A Study on the Reliability Characteristics of Non-Repairable Parallel Redundant Complex Systems via Boolean Function Analysis

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Abstract:- The study investigates the reliability characteristics of a non-parallel repairable redundant complex system, akin to a power plant. This system aims to furnish electricity, generated by three generators within a power house, to a crucial consumer via switches, wires, and associated connections. Employing the Boolean Function methodology, the dependability of the critical consumer's power supply has been assessed. To underscore the key outcomes, a numerical illustration accompanied by a graphical representation is presented in the conclusion.

Keywords:- *Boolean Function, Reliability, Non-Repairable, Weibull Distribution, Exponential Distribution .*

I. INTRODUCTION

We frequently encounter the scenario of assessing reliability utilising additional variables and the Laplace Transform approach in a variety of useful reliability systems. Several academics [1-7] have recently examined a wide range of approaches to find symbolic reliability statements for complicated systems. Two additional factors have increased the significance of symbolic reliability statements for complex systems:

- A complex system must undergo many system reliability evaluations during the design and development phase.
- In a number of engineering systems, module reliabilities fluctuate over an extended length of time while the system configuration stays constant.

Symbolic reliability expressions would significantly minimise the calculations required to assess the system dependability through a recursive procedure in each of these scenarios.

Agarwal and Pal [3] have explored the application of the Boolean function methodology in evaluating various reliability parameters concerning a gas separation plant. Similarly, Agarwal, Deepika, and Sharma [4] have extended this approach to investigate the reliability aspects of sugar production facilities. Thus far, limited attention has been given to employing the Boolean function technique for analyzing the dependability of intricate systems.

The B.F. method, as formulated in this study, is applied to assess the dependability of a symbolic expression representing a parallel redundant complex system. This analysis is specifically tailored for the system under investigation, which comprises a main switchboard, a sub-main switchboard, an output switchboard, three generators, and associated wiring. With the system's primary function being a power plant, the focus lies on evaluating the dependability of the power supply from the output switchboard, particularly for critical users. This calculation aligns with the broader context of the complex system's operation and its significance in delivering reliable power to essential consumers.

Reliability has been assessed mostly using Weibull and exponential distributions. The significant findings are further illustrated by a graph that is added and provides a numerical illustration.

The system that is being examined is shown in Figure 1.

II. ASSUMPTIONS

- No service facility is present.
- All component failure times are arbitrary.
- All of the system's component parts have recognised dependabilities beforehand.
- At first, every part of the system functions well.
- The system as a whole and each of its parts are either in excellent (functioning) or bad (failing) condition.
- Every component has an autonomous state.
- Only one component's state can alter at any given time.
- Every component is constantly in operation; there is no switched redundancy or backup.

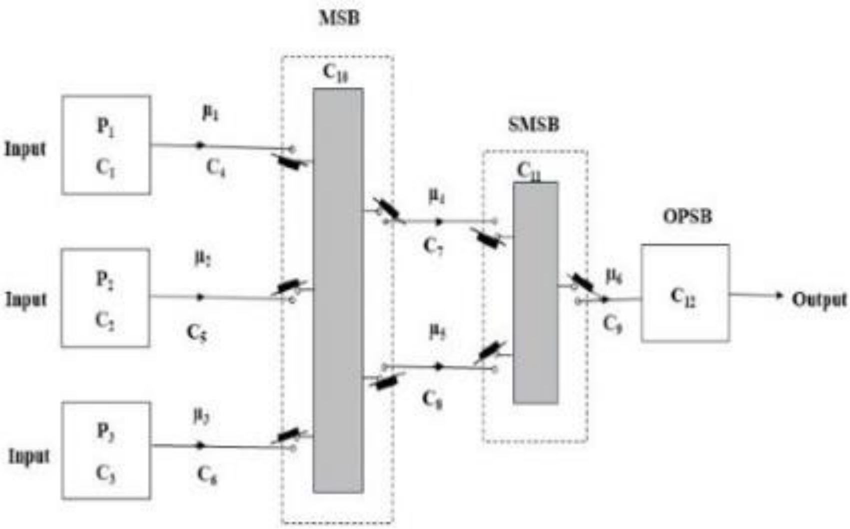

Fig. 1 System configuration

- *NOTATION*

MSB, SMSB, OPSB main switchboard, sub-main switchboard and output switchboard

C_1, C_2, C_3 States of generators P_1, P_2, P_3
C_{i+3} State of the cable $\mu_i (i = 1,2,3,\ldots,6)$

