



# Statistical Analysis of Compressive Strength of Concrete

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## **CERTIFICATE**

**Certificate that Sujeet Kumar Mahto (210240714010) has conducted the research described in the thesis for the Master of Technology in Structural Engineering and Construction award, "STATISTICAL ANALYSIS OF COMPRESSIVE STRENGTH OF CONCRETE." The work was done at Roorkee Institute of Technology, which is affiliated with Uttarakhand Technical University, and was overseen and guided by me. The thesis' subject matter has never been submitted for consideration for a degree or certificate anywhere.**

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( **Sujeet Kumar Mahto** ) ( 210240714010)

## ABSTRACT

Concrete is used in the construction practice as it is easily available, cheap, has flexibility of handling and can be shaped in any form desired. The use of concrete in the construction industry is now being done for a very long time. It is the most used material in the parts of structure where compressive strength is required. So in the construction process evaluation of the compressive strength of concrete plays a vital role. There is no universal method for the evaluation of the compressive strength. In India, the method prescribed by IS 456:2000 is used for checking whether the concrete made is fulfilling the requirement. An attempt is made to analyse this method statistically. The characteristic strength of the concrete is represented by test results of concrete cubes at 28 days. The compressive strength of concrete in India is defined based on this characteristic compressive strength. The sampling process of the concrete which is based on the amount of work of concrete to be done is studied and a trend between the number of samples and the failure percentage of samples in terms of compressive strength is studied. The concept of population and sample in terms of the concrete mix design is studied. Hypothesis testing is performed on various groups of samples and their deviation from the population is calculated. An attempt to calculate optimal number of samples for testing and an improvised method using statistics as a tool for checking the compressive strength of concrete is made.

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## CHAPTER ONE INTRODUCTION

### ➤ *Concrete*

The concrete as a material is a mixture of aggregates, coarse and fine mixed with cement paste which becomes hard after a period of time. It is a composite material. It is one of the most commonly used man made material in the construction industry all over the world. Its usage worldwide in the construction industry is about twice that of steel, wood, aluminium and plastic combined. One of the reasons for wide use of concrete in the construction is because of its sustainability.

- *Concrete Sustainability*

Concrete is a natural choice for sustainable construction as it is environment friendly than other construction materials throughout its life span, starting from raw material production up to demolition. The following are some of the factors given by the Environmental Council of Concrete Organizations and the Portland Cement Association that shows the sustainability of concrete [1]:-

- *Resource Efficiency*

“The principal raw material in the manufacturing of Portland cement is limestone, and it is a mineral that exists in huge quantities naturally. As a substitute of cement, fly ash, silica fumes, slag, etc. is also being used widely. All the waste by products from the steel mills, power plants, and other manufacturing facilities can also be put to use.” [1]

- *Durability*

The structures built from concrete material are long lasting and durable which will not burn rot or rust very easily. The life span of the concrete structures is much longer than the other common building structures [1].

- *Storm Water Retaining Capacity*

“Concrete overlays on pavement prevent water entering into the ground surface because of their can be reduced because of the water-resistive characteristics. This tends to create problems such as water table depletion, erosion, flash floods and pollution which creates imbalance in natural ecosystem. Pervious concrete is a unique concrete possessing inter-connected pores within itself that allows free flow of solutions. The structures like parking lots, driveways, pavements, sidewalks, etc. use this type of concrete that helps in the detainment and restoration of water.” [1]

Prestressed Concrete “The production of concrete can be done as per the quantity needed whenever required for a particular work which reduces the wastage. The recycled concrete aggregates, which are obtained from old fallen structure, are utilized for making new pavements or for filling applications in base or sub-base layers.” [1]

The above factors shows some of the reasons why the concrete is extensively used in, bridges, buildings, dams and roads with its usage ranging from structural applications pipes, kerbs, drains, etc. The concrete used in these structures can also be of varying types. The following are few of the types of commonly used concrete that are used.

### ➤ *Types of Concrete*

- *Plain Cement Concrete (PCC)*

Plain Cement Concrete or PCC is a mixture of fine aggregates (sand), coarse aggregates and cement along with water. A small quantity of admixture like water proofing agents, air entraining agents, etc. can also be added based on the requirement. It is an important component for building construction which is laid on soil surface to avoid the direct contact of reinforced concrete with water and soil. The proportioning of this concrete is done based on the Indian Standard codes or as per requirement. The material used is measured using weight batching or volume batching. In the weight batching method, the materials are taken based on weights. On the contrary in volume batching, the volume of individual material required is calculated and the mixing is done on volume basis. The mixing of the materials in PCC is done by hand mixing or machine mixing. The following are a few applications of PCC [2].

- ✓ Concrete sills to create a level, robust surface at window and ventilator sills.
- ✓ Concrete in the bed below the footings of columns, walls, and walls below beams.
- ✓ Coping concrete on compound walls and parapet.
- ✓ For making pavements.
- ✓ For flagging area around the building.
- ✓ For making basketball courts, tennis courts, etc.

So as we can see the plain cement concrete has wide range of uses, but as every material PCC also has some of its own advantages and disadvantages. The following are the list of some major advantages and disadvantages of PCC [2].



➤ *Advantages*

- Easy availability of ingredients.
- Concrete can be cast to different sizes and designs, with less efforts during the preparation process
- Transportation of concrete from the place where it is mixed and placing is easy.
- It has the ability to be easily pumped to fill up the cracks and linings of tunnels.
- The monolithic character gives a better appearance to the structure also offering better rigidity.
- Concrete is strong in compression, cheaper and hence, preferred more than steel in structures

➤ *Disadvantages*

- Reinforcement is required to avoid cracks due to low tensile strength.
- Due to shrinkage and expansion, construction joints are to be provided to avoid cracks
- Expansion joints are to be provided in large structures if the area has large temperature variance.
- Certain dissolved calcium hydroxide compounds in concrete are leached due to movement of water inside concrete leading to efflorescence phenomena.
- Sustained loading causes creep in the structure.
- Concretes made with OPC or ordinary Portland cement can get integrated due to presence of sulphates, alkali, etc.

➤ *Reinforced Cement Concrete (RCC)*

“The tensile capacity of cement concrete is very low as compared to its compressive strength. This inability of concrete is subdued by adding reinforcements in the concrete structure whenever required. This composite form of concrete is termed as RCC or Reinforced Cement Concrete. The steel bars provided must have appropriate deformations to have good interlocking to provide strong bond between the two materials. The reinforcing bars are usually circular in shapes, and the commonly used bars are graded as Fe250, Fe415, Fe500 and Fe550 where the Fe stands for ferrous metal and the number refers to the yield stress in MPa. The designing of the members of reinforced cement concrete are majorly done by two methods viz. Working stress method (WSM) and Limit state method (LSM). As the reinforced cement concrete is a heterogeneous material, there are a few assumptions that are taken before designing a structural member as a reinforced cement concrete member. The assumptions are as follows [3]:-

- The strain developed in concrete and steel at a particular level is the same if sufficient bond strength exists between them.
- The strain produced in concrete is linearly proportional to the distance from the neutral axis.
- The stress developed is directly proportional to strain with modulus of elasticity being constant.
- After bending, the cross section's plane stays flat.
- There is very little tensile strength to concrete.
- The cracked section is completely ineffective and the entire section is resisting the external moments before cracking.
- When the stresses exceed, the non-elastic and realistic behaviour of steel and concrete composite are approximately the same.

So we can see that Reinforced cement concrete or RCC is widely used and an efficient material of building construction.” [3]  
The following are some of the advantages and disadvantages of RCC

➤ *Advantages*

- Compared to steel, it is more fire resistant.
- It has relatively high compressive strength.
- It has low maintenance cost with long service life.
- Various large structures like dams, piers, and foots can be constructed cost-effectively
- It can be easily casted to take the required shape which makes it a better option in precast structures.
- The use of steel helps in reducing the cross sectional dimensions of the structural member
- e.g. beam with lower depth.
- It makes the structural member rigid reducing the apparent deflection

➤ *Disadvantages*

- The final strength of concrete is affected by the requirements for mixing, casting, and curing.
- The cost of formwork for casting is relatively higher
- The compression strength of concrete compared to steel is lower which leads to large section of beams and columns in multi-storeyed buildings
- Cracks are developed in due to shrinkage and application of live load.

### ➤ *Prestressed Concrete*

“Despite the various advantages of using reinforced concrete, the design compressive stress in concrete is lower. In addition, the tensile stress in concrete is completely neglected. For applications demanding higher compressive and tensile strength of concrete, the reinforced concrete cannot cater to the requirements. Therefore, the researchers with an alternative that can impart certain level of pre-compressive force during the casting stage itself. This led to the invention of prestressed concrete, which is defined as the concrete in which the compressive stresses are pre applied in such a way so as to counteract the tensile stresses up to a desirable limit based on load application. The prestressed concrete utilizes the best properties of both, the compressive strength of concrete and tensile strength of steel. Therefore, the prestressed concrete is more susceptible to complicated loadings such as vibration and shocks. Prestressed concrete is now-a-days commonly used in the design of decks of bridges, water tanks, railway sleepers, floorbeams, etc. The steel used for prestressing can be in form of wires or tendons which can be grouped to form cables. In some cases solid bars can also be used. This method is employed to make precast members that can be assembled directly in site or to cast the entire member at the site itself. There are two methods by which the concrete can be Prestressed [4].

#### • *Pre-Tensioning*

In pre-tensioning the wires are initially stretched to the amount of predetermined stresses with the help of anchors. The concrete is poured in the series of formwork in which the high strength steel tendons are prestressed. These tendons are bonded with the concrete as the concrete progresses through hardening or hydration. After the instant the concrete attains sufficient strength, these steel tendons are cut and the compressive stresses are transferred through concrete. The tendons are generally stretched using hydraulic jacks and the stress in the concrete has to be maintained during placing and curing of concrete. The method is used for cast in place as well as precast construction. [4].

#### • *Post-Tensioning*

In case of post tensioning the concrete members are casted with PVC pipes introduced at predefined locations. After the concrete attains sufficient strength, the high strength steel tendons are introduced inside the PVC pipes and tightened at the ends of the prestressed concrete through one of the following jacking mechanisms:

- ✓ Lee-McCall system.
- ✓ Freyssinet system.
- ✓ Gifford-Udall system
- ✓ Leonhardt system.
- ✓ Magnel system.

The tendon wires are also coated with bituminous material to avoid the frictional losses between the concrete and tendons during the jacking mechanism. In other cases metal hoses known as ducts or sheath are placed in the structure from which tendons pass. After the tendon is stretched the ducts may be filled with grout. This procedure has to be done on site while placing members. It is done in cases where the members of structures are large and transportation of pre-tensioned members is not possible.” [4]

There are also various advantages along with few disadvantages of prestress concrete [4]. The advantages and disadvantages are given as follows:-

#### • *Advantages*

- ✓ compressive strength potential inherent in concrete is used to the fullest.
- ✓ The tension cracks in concrete are eliminated, which helps in reducing the risk of corrosion of steel component.
- ✓ The stresses induced due to shear are reduced.
- ✓ The percentage of carbon in steel can be significantly changed to produce strands that can impart high strength in the prestressing process. Such tendons are utilized and preferred as much as possible in the design purpose.
- ✓ It is possible to achieve a component with a smaller cross section and lower weight for a given span and loading condition.

