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An Optimal Inventory Planning for Deteriorating Items with Price Dependent Demand and Shortages Arising from Supplier Lead Time

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Abstract:- The determination of the accurate reorder point is a critical aspect of sustainable inventory management, particularly when dealing with deteriorating items. The reorder point signifies the inventory threshold at which initiating a new order is necessary to restock before depletion occurs.For deteriorating items, where the quality or usability of the product diminishes over time, precisely establishing the reorder point is of supreme importance.In this article an inventory model is developed to address the complex challenges of determination of perfect order size and reorderpoint associated with the optimal inventory planning of deteriorating items, under price-dependent demand and shortages resulting from supplier lead time. The proposed model integrates convex optimization techniques to develop a comprehensive framework for inventory control, considering the interplay of various factors affecting inventory levels.The research explores the trade-offs between ordering costs, holding costs, and stockouts, aiming to identify the optimal inventory policies that minimize overall costs while ensuring adequate service levels. To achieve this, the study employs convex optimization theory and sensitivity analysis. The suggested model undergoes validation using a numerical illustration. Ultimately, the model's behavior is visually depicted.

Keywords:- *Deterioration; Price-Dependent Demand; Reorder Point; Lead Time; Stockouts.*

I. INTRODUCTION

In the current dynamic business landscape,effective management of inventory is crucial in ensuring the smooth operation of supply chains and sustaining competitive advantages. The intricate interplay of factors such as deteriorating item characteristics, price-dependent demand dynamics, and the uncertainties associated with supplier lead times presents a challenging landscape for organizations striving to strike a balance between cost-effectiveness and customer satisfaction. Shortages arising from delays in the procurement process can lead to stockouts, resulting in dissatisfied customers and potential revenue loss. This paper seeks to fill the current void in the literature by putting forth a comprehensive inventory model.that considers both the deteriorating nature of items and the influence of price-dependent demand, while also addressing the challenges presented by supplier lead time uncertainties.

Deteriorating items, characterized by a limited shelf life or perishable nature, demand a unique approach to inventory control to minimize losses and enhance overall operational efficiency. Moreover, the incorporation of price-dependent demand, which reflects the sensitivity of customer demand to changes in the product's pricing, introduces an additional layer of complexity. This necessitates the formulation of a model that not only accommodates the perishability aspect but also adapts to the dynamic nature of consumer behaviour. Additionally, the incorporation of supplier lead time considerations is crucial, as delays in procurement can lead to stockouts, disrupting the delicate balance between supply and demand.

The reorder point (ROP) is the inventory threshold at which initiating a new order becomes necessary to replenish stock before depletion.For deteriorating items, the ROP becomes critical because the item's condition may worsen over time, leading to potential obsolescence or loss of value. Setting an accurate ROP helps avoid stockouts, ensuring that the item is available for customers while minimizing excess inventory carrying costs. Furthermore, under price-dependent demand which increases with a lower price, the ROP needs to account for variations in pricing to meet market demand efficiently.

Choosing the right order size is another crucial aspect for minimizing holding costs, which can be substantial for deteriorating items. Smaller order sizes may reduce holding costs but increase ordering costs, while larger order sizes may lead to higher holding costs but lower ordering costs. Striking a balance is essential. For items with price-dependent demand, order size needs to be aligned with pricing strategies. If demand is sensitive to price changes, adjusting order sizes based on pricing fluctuations can optimize the overall cost structure.

This paper adds to the current knowledge baseby synthesizing theoretical foundations and practical insights, offering a complete perspective on the challenges posed by items experiencing degradation with demand influenced by price sensitivity and lead time-induced shortages.

Ultimately, our research aims to empower organizations to navigate the complexities of inventory management with a heightened ability to meet customer demands efficiently while maintaining a judicious control over costs.

