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Cairo Techniques Solution of Schrödinger's Equation – Time Dependence

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Abstract:- **We have shown in previous papers that B-matrix string mechanics, which is the product of a numerical statistical method called the Cairo technique, can be successfully applied to solve the most general form of the diffusion equation of heat as well as the time-independent equation. Schrödinger equation.**

In this paper we examine the extension of matrix chains from the Cairo technique to the solution of the time-dependent Schrödinger equation.

Numerical results for a quantum particle in a three-dimensional box are presented and are intended to be strikingly precise.

I. INTRODUCTION

We have shown in previous papers that B-matrix string mechanics, which is the product of a numerical statistical method called the Cairo technique, can be successfully applied to solve the most general form of the diffusion equation of heat as well as the time-independent equation. Schrödinger equation. [2,3,4,5,6].

The mechanical product of the B matrix of the Cairo techniques predicts that the solution of the heat diffusion equation is forced by the boundary conditions and physical properties of the material involved, while the solution of the quantum mechanical system is spontaneous and function of the size of the quantum object as well as the external potential if applicable.

One of the important reasons for replacing the Schrödinger equation with chains of statistical transition matrices is that you are moving from a field of SE where many questions remain unanswered to the field of modern statistical physics where almost all the questions have adequate answers.

The question arises as to why the quantum mechanics solution and the classical physics solution emerge from the same transition matrix B?

The answer is that Mother Nature only has one face to show, in both classical physics and quantum mechanics.

Note that the Schrödinger wave function is just a mathematical representation of the probabilistic potential for an electron to appear at a given location or time.

It is worth mentioning that the Schrödinger wave equation treats time t as a non-woven external controller in the geometric space of the three Ds.

The unique fundamental property of Cairo techniques is that they treat time t not as an external controller but rather as a rhythm subject to external control via boundary conditions (BC) and the source term (S).

Transition Matrix B acts as a sort of mayestro leader.

Therefore, B-matrix chains inherently include time and evolve over time in a 4D unit space with the source/sink vector and the boundary conditions vector (S and BC) acting as external controllers.

The variable separation method W(x,y,z,t)=X(x).Y(y).Z(z).f(t) is not necessary because it exists intrinsically in the strings of the matrix B.

By applying the chains of the matrix B Both in classical physics and in quantum mechanics, the classical time t is completely lost and replaced by an integer N which is a kind of quantification of time (not to be confused with the unit of time chronon).

The fundamental assumption of the numerical statistical solution via matrix chains B is that a statistical physical transition matrix (B) exists and is defined by the matrix equation,

U(x,y,z,t+dt)= U(x,y,z,t) + [B(x,y,z,t)] . (BC+S)…… (1)

Or,

U is the energy density in the object considered.
BC and S are respectively the vectors of the boundary conditions and the source/sink term.

The entries of matrix B itself are well defined through four conditions explained in references [2,3].

➢ *It Follows That,*

- In the classic physical process which is stimulated,

U(x,y,z,t)=[B^0+B+B^2+ . . . . B ^ N] (vector BC + vector S) + IC .[B^N]……(2)

IC is the vector of initial conditions and B^0 =I the unit matrix.

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Note that B^N goes to zero as N(time) goes to infinity since the norm of [B] is less than 1.

- In the time-independent solution of the quantum physical system which is spontaneous,

([B] + Constant. V(x,y,z).[I]) = λ. Constant. V(x,y,z). . . . (3)

It is clear that equation 3 is an eigenmatrix problem[1] with λ the eigenvalue and V(x,y,z) its eigenvector. Equation 3 can be derived via the following revolutionary hypothesis:

S(x,y,z,t)=Constant. V(x,y,z,t)…..……...(4)

And,

Ψ^2(x,y,z,t) = Constant .V(x,y,z,t)…….(5)

The physical meaning of the expressions 3,4 is that energy can be transformed into a quantum particle and vice versa in an isolated system located in infinite free space.

The question arises Can we extend Equation 3 which is the solution of the time-independent Schrodinger equation to the solution of of time-dependent Schrodinger PDE that should satisfy the principle of least action and the normalization condition without going through SE or any of its derivatives? This is the subject of the present article.