#### • *Disadvantages*

- ✓ Expensive equipment's along with high amount of safety measures are required.
- ✓ The composition of conventional steel can be suitably changed using other elements or alloys to meet the requirements of higher strength and cost-effectiveness.
- ✓ Since the tendons in prestressed concrete are subjected to high stresses, workers should be highly skilled and efficient during the assembling of components.

### ➤ *Precast Concrete*

“This high strength concrete is specially manufactured in construction industries under controlled environment with the help of skilled workmanship which prevents the negligence in mixing or casting or curing of concrete specimen. These pre-cast concrete components are carried to the construction site and assembled together to build the entire structure. Although huge cost is associated

with the precast concrete but the construction period is highly minimized. It is primarily used as columns, beams, staircases, wall panels, pipes, tunnels, floors, etc. Due to less cross sectional area of steel members used in steel structures, it is unable to satisfy the 'Limit state of serviceability' criteria economically. But the prestressed concrete includes more cross sectional area and higher strength parameters than normal strength concrete. Therefore it helps in achieving the best properties of concrete structures and steel structures. However in the modern era, the engineers suggest to utilize the cast in situ concrete elements in steel structures which make the structure economical. The precasting is mainly utilized over the construction site where we require same size and kind of concrete components which can be manufactured in industries economically. The precasting can be an efficient method for casting of walls slabs for the apartments. As the components are made beforehand, the construction process can be speeded up. The precasting can be carried out at a casting yard, on or near the site or in a factory. The main factor in deciding if to use site as casting yard or factory is the transportation cost. The factory work offers superior quality due to better quality control.”[5] The following are a few advantages and disadvantages of using precast concrete:-

- *Advantages*

- ✓ The construction can be done in climate controlled environment which eliminates the problems occurring due to heat, cold, rain or dust.
- ✓ The construction happens on ground instead of at a height which gives more control.
- ✓ Specialized equipments can be used to pour, make and move concrete.
- ✓ Special moulds or formworks can be built for doing repetitions of same components.
- ✓ Curing takes place in a controlled environment.
- ✓ The quality of precast concrete is very high due to above reasons.

- *Disadvantage*

- ✓ Structural discontinuity is caused in the structure due to joints between different pieces.
- ✓ As the structural components are joined to each other, the structure no longer remains monolithic.
- ✓ The joints between the components have to be sealed properly making them waterproof.
- ✓ The precast components are heavy and large, so cranes are required to lift, move, position and place them making the job critical.

- *Heterogeneity of Concrete*

The definition of concrete suggests the concrete as a mixture of cement aggregates and water. This tells us about the heterogeneous structure of concrete. The figure 1 shows the difference in two types of heterogeneity. The heterogeneity of concrete can be divided into two types.

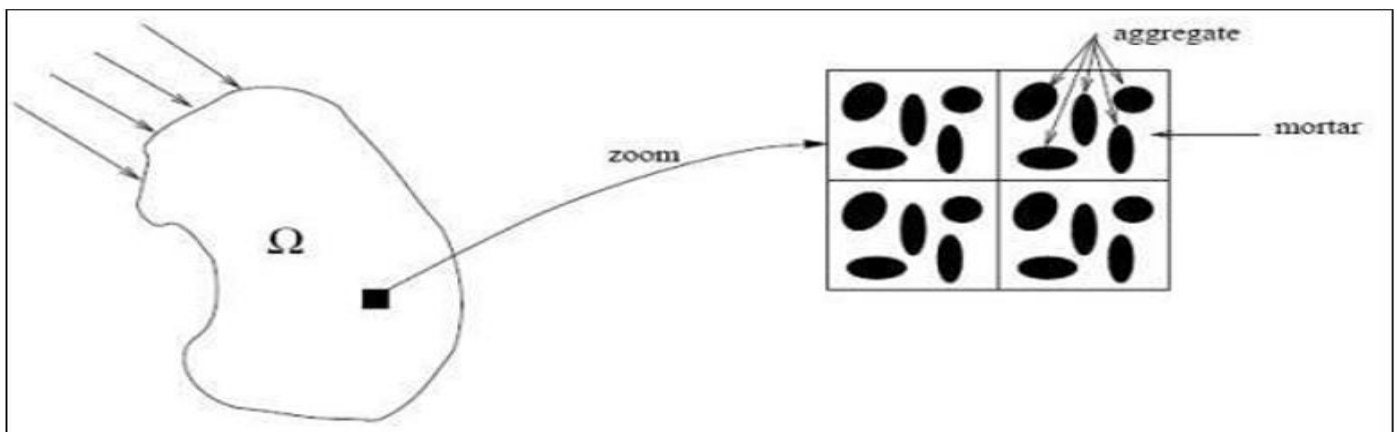


Fig 1 Macro and Micro Structure of Concrete Material [6]

- *Microscopic Heterogeneity*

The concrete has a heterogeneous structure and is considered as a brittle or quasi-brittle material. One of the important scales in the concrete specimen is considered to be its scale of heterogeneity due to present of defects such as micro-cracks, voids and ITZ. Voids are cavities or little holes that occur inside or on surface of concrete. Micro-cracks are small discontinuities in the concrete material. The ITZ or Interfacial Transition Zone is the region of cement paste around the aggregates which is perturbed or unsettled due to the presence of aggregates. The figure 2 shows the various components of concrete and the evidence of Fractured Process Zone (FPZ) is highlighted. The coarse aggregates comprise of almost one-third volume of concrete so the changes in coarse aggregates can have changes in fracture and strength properties of concrete. The micro cracking is induced by thermal and autogenous shrinkage. The ITZ also has an important effect on concrete properties [7].

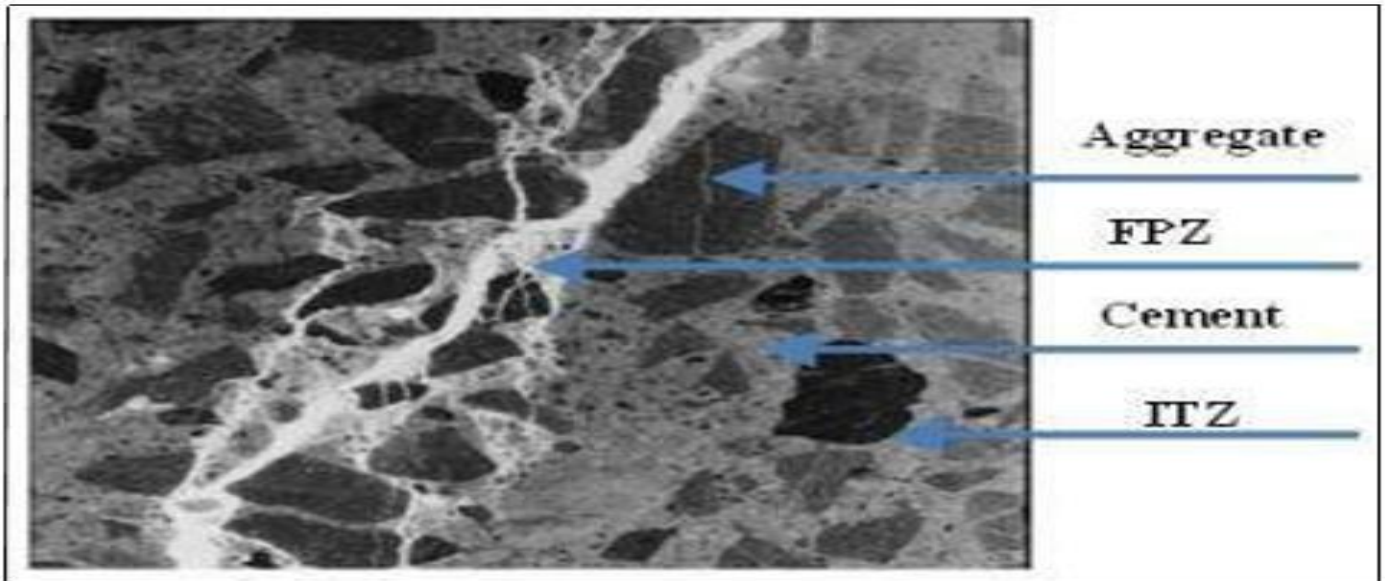


Fig 2 Different components of concrete and the occurrence of cracking

- *Influence of ITZ*

The REV scale or Representative Elementary Volume scale is considered as a small volume of heterogeneous material which is capable of obtaining appropriate homogenized behaviour. Among the properties ITZ acts as a weak link in chain compared to aggregate particles and bulk cement paste. In the various phases composing concrete, the ITZ is often more porous and weak where the initial micro-cracks generally occur [7].

- *Influence of Aggregate Size*

The increase in aggregate size for a constant fraction increases the permeability of concrete. The size of aggregates is important for the prediction of service-life and durability. The size is direct associate to the length of micro crack and average crack width [7].

- *Macroscopic Heterogeneity*

The concrete possess various non-hydrated granules, crystalline solids and numerous capillary porosities as a typical heterogeneous material. For the purpose of simplicity, the concrete at macro-scale is taken as a homogenous material without considering the heterogeneity which is convenient for engineering applications. But even at the macroscopic level there are various factors that can cause an effect on properties of concrete. Some of them are as mentioned below.

- *Positioning of Aggregates*

The placement or the position of aggregates can cause change in the properties of concrete. To give an example, if the aggregates are placed on surfaces the strength of concrete can be directly affected.

- *Spacing of Aggregates*

The layer of ITZ is directly affected by the spacing between the aggregates. This can lead to changes in the cracking pattern or development of micro-cracks in the concrete structure.

- *Size of Aggregates*

”The changes in the size of aggregates or the varying use of aggregates causes the changes in the factors like entrapped or entrained air. These factors can directly affect important properties of concrete like workability.”[8]

“So we can see that the heterogeneity in concrete is an important factor in use of concrete material. The type of concrete used depends on various factors one of which is the type of loading. The loading on concrete defines what type of load is to be considered while designing for concrete. Tension, compressive or compression, flexure or bending, shear and torsion are various types of stresses that can be applied on the concrete structure and must be resisted by the concrete under the given conditions. Sometimes more than one type of stress can be applied on the same structure of concrete. But, when we talk about the strength of concrete, it is generally assumed that the compressive strength or compression is being consideration.”[8]

- *Importance of Compressive Strength of Concrete*

Due to its brittle nature, concrete cannot withstand high tensile stresses, hence its compressive strength is the primary criterion used to evaluate its quality. The strength of concrete is affected by a wide range of factors because of its heterogeneous nature. Here are some of the contributing elements. In [9]

- *Water / Cement Ratio (W/C Ratio)*

The primary factor influencing concrete's compressive strength is its weight-to-cement ratio. It is the proportion of cement weight to water weight that will be added to the mixture. The variation of w/c ratio with the 28 day strength of concrete is as shown in the figure 1.3.

