FALL and RISE of Matrix Mechanics

Dr. Ismail Abbas

Abstract:- In 1925, W. Heisenberg, Max Born and Pascual Jordan introduced the first so-called matrix mechanics (HMJ theory) to study the fine structure of the Bohr hydrogen atom.

However, in the early 1930s, the equivalence between the HMJ theory and the Schrödinger equation was denied and the HMJ theory fell.

In 2020, a new theory of matrix mechanics emerged, called b-matrix chains, and has been successfully applied to different 3D situations in classical physics as well as quantum mechanics.

In this paper we study the application of new matrix theory to the initial value problem in the 3D heat diffusion equation as well as to quantum particles in a 3D cube where the numerical results are strikingly accurate.

The similarity of the matrix techniques applied in both cases suggests that nature has only one face to show in classical and quantum physics.

I. INTRODUCTION

In 1925, W. Heisenberg, Max Born and Pascual Jordan introduced so-called matrix mechanics (HMJ theory) to study the fine structure of Bohr hydrogen atom. HMJ theory succeeded in finding quantum numbers such as l,m,s in addition to the principal quantum number n.

HNJ proposed a square matrix generally having complex entries which was considered a matrix formulation of QM and obviously the first mathematical formulation of quantum mechanics. It is conceptually autonomous and logically coherent [2].

In 1927 the well-known Schrödinger equation SE appeared with the Bohr/Copenhagen interpretation.

SE, with the Copenhagen interpretation of quantum mechanics, has dominated the field since the 1930s.

In the years 1927 to 1930, the equivalence between HMJ theory and SE theory was supported by rigorous mathematical proofs, with the exception of the extended concept regarding the quantum superposition of the wave function resulting from the interpretation of Bohr/Copenhagen where the HBJ takes late.

Unlike HBJ, Schrödinger equation perceived the field-like continuity of some key microphysical phenomena, such as the interference phenomena of a coherent electron beam in double-slit experiments.

In the late 1930s there was much debate about the equivalence of HMJ's and Schrödinger's equation until the performance of a double-slit interference experiment which clearly showed that the theory of HMJ fell short of SE theory with respect to the principle of superposition.

Furthermore, HBJ theory has never addressed the description of macroscopic physical phenomena and is therefore considered incomplete.

In other words, HMJ theory is incomplete because it cannot solve classical physics problems such as the thermal diffusion equation.

As a result, the HMJ and SE equivalence has been debunked and considered a myth in the interpretation of double-slit interference experiments.

This is what we call the fall of Matrix Mechanics almost a century ago.

However, In 2020, a new theory of matrix mechanics MM emerged and successfully applied to different areas of classical physics in addition to quantum mechanics.

This is what we call the renaissance or rise of matrix mechanics.

The new matrix mechanics (MM) procedure is called B-matrix chains and is abbreviated as BMM.

The important inherent depth difference between HBJ and BMM matrix mechanics is that BMM theory is more comprehensive.

HBJ is a lifeless mathematical description in 3D+t space while matrix chain technique B is a living natural statistical system capable of evolving natural situations in 4D x-t unit space.

In other words, in the HBJ technique, the time step dt is fixed arbitrarily and not quantified as in the case of the BMM technique.

The Schrödinger equation and HBJ matrix mechanics operate in a separable 3D+t space, so HBJ and SE theories can be seen, in some way, as a subset of SE and not the other way around.

In this article, we explain how to apply the new BMM technique to show the temporal evolution of initial value problems in two 3D situations namely:
- Heat diffusion in a metal cube,
- Energy states of a quantum particle in a closed box.

The analysis and numerical results show that the same inputs of 3D matrix chains are used in both classical physics (i) and quantum mechanics (ii).

The similarity between classical physics and quantum physics has also been demonstrated in previous articles[3,4,5].

This means that nature has a face to show in classical and quantum physics.