II. LITERATURE REVIEW

Harris (1913) first introduced the classical Economic Order Quantity (EOQ) model, presupposing a constant demand rate.Later, Resh et al. (1976) expanded this model to account for a linearly increasing demandKim et al. (1995) advanced the concept by incorporating a demand rate dependent on price, where a retailer optimizes both unit retail price and order size to maximize profit, accounting for trade credit from the supplier. Subsequently, Giri et al. (1996) explored stock-dependent demand and formulated an inventory model tailored for deteriorating items.Teng and Chang (2005) introduced an economic production quantity inventory model, taking into account demand, including not just the selling price unit but also the on-display stock level. They identified optimal solutions to maximize profit within the economic production quantity model. In a similar vein, Hou and Lin (2006) formulated an inventory model that considers demand as dependent on both price and stock levels for selling rates. They applied a discounted cash flow approach to analyze a replenishment problem over a finite planning horizon and derived the optimal solution for the economic ordering quantity model. Additionally, Alfares (2007) proposed an inventory model with stock-dependent demand and variable holding costs based on the storage time of products, determining optimal solutions for the model.Dye and Hsieh (2011) formulated an inventory model considering a demand pattern dependent on both price and stock, where shortages are partially backlogged. They utilized particle swarm optimization to solve the proposed model and obtain the optimal solution. Shaikh et al. (2017) examined a model with demand dependent on both price and stock under inflation. More recently, Mashud et al. (2018) expanded upon the model proposed by Shaikh et al. (2017), introducing distinct deterioration rates and partial backlogging of shortages. Panda et al. (2018) investigated an inventory model in a two-storages environment with price and stock-dependent demand, incorporating an alternative trade credit policy.

In recent years, there has been notable focus on inventory models related to lead time. The primary objective of minimizing lead time is to reduce safety stock, mitigate losses from stockouts, enhance customer service levels, and gain a competitive advantage in the business. Ben-Daya and Raouf (1994) introduced more comprehensive models that incorporated both lead time and order quantity as decision variables. Building on the work of Ben-Daya and Raouf, Ouyang et al. (1996) addressed the item shortage issue and considered total stockout as a combination of backorders and lost sales. Hariga and Ben-Daya (1999) extended the model by relaxing the assumption of a given service and treating the reorder point as a decision variable.

Ouyang and Wu (1998) assumed that lead time demand follows any cumulative distribution with known and finite first and second moments. They developed a procedure to determine the optimal lead time and order quantity for an inventory problem involving a mixture of backorders and lost sales. Kim and Benton (1995) established a linear relationship between lead time and lot size in the classical stochastic continuous review model. They demonstrated that significant cost savings could be achieved if companies considered the impact of lot size on lead time and safety stock requirements.

Hariga (2000) made modifications to the model proposed by Kim and Benton (1995) by rectifying the expression for the annual backorder cost. Additionally, Hariga proposed an alternative relation for the revised lot size, leading to a smaller lot size compared to Kim and Benton's model. Furthermore, Hariga (2000) extended the study by incorporating the consideration of investment in setup time reduction and investigating the interplay between lead time, lot size, and setup time.

Pan et al. (2002) recommended that transportation expenses, overtime wages, and additional inventory holding costs associated with expedited orders were directly proportional to the quantities of the rushed items. They put forward the idea that the crashing cost could be expressed as the combination of a fixed component and a variable component proportionate to the quantities in the expedited order.

In situations of stockouts, where there is an inadequate supply to meet a replenishment order, the outcome is typically either backordering all demand or losing all demand. Nevertheless, in actual inventory systems, demand may be partially captive, permitting some customers to wait for fulfillment while others, with more immediate requirements, turn to alternative sources. The cost associated with a lost sale can range from a loss in profit to an unspecified loss of goodwill. Backordering involves handling costs, expediting costs, and frequently additional shipping costs to expedite delivery and reduce lead time.

Pan and Yang (2001) introduced a comprehensive supplier-purchaser model incorporating controllable lead time, demonstrating significant cost savings when lead time is manageable. Chen et al. (2001) presented a continuous review inventory model considering ordering costs dependent on lead time. Subsequently, Pan and Yang (2002) conducted a study on an integrated inventory model with controllable lead time. Ouyang et al. (2004) later expanded on Pan and Yang’s (2001) model by allowing for shortages.

More recently, Priyan and Uthayakumar (2014) have developed optimal inventory management strategies tailored for pharmaceutical companies and hospital supply chains operating within a fuzzy-stochastic environment. In this particular context, the author employed the exponential function of lead time crashing cost.

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M. Vijayashree and R. Uthayakumar (2015) have formulated an integrated inventory model featuring controllable lead time, incorporating investments for enhancing quality within the supply chain system.This work, scrutinizes the impacts ofshortages occurred due to supplier's lead time in an inventory planningfor a deteriorating item. The consumption rate is assumed as dependent upon the selling price andshortages arefully backlogged.