II. THEORY

A. *Least Action Principle*

Any time-dependent physical process must conform to the principle of least action which is a universal law.

The principle of least action is satisfied in the solution of the time-dependent Schrödinger equation when the principal quantum eigenvalue λ is equal to 1.

The geometric and physical meaning of the eigenvalue λ is equal to 1 is that the time-dependent solution goes straight to the stationary time-independent solution.

The solution follows a straight line in unit space x-t towards its final stationary vector solution.

B. *Normalization Condition*

The sum of the areas under the curves of Ψ^2 which are one or more triangles is equal to 1

What is the normalization condition.

Equations 3,4,5 imply a quantum statistics transition matrix Q defined by,

[Q]=[B] + C[I} . . …… . . . (6)

Where I is the unit matrix and C is an arbitrary scalar constant.

And,

[Q]. ψ(r,t)^2= λ. ψ(r,t)^2. …….. . . . (7)

With λ=1.

Note that the definition of the quantum matrix Q by two independent equations 6,7 is not over specified because the matrix B has such a property.

The arbitrary constant C is found by the method of trial and error such that the following conditions are satisfied:

- The determinant of matrix Q is equal to zero.
- The proper matrix of equation 3 is true, that is to say

([B] + Constant. V(x,y,z).[I]) = λ. Constant. V(x,y,z). . . . (8)*

In order not to worry too much about the details of the theory, let's move on to illustrative numerical applications.

III. NUMERICAL RESULTS

Consider a quantum particle inside a three-dimensional geometric cube (box) discretized into 27 equally spaced free nodes, as shown in Figure 1.

Note that the quantum box (black) is placed inside a larger box (blue) where the boundary conditions ψ = ψ^2=0 are satisfied at all eight corners.

Fig.1 A geometric three-dimensional cube discretized into 27 equally spaced free nodes.

Here the 3D quantum matrix Q 27x27 is expressed as a 3D matrix B 27x27 + C I and is given by,

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We can also show that the determinant of this matrix Q is equal to zero and that the potential V(x,y,z) is given by,

(1/8	1/4	1/8	1/4	3/8	1/4	1/8	1/4	1/8	1/4	3/8	1/4	3/8	1/2	3/8
	1/4	3/8	1/4	1/8	1/4	1/8	1/4	3/8	1/4	1/8	1/4	1/8)		

Additionally, we can also show that Q matrix multiplied by V^T gives the stationary vector solution of the quantum particle in a 3D box which is,

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35156251/250000000	14322917/62500000	24414063/62500000	14322917/62500000
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Furthermore, λ is equal to the principal eigenvalue, i.e. λ=1.

Note that the solution vector (Total Energy E(x,y,z)) and the potential vector V(x,y,z) are almost equal as expected.

Here It is clear that this process conforms to the principle of least action.

In Figure 1, we present the Ψ^2 (Blue Triangle) solution for a quantum particle in a 1D infinite potential well. The 1D potential well is assumed to have a small width of [a](meter).