The transition matrix B and hence BMM is well defined and has been successfully applied to different areas of classical physics such as Poisson and Laplace PDE, heat diffusion equation, theory and design of the audio. rooms, Limited integration in 1D, 2D, 3D , etc. [6,7,8,9].

Furthermore, the extension of BMM to find a numerical statistical solution of the time-independent Schrödinger equation in 1D, 2D and 3D was also recently described and the numerical results were surprisingly accurate [3,4,5].

An important reason to replace the Schrödinger equation with the equivalent B-matrix mechanics is that you move from the domain of SE where many questions remain unanswered to the domain of statistical matrix equations where almost all questions have answers.

The extension of physical matrix mechanics to the solution of Schrödinger equation requires strict adherence to certain fundamental limits that apply to the entire BMM theory.

Additionally, the application of the concept of physical B-transition matrix chains (B-matrix mechanics) to the solution of both classical physics problems and Schrödinger equations requires the introduction of certain physical terms or concepts and basic mathematics which we leave briefly explained in the following hypotheses:

- Square matrices are a subset of mathematical matrices, and physical square matrices that have physical meaning (such as the transition matrix B) and favored for defining matrix mechanics are a subset of square matrices.
- What is a numerical and/or statistical solution!

The numerical solution replaces the analytical solution of the time-dependent PDE by discretizing space and time into dx and dt and replacing the differentials dy/dx by $[y+dy-2y +y-dy]/2dx$ and $d^2 y/dx^2$ by $[y+dy-2y +y-dy]/dx^2$. etc.

In other words, the numerical solution method reduces the PDE to a system of algebraic equations via the finite difference method FDM.

On the other hand, computational methods such as (FDM) are not necessary in transition matrices of statistical solutions since FDM techniques inherently exist in statistical chains of transition matrices.

- Statistical transition matrices B and chains of statistical transition matrices exist and define a distinguished kind of matrix mechanics. Its modeling works effectively to find the evolution of energy density in partial differential equations.

We currently know two, namely the mathematical and statistical transition matrix of Markov and the physical transition matrix B which is the subject of this article.

It is worth mentioning that in Markov matrix chains we do not care about the energy density, boundary conditions, source term, average properties of the medium, etc., whereas in the case of matrix chains B, we do it.

- When a physical statistical transition matrix chain B for the energy density U(x,y,z,t) exists then it can be defined by the recurrence relation,

U (x, y, z, t+ dt) = B. U(x,y,z,t)

Note that the transition matrix B should have a place for the boundary conditions BC and the source term S which are essential in the solution of the heat diffusion equation as well as the Schrödinger equation.

As a result, a chain transition matrix B emerges and must be able to describe the solution trajectory for the energy density U through its own solution space for any given time evolution in 4-D unitary xt space. Also note that the string transition matrix B describes the energy density in classical physics problems and the square of the wave function $\psi 2(r)$ in quantum mechanics problems.

- Classical macroscopic statistics and quantum microscopic statistics are subject to the same physical transition matrix B. which means that nature only has one face to show.
- In the matrix chain solution for the time-dependent energy density U(x,y,z,t) the real time t is completely lost and replaced by N.dt where N is an integer describing the number of iterations and dt is an inherent time step or jump in time.

Time is replaced by the number of repetitions of the physical process N.

It is also worth mentioning that discretizing time t into forbidden and allowed where t = N dt and N is an integer is itself a quantification of time. Apparently, this condition replaces Bohr's hypothesis of energy quantification in 1913.

- The matrix solution for the energy density U(x, y, z, t) in the time-dependent and time-independent PDEs appears as a matrix equation.

This is exactly what one would expect from a matrix mechanics technique.