The remainder of this research is organized as follows: Section 2 furnishes a depiction of the problem, accompanied by the notation and assumptions. Section 3 articulates the mathematical model, succeeded by the introduction of some theoretical findings in Section 4. In Section 5, a numerical example is solved, illustrating graphically the convex nature of the cost function. Section 6 carries out a sensitivity analysis of various parameters, presenting and discussing the outcomes.Managerial insights are delineated in Section 7, and finally, recommendations and concluding remarks are presented in Section 8.

III. DESCRIPTION OF THE PROBLEM

This paper addresses an inventory planning challenge faced by a company dealing with perishable goodslike fruits, food grains, vegetables, seafoods etc. which are subject to deterioration over time. The analysis takes into account that the consumption rate of the stored items decreases as the selling price increases. Delays in item delivery from the supplier, attributed to factors like item scarcity or transportation issues, may lead to stock-outs for the company. The current study formulates the problem in situations where stock-outs occur due to delays in product delivery or lead time from the supplier. The primary objective of this research is to assist the company in minimizing the overall inventory cost during stock-out scenarios by offering precise guidelines for inventory planning concerning deteriorating items.

To express the inventory planning issue mentioned above in mathematical terms, the subsequent symbols and assumptions are employed throughout the remainder of the study.

A. Notation

A :	Company's ordering cost (\$/order)
a :	Constant component of the demand rate $(a > 0)$
b :	Price coefficient in the demand rate $(b > 0)$
c_j :	Cost of purchasing per unit item (\$/unit)
p :	Price of selling per unit (\$/order)
$D(p)$:	Price dependent demand function (units/unit of time)
θ :	The rate of deterioration for positive stock within the warehouse($0 < \theta << 1$)
h :	Cost per unit of time for holding one unit (\$/unit/unit of time)
d_c :	Deterioration cost (\$/unit)
η	Shortage cost parameter
c_s :	Shortage cost (\$/unit/unit of time)
t_1 :	The moment at which reorder is initiated(unit of time)
T :	Duration of the stock available inventory (time unit)
L :	Lead time $(L > 0)$ (unit of time)
$T_1 = t_1 + L$:	Overall duration of the business cycle (time unit)
Q :	Quantity purchased by the company for each cycle (units)
R :	Quantity for backordering (units)
S :	Quantity of stock at the beginning of each cycle after backordering (units).
S_1 :	Reorder point (Stock level at time $t = t_1$) (units)
$I_1(t)$:	Inventory level at any given time $t \in [0, T]$
$I_2(t)$:	Inventory level at any time $t \in [T, t_1 + L]$
$TC(t_1, T)$:	Total cost of inventory (\$ / unit of time)

B. Decision variables

t_1 :	The moment at which reorder is initiated(unit of time)
T :	Duration of the stock available inventory (time unit).

C. Assumptions

- The model is designed for a solitary deteriorating item, considering a demand pattern $D(p) = a - bp$ which is linearly dependent on the price p where $p < \frac{a}{b}, a, b > 0$.
- In other words, demand is solely influenced by the product's price. Furthermore, we presume that $a - bp > 0$ andwhen both the demand parameters b and p are set to zero, the demand remains constant throughout the cycle.
- Items deterioration is instantaneous. The quantity of deteriorated items is contingent on the existing stock level, and the deterioration rate θ is a constant where $0 < \theta << 1$.
- No substitution of degraded items occurs within the specified time frame. Lead time *L* is constant and knownwhere *L >0*. Holding cost *h*per unit time is constant.
- Shortages are induced due to supplier's lead time and fully backlogged.The cost of Shortages c_s per unit per unit of time, is directly proportional to the duration of the lead time *L*,assuming $c_s = \eta L$ with $\eta \geq 1$.

IV. MODEL FORMULATION

Let's suppose that a retail company initiates its operations with an initial inventory of $Q = S + R$ units of an instantaneous deteriorating item. In this scenario, *R* units are transferred to fulfil the overall accumulated backlogged demand, resulting in a remaining on-hand inventory level *S*. The reduction in inventory level occurs due to the combined impact of deterioration and customer demand throughout the specified period $[0, T]$.Let at time $t = t_1$ the inventory reaches the level S_1, which is the reorder point. At this point the retailer issues a new order to the supplier for Q units of the item. The inventoryfinally drops to the zero level at $t = T$. Afterwards due to persisting customer's demand and supplier's lead time *L*, shortages occur which is accumulated throughout the period $[T, t_1 + L]$and fully backlogged at the arrival of the next replenishment at $t = t_1 + L$. In the next period entire inventory system is repeated.