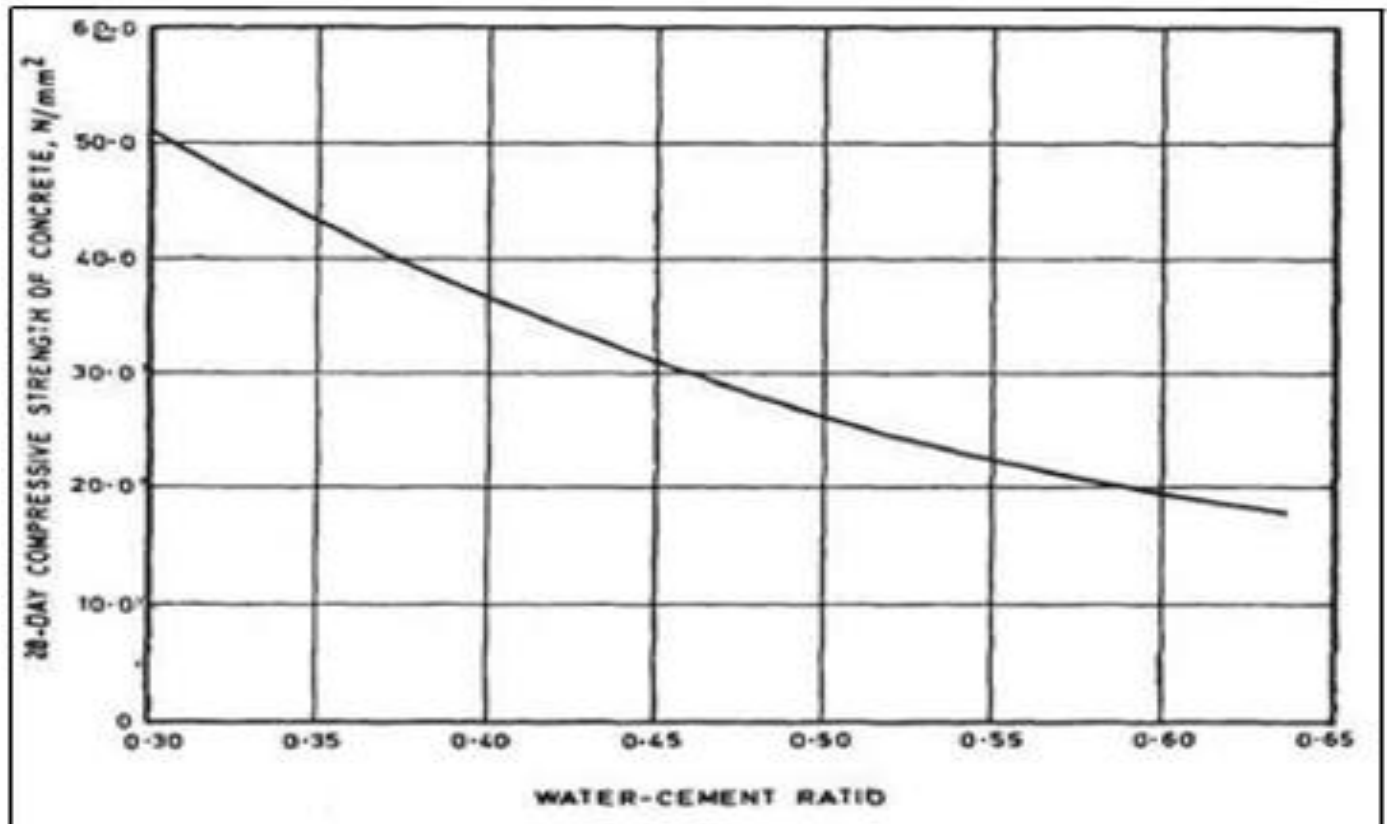


Fig 3 Graph of Water/Cement Ratio Vs. 28 Days Compressive Strength of Concrete [8]

The figure 3 graph illustrates the inverse relationship between the compressive strength and the w/c ratio. The strength of concrete decreases as the w/c ratio rises. [9]

- *Quality of Raw Material*

"The type of cement to be used should conform to Cl.5.1 of IS 456:2000. The cement should also be tested for various physical, chemical and laboratory tests. The aggregate used in manufacturing of concrete are much harder than the cement paste utilized in manufacturing of concrete. Therefore the concrete strength is primarily governed by the strength of cement phase or the ITZ (Inter transition zone). Since the ITZ is dependent over the size, texture and shape of the aggregate, due to the formation of bonding between the aggregates and binder which holds together all the aggregates, therefore the gradation of aggregates is necessary. The bonding at ITZ is also affected by the presence of salts which causes durability issues in concrete and ultimately decreases the strength of concrete. The permissible values for the chemical composition such as normality, presence of solids are given in Cl.5.4 IS 456:2000." [9]

- *Coarse / Fine aggregate ratio*

"The aggregates' total surface area will increase as the proportion of fine to coarse aggregates increases. This will cause the demand for water to rise immediately. The increase in the water demand will increase the w/c ratio. This will decrease the compressive strength of concrete." [9]

- *Grading of Aggregates*

"Due to the significance of gradation in manufacturing of concrete, these are primarily divided into three categories.

- Gap Graded
- Uniformly Graded
- Well Graded

The use of well graded aggregates is suggested due to its better packing efficiency and low w/b ratio which ultimately induces the higher strength in concrete." [9].

- *Compaction of Concrete*

“The irregular or improper compaction of concrete is unable to remove all the excess air present in the fresh concrete. The trapped air causes the concrete to become less capable of supporting interparticle loads, which lowers the concrete's compressive strength and causes voids to form in the material. Air voids content of about 5% and 10% results in a loss of strength, to about 30% and 55% respectively.” [9]

- *Age of Concrete*

“The hydration of concrete depicts that the concrete strength gain is a long term process although 67% strength is achieved in 7 days and 98% strength is achieved in 28 days in case of normal strength concrete if the normal environmental conditions are prevailing such as temperature, curing regime etc. Therefore a parameter known as ‘Age of concrete’ is synonymously used with the term ‘Hydration of concrete’, until the period of time at which the concrete is dried out and no more hydration is taking place.” [9]

- *Temperature*

“The hydration process is dependent over the factors like the environmental temperature, curing regime, radiation, humidity, pressure etc. in case if the temperature rises then the rate of reaction also increases which directly affects the hydration process i.e. the concrete member subjected to higher temperature and higher pressure will attain the strength earlier than the concrete subjected to normal environmental conditions. The environmental conditions also affect the ultimate strength of concrete. The ultimate strength of concrete subjected to higher temperature is lesser than that subjected to normal regime because the constituents of particles subjected to higher temperature are not well structured.” [9]

- *Gel to Space Ratio*

“The gel to space ratio is given by the ratio of hydration products to the total volume occupied by cement and pores available in the concrete. It is expressed as the ratio of volume/volume.” [9]

Thus, even the smallest change in the characteristics of concrete can have an impact on its compressive strength. There are several approaches to estimating compressive strength that are used globally. The American Society for Testing and Materials (ASTM) C39/C39M-18 test method is provided. Test specimens must be cylindrical in accordance with ASTM procedures. Concrete's compressive strength is tested in India using the procedure outlined in IS 456:2000. According to IS 456:2000, the compressive strength is expressed in terms of the characteristic compressive strength. The foundation for the compressive strength of concrete, which is based on the cube tested at 28, is the characteristic compressive strength of concrete ( $f_{ck}$ ). According to IS 456:2000, the characteristic strength is the value at which no more than 5% of the test results are predicted to fall. [10]

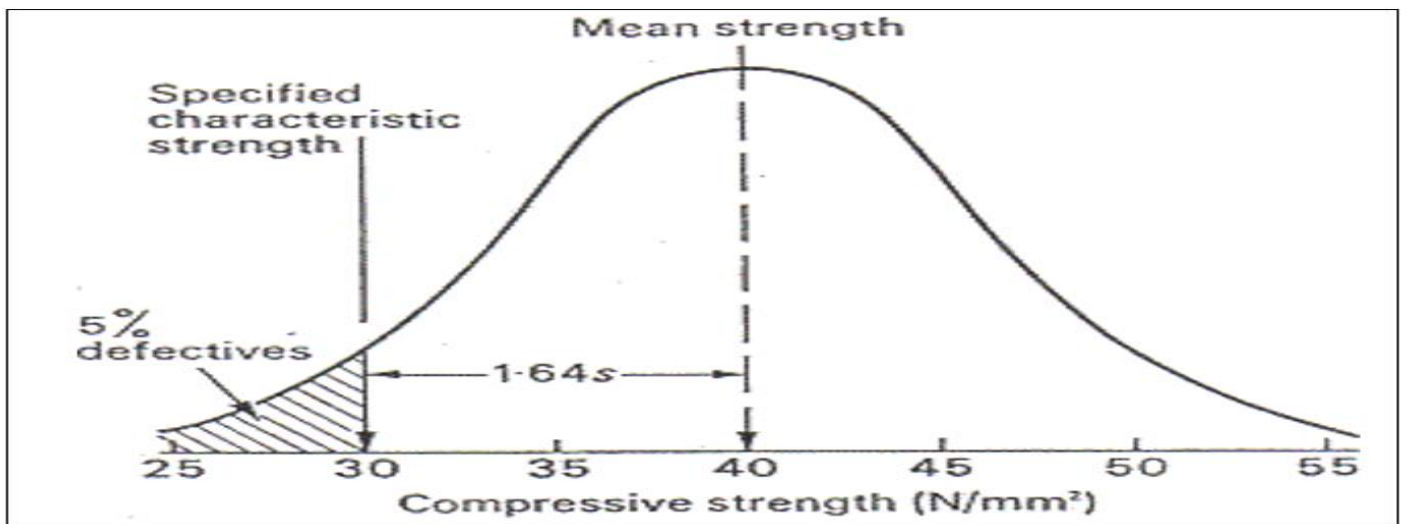


Fig 4 Normal Distribution Curve for Characteristic Compressive Strength of Concrete [11]

The characteristic compressive strength of concrete ( $f_{ck}$ ) is one of the most important values when it comes to designing of any concrete structure. The formulas, equations and concepts of designing concrete are based on the value of  $f_{ck}$ . Even the formulas of stresses of concrete other than compression such as tensile stress and shear stresses as per IS 456:2000 are given in terms of the characteristic compressive strength ( $f_{ck}$ ). So the calculation of the value of characteristic compressive strength ( $f_{ck}$ ) is very important. This value as per Indian standards is generally calculated by testing concrete cubes of size 150 mm x 150 mm x 150 mm. This test results depend on various parameters. Some of the factors affecting the test results are as follows:-

- *Size and Shape of Test Specimens*

The concrete's strength test results are significantly influenced by the size and shape of the specimens. When two identically

prepared cubes with varying sizes are put through testing, the test results will differ. For instance, the strength of a cube specimen with a size of 10 cm is 10% lower than the strength of a cube specimen with a size of 15 cm. When two cubes with distinct shapes—like a cube and a cylinder—are put to the test, the results will differ. According to an experiment, the strength of a cylinder with a diameter of 15 cm and a length of 30 cm is equivalent to 0.8 times the strength of 15 cm cubes [12].