C_{10}, C_{11}, C_{12} States of MSB, SMSB, OPSB
C_i' negation of C_i
$\wedge$ conjunction

$$C_i = \begin{cases} 0, & in\ bad\ state \\ 1, & in\ good\ state\ (i = 1 - 12) \end{cases}$$

III. DEVELOPMENT OF THE MATHEMATICAL MODEL

By using the B.F. technique, the condition of capability for the successful operation of the system in terms of logical matrix are expressed as

$$f(C_1, C_2 \ldots, C_{12}) = \begin{vmatrix} C_1 & C_4 & C_{10} & C_7 & C_{11} & C_9 & C_{12} \\ C_1 & C_4 & C_{10} & C_8 & C_{11} & C_9 & C_{12} \\ C_2 & C_5 & C_{10} & C_7 & C_{11} & C_9 & C_{12} \\ C_2 & C_5 & C_{10} & C_8 & C_{11} & C_9 & C_{12} \\ C_3 & C_6 & C_{10} & C_7 & C_{11} & C_9 & C_{12} \\ C_3 & C_6 & C_{10} & C_8 & C_{11} & C_9 & C_{12} \end{vmatrix}. \quad (1)$$

- *SOLVING THE MATHEMATICAL FORMULATION*

By the application of algebra of logic, equation (1) may be written as

$$f(C_1, C_2, \ldots C_{12}) = |C_9 C_{10} C_{11} C_{12} H(C_1, C_2, \ldots. C_8)| \quad (2)$$

where

$$H(C_1, C_2, \ldots C_8) = \begin{vmatrix} C_1 & C_4 & C_7 \\ C_1 & C_4 & C_8 \\ C_2 & C_5 & C_7 \\ C_2 & C_5 & C_8 \\ C_3 & C_6 & C_7 \\ C_3 & C_6 & C_8 \end{vmatrix} \quad (3)$$

Substituting

$£_1 = C_1 C_4 C_7$
$£_2 = C_1 C_4 C_8$
$£_3 = C_2 C_5 C_7$
$£_4 = C_2 C_5 C_8$

$£_5 = C_3C_6C_7$
$£_6 = C_3C_6C_8$
in equation (3), one can obtain

$$H(C_1, C_2, \ldots C_8) = \begin{vmatrix} £_1 \\ £'_1 & £_2 \\ £'_1 & £'_2 & £_3 \\ £'_1 & £'_2 & £'_3 & £_4 \\ £'_1 & £'_2 & £'_3 & £'_4 & £_5 \\ £'_1 & £'_2 & £'_3 & £'_4 & £'_5 & £_6 \end{vmatrix} \qquad (4)$$

Using algebra of logic, one can determine the following

$$£'_1 = \begin{vmatrix} C'_1 \\ C_1 & C'_4 \\ C_1 & C_4 & C'_7 \end{vmatrix} \qquad (5)$$

$$£'_2 = \begin{vmatrix} C'_1 \\ C_1 & C'_4 \\ C_1 & C_4 & C'_8 \end{vmatrix} \qquad (6)$$

$$£'_3 = \begin{vmatrix} C'_2 \\ C_2 & C'_5 \\ C_2 & C_5 & C'_7 \end{vmatrix} \qquad (7)$$

$$£'_4 = \begin{vmatrix} C'_2 \\ C_2 & C'_5 \\ C_2 & C_5 & C'_8 \end{vmatrix} \qquad (8)$$

$$£'_5 = \begin{vmatrix} C'_3 \\ C_3 & C'_6 \\ C_3 & C_6 & C'_7 \end{vmatrix} \qquad (9)$$

$$£'_1£_2 = \begin{vmatrix} C'_1 \\ C_1 & C'_4 \\ C_1 & C_4 & C'_7 \end{vmatrix} \wedge |C_1C_4C_8| = |C_1C_4C'_7C_8| \qquad (10)$$