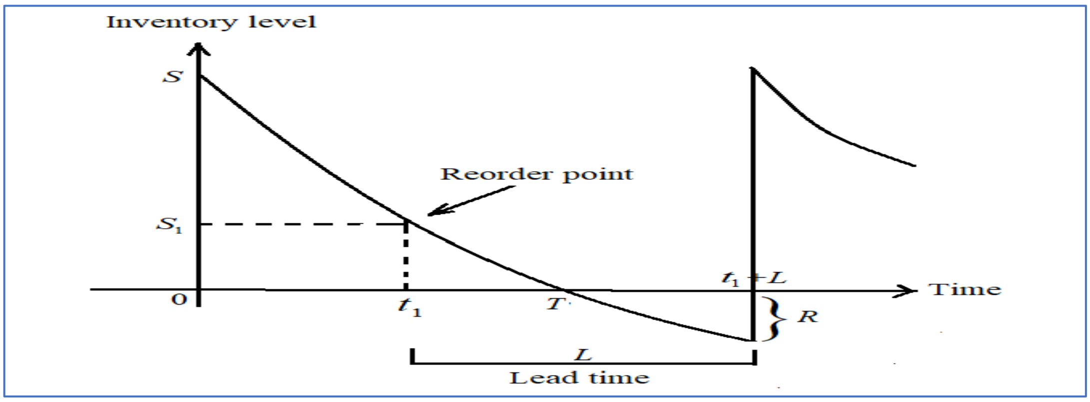

Fig. 1: Visual representation of the proposed inventory system.

Now at any instant the status of the inventory level can be described by the following differential equations:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -(a - bp), 0 \leq t \leq T \quad (1)$$

With $I_1(0) = S, I_1(t_1) = S_1$ and $I_1(T) = 0$

And

$$\frac{dI_2(t)}{dt} = -(a - bp), \; T \leq t \leq t_1 + L \quad (2)$$

With $I_2(T) = 0$ and $I_2(t_1 + L) = \text{-}R$

The solution of the differential equation (1) in conjunction with the given boundary conditionsis given by

$$I_1(t) = k\left[(T - t) + \frac{\theta(T - t)^2}{2}\right] \quad (3)$$

Where $k = a - bp$

Applying boundary condition $I_1(0) = S$ in (3) , we get

$$S = k\left[T + \frac{\theta T^2}{2}\right] \quad (4)$$

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Applying boundary condition $I_1(t_1) = S_1$ in (3), we get

$$S_1 = k\left[(T - t_1) + \frac{\theta(T - t_1)^2}{2}\right] \tag{5}$$

Solving equation (3), with $I_2(T) = 0$ we get ,

$$I_2(t) = k(T - t) \tag{6}$$

Applying the boundary condition $I_3(t_1 + L) = -R$ in equation (9), one finds

$$R = k(t_1 + L - T) \tag{7}$$

Accordingly, the total number of requiredamounts early in a business cycle is given by

$$Q = S + R$$

or, $$Q = k\left[t_1 + L + \frac{\theta T^2}{2}\right] \tag{8}$$

From equation (8),one finds the duration of stock available inventory as

$$t_1 = \frac{Q}{k} - \frac{\theta T^2}{2} - L \tag{9}$$

The total duration of the stock available inventoryperiod is represented as

$$T = \sqrt{\frac{2}{\theta}\left(\frac{Q}{K} - t_1 - L\right)} \tag{10}$$

And the overall duration of the business cycleis

$$T_1 = t_1 + L = \frac{Q}{K} - \frac{\theta T^2}{2} \tag{11}$$

The total cost of the entire inventory system per renewal cycle comprises the following elements:

- *Ordering cost (OC):* A

(2) **Purchasing cost (**PC**)**:

$$c_j Q = c_j(S + R) = c_j k\left\{t_1 + L + \frac{\theta T^2}{2}\right\}$$

- *Holding cost (HC):*

$$h\int_0^T I_1(t)dt$$

$$= hk\left[\int_0^T \left\{(T - t) + \frac{\theta}{2}(T - t)^2\right\}dt\right]$$

$$= hk\left(\frac{T^2}{2} + \frac{\theta T^3}{6}\right)$$

- *Deterioration cost (DC):*

$$d_c\left\{Q - \int_0^T (a - bp)dt\right\}$$

$$= d_c\left\{kT + \frac{k\theta T^2}{2} - kT\right\}$$

$$= \frac{1}{2}d_c k\theta T^2$$

- *Shortage cost (SC):*

$$-c_s\int_T^{t_1+L} I_2(t)dt$$

$$= -c_s\int_T^{t_1+L} k(T - t)dt$$

$$= \frac{1}{2}c_s k\{(t_1 + L) - T\}^2$$

Hence the total cost function per unit time is

$$TC(t_1, T) = \frac{1}{T_1}[OC + PC + HC + DC + SC]$$

$$= \frac{1}{t_1 + L}\left[A + k\left\{c_j\left(t_1 + L + \frac{\theta T^2}{2}\right) + h\left(\frac{T^2}{2} + \frac{\theta T^3}{6}\right) + \frac{c_s}{2}\right.\right.$$

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Now, the retailer aims to determine the optimal time t_1^* for reordering and the optimal duration for the stock available inventory T^* in order to minimize the company's cumulative cost per unit of time. The subsequent section focuses on the theoretical findings to attain the company's optimal inventory planning.

V. THEORETICAL RESULTS

The initial focus is on examining the curvature of the total cost function (Equation 12) pertaining to the inventory system in the presence of stock-out scenarios. This investigation relies on results from Theorems 3.2.9 and 3.2.10 found in Cambini and Martein's work (2009).These results expose that any function in the following form

$$\varphi(t)=\frac{M_1(t)}{M_2(t)},\, t=\left(t_1,t_1,...,t_n\right)\in R^n \quad (13)$$

is pseudo-convex, when both $M_1(t)\geq 0$ and $M_2(t)>0$ are differentiable and the convexity condition is satisfied by $M_1(t)\geq 0$ while the concavity condition is preserved by $M_2(t)>0$. Based on this significant result, an analysis is conducted to explore the joint pseudo-convexity of the company's total cost function (equation 12) for the inventory system with respect to t_1 and T .Applying the noteworthy result mentioned earlier, the subsequent theorem is introduced to study the curvature of the cost function presented in equation (12).

- *Theorem*

The total cost function $TC(t_1,T)$ of the company, exhibits joint pseudo-convexity with respect to both t_1 and T. Consequently, the global minimum value of $TC(t_1,T)$ is attained at the specific point (t_1^*,T^*). Additionally, the point (t_1^*,T^*) is unique.

- **Proof:**

For computational convenience, define the following supporting functions from equation (12):

$$f(t_1,T)=A+k\left\{c_j\left(t_1+L+\frac{\theta T^2}{2}\right)+h\left(\frac{T^2}{2}+\frac{\theta T^3}{6}\right)+\frac{c_s}{2}\left(t_1+L-T\right)^2+\frac{d_c\theta T^2}{2}\right\} \quad (14)$$

and $g_2(t_1,T)=T$ (15)

To ensure the joint convexity of $f(t_1,T)$, it is essential to construct the Hessian matrix for the function $f(t_1,T)$ concerning both t_1 and T. Subsequently, calculate the required first and second-order partial derivatives of $f(t_1,T)$ as follows:

$$\frac{\partial f(t_1,T)}{\partial t_1}=k\left\{c_j+c_s(t_1+L-T)\right\} \quad (16)$$

$$\frac{\partial^2 f(t_1,T)}{\partial t_1^2}=kc_s>0 \quad (17)$$

$$\frac{\partial f(t_1,T)}{\partial T}=k\left[c_j\theta T+h\left(T+\frac{\theta T^2}{2}\right)-c_s\left(t_1+L-T\right)+d_c\theta T\right] \quad (18)$$

$$\frac{\partial^2 f(t_1,T)}{\partial T^2}=k\left[c_j\theta+h(1+\theta T)+c_s+d_c\theta\right]>0 \quad (19)$$

$$\frac{\partial^2 f(t_1,T)}{\partial T\,\partial t_1}=-kc_s=\frac{\partial^2 f(t_1,T)}{\partial t_1\partial T} \quad (20)$$

Subsequently, the Hessian matrix for $f(t_1,T)$ is given by:

$$H_{ii}=\begin{bmatrix}\frac{\partial^2 f(t_1,T)}{\partial t_1^2} & \frac{\partial^2 f(t_1,T)}{\partial T\,\partial t_1}\\ \frac{\partial^2 f(t_1,T)}{\partial t_1\,\partial T} & \frac{\partial^2 f(t_1,T)}{\partial T^2}\end{bmatrix},\ \forall i=1,\ 2.$$

(21)

Here the first principal minor is

$$\left|H_{11}\right|=\frac{\partial^2 f(t_1,T)}{\partial t_1^2}=kc_s>0$$

Therefore, $\left|H_{11}\right|>0$.