- *Application Rate of Load*

The strength test results are significantly influenced by the rate at which the load is applied. If there is a temporal lag or a slow rate at which the load is applied, lower strength values will follow as a result. The specimen will experience some creep as a result of the load being applied more slowly, increasing the strain. The reduction in strength obtained by testing the concrete samples is caused by the increase in strain resulting through the above situation.” [12]

- *Height to Diameter Ratio*

“ASTM C39-14 specifies the standard procedure for conducting the compressive strength test over the concrete cylinders. As a thumb rule, the height of that cylinder is kept the twice of the diameter of the cylinder. Although maintaining the aspect ratio exactly equal to 2 is not always possible like in the case when the concrete core is cut from the pavement surface. However if the aspect ratio is more than 2, in that case the standard aspect ratio can be maintained for 2 by trimming the concrete specimen but if the aspect ratio is already lesser than 2, in that case a correction factor is required to consider the error introduced in the results.”[12]

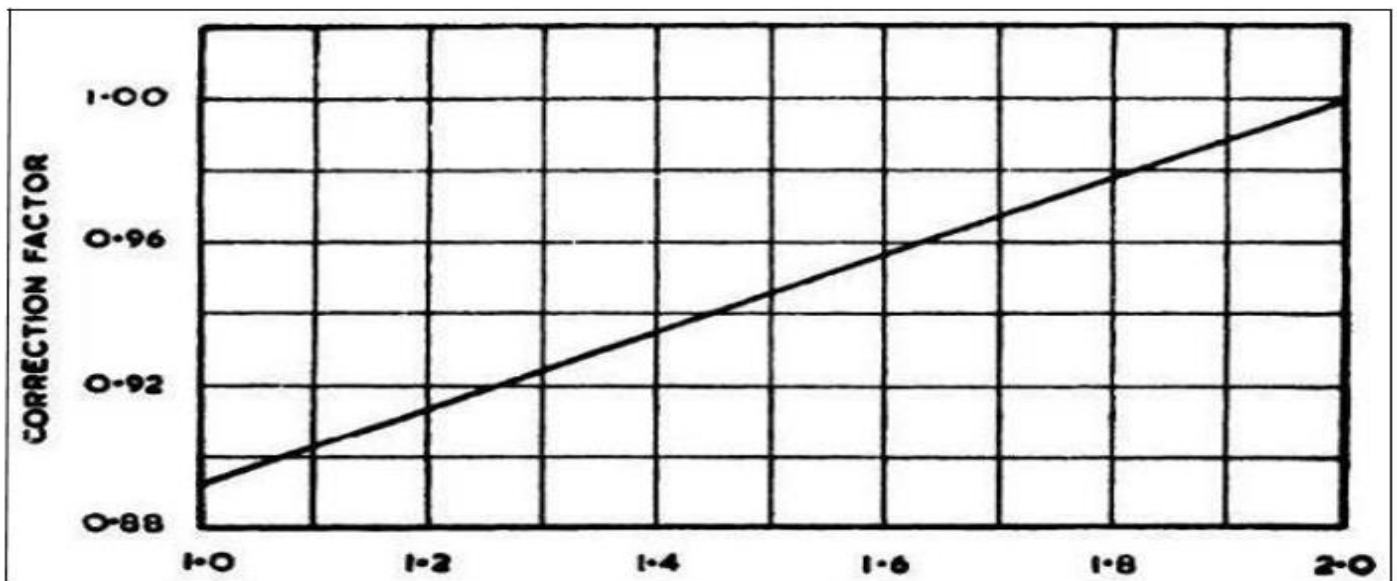


Fig 5 Graph of Correction Factor Vs. Height/Diameter Ratio [12]

Table 1 Correction Factor for the Height to Diameter Ratio of a Core [12]

L/D Ratio	1.00	1.25	1.5	1.75	2.00
Factor	0.87	0.93	0.96	0.98	1.00

“Interpolation can be used to find the correction factor for L/D ratio values between those given in table. Then the correct strength of concrete specimen is obtained by multiplying the correction factor and the strength of concrete specimen obtained in testing.” [12]

- *Moisture Content in the Specimen*

“The concrete specimens of same age when tested, the cube which is dry will give a higher strength value in comparison to the wet cube. If we consider the two cubes of concrete, the one is undertaken in dry condition while the other is wet, then the dry cube of concrete will show higher compressive strength. The reduced strength obtained in wet concrete cube specimen is attributed to the cohesive forces acting between the constituents of concrete. The presence of water inside the concrete specimen reduces the effective cohesion between different particles of concrete. Since the curing regime adopted for the testing of concrete cubes affects the compressive strength results therefore it is suggested to test the concrete specimens directly after removing them from the water (after drying the surface with the help of tissue paper or cloth) to avoid the errors in results.” [12]

- *Material used for Capping*

“Capping of concrete specimen is defined as the process of making the surface of concrete specimen smooth during the testing. Sulphuric capping and gypsum capping are some of the methods adopted for the same. Since the kind of capping also has significant effect on the results obtained for compressive strength of concrete therefore a specified kind of capping should be adopted for the entire batch and no change should be made to avoid the errors in results.” [12]

➤ *Importance of Statistical Analysis*

The concrete can be concluded to be a versatile and a heterogeneous material both on macro and microscopic level. We can also see that When designing concrete structures, the characteristic compressive strength calculation is crucial. The value of characteristic compressive strength is based on the normal distribution. Additionally, the standard deviation is essential to the concrete's compressive strength acceptance criteria. These arguments demonstrate the significance of conducting a statistical analysis of concrete's compressive strength. "Statistics is a science of obtaining information from the data, and of measuring, controlling and communicating uncertainty. The knowledge of statistics provides an important tool for the conceptual foundations of reasoning quantitatively and for extracting information from a set of data. The researchers utilize the statistical methods to transfer the information and mutual communication. A key element of data analytics is statistical analysis, which entails gathering and examining each data sample in a set of items from which samples can be taken. Sampling is defined as the random selection of a data set from a large set of population which is further used to derive the characteristic features of the entire population through mathematical models. These models are generally based over the theory of probability. An error in sampling can lead to misleading conclusions regarding the populations, so it is important that the sampling is done at a proper qualitative and quantitative basis." [12] Various studies have also been conducted that relate the statistical analysis with various different properties of concrete. Some of those are mentioned in the second chapter. Along with studies of statistical analysis to concrete properties, some concepts of statistics are very important when the study of properties of concrete is involved. Some of them are as follows:-

• *Normal Distribution*

"The normal distribution or Gaussian distribution is the most common distribution that occurs in statistics. The various continuous variables that occur in the estimation of characteristics of concrete has a normal distribution-like appearance. It can also be used to approximate various discrete probability distribution functions. The normal distribution is also closely related to the central limit theorem. It states that as the number of increase any probability distribution converges to normal distribution. The data sets in normal distribution curve are so arranged such that the statistical terms or mathematical terms like mean, mode and median coincide at one magnitude. The shape of curve is mirror image on both sides of the mean which seems like a bell shape. A graphical representation of normal distribution curve is given in figure 1.6." [13]

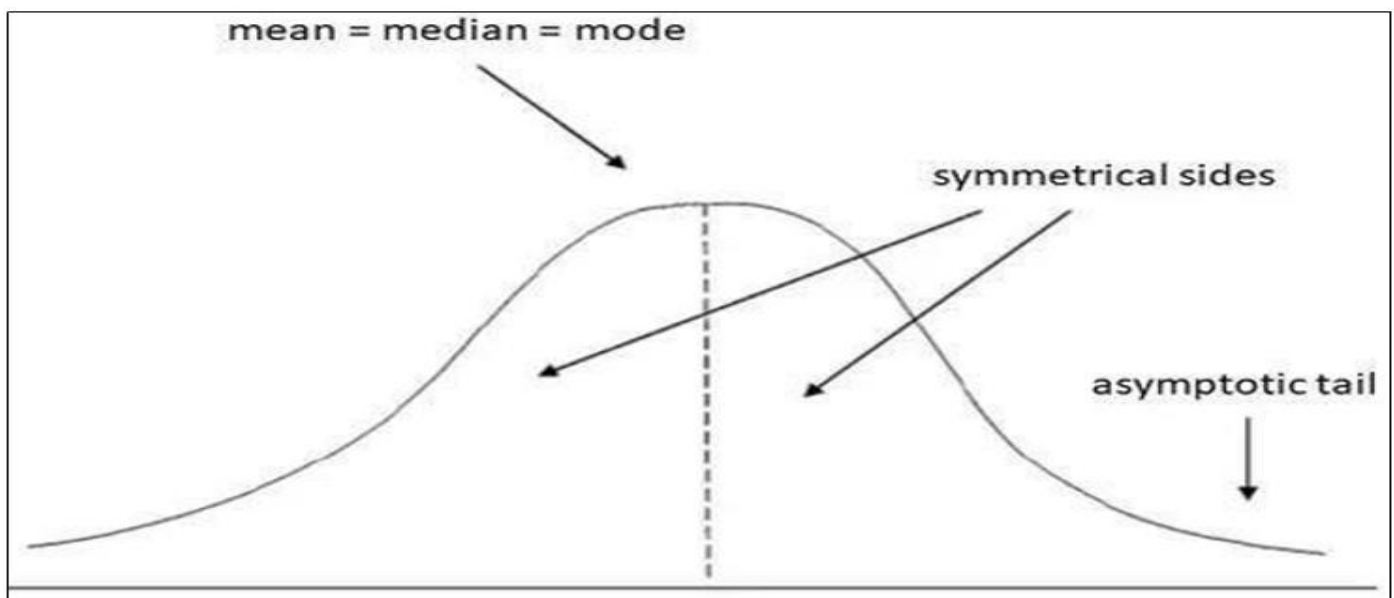


Fig 6 Bell Shaped Curve [14]

• *Moving Average Method*

In order to calculate and analyze the data points and produce a series of averages of different subsets from a full data set, the moving average method is employed. To determine the moving average of the first element of the series, a fixed subset of the series is utilized. The moving average method is generally undertaken to smoothen a fluctuating curve. This method is based over the moving subset of the series. To determine the average of next term, the moving forward term is used while one of the previous terms is excluded from the averaging subset. The threshold of the short term, long term or the number of values in the subset depends on the type of data in the series or the reason for which the trend is required. The example of graph of moving average is shown in figure 1.5." [13].



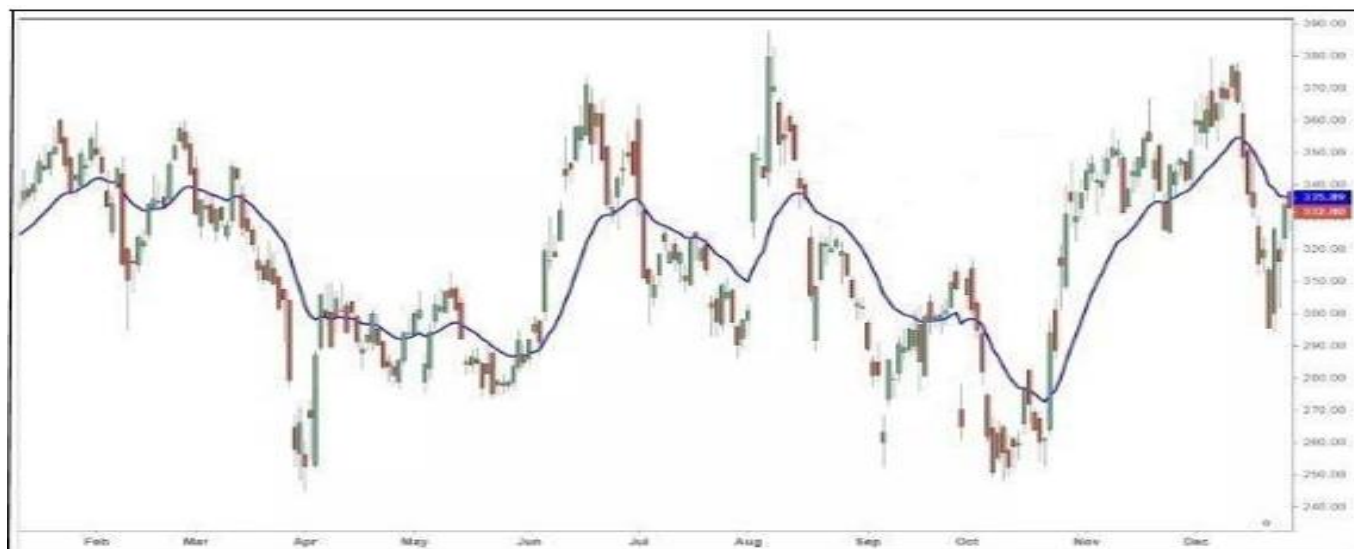


Fig 7 Graph of Moving Average [15]

- Various such methods of statistics are used for analysis of concrete. The study of statistical analysis in the context of compressive strength of concrete is the need of the hour.
- *Distribution of Work*  
The work to be done is distributed as shown in the table 2

➤ *Initial Phase*

The detailed study of the various properties of concrete along with the compressive strength of concrete was to be studied. Along with these basic statistical methods and its application to the study of concrete properties is done. The many commonly used techniques for determining the compressive strength of concrete should be examined, as well as how they relate to one another. The study of the methods used for the calculation compressive strength of concrete along with the up gradation done in the codes if any. Studying and determining the statistical methods that can be used for a more detailed study of the compressive strength method as per the Indian standard given in IS 456:2000.

