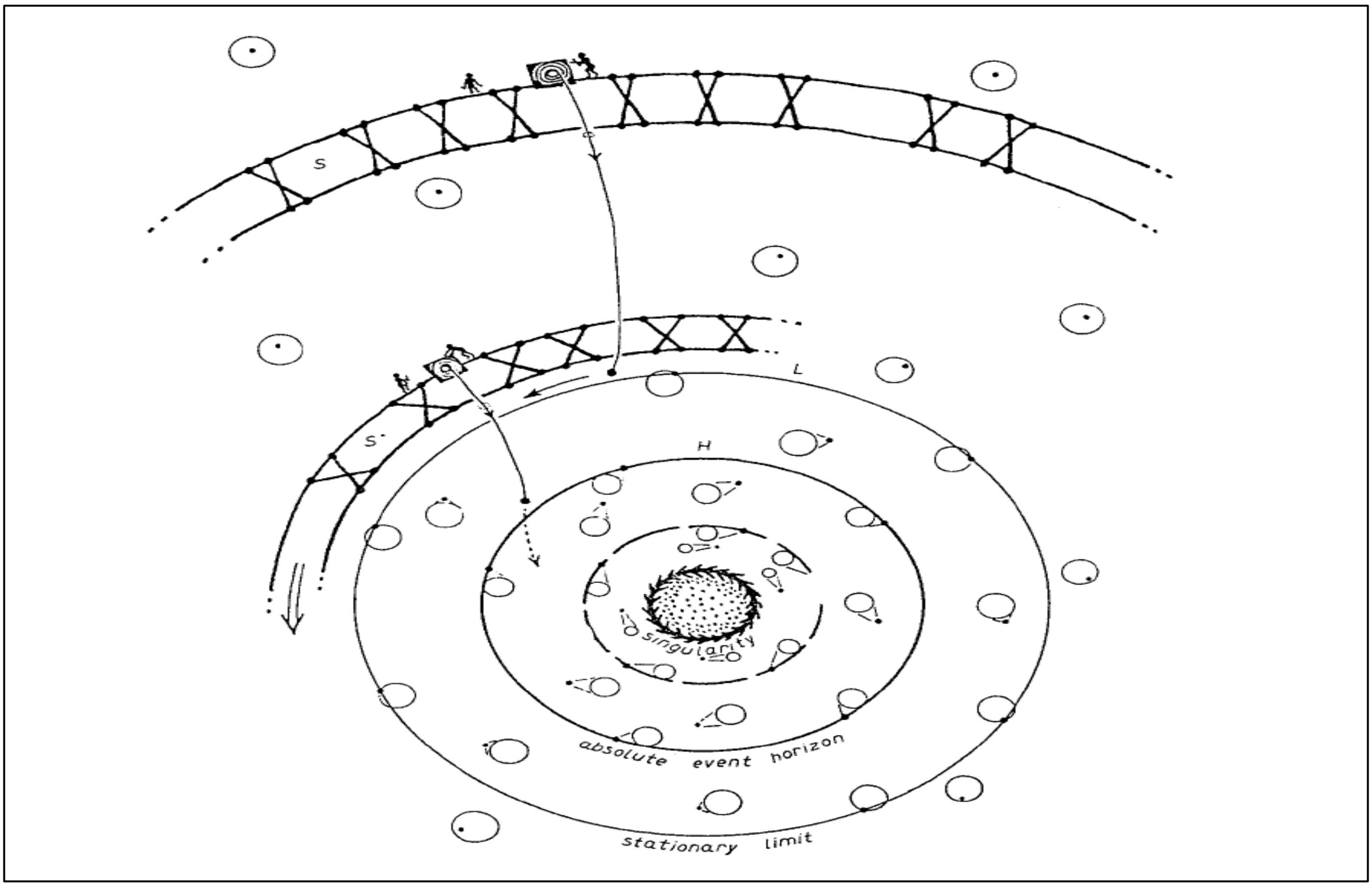

Fig 6: Spinning Black Hole at the Centre of the Picture, around which Residents of the Region S and S^* are Extracting Energy from the Rotation of the Black Hole (Penrose 1969).

VI. CONCLUSION

The Minkowski, Schwarzschild, and Kerr metrics are essential in understanding the structure of spacetime under different physical conditions. The transition from the Minkowski metric to the Kerr metric provides a profound physical interpretation of spacetime, representing significant theoretical advancements in general relativity. The curvature induced by mass and rotation is described through the transition from the Minkowski metric to the Schwarzschild and Kerr metrics. Spacetime is like a bed sheet, with the Minkowski metric being the flat sheet and the Schwarzschild metric illustrating the curvature due to the heaviness of the body. The Kerr metric, on the other hand, demonstrates the frame-dragging effect of the Kerr geometry. The dynamics of space beyond Earth are reflected in our daily lives: the absence of mass representing the morning sheet, the spacetime around regular mass like Earth and the Sun as the evening sheet, and the night sheet reflecting the areas around supermassive objects like quasars or pulsars. These metaphorical interpretations are not only mere theory anymore, observations of Twin Quasar, Crab Nebula Pulsar, Gravitational waves and so on have proved the rotational effect of spacetime to be real indeed.

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