Similarly,

$$£'_1£'_2£_3 = \begin{vmatrix} C'_1 & C_2 & C_5 & C_7 \\ C_1 & C_2 & C'_4 & C_5 & C_7 \end{vmatrix} \qquad (11)$$

and

$$£'_1£'_2£'_3£_4 = \begin{vmatrix} C'_1 & C_2 & C_5 & C'_7 & C_8 \\ C_1 & C_2 & C'_4 & C_5 & C'_7 & C_8 \end{vmatrix} \qquad (12)$$

$$£'_1£'_2£'_3£'_4£_5 = \begin{vmatrix} C'_1 & C'_2 & C_3 & C_6 & C_7 \\ C_1 & C'_2 & C_3 & C'_4 & C_6 & C_7 \\ C'_1 & C_2 & C_3 & C'_5 & C_6 & C_7 \\ C_1 & C_2 & C_3 & C'_4 & C'_5 & C_6 & C_7 \end{vmatrix} \qquad (13)$$

and

$$£'_1£'_2£'_3£'_4£'_5£_6 = \begin{vmatrix} C'_1 & C'_2 & C_3 & C_6 & C'_7 & C_8 \\ C_1 & C'_2 & C_3 & C'_4 & C_6 & C'_7 & C_8 \\ C'_1 & C_2 & C_3 & C'_5 & C_6 & C'_7 & C_8 \\ C_1 & C_2 & C_3 & C'_4 & C'_5 & C_6 & C'_7 & C_8 \end{vmatrix} \qquad (14)$$

Making use of equations (10) - (14) in equation (4), one can obtain

$$H(C_1, C_2, \dots C_8) = \left| \begin{array}{llllllll} C_1 & C_4 & C_7 & & & & & \\ C_1 & C_4 & C_7' & C_8 & & & & \\ C_1' & C_2 & C_5 & C_7 & & & & \\ C_1 & C_2 & C_4' & C_5 & C_7 & & & \\ C_1' & C_2 & C_5 & C_7' & C_8 & & & \\ C_1 & C_2 & C_4' & C_5 & C_7' & C_8 & & \\ C_1' & C_2' & C_3 & C_6 & C_7 & & & \\ C_1 & C_2' & C_3 & C_4' & C_6 & C_7 & & \\ C_1' & C_2 & C_3 & C_5' & C_6 & C_7 & & \\ C_1 & C_2 & C_3 & C_4' & C_5' & C_6 & C_7 & \\ C_1' & C_2' & C_3 & C_6 & C_7' & C_8 & & \\ C_1 & C_2' & C_3 & C_4' & C_6 & C_7' & C_8 & \\ C_1' & C_2 & C_3 & C_5' & C_6 & C_7' & C_8 & \\ C_1 & C_2 & C_3 & C_4' & C_5' & C_6 & C_7' & C_8 \end{array} \right| \qquad (15)$$

in view of equation (15), equation (2) implies that

$$f(C_1, C_2, \dots, C_{12}) = \left| \begin{array}{llllllllllll} C_1 & C_4 & C_7 & C_9 & C_{10} & C_{11} & C_{12} & & & & & \\ C_1 & C_4 & C_7' & C_8 & C_9 & C_{10} & C_{11} & C_{12} & & & & \\ C_1' & C_2 & C_5 & C_7 & C_9 & C_{10} & C_{11} & C_{12} & & & & \\ C_1 & C_2 & C_4' & C_5 & C_7 & C_9 & C_{10} & C_{11} & C_{12} & & & \\ C_1' & C_2 & C_5 & C_7' & C_8 & C_9 & C_{10} & C_{11} & C_{12} & & & \\ C_1 & C_2 & C_4' & C_5 & C_7' & C_8 & C_9 & C_{10} & C_{11} & C_{12} & & \\ C_1' & C_2' & C_3 & C_6 & C_7 & C_9 & C_{10} & C_{11} & C_{12} & & & \\ C_1 & C_2' & C_3 & C_4' & C_6 & C_7 & C_9 & C_{10} & C_{11} & C_{12} & & \\ C_1' & C_2 & C_3 & C_5' & C_6 & C_7 & C_9 & C_{10} & C_{11} & C_{12} & & \\ C_1 & C_2 & C_3 & C_4' & C_5' & C_6 & C_7 & C_9 & C_{10} & C_{11} & C_{12} & \\ C_1' & C_2' & C_3 & C_6 & C_7' & C_8 & C_9 & C_{10} & C_{11} & C_{12} & & \\ C_1 & C_2' & C_3 & C_4' & C_6 & C_7' & C_8 & C_9 & C_{10} & C_{11} & C_{12} & \\ C_1' & C_2 & C_3 & C_5' & C_6 & C_7' & C_8 & C_9 & C_{10} & C_{11} & C_{12} & \\ C_1 & C_2 & C_3 & C_4' & C_5' & C_6 & C_7' & C_8 & C_9 & C_{10} & C_{11} & C_{12} \end{array} \right| \qquad (16)$$