➤ *Final Phase*

The calculation of critical values of the compressive strength of concrete in accordance with the Indian Standard Code by determining the test results of the samples' minimum values. The graphs are also to be plotted for the critical values of concrete's compressive strength to gain a deeper comprehension of the figures in relation to one another. Based on the presumptive grade of concrete, 100 random values between predetermined numbers must be generated. Taking these numbers as the compressive strength values of one hundred cast concrete cubes in accordance with Indian standards the values are plotted. The plot is then checked for normality. These values are then divided into groups of specimens with and hypothesis testing is done to check whether the values belong to the population. Then the number of specimens in each group is then reduced and the procedure is repeated. This is done till a value of number of specimens is obtained for which a large number of samples is failing the hypothesis test. The relation between the standard deviation of all the samples from the group of specimens and the population is obtained using various statistical methods. The entire procedure is then repeated by actually casting 100 cubes of concrete and testing them for compressive strength as per the Indian standards.

Table 2 Distribution of Work

Initial Phase (May 2019 to December 2019)	Final Phase (January 2020 to June 2020)
Basic study of various properties of concrete Basic study of statistics Study the methods used for determining the compressive strength of concrete Studying the method used for used for determining compressive strength as per Indian Standards Studying various statistical approaches used in terms of concrete mixes Determining the statistical methods relevant for using for concrete compressive strength	Finding the critical values of compressive strength for IS 456:2000 Assuming 100 random data as the values of compressive strength and plotting them. Dividing the specimens into groups and analyzing each group for its relation to the population. Casting 100 cubes as per Indian Standards and doing similar analysis as done before Determining the relation between sample and population of concrete mixtures to give better understanding of compressive strength of concrete.

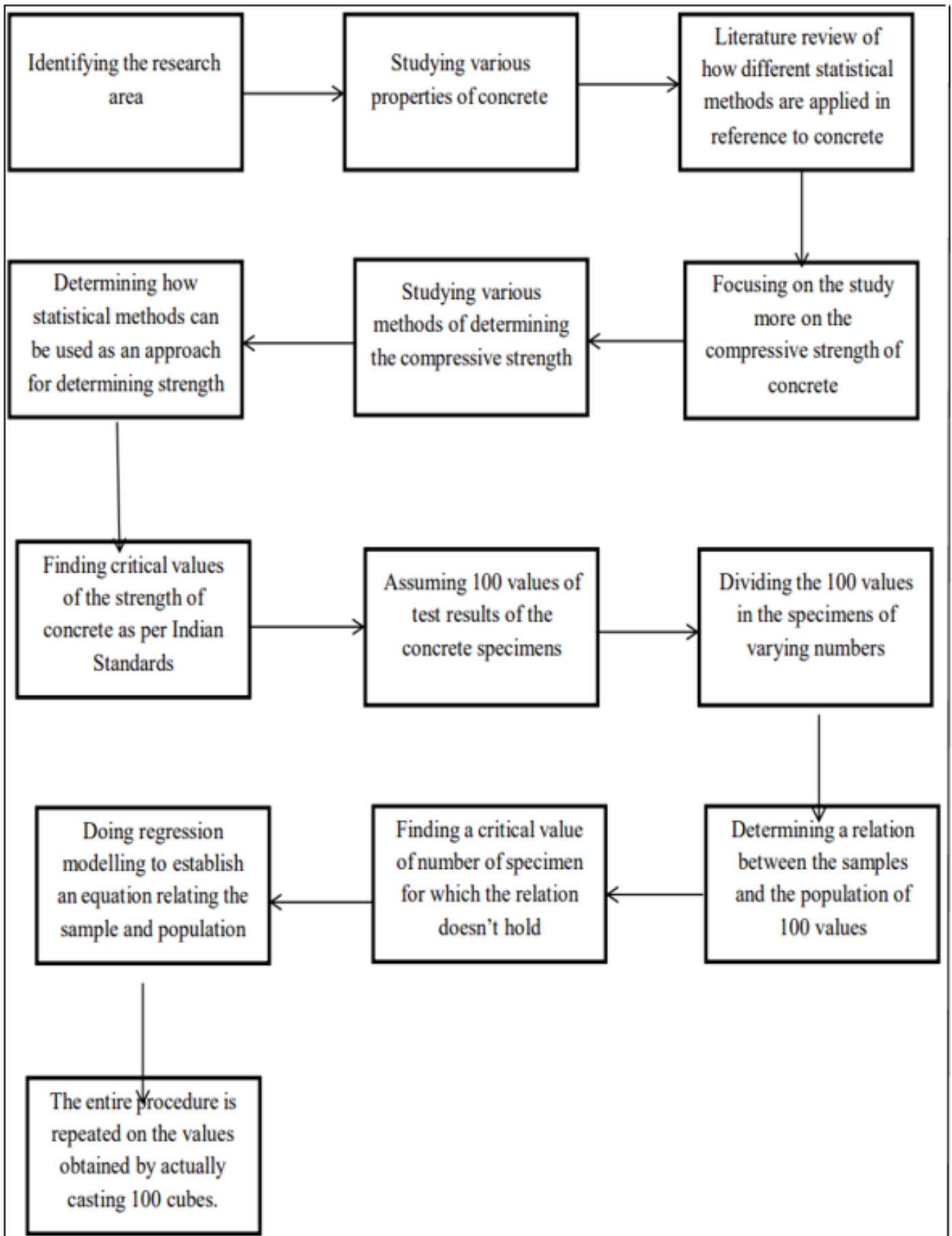


Fig 8 Flowchart of Work Distributions

## CHAPTER TWO LITERATURE REVIEW

In this chapter a detailed literature review is presented on various methods of statistics which are used for the analysis of concrete mixtures.

### ➤ *Analysis of Variance (ANOVA)*

Analysis of variance or ANOVA is a statistical method which has statistical models and various estimation procedures associated with it and is used for analysing the difference among group means in a sample. This method was developed by Ronald Fisher who is a renowned statistician. The law of total variance is the basis for this method, where a given variable's observed variance is broken down into different components, each of which is then assigned to a different source of variation. In its most basic forms, the ANOVA is a statistical test that determines whether or not the means of two or more populations are equal. When it comes to actual testing, it tells us whether a particular property of concrete is being affected by variability in mix proportions, exposure conditions, curing conditions, etc. The individual effect of the variable as well as a combined effect of two or more variables on the property of concrete can also be determined.

In one of the research conducted, the use of ANOVA was done to see the effect of varying proportion of silica fumes and polypropylene fibres along with the variation in curing temperature on both concrete's flexural and compressive strengths. The percentage contribution of variation for each property to the compressive and flexural strengths was determined using ANOVA. Table 3 displays the variable properties that were taken, and Tables 2.2 and 2.3, respectively, display the ANOVA results [16].

Table 3 Levels of Variables used in the Experiments [16]

Variables	1 <sup>st</sup> level	2 <sup>nd</sup> level	3 <sup>rd</sup> level	4 <sup>th</sup> level
Polypropylene fiber (%)	0	0.5	1	2
Curing Temperature (C°)	21	410	610	810
Silica Fumes (%)	0	10	-	-

Table 4 Results of ANOVA for Compressive Strength [16]

Control Factor	Degree of Freedom (DOF)	Sum of Square (SSA)	Variance (VA)	Fratio	Contribution (%)
Curing Temperature (°C)	3	108.40	36.79	55.22	75
Silica Fumes (%)	1	5.66	5.69	8.84	4
Polypropylene Fibres (kg/m <sup>3</sup> )	3	22.11	7.39	11.34	17
Error	8	5.57	0.67	-	4
Total	15	139.44	-	-	110

Table 5 Results of ANOVA for Flexural Strength [16]

Control Factor	Degree of Freedom (DOF)	Sum of Square (SSA)	Variance (VA)	Fratio	Contribution (%)
Curing Temperature (°C)	3	66.16	21.05	9.18	43
Silica Fumes (%)	1	9.28	9.29	3.73	6
Polypropylene Fibres (kg/m <sup>3</sup> )	3	54.44	17.71	7.44	37
Error	8	20.42	2.33	-	14
Total	15	150.30	-	-	100

In another literature, Jigar Bipinchandra Desai has used the using an analysis of variance (ANOVA) to ascertain whether the mean of samples made of slag and flash at various dosages and aggregate types differs significantly. The assumption that all sample means are equal served as the null hypothesis (H<sub>0</sub>) for the ANOVA. The critical F value was compared with the static, or observed, value of the F-test. Based on the degrees of freedom and the level of significance, which in this instance was taken to be 0.05, the critical F value was calculated. Because the observed F static value was higher than the critical F value, it cannot be concluded that there is enough evidence to support the null hypothesis

### ➤ *Hypothesis Testing*

“It is a confirmatory data analysis method which is used to check whether or not the assumed hypothesis is true or false. In a hypothesis test, two statements one known as null hypothesis and other as alternate hypothesis about the population are evaluated and the statement which supports the sample data is determined. To compare the accuracy of the data obtained through hypothesis, we either need two different data sets to compare them mutually or we need a data set obtained

From synthetic set, which is verified against the data set calculated through hypothesis. The synthetic data set is actually based over the idealized model. The testing is done by comparing the critical and calculated p-value of the statistical data according to the given distribution. Confidence intervals can be expressed in a way by the hypothesis tests which are based on statistical significance. So in a way, every confidence interval can be obtained by doing significance based hypothesis testing and based on significance every hypothesis test can be obtained via a confidence interval. For the framework of statistical hypothesis testing, significance based hypothesis testing is most commonly used.” [18]

The use of hypothesis testing can be widely found in literatures present. To give an example, a study on the effects of the variation of natural aggregates and recycle aggregate on compressive strength was done. To determine whether the test results of recycled aggregate concrete, regular aggregate concrete, and mixtures containing various proportions of natural and recycled aggregates follow a normal distribution, hypothesis testing was conducted. It was assumed that the distribution is normal, which is the null hypothesis. The results of the hypothesis are displayed in table 2.4. Since the value determined through statistics is lesser than that given by critical value in hypothesis, therefore the null hypothesis undertaken to determine the value statistically is acceptable. [19].