Furthermore, the second principal minor is

$$\left|H_{22}\right|=\frac{\partial^2 f(t_1,T)}{\partial t_1^2}.\frac{\partial^2 f(t_1,T)}{\partial T^2}-\left\{\frac{\partial^2 f(t_1,T)}{\partial T\,\partial t_1}\right\}^2$$

$$=k^2\left[c_s\left\{c_j\theta+h(1+\theta T)+d_c\theta\right\}\right]>0$$

Therefore, the Hessian matrix of $f(t_1,T)$ is positive definite. Therefore $f(t_1,T)$ is non-negative, differentiable and (strictly) convexfunction in t_1 and T simultaneously. In

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addition, the denominator $g_2(t_1,T)=T$ is always positive, differentiable, and affine. As a result, the company's total cost function $TC(t_1,T)$ is jointly pseudo-convex in t_1 and T, and therefore, $TC(t_1,T)$ holds the global minimum value at (t_1^*,T^*). Furthermore, the point (t_1^*,T^*) is unique. This finishes the proof.

For achieving the best inventory planning to minimize the company’s total cost $TC(t_1,T)$, compute the first-order partial derivatives of $TC(t_1,T)$ with respect to t_1 and T, then set equal to zero.

That is, $\frac{\partial}{\partial t_1}\left[TC(t_1,T)\right]=0$

Or,

$$A+k\left[\frac{c_j\theta T^2}{2}+h\left(\frac{T^2}{2}+\frac{\theta T^3}{6}\right)+\frac{c_s}{2}\left\{T^2-(t_1+L)^2\right\}+\frac{d_c\theta T^2}{2}\right]=0 \quad (22)$$

and $\frac{\partial}{\partial T}\left[TC(t_1,T)\right]=0$

$$\text{i.e.,}\left[c_j\theta T+h\left(T+\frac{\theta T^2}{2}\right)-c_s(t_1+L-T)+d_c\theta T\right]=0 \quad (23)$$

By concurrently solving equations (22) and (23), the optimumtime t_1^* for reordering and optimal period for stock available inventory T^* is achieved to minimize the company’s cumulative cost per unit of time.

VI. NUMERICAL EXAMPLE

To explore our theoretical findings and gain managerial insights into the proposed problem and validate the model we provide a numerical illustrationbelow.For the proposedmodel,we considerthe following numerical values of the parameters, some of which are taken from Rukonuzzaman et. al (2023) with some additional data as follows:

$A=300,\ L=1.5,\ a=100,\ b=1.5,\ h=.03,\ \theta=.05,\ p=$

Using the values of $A, L, a, b, h, \theta, p, c_j, d_c, c_s$ and k calculate $t_1^*=3.989$ and $T^*=5.090$ Upon solving equations (22) and (23), use the obtained values of t_1^* and T^* in equation (12) to ascertain the company's minimum cost per unit time $TC_{\min}(t_1^*,T^*)=472.77\,\$$. Therefore, the minimum cost of thecompany per unit time is $TC_{\min}(t_1^*,T^*)=472.77\,\$$ with the optimum $t_1^*=3.989$, $T^*=5.090$, $T_1^*=t_1^*+L=5.489$ and $Q^*=558$ units. From the purchased quantity amount, the best backordering quantity is $R^*=36$ units, while theoptimum quantity of stock at starting of each cycle after backordering is $S^*=522$ and optimal reorder point is $S_1^*=103$ units.

