Noise Filtering by Fourier Series and NHA

ADITI RAI[1], TOIBA SHEIKH[2], G. S. G. N ANJANEYULU[3]*
[1,2]School of Mechanical Engineering, [3]School of Advanced Sciences,
VIT University, Vellore-632014, Tamil Nadu, India

Abstract:- Noise interference poses a significant challenge in various fields reliant on accurate signal processing such as telecommunication, audio analysis, and medical imaging. Traditional noise filtering methods, such as Spectral Subtraction method or using Notch Filters, often fail to address the complex nature of real world noise which exhibits non-linear characteristics and lacks stationary properties. This research paper adds to an already existing analytical method-–NHA (Non-linear Harmonic Analysis) - which is one of the most effective in our time. Then, using Fourier Transform we analyze how to filter out noise more effectively.

Keywords:- *Real world audio analysis, non-linear characteristics, Non-linear Harmonic Analysis.*

I. INTRODUCTION

In today's digital era the acquisition, transmission, and the analysis of signals are essential for various applications. However, real world signals are often corrupted by noise, which can arise from numerous sources including Electromagnetic interference, Thermal Fluctuations or Sensor Imperfection. Noise is an undesirable or unwanted signal that gets randomly added to the actual information carrying signal, thereby hindering accurate analysis and interpretation of signal. The removal of such unwanted signals, or the reduction in the unwanted ambient sounds using active noise control is known as Noise Filtering. Noise Filtering plays a crucial role in numerous disciplines, including telecommunication, audio processing, image and video analysis, biomedical signal processing and more.

In order to achieve a less noise corrupted signal, we generally use a frequency spectrum to remove noise from the input waveform. Discrete Fourier Transform is an effective way of attaining noise filtering. The DFT is based on the principles of Fourier analysis, which has several useful properties. One of the key properties is that complex signals can be represented as a sum of simpler sinusoidal components, each associated with a specific frequency and amplitude. This property makes it easier to identify and separate noise components from the desired signal.

A. FOURIER TRANSFORM

Fourier Transform, named after the French Mathematician Jean Baptiste Joseph Fourier, are mathematical operations that decompose a function into its frequency components and they provide a way to analyze complex signals and study their frequency content. By converting a signal from the time domain to the frequency domain, Fourier Transforms enable the identification of specific frequencies and their respective magnitudes in the original signal.

Fourier Transforms decomposes a function into coefficients of a linear combination of trigonometric functions. Fourier Transform for a function f(t) can be given as –

$$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt$$

Where $\boldsymbol{f}$ denotes a function dependent on time and the Fourier Transform of $\boldsymbol{f}$ is given as $\hat{f}$dependent on frequency $\boldsymbol{\omega}$.The corresponding inverse Fourier Transform is:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega t}d\omega$$

In practice, we only have finite sequences of discrete measurements. For finite evenly spaced sequences, we apply the discrete Fourier transform (DFT) given as:

$$Y_{k=}\sum_{k=1}^{N-1} X_{n.e}{}^{\frac{-2\pi i}{N}kn}$$

Where Y_k denotes the k-th element in the discrete Fourier transform vector and X_n denotes the n-th element in the original time sequence. N is the number of elements in the time series. Similarly we have the inverse discrete Fourier transform (IDFT) defined as:

$$X_{n=\frac{1}{N}}\sum_{k=1}^{N-1} Y_{k.e}{}^{\frac{-2\pi i}{N}kn}$$

The DFT is generally used for frequency analysis which forms the basis of the NHA method of noise filtering.

B. NON LINEAR HARMONIC ANALYSIS

Non Linear Harmonic Analysis is a tool used in noise filtering techniques in the case of signal processing. Simply put, it helpsin removing noise and unwanted sounds from signals by identifying and filtering out random and unwanted fluctuations. The NHA is designed to take an input signal and detect any interference in it, then one applies a nonlinear filter to the signal and removes the unwanted fluctuations while still preserving the important signal information.

To fully understand the way NHA is used in analysis we need to understand what a Non Linear filter is, this is a kind of filter that uses nonlinear operations to remove noise from signals.