Not all matrix equations "resulting from the solution of PDE via the transition matrix B" are eigenvalue equations. For example, the matrix equation of the numerical solution of the heat diffusion equation results in a matrix system of non-homogeneous first-order linear algebraic equations while the matrix equation of the numerical solution of the Schrödinger equation is homogeneous and results in an eigenvalue problem. . Several eigenvalues have their corresponding eigenvectors. The time-dependent and time-independent Schrödinger equations are in-depth examples of eigenvalue equations in quantum mechanics, with their eigenvalues corresponding to the allowed energy levels of the quantum system. Generally speaking, in the statistical transition matrix B, the eigenvalue is the dominant

eigenvalue (eigenvalue of the maximum absolute value) equal to 1.

- What is the time-dependent Schrödinger equation and what is the time-independent Schrödinger equation?

The time-dependent equation is $i\hbar(d\psi/dt)= \hat{H}\psi$,

and the time-independent equation is $E\psi = \hat{H}\psi$.

Which show that the Schrödinger equation is a second order linear PDE in what is called the wave function $\psi(x,t)$ and constitutes a means of probabilistically describing the temporal evolution of energy, of the momentum and position of quantum particles in space. His time-independent equation for $\psi(x)$ describes the equilibrium state that occurs when evolutionary time tends to infinity. It should be noted that another way to describe the dynamics of quantum particles is to use statistical transition matrices (New Matrix Mechanics) which completely ignore the Schrödinger equation and the wave function ψ as if they do not had never existed in the same way as we solve the heat diffusion equation. without going through thermal EDP.

This is obviously ψ^2 and not ψ itself

- In addition, the method of separating variables $W(x,y,z,t)=X(x)Y(y)Z(z) \ f(t)$ is also not necessary because it is intrinsically included in the 4D unit space inseparable x-t from the strings of matrix B.
- The numerical method (technique) of Monte Carlo is closest to the Cairo technical method and matrix mechanics. However, the numerical Monte Carlo method is a bit old and requires generating a random numerical variable thousands of times. This makes the interpretation of its numerical results long and tedious.
- The determinant of the quantum transition matrix $Q=B+V(x,y,z)$ must be equal to zero to identify the homogeneous system which gives rise to the eigenvalue problem.

In other words, a determinant equal to zero means that a matrix is a singular matrix. A matrix is singular if it has no inverse, which means it cannot be used to solve systems of linear equations.

- In the problem of eigenvalues in matrix chains B, the dominant or maximum eigenvalue =1.
- Finally, let us emphasize again that today we only know a kind of physical transition matrix and a kind of mechanical matrix which is the transition matrix B resulting from the so-called Cairo technique.

But why does the matrix mechanics introduced by the transition matrix chains B allow a better understanding of theoretical physics?

The Schrödinger equation describes how a particle's wave function $\psi(x, y, z)$ explores 3D space as a function of time t.

Three-dimensionality is the fundamental intrinsic lack of SE.

We logically assume that SE is, in some way, a subset of matrix mechanics and not the other way around. It would be absurd to expect that the missing physical elements of the SE would be completed by the SE itself.

II. THEORY

In the Cairo techniques approach, the time-dependent solution of the PDE of energy density U(x,t) is given by [6,7,8,9], **U(x, t)=D(N) . (b + S) + IC. B^N. . . . . (1)**

Where S is the vector of the source/sink term and IC is the vector of the initial conditions.

Equation 1 is used as a time-dependent statistical equivalence matrix that can be used in the solution of classical physics problems such as thermal conduction PDE and it is also proposed to find a solution to the 3D Schrödinger equation .

It should be noted that equation 1 contains a term due to the initial state conditions described by IC. B^N which is expected to decrease exponentially with time because the modulus of matrix B is less than 1.

This term tends to zero with time in non-isolated systems of classical physics such as the heat diffusion equation, but not in isolated quantum mechanical systems such as described by the time-dependent Schrödinger equation.

For isolated quantum mechanical systems as described by the time-dependent Schrödinger equation, the matrix B is completed by the voltage matrix V(x,y,z) and therefore does not tend to zero with time.

Note that equation 1 is the solution of U(x,t) in a 4D unit space xt where the real time t is completely lost and replaced by a dimensionless integer N.

