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A Spontaneous Statistical Solution of Schrödinger's Partial Differential Equation

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Abstract:- In previous articles we have shown that the solution of the heat diffusion equation is stimulated and forced by the boundary conditions and the material tested, while the solution of the Schrödinger SE equation or quantum mechanics problems in general is generated spontaneously in the space of the isolated object. object. This object is called a quantum system.

While quantum mechanics and classical mechanics are distinct frameworks applicable to different scales and types of physical systems, only B-matrix string mechanics effectively applies to both scales.

In this article we solve SE in three different situations 1D, 2D, 3D via the spontaneous chains of the B matrix of the Cairo technique.

The validity and accuracy of numerical statistical solutions undoubtedly confirm that quantum mechanical systems operate spontaneously rather than stimulated.

I. INTRODOCTION

In previous papers [3,4,5,6,7,8,9] we have shown that B matrix chains, a product of the numerical statistical theory of Cairo techniques, can be applied to efficiently solve the equation of thermal diffusion of classical physics, as well as the time-independent Schrödinger equation.

Furthermore, the same B-matrix chains have efficiently solved classical physics problems in different fields [3,4,5,6,7,8,9] such as heat diffusion, sound intensity field in audio rooms, the electrical energy density in the partial differential equations of Laplace and Poisson in their most general form, etc.

In this article we describe via B-matrix chains how the Schrödinger equation or quantum mechanics in general is created as a spontaneous side effect of modern classical physics [classical universal laws of physics plus modern definition of probability of transition in 4D x-t unit space].

An important reason to replace the Schrödinger equation with the equivalent B-matrix mechanics is that you move from the domain of SE where many questions remain unanswered to the domain of statistical matrix equations where almost all questions have answers.

In extreme cases, it is like going from the darkness of night to the light of a sunny day.

Furthermore, the correspondence principle according to which in the limit of large quantum numbers the predictions of quantum physics become identical to the predictions of classical physics does not apply to classical physics problems such as heat diffusion and waves sound.

The extension of physical matrix mechanics to the solution of the Schrödinger equation requires strict adherence to certain fundamental limits that apply to the entire theory of matrix mechanics B.

In other words, applying the concept of chains of the physical transition matrix B (matrix B mechanics) to the solution of problems of classical physics and time-independent Schrödinger equations requires the introduction of certain terms or basic physical and mathematical concepts that we specify in the following dozen hypotheses:

- *-Square matrices are a subset of mathematical matrices, and physical square matrices that have physical meaning (such as thetransition matrix B) and favored for defining matrix mechanics area subset of square matrices.
- *-The numerical solution replaces the analytical solution of the time-dependent PDE by discretizing space and time into dx and dt and replacing the differentials dy/dx by [y+dy-2y +y-dy]/2dx and d ^2 y/dx ^ 2 by [y+dy-2y +y-dy]/dx^2. etc.

In other words, the numerical solution method reduces the PDE to a system of algebraic equations via the finite difference method FDM.

On the other hand, computational methods such as (FDM) are not necessary in the transition matrices of statistical solutions since FDM techniques themselves are intrinsically included in the statistical chains of transition matrices.

- *-Statistical transition matrices B and chains of statistical transition matrices exist and define a distinct type of matrix mechanics. Its modeling works efficiently to find the evolution of the energy density in the space-time partial differential equations.

We currently know two, namely the mathematical and statistical transition matrix of Markov and the physical transition matrix B which is the subject of this article.

It is worth mentioning that in Markov matrix chains we do not care about the energy density, boundary conditions, source term, average properties of the medium, etc., whereas in the case ofmatrix chains B, we do it.

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- *- When a physical statistical transition matrix chain B for the energy density U(x,y,z,t) exists then it can be defined by therecurrence relation, U (x, y, z, t+ dt) = B. U(x ,y,z,t)

Note that the transition matrix B should have a place for the boundary conditions BC and the source term S which are essential in the solution of the heat diffusion equation, Poisson PDE as well as the Schrödinger equation.