Unlike, linear filters that use linear operations such as adding, or averaging. Nonlinear filters use more complex operations such as ranking. The NHA algorithm uses this filter to apply a thresholding operation to the signal to remove any unwanted fluctuation below a certain threshold.

In NHA, the objective is to minimize the discrepancy between the measured and predicted harmonic amplitudes/frequencies.

Figure 1 describes the NHA algorithm and its working,

- *Fast Fourier Transform*

The Fast Fourier Transform is a way to break down a signal into its different frequency components, helping us understand the frequencies present in the signal and their respective strengths. The basic idea behind the FFT is to represent the DFT as a sum of two smaller DFTs. The algorithm recursively divides the input sequence into even-indexed and odd-indexed subsequences, computes their respective DFTs, and then combines them to obtain the final result. This process is repeated until the sequence length becomes 1, which represents the base case of the recursion.

The FFT has numerous applications in various fields, including signal processing, image processing, audio compression, data analysis, and scientific computing. It allows efficient computation of the frequency content of a signal, enabling tasks such as spectral analysis, filtering, and modulation/demodulation. The algorithm's speed and effectiveness have made it a fundamental tool in many areas of digital signal processing.

- *Steepest Descent Method*

The steepest descent method in NHA seeks to find the optimal solution that minimizes the objective function, leading to accurate predictions of harmonic amplitudes in the system under investigation. The process continues iteratively until a termination criterion is met, such as reaching a maximum number of iterations or achieving a desired accuracy level. The goal of the steepest descent method is to iteratively update the solution by taking steps in the direction of the steepest descent of the objective function. The direction of steepest descent is given by the negative gradient of the function at the current point. The gradient represents the direction of maximum increase, so negating it gives the direction of maximum decrease or descent.

- *Amplitude Convergence*

In NHA, the goal is to predict the amplitudes of harmonic components in a measured signal by iteratively updating the estimates. The algorithm starts with initial guesses for the amplitudes and refines them through successive iterations until they converge to a satisfactory solution.

It's worth noting that the specifics of amplitude convergence in nonlinear harmonic analysis can vary depending on the specific nonlinearities present in the system, the nature of the excitation, and the analytical or numerical techniques used to solve the equations of motion. Therefore, the analysis of amplitude convergence typically involves nonlinear equations and requires specialized techniques such as perturbation methods, numerical simulations, or iterative algorithms.

- *Newton's Method*

In Newton's method, we take initial guesses for the harmonic amplitude. Then, we calculate the slope and the curvature (second derivative) of the given function to find the direction of the function, its steepness and how fast the function changes. Adjust the estimates by taking the slope and its curvature in consideration. Repeat these steps until convergence is achieved or a termination criterion is achieved.

Fig. 1: The working of NHA

C. *HILBERT TRANSFORM*

Hilbert Transform is a mathematical operation used to analyze signals in time and frequency domains. It is a way to create a complex valued representation of a real valued signal, thus an analytical signal consisting of complex value is produced. Hilbert Transform is often solved by using Fourier Transform and is an efficient method on its own merit.

The Hilbert transform rotates the real-valued signal 90 degrees counterclockwise in the frequency domain. This creates a complex-valued signal that contains information about both the amplitude and phase of the original signal. The basic formula of Hilbert transform for signal x (t) is given as

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$$H(t) = {}^{1}/_{\pi}\, P \int \frac{f(t')}{t - t'} dt'$$

Where P is the Cauchy principal value which is used to handle the singularity that occurs when the denominator of the integrand is zero(when t=t'). Analytical signal A(t) combines with H(t) and f(t), where f(t) is the real part of A(t) while H(t) is the imaginary part.

Thus, A(t) can be written as

$$A(t) = f(t) + iH(t) = a(t)e^{\omega t}$$

D. SIGNIFICANCE OF HILBERT TRANSFORM

When we discuss signal processing, we need to know the importance of an envelope in a modulated signal. The envelope is a representation of the variation in the amplitude of a signal. For example, in amplitude modulation (AM), the amplitude of the carrier signal is varied in proportion to the amplitude of the modulating signal.

This envelope can be extracted using Hilbert Transform and in the context of noise filtering it helps us in distinguishing between signal of interest and noise. The basis of it being that the envelope is much smoother than the signal itself, while noise tends to be more varying and random. By extracting this envelope using Hilbert Transform, we can filter out the noise and recover a less corrupted signal.