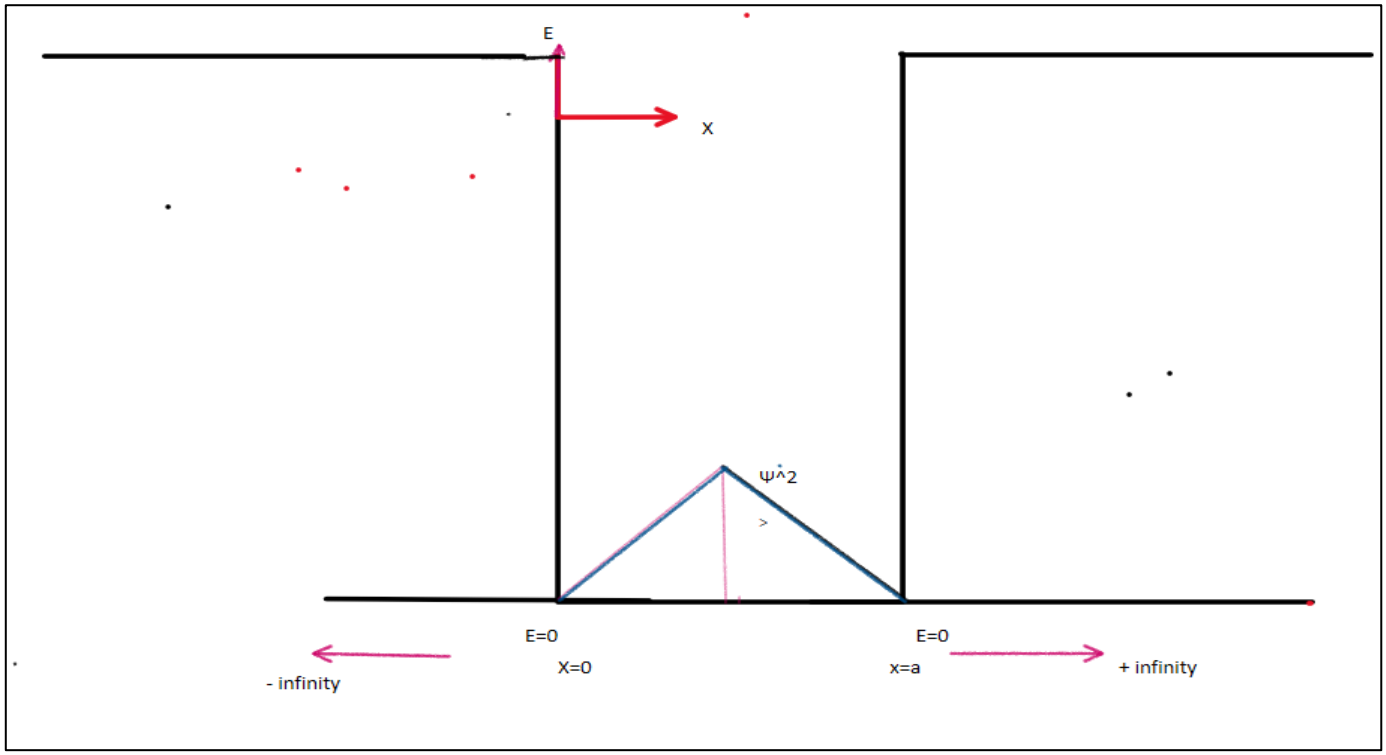

Fig 1: Triangular Solution (Blue) for the Quantum Particle Ψ^2 in a 1D Infinite Potential Well.

In Figure 2 we present the Ψ^2 (triangle) and Ψ solution for a quantum particle in a 3D box while Figure 3 from Google search is shown for comparison.