As a result, a chain transition matrix B emerges and must be able to describe the solution trajectory for the energy density U through its own solution space for any given time evolution in 4-Dunitary x-t space.

Also note that the string transition matrix B describes the energy density in classical physics problems as well as the square of the wave function ψ2(r) in quantum mechanics problems.

- *-Classical macroscopic statistical solutions and quantum microscopic statistical solutions are subject to the same physical transition matrix B, which means that nature has only one face to show.
- *- In the matrix chain solution for the time-dependent energy density U(x,y,z,t) the real time t is completely lost and replaced by N.dt where N is an integer describing the number of iterations anddt is an inherent time step or jump in time.

Time is replaced by the number of repetitions of the physical process N.

It is also worth mentioning that discretizing time t into forbidden and allowed where t = N dt and N is an integer is itself quantification of time.

Apparently, this condition replaces Bohr's hypothesis on the quantification of angular momentum in 1913, namely:

$$m\ v\ 2\ \pi\ r = n\ h$$

In normal convention.

- *The time-dependent and time-independent Schrödinger equations are instances of eigenvalue equations in quantum mechanics.

✓ The dominant (eigenvalue of the maximum absolute value) equal to 1.by assumption.

His time-independent equation for ψ(x) describes the equilibrium state that occurs when evolutionary time tends to infinity.

It should be noted that another way to describe quantum particle dynamics is to use statistical transition matrices (Matrix Mechanics) which completely ignore the Schrödinger equation and the wave function ψ as if they had no never existed in the same way as to solve the thermal problem. diffusion equation without going through the thermal partial differential equation. It is obviously ψ2 which is described by U and not ψ itself.

Furthermore, the variable separation method W(x,y,z,t) =X(x)Y(y)Z(z) f(t) is also not necessary because it is implicitly included in the 4Dunit space inseparable x-t from the strings of matrix B.

- *-The Monte Carlo numerical (technical) method is in the background, close to the Cairo technical method and matrix mechanics.

However, the numerical Monte Carlo method is a bit old and requires generating a random numerical variable thousands of times. This makes the interpretation of its numerical results long and tedious.

- *-The determinant of the quantum transition matrix Q=B+V(x,y,z).I must be equal to zero to identify the homogeneous system which gives rise to the problem eigenvalues.
- *-In the numerical solution of SE and Heisenberg matrix mechanics HMM you need a large number of nodes whereas in B matrix chains a small number of nodes can do the job.
- *-In the numerical solution of SE and HMM you additionally need assume and process the so called normalization condition whereas in B-matrix chains a normalization condition is inherent in it.

II. THEORY

The time-dependent and time-independent Schrödinger equations are instances of eigenvalue equations in quantum mechanics, with its eigenvalues corresponding to the allowed energy levels of the quantum system.

The mechanical product of the B matrix of the Cairo techniques predicts that if the solution of the heat diffusion equation is forced by the boundary conditions and physical properties of the material involved, then the solution of the quantum mechanical system is spontaneous and function of the size of the quantum object as well as the external potential if applicable.

We emphasize that the extension of physical matrix mechanics to the spontaneous statistical solution of the Schrödinger equation requires strict compliance with certain fundamental assumptions:

- 1-Ψ1= Ψn =0
  Ψn is the value of the wave function at the nth point
- Sch PDE describes the wave function of a quantum particle in an isolated quantum system or infinite free space (BC = 0) with no further particle interactions.
- Symmetry:
  V(x,y z) is symmetrical with respect to its CM
- S(x,y,z,t)= Constant C. V(x,y,z,t)

Where S (x, y, z, t) is the source/sink term intrinsic to the spatial point x, y, z, t.

It follows that the quantum transition matrix Q is given by,

$$Q=B+ V(x,y,z)\ .\ I$$

I is the unitary matrix.

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- The constant C is found by trial and error such that,
- ✓ Determinant [Q]=0.

Determinant [q]=0 to guarantee the eigenvalue problem.