Again, the integer N is the number of iterations which is the number of time steps or time jumps dt.

Equation 1 is very important because it defines the spatio-temporal evolution of the energy density U in space and time in matrix form. Furthermore, equation 1 is the solution source of matrix mechanics in classical and quantum physics.

In fact, Equation 1 is the fundamental idea of B-matrix mechanics.

The application of equation 1 to the solution of quantum mechanical problems has been successfully carried out in 1D, 2D and 3D situations [3,4,5] where the solution process must be carried out in consecutive steps.

- We first assume an intrinsic and/or extrinsic landscape potential V(x,y,z) which must be symmetrical and imply a zero BC potential.
- We assume that the matrix B(x,y,z) must be completed by a diagonal matrix V(x,y,z) representing the source term S, that is to say

S(x,y,z,t)=Constant. V(x,y,z,t) . . . .. (2)

Where S is a source/sink term.

The constant of equation 2 is found by trial and error such that the determinant of [B + V(x,y,z,t).I] is equal to zero (rule 12 of section I) and is explained in more detail in section III (numerical results).

Equation 2 is a breakthrough because it characterizes the physical domain of validity of SE and suggests a statistical solution to 3D Schrodinger equation that circumvents SE itself.

Note that there is a simple way to solve the matrix statistical equivalence of SE, i.e.

U(x, t)=D(N) . (b + S) + IC. B^N. . . . . . (1)

This involves assuming in advance the potential landscape V (x, y, z) first, then working backward to find the eigenvalues and eigenvectors of the energy. This can also be a way to resolve the SE, while it's easier to go back and look for a solution.

This is exactly what happens even when solving the 1D, 2D and 3D Schrödinger equation via B-matrix statistical chains [3,4,5], while it is better to first assume the potential landscape before solve.

We also know that the solution of matrix chains B bypasses SE and therefore may be, in some way, more informative than SE itself, a claim which will be explored in more detail when describing solutions to time dependent Schrödinger's equations in 4D.

It is worth mentioning that B-matrix string theory is not entirely new and has been working effectively in different fields of classical physics since 2020 [6,7,8,9].

The ultimate goal of B-matrix mechanics is to find the adequate numerical solution to physical problems in almost all areas of classical physics as well as those of the Schrödinger equation by expressing the relevant physical quantities in a unit x-t space.

In order not to worry too much about the details of the theory, let's move on to the following numerical results.

III. NUMERICAL RESULTS

A. III-A
A Heat diffusion equationInitial value problem,

T(x,y,z,t=0) =constant or unity.

[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]T

Required to find T(x,y,z,t=0) at any time t>0.

➤ *Step 1*
Construct the transition matrix B with fixed RO corresponding to the adequate thermal diffusion coefficient D of the material tested.

We assume here that RO = 0.2, which corresponds to the thermal diffusivity of high purity aluminum metal.

The 3D transition matrix B(27x27) for RO=0.2 is given by,

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0.0 0.00 0.00 0.00 0.00 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.00 0.0 0.0 0.6/8 0.0 0.0 0.00 0.00 0.00 0.6/8 0.00 0.6/8 0.20

We call this matrix **M1.**

➢ *Step 2*
Use equation 1 with the substitution, b=0 and S=0.

Equation 1 reduces to,
U(x,y,z,t)=B^N. U(x,y,z,t=0)

The numerical results of temperature T as a function of dimensionless time t=N are presented in Table I.