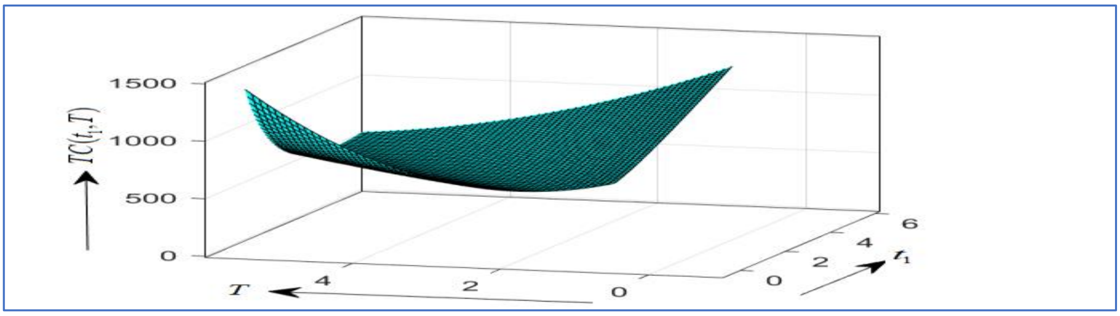

Fig. 2: Behaviour of $TC(t_1,T)$ in Example 1

Figure 2 exhibits the behaviour of the company's total cost $TC(t_1,T)$ against variables t_1 and T. It is seen in the Figure 2 that the company's cost $TC(t_1,T)$ is jointly convex in t_1 and T. Thus, $TC(t_1,T)$ achieves its global minimum value at the distinct point (t_1^*,T^*). To gain a clearer understanding of the nature of the company's cost $TC(t_1,T)$, two additional graphs (Figures 3 and 4) are included, each plotted against a single variable.

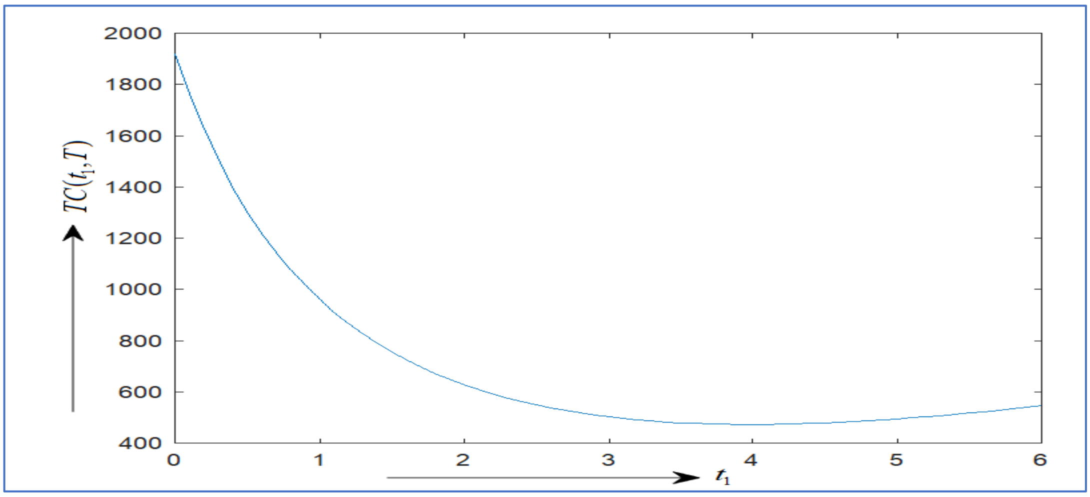

Fig. 3: Behaviour of the company's total cost $TC(t_1,T)$ with respect to t_1 in Example1