- ✓ B: [B+ V(x,y,z) . I] = λ. V(x,y,z)

With λ = 1.

- Amplitude of wave function ψ (x,y,z) =Sqrt of U(x,y,z)
- Normalization condition: The normalization condition in quantum mechanics is a mathematical expression that states that the integral of the square of the wave function over all possible values of the independent variable must equal one. In other words, the probability of finding the particle in any region of space must be equal to one.

The normalization condition is included in the transition matrix Q as will be explained in section III of applications.

The conclusion is that while the solution of the heat diffusion equation depends on the boundary conditions and physical properties of the material concerned, the solution of the quantum mechanical system is a spontaneous function of the size and number of nodes of the object quantum.

In order not to worry too much about the details of the theory, let's move directly to the following illustrative applications.

III. APPLICATION AND NUMERICAL RESULTS

A. 1D infinite potential well

The 1D matrix Q 13x13 is given by,

00.5	0	0	0	0	0	0	0	0	0	0	0
0.5	0.045	0.50	0	0	0	0	0	0	0	0	0
00.5	0.09	0.5	0	0	0	0	0	0	0	0	0
0 0	0.5	0.135	0.5	0	0	0	0	0	0	0	0
00	0	0.5	0.18	0.5	0	0	0	0	0	0	0
00	0	0	0.5	0.225	0.5	0	0	0	0	0	0
00	0	0	0	0.5	0.27	0.5	0	0	0	0	0
00	0	0	0	0	0.5	0.225	0.5	0	0	0	0
0	0	00	0	0	0	0.5	0.18	0.5	0	0	0
00	0	0	0	0	0	0	0.5	0.135	0.5	0	0
00	0	0	0	0	0	0	0	0.5	0.09	0.5	0
00	0	0	0	0	0	0	0	0	0.5	0.045	0.5
00	0	0	0	0	0	0	0	0	0	0.5	0

It is obvious that the vector V(x,y,z) is given by,

[0 0.045 0.090.1350.180.2250.270.2250.180.1350.090.045 0] T

And that the product
Q.V(x,y,z) is given by,

[9/400 1881/40000 981/10000 4329/40000 981/10000 1881/40000 9/200 1881/40000
981/10000 4329/40000 981/10000 1881/40000 9/400]T

Which is in good agreement with the hypothesis,
[B+ V(x,y,z) . i] = λ. V(x,y,z)

With λ = 1.
The above numerical results are shown in Figures 1,2 below.

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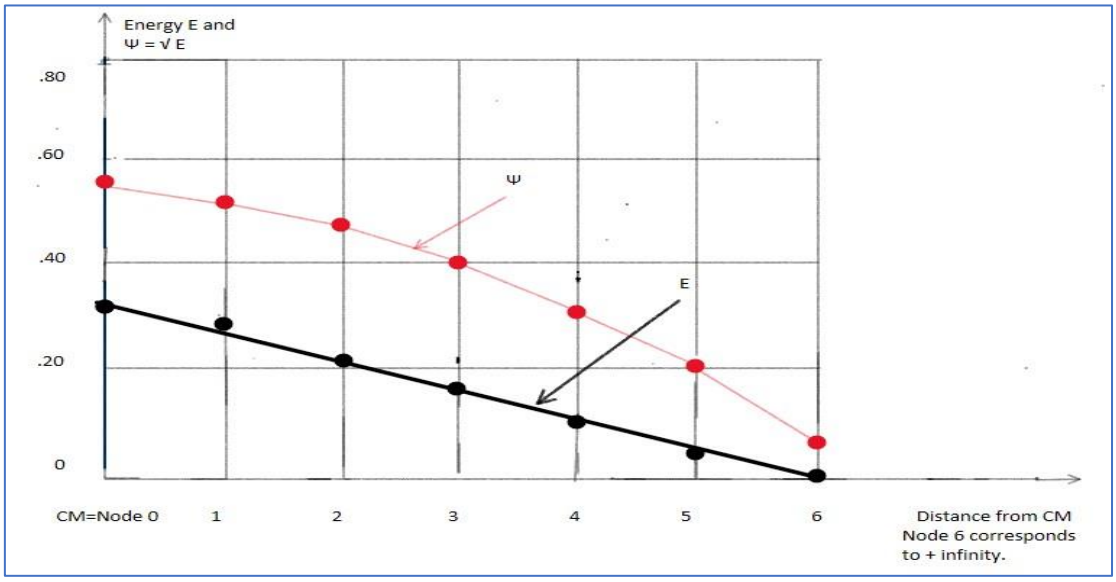