Table I. temperature T (x,y,z,t) against time N
N=0
T=
[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]

What is the temperature distribution under initial conditions t=0.
N=1
T=
{0.425, 0.5, 0.425, 0.5, 0.575, 0.5, 0.425, 0.5, 0.425, 0.5, 0.575, 0.5, 0.575, 0.65, 0.575, 0.5, 0.575, 0.5, 0.48, 0.555, 0.425, 0.5, 0.575, 0.5, 0.48, 0.555, 0.425]

N=2
T=
0.198   0.25   0.198   0.25   0.314   0.25   0.198
0.198   0.25   0.314   0.318   0.25   0.318
0.389..etc..
N=3
T=
[0.096, 0.127, 0.096, 0.127, 0.167, 0.127, 0.096 , 0.127, 0.096, 0.131, . 127 , 0.127, 0.168 , 0.229 .. etc..,

Note that throughout the previous cooling curve where all 27 nodes evolve towards zero temperature, the central node 14 is always at the maximum temperature.

Figure 1 shows the numerical results presented in Table I for the nodes on axes 5,14,23.
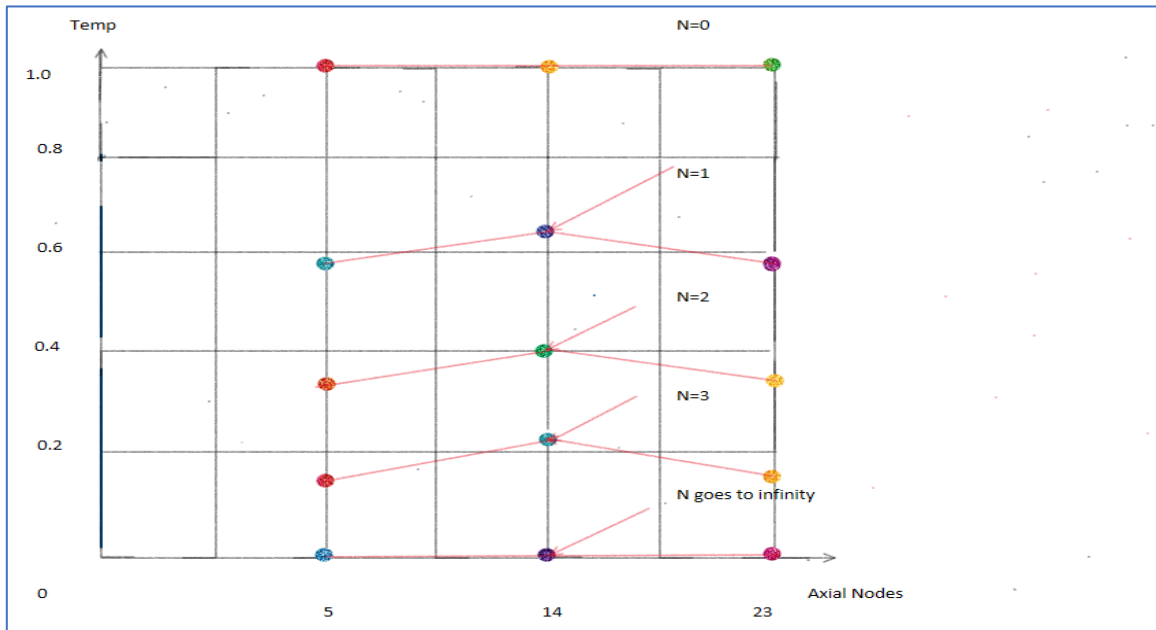
Fig. 1: Numerical results of the temperature T on the axis of the metal cube at different times N.

B. III-B: Quantum particle in the 3D dimensional box

One of the important applications in quantum mechanics is the total energy distribution of a quantum particle in the 3D dimensional box.

➢ *Step 1*

Similar to the heat diffusion equation, we start with the 3D transition matrix B with the only change that RO = 0 and not 0.2.

We call this matrix **M2**.

➢ *Step 2*

Construct the quantum transition matrix Q with fixed RO = 0 as explained in reference 5.

Again, the transition eigenmatrix Q is given by,
Q = B + C. V(x,y,z).I

The constant C is a function of the size and shape of the matrix B and can be easily found by trial and error if we know that:

• Determinant [Q}=0.
• V(x,y,z) is symmetrical with respect to the center of mass of the geometric shape which here coincides with node 14.