Finally, the probability of the successful operation (i.e.) reliability) of the complex system is given by

$$Z_s = Pr\{f(C_1, C_2, \dots, C_{12}) = 1\}$$

$$= Z_9Z_{10}Z_{11}Z_{12}(Z_1Z_4Z_7 + Z_1Z_4Q_7Z_8) + Q_1Z_2Z_5Z_7 + Z_1Z_2Q_4Z_5Z_7 + Q_1Z_2Z_5Q_7Z_8 + Z_1Z_2Q_4Z_5Q_7Z_8 + Q_1Q_2Z_3Z_6Z_7 + Z_1Q_2Z_3Q_4Z_6Z_7 + Q_1Z_2Z_3Q_5Z_6Z_7 + Z_1Z_2Z_3Q_4Q_5Z_6Z_7 + Q_1Q_2Z_3Z_6Q_7Z_8 + Z_1Q_2Z_3Q_4Z_6Q_7Z_8 + Q_1Z_2Z_3Q_5Z_6Q_7Z_8 + Z_1Z_2Z_3Q_4Q_5Z_6Q_7Z_8)$$

$$= Z_9Z_{10}Z_{11}Z_{12}(Z_1Z_4Z_7 + Z_1Z_4Z_8 + Z_2Z_5Z_7 + Z_2Z_5Z_8 + Z_3Z_6Z_7 + Z_3Z_6Z_8 - Z_1Z_4Z_7Z_8) - Z_2Z_5Z_7Z_8 - Z_3Z_6Z_7Z_8 - Z_1Z_2Z_4Z_5Z_7 - Z_1Z_2Z_4Z_5Z_8 - Z_1Z_3Z_4Z_6Z_7 - Z_1Z_3Z_4Z_6Z_8 - Z_2Z_3Z_5Z_6Z_7 - Z_2Z_3Z_5Z_6Z_8 + Z_1Z_2Z_4Z_5Z_7Z_8 + Z_1Z_3Z_4Z_6Z_7Z_8 + Z_2Z_3Z_5Z_6Z_7Z_8 + Z_1Z_2Z_3Z_4Z_5Z_6Z_7 + Z_1Z_2Z_3Z_4Z_5Z_6Z_8 - Z_1Z_2Z_3Z_4Z_5Z_6Z_7Z_8) \qquad (17)$$

Where $Z_1, Z_2 \dots, Z_{12}$ are reliabilities of the generators P_1, P_2, P_3 cables $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6$, **MSB, SMSB** and **OPSB**, respectively, and Q_i s are corresponding unreliabilities.

➢ SPECIFIC INSTANCES

Case I: If reliability of each component of the complex system is Z, equation (17) yields

$$Z_s = 6Z^7 - 3Z^8 - 6Z^9 + 3Z^{10} + 2Z^{11} - Z^{12}. \qquad (18)$$

Case II: when failure rates follow weibull distribution

Let failure rates of the generators P_1, P_2, P_3 cables $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6$, **MSB, SMSB** and **OPSB**, be $\alpha_1, \alpha_2, \dots, \alpha_{12}$respectively. Then from equation (17) reliability of the system at an instant *t* is given by