Fig. 1: Spatial distribution of the wave function ψ in a 1D infinite potential well. (maximum probability)

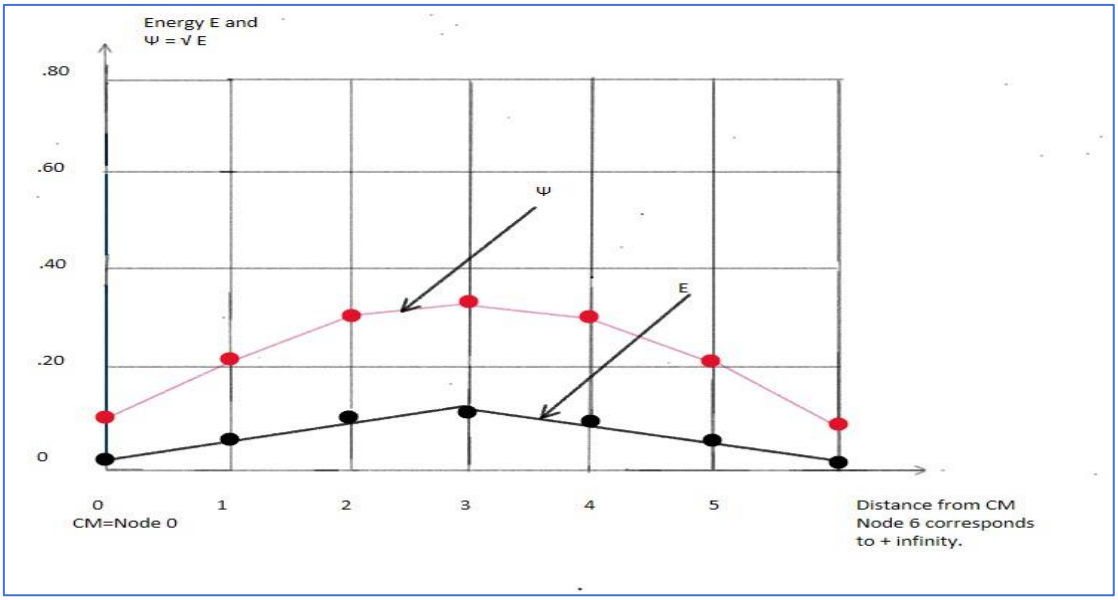

Fig. 2: Spatial distribution of the wave function ψin a 1D infinite potential well. (next to maximum probability)

If we compare Figures 1, 2 with Figure 3 below from reference 11, we find excellent qualitative and quantitative agreement.