E. DRAWBACKS OF HILBERT TRANSFORM

While Hilbert Transform is a powerful tool for noise filtering it comes with certain drawbacks including but not limited to the following.

➢ *Susceptibility to noise*

Since Hilbert transform operates on the principle of using envelopes over the entire signal, any noise present in the signal will also enter the envelope making the extracted envelope corrupted.

➢ *Limited frequency resolution*

It is not able to accurately separate signal components that are very close in frequency. This can be a problem in some noise filtering applications where the noise is close in frequency to the signal.

➢ *Complexity in computation*

Hilbert transform is a complex mathematical operation that can be computationally very expensive, especially for large signals, this is also a time consuming process. This is a limiting factor for real world problems.

➢ *Phase distortions*

There are many cases in which the extracted envelope has a different phase than the input signal. This leads to incorrect analysis of signal.

Keeping in mind these issues with Hilbert Transform we can discuss Fourier Transform as an alternative method.

F. FOURIER TRANSFORM AND ITS MERITS

- By performing a Fourier transform on the signal before applying the Hilbert filter, the frequency content of the noise can be identified and removed. We don't have to analyze the envelope filled with corruption and noise as they are already removed.
- Fourier transform can be used to correct the phase distortion by applying a phase shift to the Fourier coefficients. This can help to align the phase of the extracted envelope with the original signal.
- Fourier transform can be used to speed up the filtering process by transforming the signal into the frequency domain, where filtering can be performed more efficiently. Once the filtering is complete, the signal can be transformed back into the time domain using an inverse Fourier transform.

II. IMPROVISED MODEL OF NHA

The NHA (Nonlinear Harmonic Analysis) technique is a noise filtering method that uses Fourier series to model and remove noise from a signal.

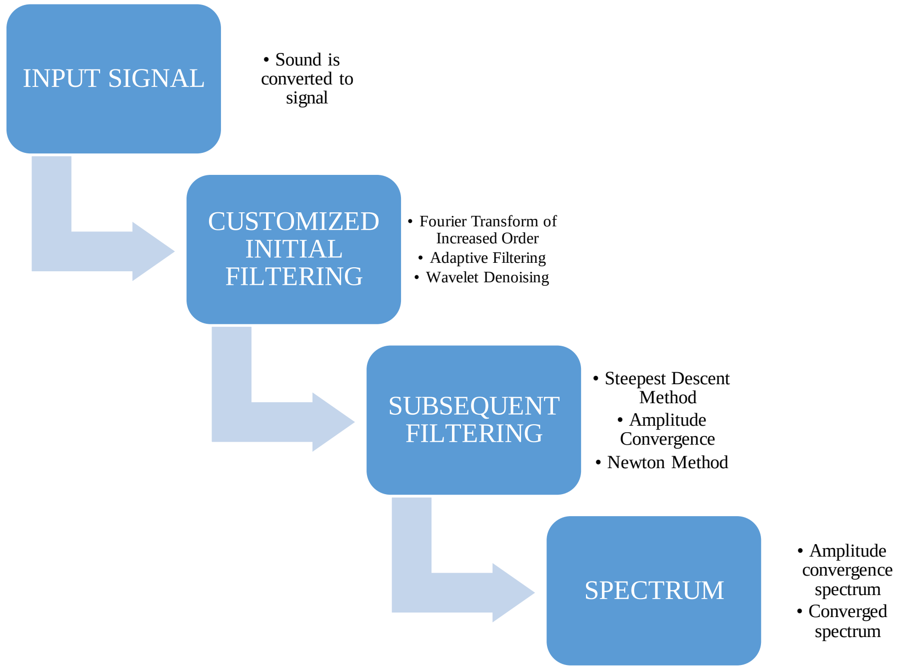

Fig. 2: Flowchart summarizing the improvised model of NHA

To improve this technique, you can consider the following steps:

- **INCREASE THE ORDER OF FOURIER SERIES**: Increasing the order of Fourier series means using more terms to approximate the given function, in this case our input signal. By increasing the order of the Fourier series used to model the signal, we can capture more complex signal patterns and noise components. This can lead to better noise filtering performance.

$$f(x) = \frac{\mathring{a}}{2} + \sum_{n=1}^{\infty}[an\cos(n\omega x) + bn\sin(nwx)]$$

Where å, an, bn are coefficient determined by the function f(x)

$\boldsymbol{\omega}$ is the angular frequency.