Here the quantum transition matrix Q is given by,

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3/16  1/6   0.0   1/6   0.0   0.0   0.0   0.0   0.0   1/6   0.0   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

1/6   4/16  1/6   0.0   1/6   0.0   0.0   0.0   0.0   0.0   1/6   0.0   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

0.0   1/6   3/16  0.0   0.0   1/6   0.0   0.0   0.0   0.0   0.0   1/6   0.0   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

1/6   0.0   0.0   4/16  1/6   0.0   1/6   0.0   0.0   0.0   0.0   0.0   1/6   0.0   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

0.0   1/6   0.0   1/6   6/16  1/6   0.0   1/6   0.0   0.0   0.0   0.0   0.0   1/6   0.0
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

0.0   0.0   1/6   0.0   1/6   4/16  0.0   0.0   1/6   0.0   0.0   0.0   0.0   0.0   1/6
0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

0.0   0.0   0.0   1/6   0.0   0.0   3/16  1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0
1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

0.0   0.0   0.0   0.0   1/6   0.0   1/6   4/16  1/6   0.0   0.0   0.0   0.0   0.0   0.0
0.0   1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

0.0   0.0   0.0   0.0   0.0   1/6   0.0   1/6   3/16  0.0   0.0   0.0   0.0   0.0   0.0
0.0   0.0   1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   4/16  1/6   0.0   1/6   0.0   0.0
0.0   0.0   0.0   1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0

0.0   1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0   1/6   6/16  1/6   0.0   1/6   0.0
0.0   0.0   0.0   0.0   1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0

0.0   0.0   1/6   0.0   0.0   0.0   0.0   0.0   0.0   0.0   1/6   4/16  0.0   0.0   1/6
0.0   0.0   0.0   0.0   0.0   1/6   0.0   0.0   0.0   0.0   0.0   0.0
```

```
0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    6/16   1/6    0.0
1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0

0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    1/6    8/16   1/6
0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0

0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    1/6    6/16
0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0

0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0
4/16   1/6    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0

0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0
1/6    6/16   1/6    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0

0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0    0.0    1/6
0.0    1/6    4/16   0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0
0.0    0.0    0.0    3/16   1/6    0.0    1/6    0.0    0.0    0.0    0.0    0

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0    0.0
0.0    0.0    0.0    1/6    4/16   1/6    0.0    1/6    0.0    0.0    0.0    0.0

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    0.0
0.0    0.0    0.0    0.0    1/6    3/16   0.0    0.0    1/6    0.0    0.0    0.0

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0
0.0    0.0    0.0    1/6    0.0    0.0    4/16   1/6    0.0    1/6    0.0    0.0

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6    0.0
0.0    0.0    0.0    0.0    1/6    0.0    1/6    6/16   1/6    0.0    1/6    0.0

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1/6
0.0    0.0    0.0    0.0    0.0    1/6    0.0    1/6    4/16   0.0    0.0    1/6

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    0.0    3/16   1/6    0.0