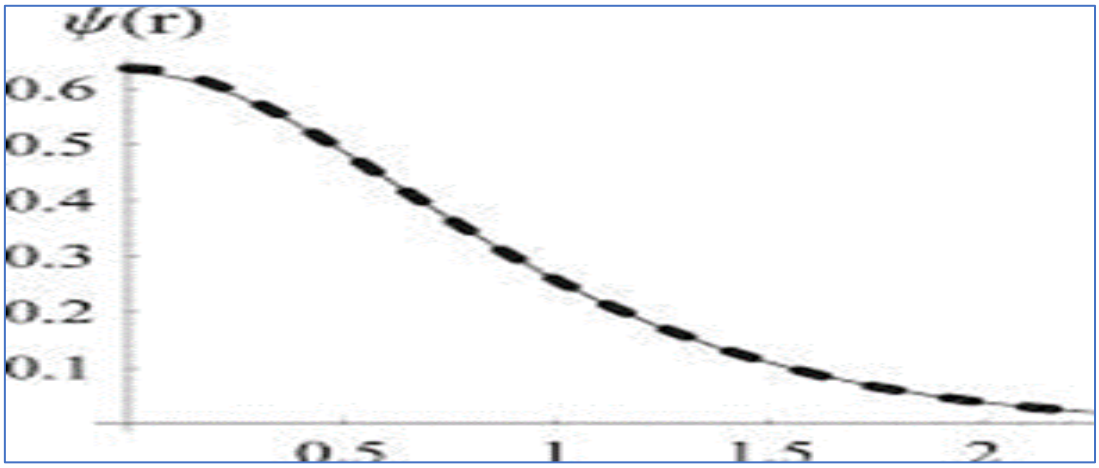

Fig. 3: Google search - Numerical solution of the time-independent 1D Schrödinger equation.

B. 2D infinite potential well

The 2D 9x9 matrix Q is given by,

0.12	1/4	0	1/4	0	0	0	0	0
1/4	0.24	1/4	0	1/4	0	0	0	0
0	1/4	0.12	0	0	1/4	0	0	0
1/4	0	0	0.24	1/4	0	1/4	0	0
0	1/4	0	1/4	0.36	1/4	0	1/4	0
0	0	1/4	0	1/4	0.24	0	0	1/4
0	0	0	1/4	0	0	0.12	1/4	0
0	0	0	0	1/4	0	1/4	0.24	1/4
0	0	0	0	0	1/4	0	1/4	0.12

Obviously, the vector V(x,y) is given by,

[3/25 6/25 3/25 6/25 9/25 6/25 3/25 6/25 3/25]T

And that the product

Q.V(x,y) is given by,

[84/625 519/2500 84/625 519/2500 231/625 519/2500 84/625 519/2500 84/625]T

Which is in good agreement with the hypothesis,
S(x,y,t)=Constant. V(x,y,t)

And,
[B+ V(x,y) . i] = λ. V(x,y)

With λ = 1.

Where S is a source/sink term.

The above numerical results are shown in Figures 4 below.

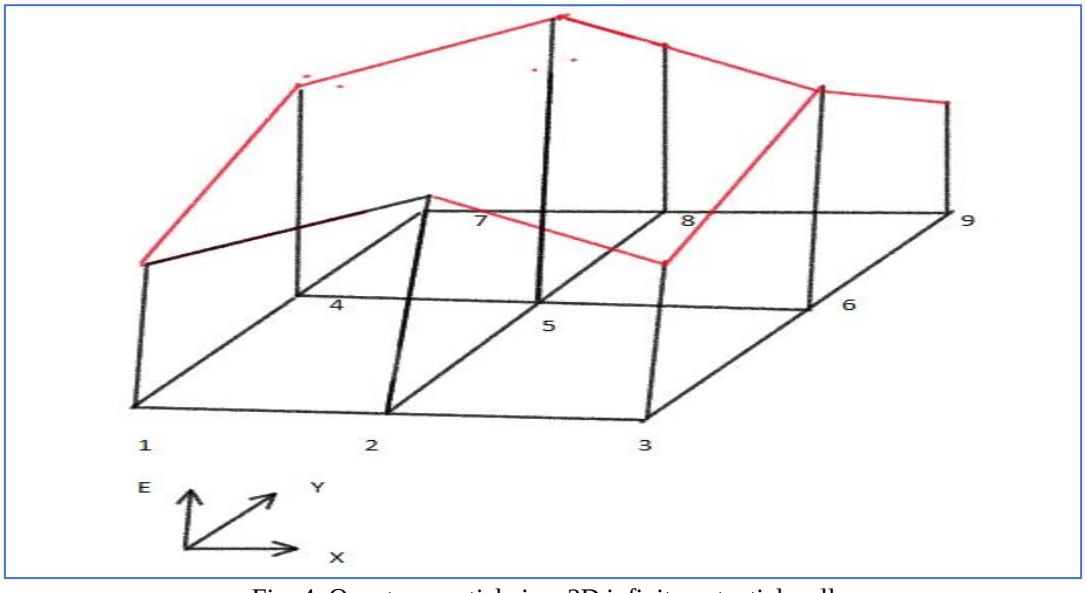

Fig. 4: Quantum particle in a 2D infinite potential well

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C. Quantum particle in the 3D dimensional box

One of the important applications in quantum mechanics is the total energy distribution of a quantum particle in the 3D dimensional box.