$Z_s(t) = exp(-j_1 t^\theta) + exp(-j_2 t^\theta) + exp(-j_3 t^\theta) + exp(-j_4 t^\theta) + exp(-j_5 t^\theta) + exp(-j_6 t^\theta) - exp(-j_7 t^\theta) - exp(-j_8 t^\theta) - exp(-j_9 t^\theta) - exp(-j_{10} t^\theta) - exp(-j_{11} t^\theta) - exp(-j_{12} t^\theta) - exp(-j_{13} t^\theta) - exp(-j_{14} t^\theta) - exp(-j_{15} t^\theta) + exp(-j_{16} t^\theta) + exp(-j_{17} t^\theta) + exp(-j_{18} t^\theta) + exp(-j_{19} t^\theta) + exp(-j_{20} t^\theta) - exp(-j_{21} t^\theta)$
(19)

Where, θ is a positive parameter and $j_i's$ are given by α
$j_1 = A - (\alpha_2 + \alpha_3 + \alpha_5 + \alpha_6 + \alpha_8)$
$j_2 = A - (\alpha_2 + \alpha_3 + \alpha_5 + \alpha_6 + \alpha_7)$
$j_3 = A - (\alpha_1 + \alpha_3 + \alpha_4 + \alpha_6 + \alpha_8)$
$j_4 = A - (\alpha_1 + \alpha_3 + \alpha_4 + \alpha_6 + \alpha_7)$
$j_5 = A - (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5 + \alpha_8)$
$j_6 = A - (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5 + \alpha_7)$
$j_7 = A - (\alpha_2 + \alpha_3 + \alpha_5 + \alpha_6)$
$j_8 = A - (\alpha_1 + \alpha_3 + \alpha_4 + \alpha_6)$
$j_9 = A - (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5)$
$j_{10} = A - (\alpha_3 + \alpha_6 + \alpha_8)$
$j_{11} = A - (\alpha_3 + \alpha_6 + \alpha_7)$
$j_{12} = A - (\alpha_2 + \alpha_5 + \alpha_8)$
$j_{13} = A - (\alpha_2 + \alpha_5 + \alpha_7)$
$j_{14} = A - (\alpha_1 + \alpha_4 + \alpha_8)$
$j_{15} = A - (\alpha_1 + \alpha_4 + \alpha_7)$
$j_{16} = A - (\alpha_3 + \alpha_6)$
$j_{17} = A - (\alpha_2 + \alpha_5)$
$j_{18} = A - (\alpha_1 + \alpha_4)$
$j_{19} = A - \alpha_8$
$j_{20} = A - \alpha_7$
$j_{21} = A = (\alpha_1 + \alpha_2 + \ldots + \alpha_{12})$

Case III: when failure rates follow exponential distribution

A specific instance of the weibull distribution for θ=1, the exponential distribution is highly helpful in many real-world scenarios. As at this instant, t, the system's dependability is provided by

$Z_s(t) = exp(-j_1 t) + exp(-j_2 t) + exp(-j_3 t) + exp(-j_4 t) + exp(-j_5 t) + exp(-j_6 t) - exp(-j_7 t) - exp(-j_8 t) - exp(-j_9 t) - exp(-j_{10} t) - exp(-j_{11} t) - exp(-j_{12} t) - exp(-j_{13} t) - exp(-j_{14} t) - exp(-j_{15} t) + exp(-j_{16} t) + exp(-j_{17} t) + exp(-j_{18} t) + exp(-j_{19} t) + exp(-j_{20} t) - exp(-j_{21} t)$
(20)

➢ MATHEMATICAL EVALUATION OF RELIABILITY

Setting $\alpha_i = 0.1$ for $i = 1 - 12$ and $\theta = 2$ in equations (19) and (20), one can compute following table.

Table 1:- Reliability Over Time : Exponential vs. Weibull distributions

Sl. No.	Time t	$Z_s(t)$	
		Exponential distribution	Weibull Distribution
1	0	1.000000	1.000000
2	2	0.418984	0.149910
3	4	0.149910	7.07968×10^{-5}
4	6	0.047595	6.72469×10^{-11}
5	8	0.013963	2.09595×10^{-19}
6	10	0.003888	2.38520×10^{-30}