Order of the series is determined by 'n'.

- **USE ADAPTIVE FILTERING**: Rather than using a fixed Fourier series order, you can use an adaptive filtering approach that adjusts the Fourier series order based on the characteristics of the input signal and noise. This can help to improve the accuracy of the noise filtering. For example, if the input signal contains a lot of high-frequency noise, the filter coefficients can be adjusted to attenuate the high-frequency components of the signal.

  Adaptive filtering using NHA and Fourier analysis can improve the noise filtering performance in situations where the noise characteristics are changing over time.

- **INCORPORATE SIGNAL PRIORS**: If you have prior knowledge about the signal, such as its frequency components or other statistical properties, you can use this information to improve the noise filtering performance. For example, if you know that the signal has a sparse representation in the Fourier domain, you can use a sparse approximation method to better capture the signal and remove the noise.

- **USE MULTIPLE FOURIER MODELS**: Instead of using a single Fourier series model to represent the signal, you can use multiple models with different orders or parameters. This can help to capture different aspects of the signal and improve the noise filtering performance.

  For example, in a signal that contains both high frequency and low frequency components, when we use multiple Fourier models, each model can be used to represent a specific frequency range. This results in a more accurate representation of the signal.

III. POST-PROCESSING TECHNIQUES

After applying the NHA technique, you can use post-processing techniques to further refine the noise filtering results. For example, you can use wavelet DE noising or total variation regularization to remove any remaining noise or artifacts.

A. SPARSE APPROXIMATION

Sparse approximation is a mathematical technique used to approximate a signal or data set using a small number of basic functions or features. The basic idea behind sparse approximation is to find a linear combination of a small number of basic functions that can closely approximate the original signal, while ignoring or discarding the contribution of other basis functions.

The term "sparse" refers to the fact that the number of non-zero coefficients in the linear combination is small compared to the total number of basic functions. This is often desirable in signal processing applications, as it can help to reduce the complexity of the model and improve the interpretability of the results.Sparse approximation

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techniques are often used in signal processing, image processing, and machine learning applications, especially when there is a time crunch involved.

B. WAVELET DENOISING

Wavelet denoising is a signal processing technique used to remove noise from a signal by applying a mathematical transform known as the wavelet transform. The wavelet transform breaks down a signal into a set of wavelet coefficients, which represent different frequency components of the signal at different scales.

The wavelet denoising process involves the following steps:

- **Decompose the signal**: The first step is to apply the wavelet transform to the signal, which decomposes it into a set of wavelet coefficients. The decomposition can be performed using various wavelet functions and decomposition methods.
- **Thresholding**: Once the wavelet coefficients are obtained, the next step is to apply a thresholding operation, which removes the coefficients that correspond to the noise components of the signal. There are different types of thresholding methods that can be used, such as hard thresholding, soft thresholding, and adaptive thresholding.
- **Reconstruct the signal:** After thresholding the wavelet coefficients, the final step is to reconstruct the signal by applying the inverse wavelet transform to the threshold coefficients. The reconstructed signal will have the noise components removed and the original signal preserved

Overall, by combining these techniques and exploring different model architectures and optimization methods, you can improve the performance of the NHA technique for noise filtering using Fourier series.