Table 6 Test Results on the Normal Distribution of Compressive Strength [19]

Type of mixture	Statistic Value	Critical Value(95% Confidence interval, 2 Tailed test)	Result of Null Hypothesis
NAC	4.2379	7.815	Accepted
RAC1	7.6089	7.815	Accepted
RAC2	4.9288	7.815	Accepted
RAC3	6.7227	7.815	Accepted

➤ *Regression Analysis*

The regression analysis is a statistical method where a relation between two or more variables of interest can be obtained. The basis of this analysis is that it examines the influence of the independent variables which can be one or many on the dependent variable. Although there are other methods available to carry out the regression analysis but the regression modelling is one of the finest methods available to describe the relationship between a dependent and multiple independent variables. The regression modelling helps in obtaining the values of dependent variable more accurately which can be again utilized in determining the other critical parameters.

Linear regression is the most common form of regression analysis which is used. In linear regression, a line that fits the data the closest according to a specific mathematical criterion is modeled. There are majorly two conceptually distinct purposes where regression analysis is used. The regression analysis is used for forecasting and prediction and the uses have a lot of applications in the field of machine learning. Secondly, the regression analysis in various situations is used to infer causal relationship between dependent and independent variable. Mainly, regression only reveals relationship in a fixed dataset between one or more independent variables and a dependent variable. Another kind of regression analysis that can be used to determine the ideal concrete mix design is the simplex centroid.

A study on the relationship between the failure strength resulting from impact loading on fiber-reinforced concrete and the initial crack strength is presented. The relationship between the first crack strength and failure strength of steel-polypropylene hybrid fiber reinforced concrete and steel fiber reinforced concrete was demonstrated by the development of a simple linear regression model. The impact strength studies are made over the SFRC and polypropylene hybrid FRC cylinders which are further used to obtain the model undertaken in the report to find the other parameters. The impact strength of SFRC and polypropylene FRC was determined with the help of cylindrical specimen over 48 cubes for each kind of FRC. The modeled equations are as shown in table 2.5 [20]

Table 7 Regression Modeling of Fiber Reinforced Concrete [20]

Steel Fiber Reinforced Concrete	Steel-Polypropylene Hybrid Fiber Reinforced Concrete
$F_s = a + b \times F_c$	$F_s = p + q \times F_c$
Where, $F_s$ is failure strength $F_c$ is first crack strength A and b are constant calculated based on known data whose values are 51.969 and 1.189 respectively The determination coefficient ( $R^2$ ) has value 0.915 for this equation	Where, $F_s$ is failure strength $F_c$ is first crack strength p and q are constants calculated based on known data whose values are 106.887 and 1.008 respectively The determination coefficient ( $R^2$ ) has value 0.843 for this equation

In one of the literatures by Sujay Math, he uses the simplex centroid method (SCM) to optimize the use of fly ash and slag as cementitious material in the concrete to obtain the desirable properties of concrete. “The simplex centroid method was introduced by Henry Scheffe in the 1960’s. The technique concentrates on fitting a mathematical equation for the prediction of its response to any mixture. It is a type of simple lattice design which uses boundary points with the exception of the centroid points seeing to it that the design points are on the boundaries of the simplex. The example of simplex centroid design with test points is as shown in the figure 9. The points on the vertices represents a pure mix, the point on the edges represents a binary blend whereas any other

point in the centre of triangle is a ternary blend. The design points in the method are associated with a simple cubic regression polynomial equation. The polynomial which is fitted at a particular point of the simplex centroid data by collecting the response needsto have the same number of parameters or terms which are to be calculated with the points in associated in the design.” [21]

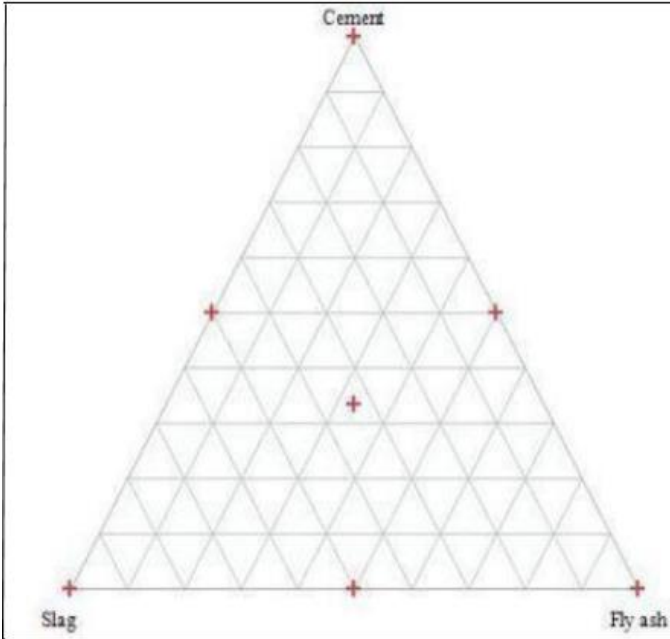


Fig 9 A Standard

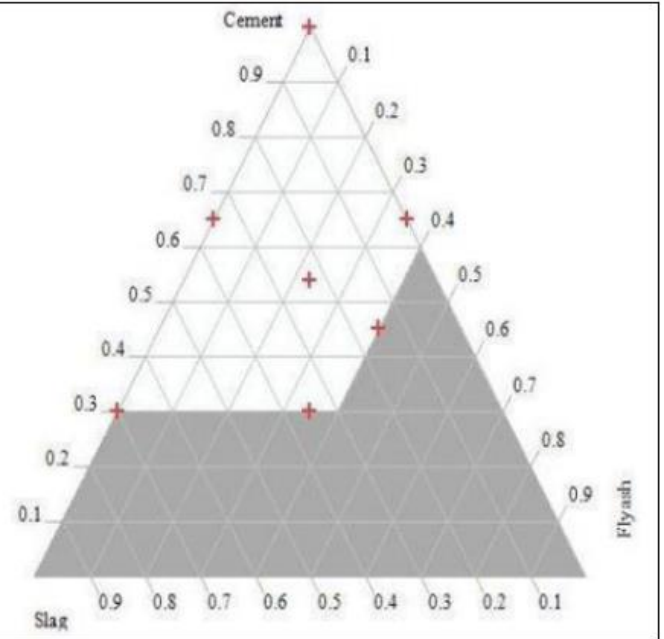


Fig 10 B with Constraints

- *Normality test*

The normality test in statistics is used to check if the obtained data set is modelled properly by normal distribution and to compute whether it is likely for a data set underlying a random variable to be normally distributed. The K-S test or Kolmogorov–Smirnov test is a commonly used hypothesis test used to find whether the given set of data follows normal distribution. The p-value for the normal distribution is calculated and if the obtained value is less than the significance level or the critical p value the data is said to follow the normal distribution [22].

In the research mentioned above for the fibre reinforced concrete, the Kolmogorov–Smirnov normality test was also conducted. The table 8 shows the analysis done for the failure strength data. Also figure 11 shows the graph of distribution [20].

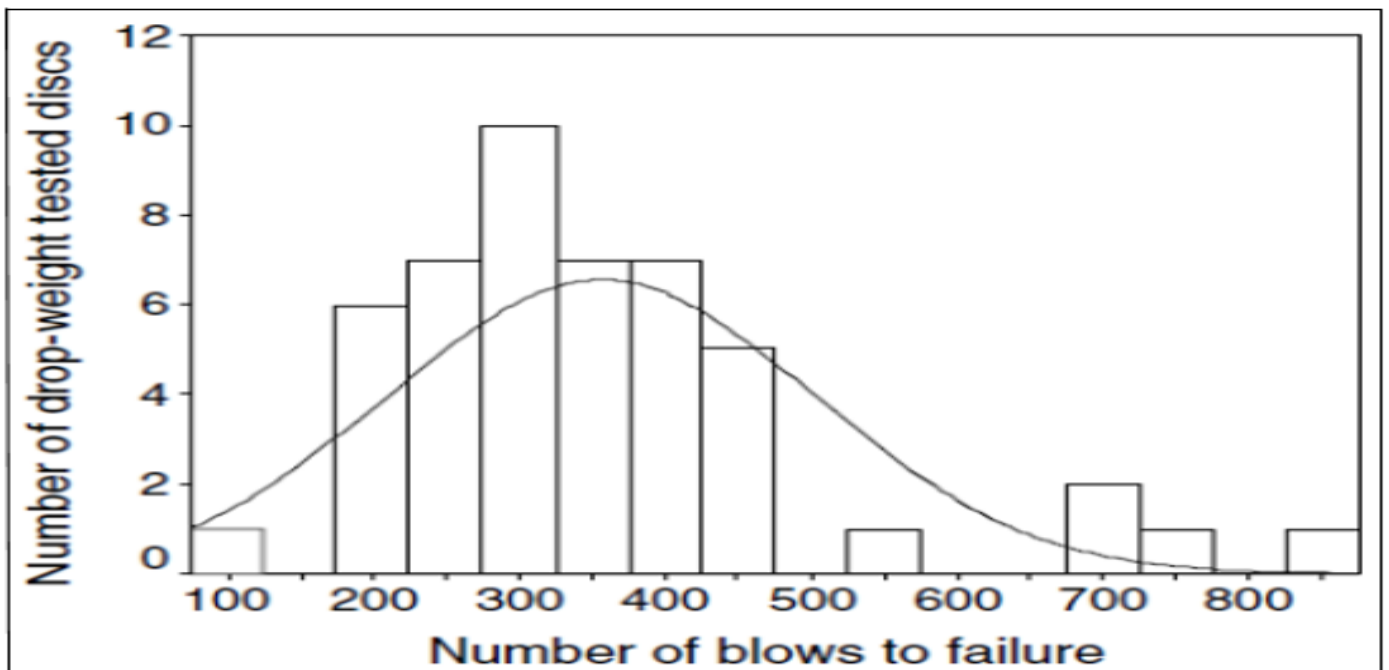


Fig 11 Frequency Histogram and Fitted Normal Curve for Failure Strength [20]

Table 8 Statistical Analysis for failure of discs [20]

<b>Minimum number of blows required</b>	<b>124</b>
Maximum number of blows required	846
Mean number of blows required	356
Standard deviation of the blows required	146
Coefficient of variation of the blows (%)	41
95% Confidence Interval	
Upper Bound of the number of blows	399
Lower bound of the number of blows	314
p-value of K-S test	0.01

## CHAPTER THREE NEED FOR RESEARCH

A universal method for the calculation of the compressive strength is not available. Sophistication of means of analysis and interpretation of test data are constantly being done along with the modernization of method of testing and improvement of equipments. The compressive strength is defined by various countries as per their codal provisions. In India, the definition given by IS 456:2000 is considered. The provisions of acceptance criteria as per IS 456:2000 is as follows:-

➤ *Compressive strength Provisions for concrete according to IS 456:2000*

“Samples of fresh concrete are taken and cubes of size 150 x 150 x 150 mm are casted, cured for 28 days and tested in a compression testing machine. A sampling procedure which is considered to be random has to be adapted to making sure that each concrete batch has reasonable and equal chances of being tested. All the mixing units are to be covered and the sampling is to be done for the entire period of concreting. The minimum number of samples required to specify the characteristic compressive strength of concrete for each class should satisfy the criteria given in table 3.1.” [10]

Table 9 Minimum Frequency for Sampling of Concrete [10]

Amount of Concrete to be used for work (m <sup>3</sup> )	Number of Samples
1 to 5	1
6 to 15	2
16 to 30	3
31 to 50	4
51 and above	4 and one additional sample each for increase in 50 m <sup>3</sup> of work

**NOTE:** - At least one sample representing each shift has to be taken.

“One sample consists of three specimens or three cubes to be tested at 28 days. More samples can be cast for a variety of purposes, like testing the cubes after seven days, figuring out how long it takes to cure, striking the formwork, examining testing errors, etc. Each test result is considered as the average of three specimens. The strength value of each cube or specimen should be within  $\pm 15$  percentage of the average of the three cubes otherwise the test results of the sample are not valid and the sample is discarded. As for the acceptance criteria of the strength, the samples should satisfy the following conditions.” [10]:-

- The mean value obtained from a set of 4 non overlapping consecutive samples must satisfy the strength criteria given in column 2 of table 3.2 [10].
- The value of individual sample or test result should be within the limit of values shown in column 3 of table 3.2 [10].