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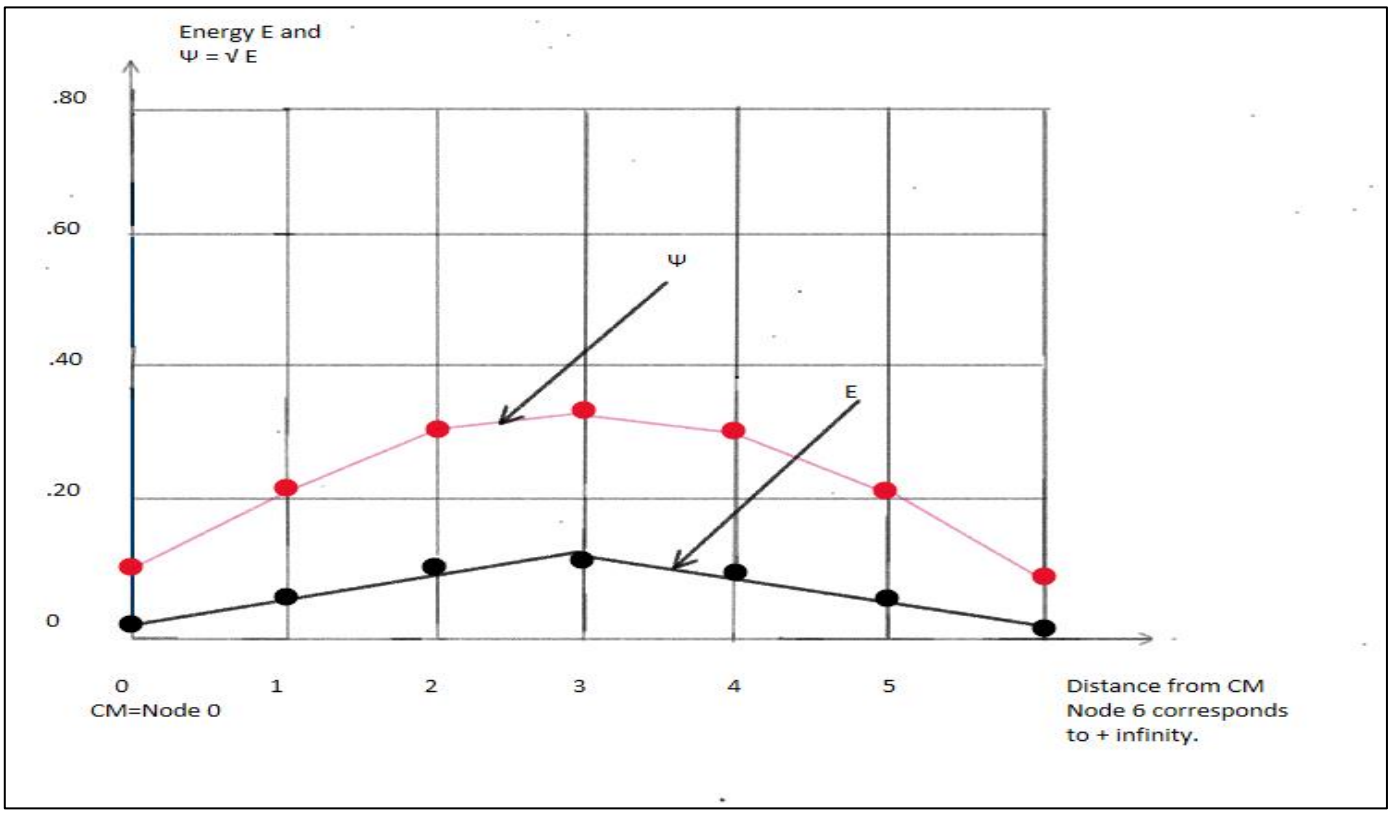

Fig 2: The solution E=Ψ^2 (triangle) and wave function Ψ for a quantum particle in a 3D box.

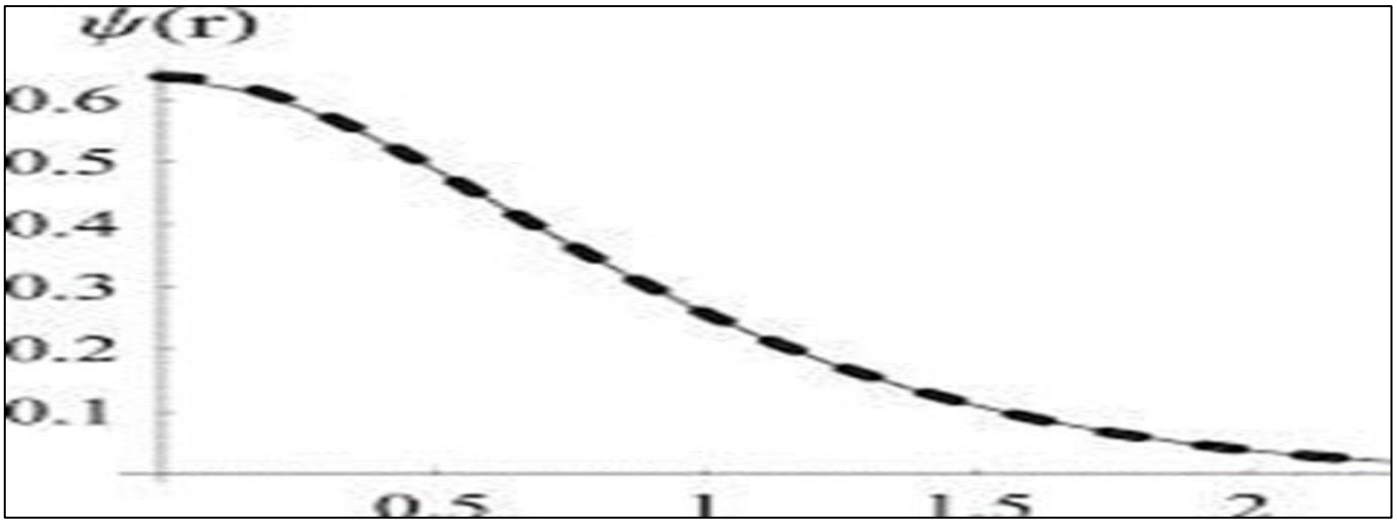

Fig 3: From Google Search is Shown for Comparison.