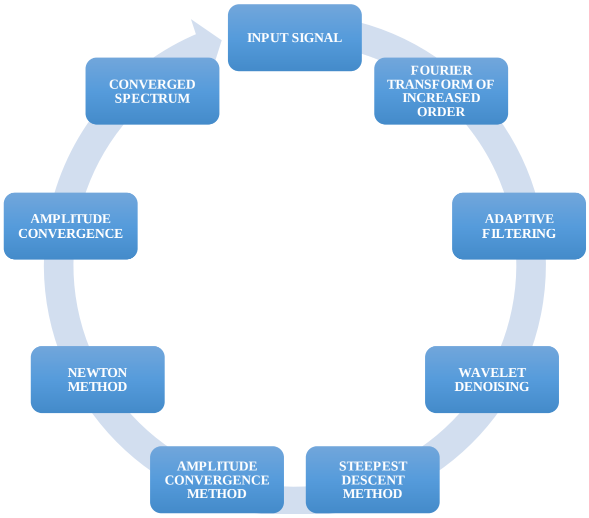

Fig. 3: Flowchart demonstrating the improvised NHA algorithm

IV. EXPERIMENTAL RESULTS

A. Analysis with respect to frequency

We have used Audacity, which is an open source software used for editing sounds and separating individual notes from a pre-recorded audio file. A whistle is recorded from the stereo microphone. If properly recorded in an appropriate environment, a whistle will form a sine wave.

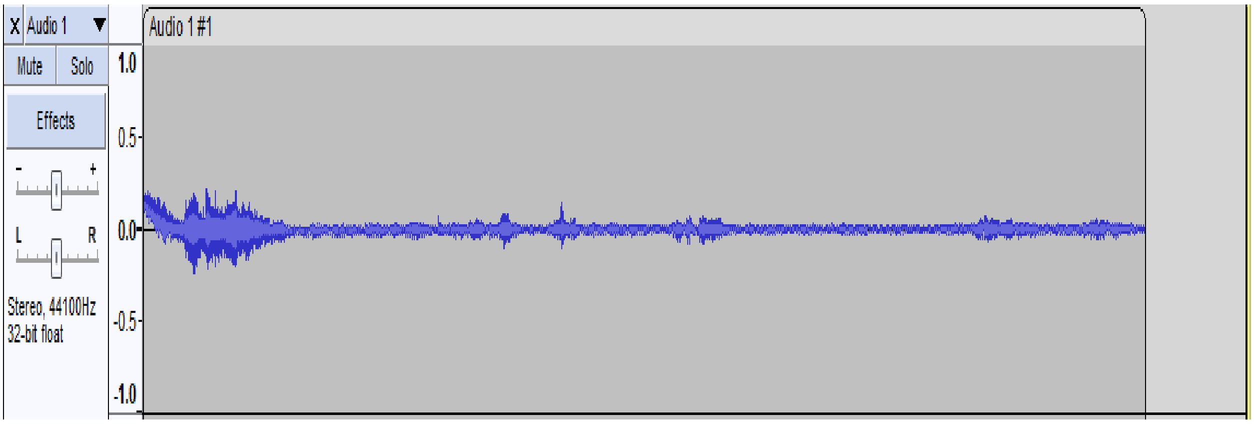

Fig. 4: A whistle recorded from microphone

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Plots are made using Fast Fourier Transform or FFT. This helps us in providing a value for every narrow band of frequency present in the wave. All the values are then interpolated to create the graph given below.