Table 10 Requirement of Characteristic Compressive Strength [23]

Specified Grade	Average of the Group of 4 Consecutive and Non-Overlapping Test Samples (N/mm <sup>2</sup> )	Individual Test Samples (N/mm <sup>2</sup> )
M 15 and above	$\geq f_{ck} + 0.825 \times \text{established standard deviation}$ OR $\geq f_{ck} + 3$ whichever of the two greater	$\geq f_{ck} - 3$

If the quantity of concrete is less than 30m<sup>3</sup> the criteria given in the table above is not applied. The average or mean strength of the group of samples shall be more than  $f_{ck} + 4$  and the requirement of minimum value of individual sample or test result should be  $f_{ck} - 2$ . If the standard deviation of the samples is not already established, the values of assumed standard given in table 2.3 must be used and casting of 30 samples is to be done as soon as possible for establishing the value of standard deviation [10][23].

Table 11 Assumed Standard Deviations [23]

Concrete Grade	Assumed Standard Deviation (MPa)
M-10	3.5
M-15	
M-20	
M-25	4.0
M-30	
M-35	
M-40	
M-45	
M-50	
M-55	5.0
M-60	

The strength criteria given in the IS 456:2000 which is used to specify the characteristic compressive strength of concrete is primarily dependent on the magnitude of standard deviation obtained during the testing of 150x150x150mm cubes of concrete. There is no basis for these given values of standard deviation or for the equations of the acceptance criteria. Also it is not always possible to cast 30 samples of cube for the standard deviation.

➤ *Objectives*

- A detailed study of various statistical methods applicable to concrete material.
- Statistical analysis of the compressive strength of 100 cubes of M30 grade of concrete.
- Determining the optimal number of cubes required to check whether the required amount of strength of concrete is achieved or not.
- Performing analysis for obtaining an equation for determining the compressive strength of the concrete of an obtained mixture.



### CHAPTER FOUR COMPRESSIVE STRENGTH FOR CRITICAL VALUES

The values of the samples and the group of samples below which the test results are not expected to fall are known as the critical values of compressive strength. These figures are determined using the codal rules specified in IS 456:2000. "In accordance with IS 456 for the scenario where the value of standard deviation is taken from the table of assumed standard deviations, the values given in tables 12 and 13 along with the criteria that the value of compressive strength of individual specimens should be within 15% of the average of the specimens which is the sample value, is sufficient to check the compressive strength of concrete mixture. The values in the table 14 and table 15 are subjected to change if a standard deviation other than the assumed standard deviation is established by actually casting 30 samples. The table 4.1 shows the minimum value of individual test result of compressive strength (in MPa) that can be obtained according to the grade of concrete for varying volume of quantity of concrete work as per the codal provision of IS 456:2000." [10][23]

Table 12 Minimum Value of test Results of Each Sample of Concrete

Grade of Concrete	Values of strength for varying volume of concrete (MPa)		
	1 m <sup>3</sup> to 5 m <sup>3</sup>	6 m <sup>3</sup> to 30 m <sup>3</sup>	31 m <sup>3</sup> and above
M15	19	13	12
M20	24	18	17
M25	29	23	22
M30	34	28	27
M35	39	33	32
M40	44	38	37
M45	49	43	42
M50	54	48	47
M55	59	53	52
M60	64	58	57

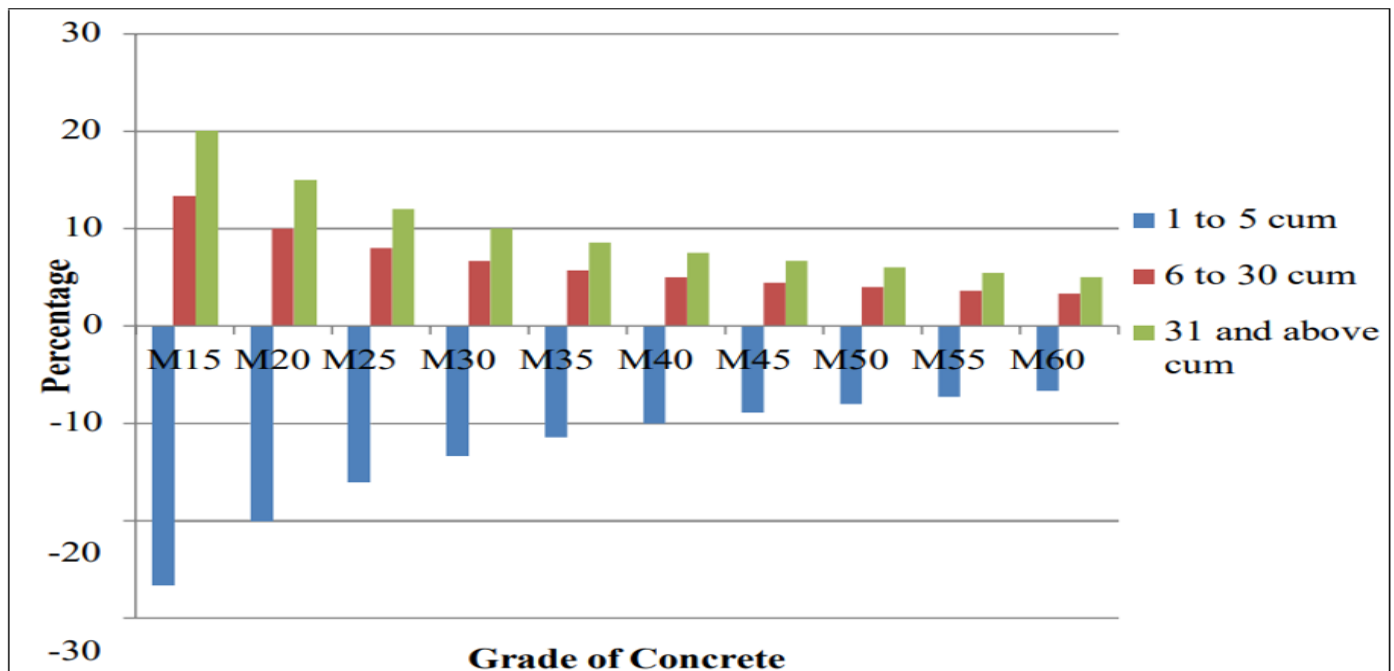


Fig 12 Grade of Concrete Vs. Percentage Difference Between fck and Minimum Value of test Result

The figure 12 shows the percentage difference between the characteristic compressive strength (f<sub>ck</sub>) of concrete and the minimum value of individual test result of compressive strength for varying volume of quantity of concrete work. The table 4.2 shows the minimum value of mean of group of all the samples that can be obtained according to the grade of concrete as per the codal provision of IS 456:2000.

Table 13 Minimum Value of Mean of Group of all Sample of Concrete (in Mpa)

Grade of Concrete	Values of strength for varying volume of concrete		
	1 m <sup>3</sup> to 5 m <sup>3</sup>	6 m <sup>3</sup> to 30 m <sup>3</sup>	31 m <sup>3</sup> and above
M15	19	19	18
M20	24	24	23.3
M25	29	29	28.3
M30	34	34	34.1
M35	39	39	34.1
M40	44	44	44.1
M45	49	49	49.1
M50	54	54	54.1
M55	59	59	59.1
M60	64	64	64.1

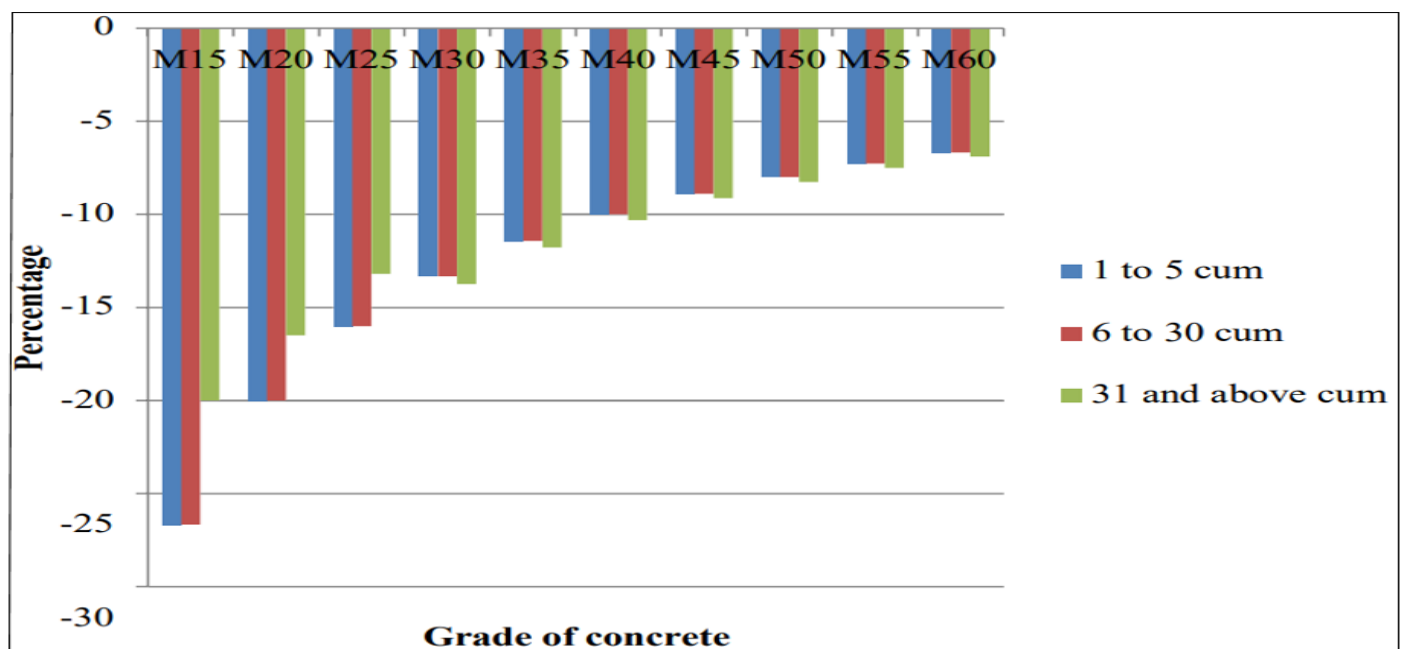


Fig 13 Grade of Concrete Vs. Percentage Difference Between fck and Mean of all Samples

The figure 13 shows the percentage difference between the concrete's characteristic compressive strength (fck) and the lowest mean value of the group of all samples for different amounts of concrete work. Thus, even though the test results show that the compressive strength obtained is higher than the characteristic compressive strength, it is evident that in the majority of cases, the concrete mixture is rejected. The assumed standard deviation's value is basically the basis for the calculation of these values. There is no proper basis for the values of assumed standard deviations as well as the equations of the acceptance criteria. A relation between the entire concrete mixture and the casted samples needs to be established. This can be done by the analysis of population and samples.