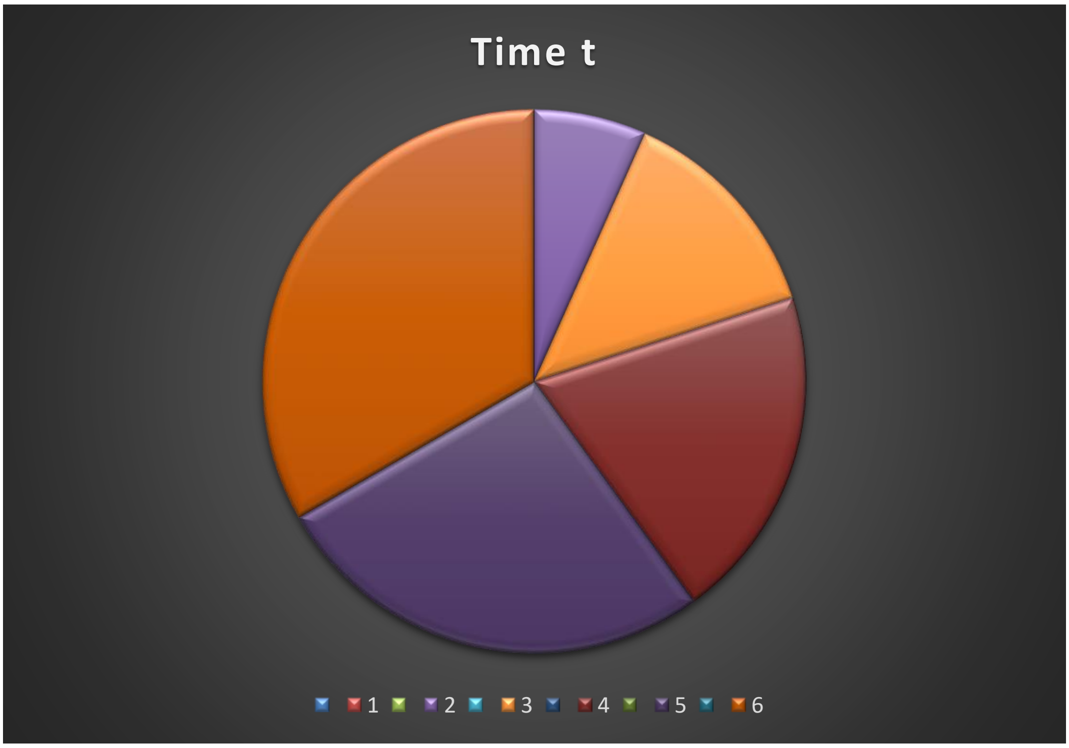

Chart -1 : Time- based Reliability Evaluation by Pie chart

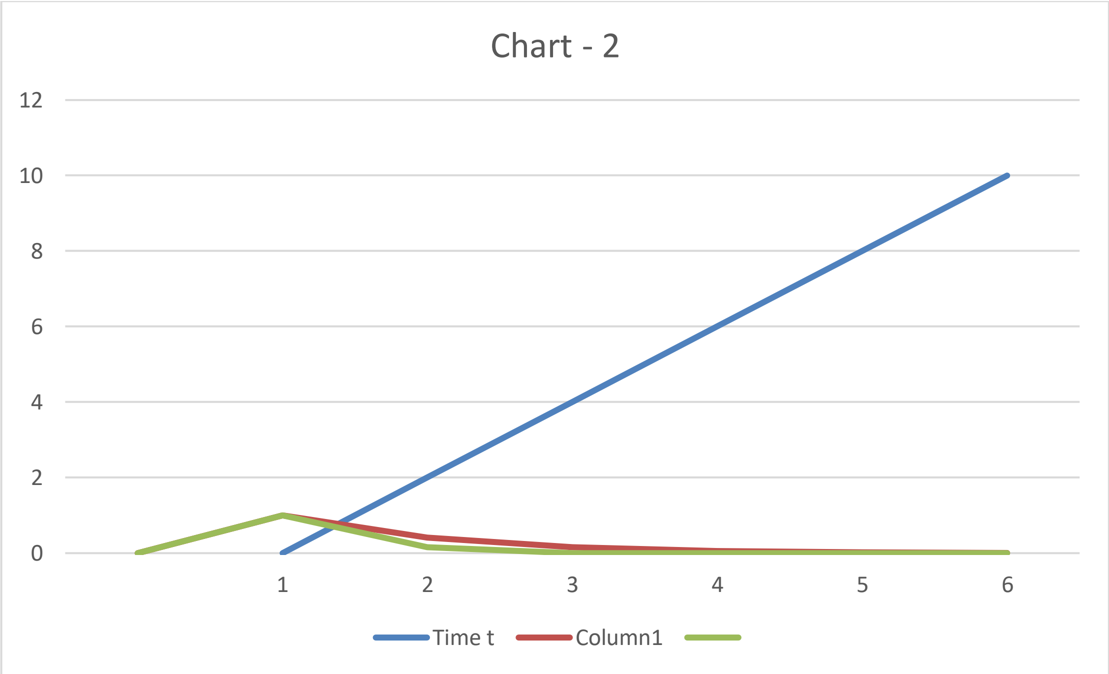

Chart -2 Reliability vs time.

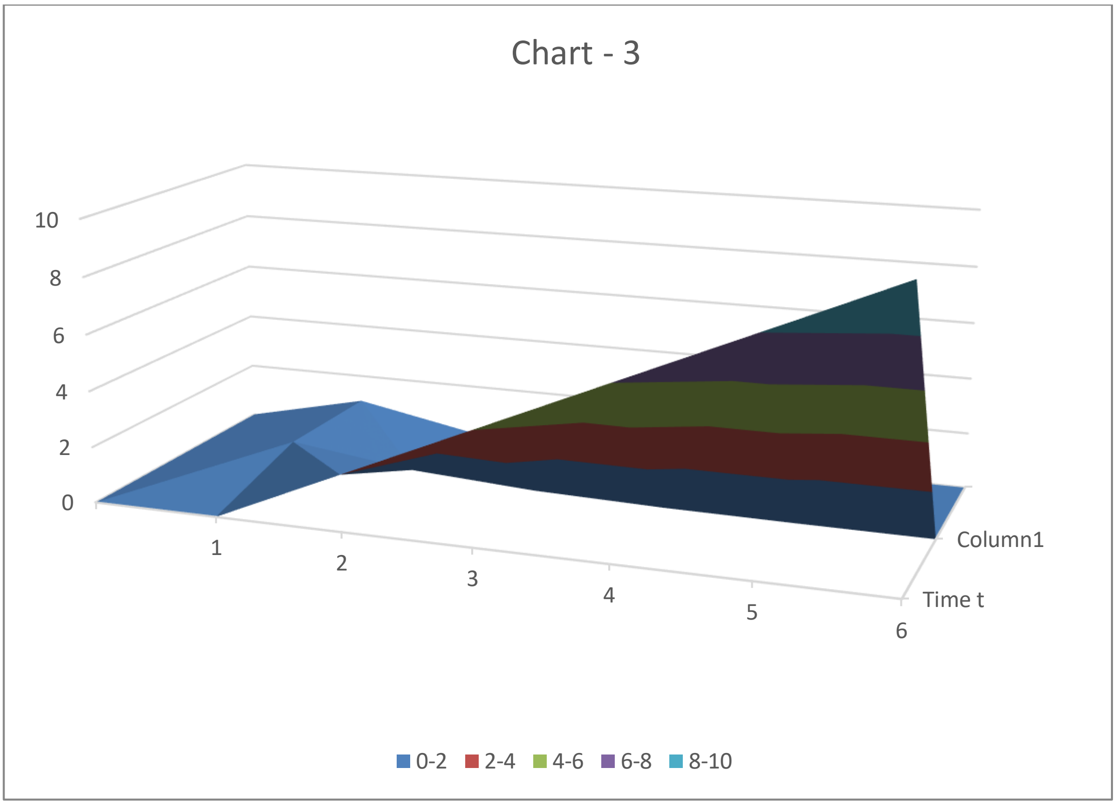

Chart -3 : Time – based Reliability Evaluation by Surface chart

IV. CONCLUSION

An exponential and Weibull distribution for failure are used to calculate the variation in dependability with respect to time in the table. An analysis of chart.(1 & 2 & 3) critically shows that when failure follows the Weibull distribution, system dependability reduces extremely quickly, but when failure follows an exponential distribution, it declines somewhat at a uniform pace. A non-repairable redundant complex system's dependability may therefore be estimated well in advance for a given failure rate, allowing one to predict the system's behaviour.

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