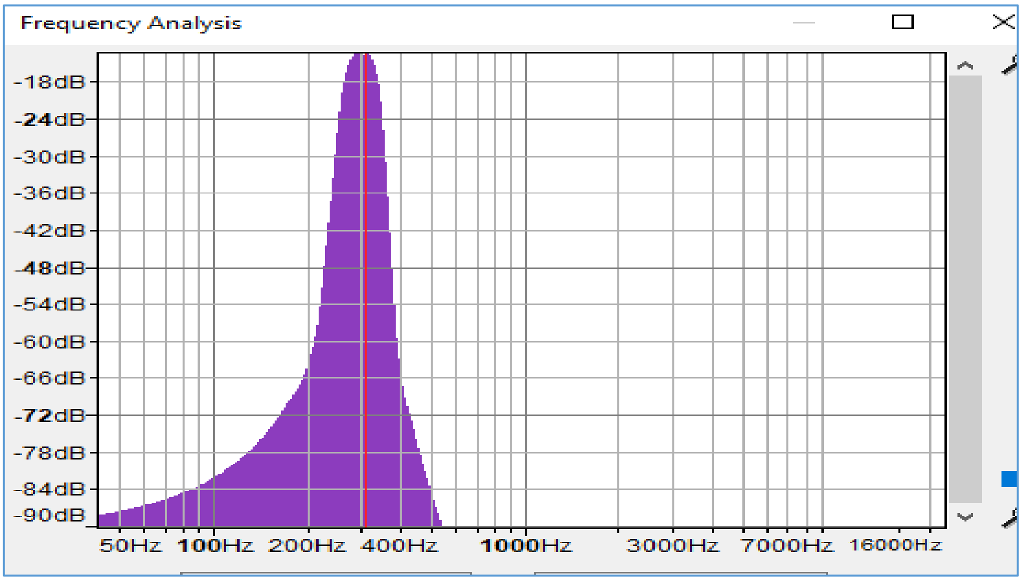

Fig. 5: Frequency of waveform

Now, audacity applies Fourier Transform on figure 4 along with the other post processing techniques. Please note that while Audacity allows a way to visualize the spectrum using Fourier Transform for more direct control and advanced analysis we can use specialized software or programming languages such as Python with libraries like NumPy or SciPy.