Here the 3D transition matrix Q 27x27 is given by,

3/16	1/6	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
1/6	4/16	1/6	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	1/6	3/16	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
1/6	0.0	0.0	4/16	1/6	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	1/6	0.0	1/6	6/16	1/6	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	0.0	1/6	0.0	1/6	4/16	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	0.0	0.0	1/6	0.0	0.0	3/16	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	1/6	0.0	1/6	4/16	1/6	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	1/6	0.0	1/6	3/16	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4/16	1/6	0.0	1/6	0.0	0.0
	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	6/16	1/6	0.0	1/6	0.0
	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	4/16	0.0	0.0	1/6
	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	0.0		
0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	6/16	1/6	0.0
	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	1/6	8/16	1/6
	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	1/6	6/16
	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0
	4/16	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0
	1/6	6/16	1/6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6
	0.0	1/6	4/16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	3/16	1/6	0.0	1/6	0.0	0.0	0.0	0.0	0		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	1/6	4/16	1/6	0.0	1/6	0.0	0.0	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	1/6	3/16	0.0	0.0	1/6	0.0	0.0	0.0		

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0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0
	0.0	0.0	0.0	1/6	0.0	0.0	4/16	1/6	0.0	1/6	0.0	0.0		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6	0.0
	0.0	0.0	0.0	0.0	1/6	0.0	1/6	6/16	1/6	0.0	1/6	0.0		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1/6
	0.0	0.0	0.0	0.0	0.0	1/6	0.0	1/6	4/16	0.0	0.0	1/6		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	0.0	3/16	1/6	0.0		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.00	1/6	4/16	1/6		
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	1/6	0.0	0.0	0.0	0.0	0.0	1/6	0.0	1/6	3/16		

It is clear that the potential vector V(x,y,z) is given by,

[3/16	1/4	3/16	1/4	3/8	1/4	3/16	1/4	3/16	1/4	3/8	1/4	3/8	1/2	3/8
	1/4	3/8	1/4	3/16	1/4	3/16	1/4	3/8	1/4	3/16	1/4	3/16]T		

And that the product

Q.V(x,y,z) is given by,

[41/256	1/4	41/256	1/4	25/64	1/4	41/256	1/4	41/256	1/4	25/64	1/4	25/64	5/8
25/64	1/4	25/64	1/4	41/256	1/4	41/256	1/4	25/64	1/4	41/256	1/4	41/256]T	

Which is in good agreement with the hypothesis,
S(x,y,z,t)=Constant. V(x,y,z,t)

And,
[B+ V(x,y,z) . i] = λ. V(x,y,z)
With λ = 1.

IV. CONCLUSION

It is possible to extend physical transition matrix chains or B-matrix mechanics to the solution of the time-independent Schrödinger equation.

The present study shows that the same statistical chains of matrix B can be applied to the spontaneous solution of the time-independent Schrödinger equation 1D, 2D, 3D.

This means that nature only has one face to show in classical and quantum physics.

We present the numerical statistical solution for three specific situations, namely a 1D, 2D infinite potential well as well as a quantum particle in a three-dimensional box where the numerical results are of excellent precision.

The validity and accuracy of numerical statistical solutions confirm beyond doubt that quantum mechanical systems operate spontaneously rather than stimulated.

Note: In the previous calculations, the author used his own double precision algorithm as explained in ref. 12.

No ready-made algorithms such as Python or MATLAB are required.

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