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.00   1/6    4/16   1/6

0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0
0.0    0.0    1/6    0.0    0.0    0.0    0.0    0.0    1/6    0.0    1/6    3/16
```

We call this matrix **M3**.

➤ *Step 3*

Use equation 1 with the substitution, b=0 and S=0.

Equation 1 reduces to,

U(x,y,z,t)=Q^N. U(x,y,z,t=0)

The numerical results of the total quantum energy as a function of time N are presented in Table II.

Table II. Energy density U (x,y,z,t) against time N

N=0

U(x,y,z)=

[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]

What is the temperature distribution under initial conditions t=0.

N=1

U(x,y,z)=

[11/16 11/12 11/16 11/12 29/24 11/12 11/16 11/12 11/16 11/12 29/24 11/12 29/24 3/2 29/24 11/12 29/24 11/12 11/16 11/12 11/16 11/12 29/24 11/12 11/16 11/12 11/16]

N=2

U(x,y,z)=

930611/1769472 284309/331776 930611/1769472 284309/331776 487973/331776 284309/331776 930611/1769472 284309/331776 930611/1769472 284309/331776 487973/331776 284309/331776 487973/331776 35113/13824 487973/331776 284309/331776 487973/331776 284309/331776 930611/1769472 284309/331776 930611/1769472 284309/331776 487973/331776284309/331776 930611/1769472 284309/331776 930611/1769472

N=3

U(x,y,z)=

0.526, 0.857, 0.526, 0.857, 1.471, 0.857, 0.526, 0.857, 0.526,0.857, 1.471, 0.857, 1.471, 2.540, 1.471, 0.857, 1.471, 0.857,0.526, 0.857, 0.526, 0.857, 1.471, 0.857, 0.526, 0.857, 0.526

(PDF) FALL and RISE of Matrix Mechanics. Available from: https://www.researchgate.net/publication/377220714_FALL_and_RISE_of_Matrix_Mechanics [accessed Jan 08 2024].

When the above results are validated via the conditions, i,ii, they showed excellent agreement, which means that the mechanics of matrix B are almost exact.

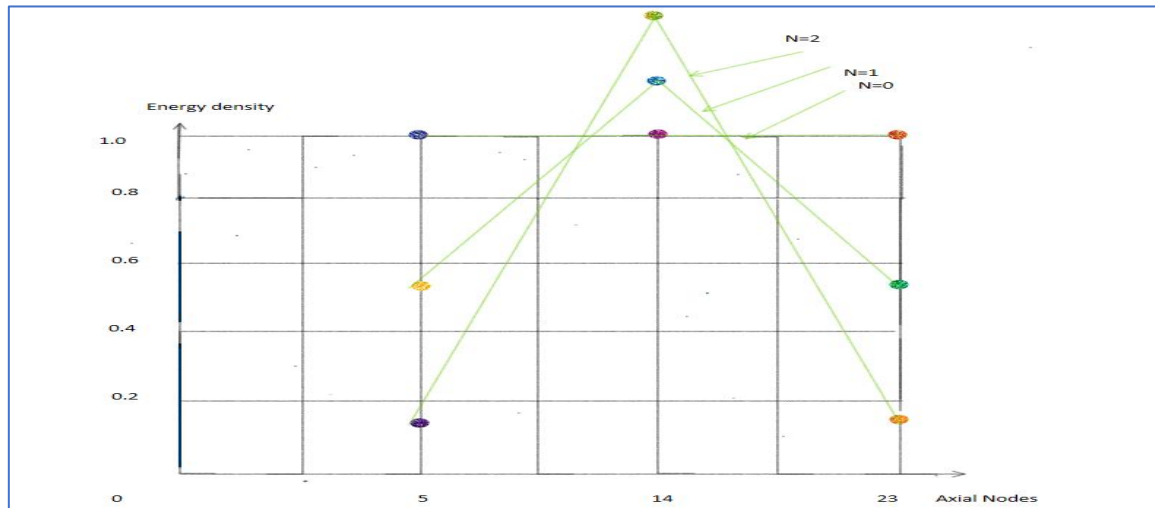Figure 2 shows the numerical results presented in Table II.

Fig. 2: Numerical results for the energy of quantum particles in a box at different times N.

IV. CONCLUSION

It is possible to extend physical transition matrix chains or B-matrix mechanics to the solution of the time-dependent Schrödinger equation.

The present study shows that the same statistical chains of the B matrix can be applied to the solution of the 3D heat equation as well as to that of the Schrödinger equation.

This means that nature only has one face to show in classical and quantum physics.

We present the solution for two illustrative situations, namely the initial-valued thermal diffusion problem and the equilibrium energy distribution for a given initial state of a quantum particle in a three-dimensional box where the numerical results are excellent precision.

Note: In the previous calculations, the author used his own double precision algorithm as explained in ref. 12.

No ready-made algorithms such as Python or MATLAB are required.

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