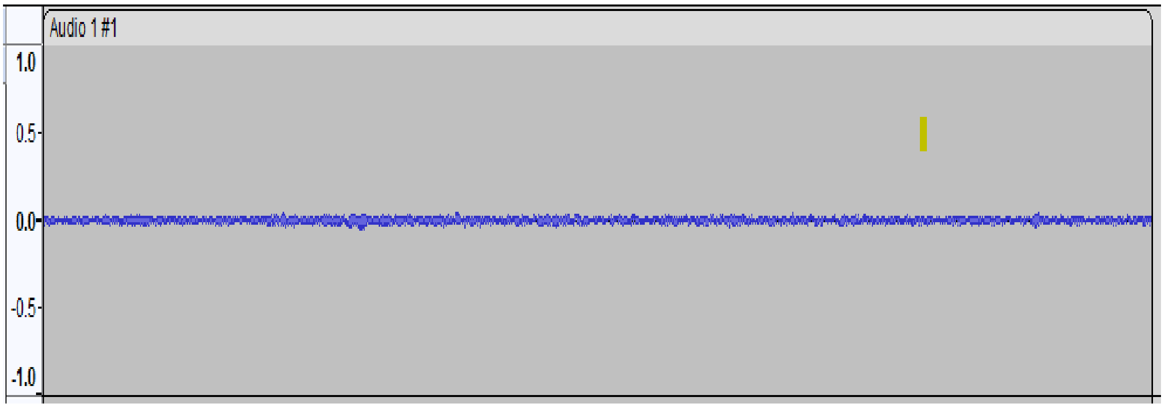

Fig. 6: Post NHA wave

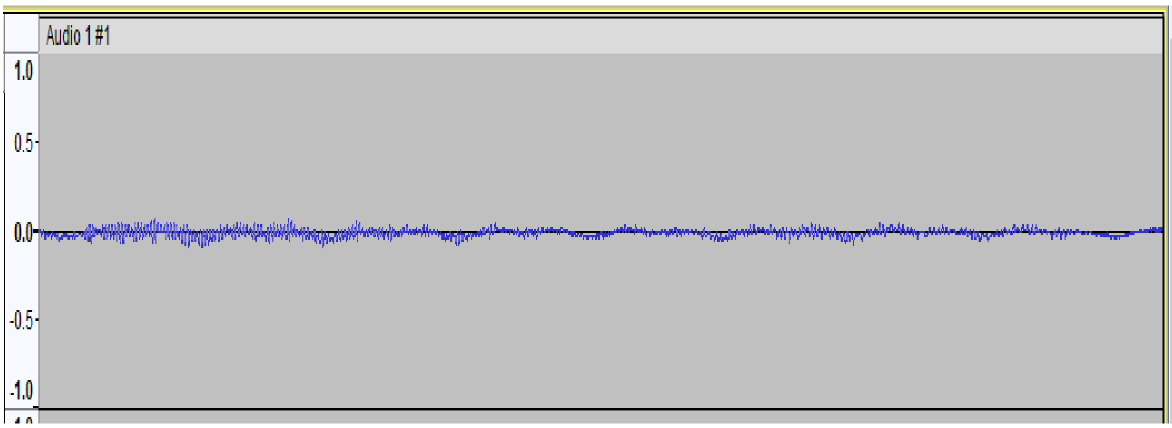

Fig. 7: Improvised NHA

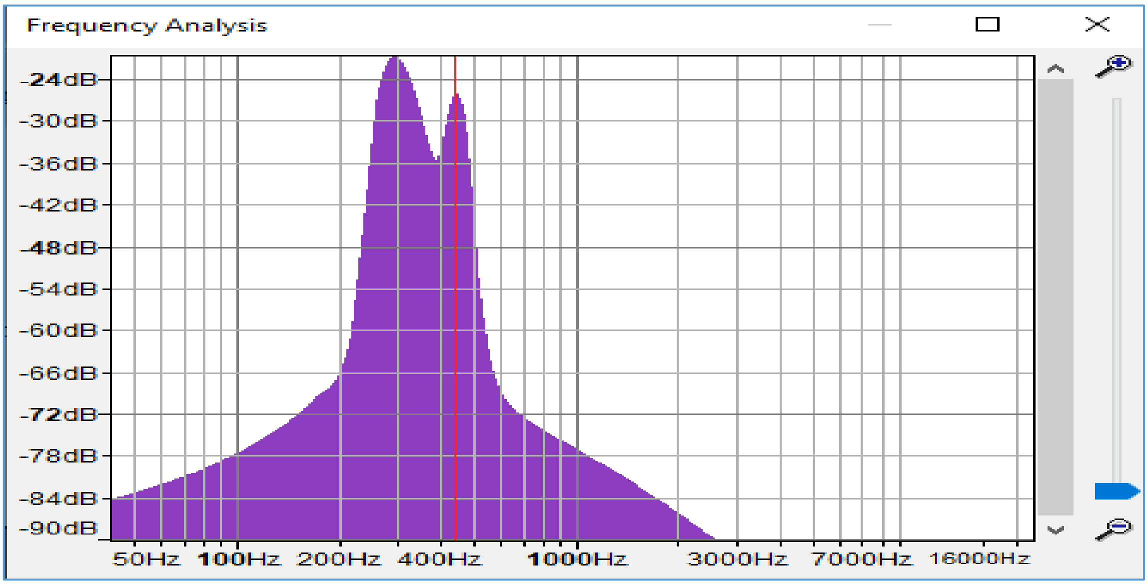

Fig. 8: Frequency spectrum of improvised NHA

In the frequency analysis graphs above, the x-axis represents the frequency in Hertz (Hz), and the y-axis represents the amplitude of each frequency component present in the whistle sound.

Before applying the filter: The original whistle sound contains energy across a wide range of frequencies, represented by the peaks and variations in amplitude at different frequencies. It includes both the desired frequency components of the whistle and any potential noise that might be present.

After applying the filter: When the low-pass filter is applied, it allows frequencies below a certain cutoff point to pass through while attenuating frequencies above that cutoff point. As a result, the higher-frequency noise is reduced or removed, leaving primarily the lower-frequency components of the whistle.

In summary, the frequency analysis after applying the low-pass filter should show reduced high-frequency noise, resulting in a cleaner representation of the whistle's lower-frequency components. The whistle's core characteristics and pitch should remain relatively unchanged, while unwanted high-frequency noise is suppressed.

B. Analysis with respect to time

Fig. 9: Sound wave containing noise. In the time domain, a sound wave with noise will appear as a jagged and irregular waveform