IV. TIME DEPENDENCE

It is easy to show that equations 3 to 6 predict a solution for the time-dependent Schrödinger equation in the form:

Ψ^2(x,y,z,N)=Ψ^2(x,y,z,0) . [1.-Exp -C N]

Where the real time t is discretized in Ndt.

Here we present the following expressions for the Ψ^2 solution of the time-dependent Schrödinger equation:

Ψ^2(1)=Q. V
Ψ^2(2)=Q^2 . V
Ψ^2(3)=Q^3 . V . . . . . .

Etc.

Therefore, assuming that the value of vector Ψ^2 1x27 is given by,

(1 , 1 ,1 , . .1 ,1 ,. . . .1 , 1 ,1)^T

Then the solution state vectors are given by,
Ψ^2(1dt)=
(0.1111 , 0.1778 , 0.1111 , 01944 , 0.3167 , 0.1944 , .01111, 0.1778 , 0.1111 , 0.1944 , 0.3167 , 0.1944 , 0. 3500 , 0.5556...etc. .)^T

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And,

Ψ^2(2dt)=
(0.1068 , 0.1821 , 0.1968 , 0.1914 , 0.3222 , 0.1914 , 0.1068 , 0.1821 , 0.1068 , 0.1914 , 0.3222 , 0.1914 , 0.3389, 0.5747 . .etc . .)^T

➢ *Note That:*

- The time-dependent solution for Ψ^2(t) is formed from equilateral triangles as expected.
- The time dependent solution is heading towards the stationary time independent solution describe before

V. CONCLUSION

We have shown in previous papers that B-matrix string mechanics, which is the product of a numerical statistical method called the Cairo technique, can be successfully applied to solve the most general form of the diffusion equation of heat as well as the time-independent equation. Schrödinger equation.

In this paper we examined the extension of B-matrix chains from the Cairo technique to the solution of the time-dependent Schrödinger equation.

Numerical statistical equations for a quantum particle in a three-dimensional box are presented and the numerical results of the solution are surprisingly accurate.

NB: The numerical results presented in this article are obtained via the author's double precision algorithm as explained in Ref (11).

No MATLAB or Python ready algorithms are needed.

REFERENCES

[1]. John H. Mathews, Numerical methods for Mathematics, Science and Engineering,1994, pp. 540-550.
[2]. I. M. Abbas, I.M. Abbas, IJISRT review, A Numerical Statistical Solution to the Laplace and Poisson Partial Differential Equations, Volume 5, Issue11, November – 2020
[3]. I. M. Abbas, How Nature Works in Four-Dimensional Space: The Untold Complex Story. Researchgate , IJISRT review, may 2023.
[4]. I. M. Abbas, IJISRT, Time Dependent Numerical Statistical Solution of the Partial Differential Heat Diffusion Equation, Volume 6, Issue 1,January - 2021
[5]. Cairo Techniques Solution of Schrödinger's Partial Differential Equation – Time Dependence
[6]. I. Abbas, A statistical numerical solution for the time-independent Schrödinger equation, Researchgate, Nov 2023.
[7]. Marc Baldo, The time-independent Schrödinger equation, Massachusetts Institute of Technology, 2022.
[8]. Cairo Techniques Solution of Schrödinger's Partial Differential Equation – Time Dependence, Researchgate, IJISRT review, Mars 2024.
[9]. Google search, Numerical solution of time-independent 1D Schrodinger equation
[10]. G.K NCERT, Particle In 3 Dimensional Box, Rayagada College, October 07, 2021.
[11]. I. M. Abbas, IEEE.1996, Pseudo-discharge spark. Plasma ScienceTransactions24(3):1106 - 1119, DOI: 10.1109/27.533119.