## CHAPTER FIVE ANALYSIS OF POPULATION AND SAMPLE

Statistical population is the entire set of similar items like in case of compressive strength of concrete, the strength of the entire mixture can be termed as population. “A sample is a subset of population which represents the entire population. Sampling is the method which is used in the estimation of the population characteristics from a subset taken from the same population. An error in sampling can lead to misleading conclusions regarding the populations, so it is important that the sampling is done at a proper qualitative and quantitative basis.”[12] Hypothesis testing is a method in statistical analysis where a relation between sample and population can be established. So for the analysis, considering M30 grade of concrete a random data of 100 numbers between 27 and 45 was generated using MATLAB coding software, details of which are given in table 5.1. These 100 values which were rounded off to the second decimal point were considered as the compressive strength of 100 cubes and the data was further evaluated.

➤ *Matlab Code for Random Data*

```
clear clc close all
value=[27:0.01:45];
rd=datasample(value,100);
xlswrite('randomdata.xls','rd');
```

Table 14 Details of Compressive Strength Values of 100 Cubes

<b>Characteristic Compressive Strength (fck)</b>	<b>30 MPa</b>
Mean (favg)	35.82 MPa
Standard Deviation ( $\sigma$ )	3.63 MPa

These values were then plotted to see whether they follow a normal distribution. The figure 4.1 shows the plot of 100 values, and as we are getting a smooth bell shaped curve, it can be assumed that the values follow normal distribution.

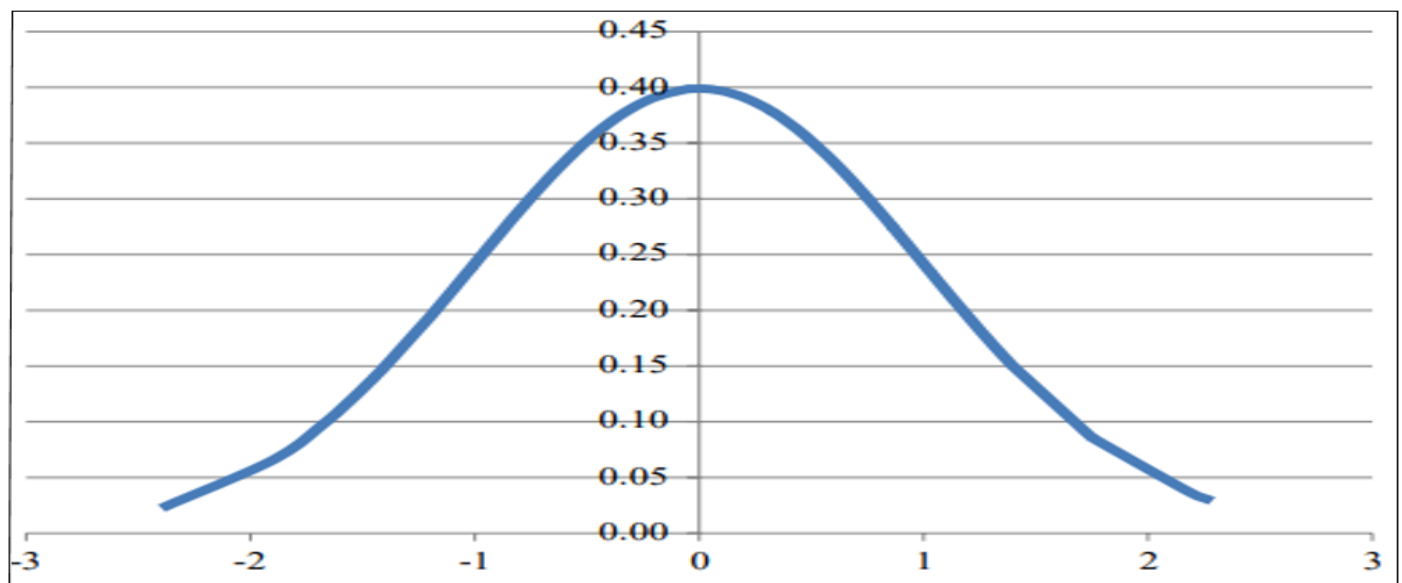


Fig 14 Distribution of Assumed Random Data

The cubes are then divided in specimens of 75, 50, 30, 25, 20, 18, 16, 15, 14, 13, 12, 11, 10 and 9. Hypothesis testing is then performed on each group of specimens to check whether they belong to the population. 5000000 such different combinations from each group were tested for 99% confidence interval. The table 5.2 shows the percentage of total specimens tested. The numbers of combinations from each group that passed and failed the hypothesis test were calculated and are shown in the table 5.3. The interval for the number of specimen in each sample is decreased as the rate at which the number of samples that are failing is increasing.

Table 15 Percentage of the Tested Combinations from the total Number of Combinations

Number of Specimens	Percentage of total combinations taken
75	$2.06 \times 10^{-14}$
50	$4.96 \times 10^{-20}$
30	$1.70 \times 10^{-16}$
25	$2.06 \times 10^{-14}$
20	$9.33 \times 10^{-12}$
18	$1.63 \times 10^{-10}$
16	$3.72 \times 10^{-9}$
15	$1.97 \times 10^{-8}$
14	$1.13 \times 10^{-7}$
13	$7.03 \times 10^{-7}$
12	$4.76 \times 10^{-6}$
11	$3.53 \times 10^{-5}$
10	$2.89 \times 10^{-4}$
9	$2.63 \times 10^{-3}$

Table 16 Results of Hypothesis Testing

Number of Specimens in each sample	Number of Samples Passing the Hypothesis	Number of Samples Failing the Hypothesis
75	50000000	0
50	49989632	10368
30	49906940	93060
25	49866036	133964
20	49818189	181811
18	49796987	203013
16	49772951	227046
15	49761279	238721
14	49750593	249407
13	49737452	262548
12	49725967	274033
11	49713861	286139
10	49702150	297850
9	49690287	309713

The numbers of combinations from each group that passed and failed the hypothesis test were calculated and are shown in the table 5.3. The interval for the number of specimen in each sample is decreased as the rate at which the number of samples that are failing is increasing. The graph showing of percentage failure of each group which is the percentage of combinations failing the hypothesis test over the total combinations taken for each group is plotted as shown in the figure 5.2. The graph shows that as the number of specimens is decreasing the percentage of failure increases

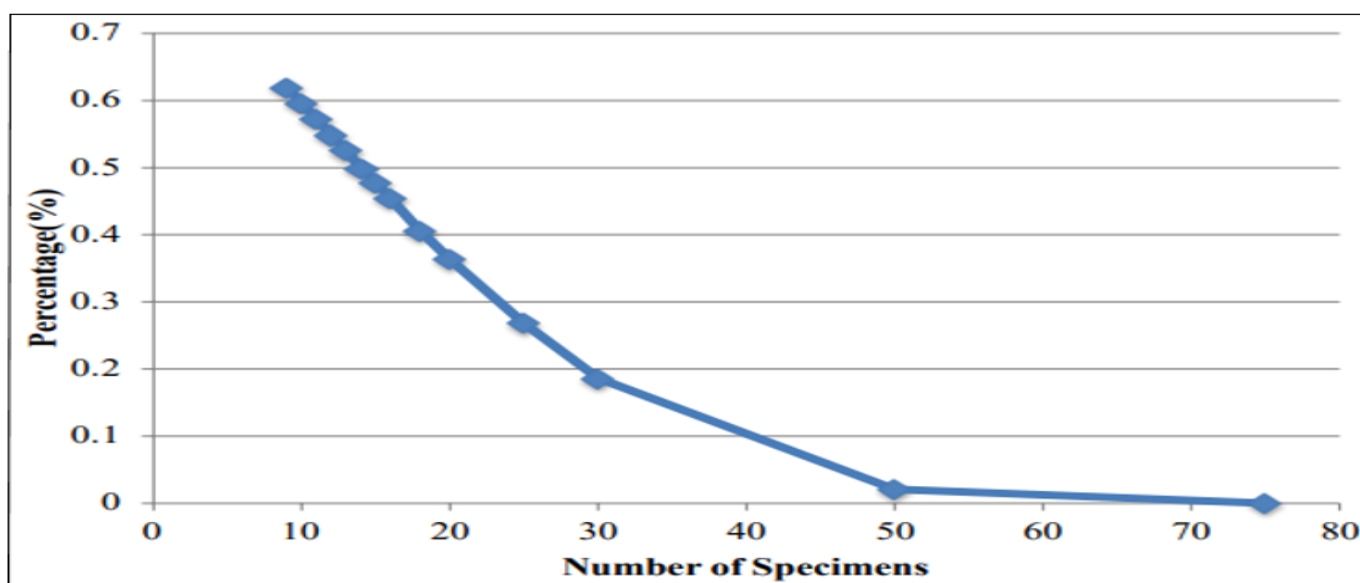


Fig 15 Percentage Failure Vs. Number of Specimens

➤ *Matlab Code for Hypothesis testing of 75 specimens*

```
clear clc close all

data=xlsread('randomdata.xlsx');v=data(:,2);

s75 = zeros(75,1);pass=0;fail=0; for i=1:50000000

s75= datasample(v,75,'Replace',false);X = mean(s75);

Z=(X-35.8198)*(3.6342/sqrt(75));if Z>=-2.576 && Z<=2.576

pass=pass+1;else

fail=fail+1;

end

clear s75;

end

disp('Pass=');disp(pass); disp('Fail=');disp(fail);
```

## CHAPTER SIX FURTHER WORK

The number of samples at which the failure is high was to be calculated. The standard deviation for each combination of that sample is calculated. The relation between the standard deviation of the population and that of the samples is found. Then a regression analysis is done to find the equation relating the samples to the population for any number of given specimens. The entire analysis is to be done for the data obtained by casting 100 cubes of concrete of grade M30. The mix design for the cubes was done as per the Indian Standard code as well as various literatures were studied. The mix design as shown in table 6.1 was finalized.

Table 17 Summary of Mix Design

<b>Total number of cubes to be casted</b>	<b>120 cubes</b>
Volume of cubes to be casted	0.405 m <sup>3</sup>
Total amount of cement required	270 Kg
Total amount of fine aggregates (sand) required	294 kg
Total amount of coarse aggregates (gravel) required	438 kg
Total amount of water required	132 liter

## **CHAPTER SEVAN CONCLUSIONS**

➤ *The Conclusions that Could be Derived from the work Done are as Follows: -*

- The calculation of compressive strength of concrete of the given grade as per Indian Codal Provisions given in IS 456:2000 is largely based on the table of assumed standard deviations and equations which do not have a strong basis.
- In some cases even the samples in which values obtained more than the required characteristic compressive strength are rejected.
- As the number of specimens in the sample decrease, the number of samples passing the hypothesis of being from the population decreases
- The work can be useful in the following ways

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