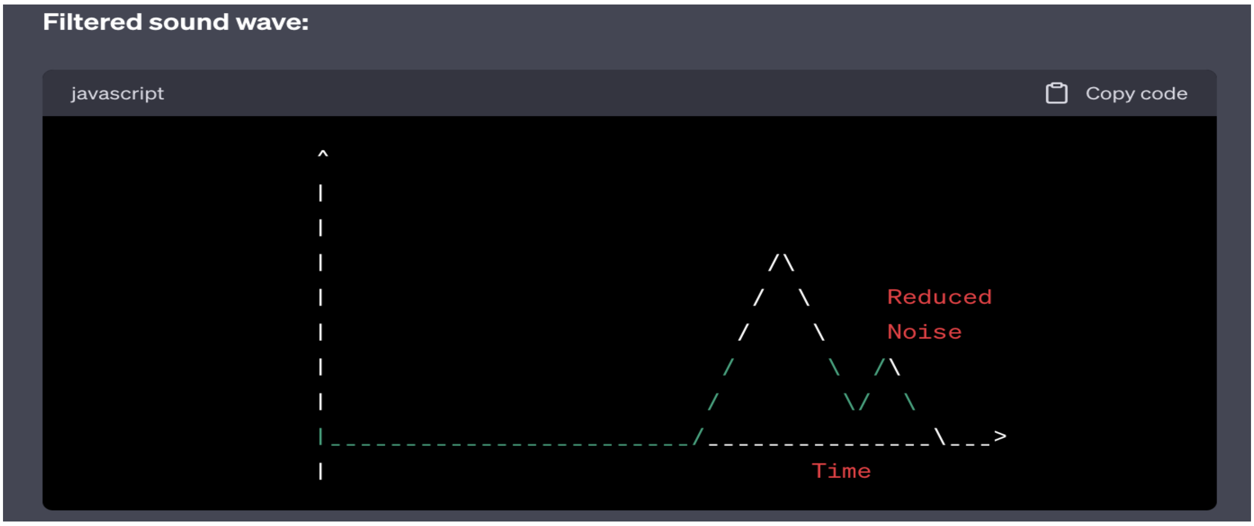

Fig. 10: Filtered sound wave using improvised NHA.After filtering, the resulting sound wave has reduced noise and a smoother waveform in the time domain

The waveform of a sound wave with noise often appears irregular and jagged. The noise introduces random variations in the amplitude of the signal, causing the waveform to exhibit rapid and unpredictable fluctuations. This can result in a messy and complex waveform pattern that can be difficult to predict or analyze.

After applying a filter to reduce the noise, the waveform of the filtered sound wave becomes smoother and more regular. The filter removes or attenuates the high-frequency noise components, resulting in a more coherent and controlled waveform pattern. The peaks and troughs in the waveform correspond more closely to the intended signal, and the overall shape becomes clearer and more recognizable.

In essence, the key difference lies in the complexity and irregularity of the waveform pattern. A sound wave with noise will have a more erratic pattern due to the interference of noise, while a filtered sound wave will have a cleaner and more organized pattern after the unwanted noise has been reduced or removed.

In summary, in the filtered sound wave graph, the noise is significantly reduced compared to the original sound wave. The result is a smoother waveform with less random variations, improving the overall quality and clarity of the sound. Keep in mind that the actual appearance of the filtered waveform will depend on the specific filtering process and the characteristics of the noise present in the original sound wave.

C. Advantages of the improvised model of NHA

- **Enhanced noise filtering performance:** The refined modeling and adaptive techniques can lead to improved accuracy in separating the signal from the noise, resulting in better noise reduction.
- **Increased adaptability**: By introducing adaptive mechanisms, the model can adapt to various signal and noise characteristics, making it more versatile and applicable to different types of noise sources.
- **Reduced artifacts and signal distortion:** The improved model can help mitigate artifacts or distortions that may occur during the filtering process, resulting in a cleaner and more faithful representation of the underlying signal.
- **Flexibility for different noise sources:** By incorporating advanced noise models, the improved NHA model can effectively handle a wide range of noise sources, including non-Gaussian noise or noise with time-varying characteristics.
- **Improved applicability to real-world scenarios:** The enhanced NHA model can better address the challenges posed by complex noise environments, making it real world applications where noise is often non-stationary, non-linear, or contains various sources.

V. CONCLUSION

In this paper we try to understand Noise Filtering through Fourier Series and its applications in signal analysis. Through a comprehensive analysis of the Fourier series theory and its application in noise reduction, we have demonstrated its effectiveness in separating and attenuating unwanted noise components from signals. Justification of why other methods namely Hilbert Transform won't be as effective for the same purposes is also provided.

This research paper has examined and explored the concept of nonlinear harmonic analysis, shedding light on its significance in understanding complex nonlinear systems and phenomena. An improvised NHA algorithm is introduced which aims for higher accuracy and faster results along with better reliability in understanding the complicated nature of signals in the real world. Through studying the algorithms behind NHA such as -the Fourier transform, wavelet denoising and sparse approximation, we have demonstrated its effectiveness in characterizing and modeling nonlinear dynamics.

In conclusion, this research paper contributes to the body of knowledge in the field of noise filtering by Fourier series and the contribution of NHA in the same.

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