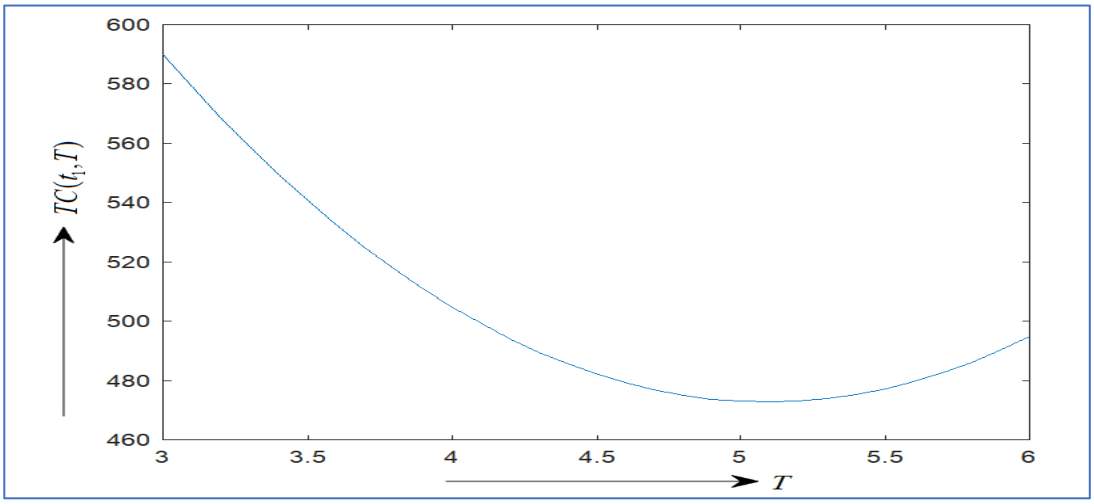

Fig. 4: Behaviour of the company's total cost $TC(t_1,T)$ with respect to T in Example 1

VII. SENSITIVITY ANALYSIS

Sensitivity analysis serves as a robust tool in the realm of inventory management, facilitating the evaluation of how alterations in various parameters impact the performance of an inventory model. This method entails systematically adjusting the input variables of the model to observe the extent to which the optimal solution, such as order quantity, reorder point, and costs, is sensitive to these changes. Itprovides valuable insights into the interrelationships among key variables, supports risk assessment and decision-making, aids in process optimization and continuous improvement efforts, and contributes to cost optimization. Organizations can harness sensitivity analysis to improve their inventory management practices, mitigate risks, and achieve superior financial and operational outcomes.

To investigate the effects of changes in various inventory parameters on optimal solutions, sensitivity analyses were carried out for the proposed inventory model. The computational findings, derived by individually altering each inventory parameter within a range of -20% to +20%, are outlined in Table 1, using numerical data from Example 1 in Section 5.Examining the evolving trends of the company's optimal inventory strategy through the sensitivity analysis table, several significant observations are made. Subsequently, guidelines for reducing the overall cost incurred by the company are proposed in the managerial insights section in Section 7.

Table 1: Consequences of system parameters on the optimum result for the suggested inventory procedure:

Parameter	**Original value**	**% of changes**	**New values**	t_1^*	T^*	T_1^*	S_1^*	S^*	R^*	Q^*	TC^*
A	300	-20	240	3.414	4.558	4.914	107	462	32	494	461.235
		-10	270	3.710	4.832	5.21	105	493	34	527	461.403
		+10	330	4.254	5.336	5.754	101	550	38	588	478.107
		+20	360	4.507	5.570	6.007	99	577	40	617	483.209
a	100	-20	80	4.706	5.754	6.206	76	467	32	499	380.140
		-10	90	4.314	5.392	5.814	90	496	34	530	426.651
		+10	110	3.713	4.834	5.213	116	547	38	585	519.256
		+20	120	3.474	4.614	4.974	130	571	40	661	564.072
b	1.5	-20	1.2	3.936	5.041	5.436	105	527	37	564	481.035
		-10	1.35	3.962	5.066	5.462	104	525	36	561	476.904
		+10	1.65	4.016	5.115	5.516	102	520	36	556	468.633
		+20	1.8	4.043	5.141	5.543	101	518	36	554	464.494
p	6	-20	4.8	3.936	5.041	5.436	105	527	37	564	481.035
		-10	5.4	3.962	5.066	5.462	104	525	36	561	476.904
		+10	6.6	4.016	5.115	5.516	102	520	36	556	468.633
		+20	7.2	4.043	5.141	5.543	101	518	36	554	464.494
Parameter	Original value	% of changes	New values	t_1^*	T^*	T_1^*	S_1^*	S^*	R^*	Q^*	TC^*
θ	0.05	-20	0.04	4.496	5.632	5.996	106	570	33	603	463.517
		-10	0.045	4.224	5.342	5.724	104	544	35	579	468.277
		+10	0.055	3.783	4.869	5.283	102	502	38	540	477.030
		+20	0.06	3.60	4.672	5.1	101	485	39	524	481.085
L	1.5	-20	1.2	4.339	5.045	5.839	65	517	45	562	471.790
		-10	1.35	4.161	5.070	5.661	85	520	40	560	472.330
		+10	1.65	3.820	5.107	5.32	121	524	33	557	473.131
		+20	1.8	3.655	5.121	5.155	138	526	30	556	473.434
h	0.03	-20	.024	4.065	5.172	5.565	104	531	36	567	471.358
		-10	.027	4.027	5.131	5.527	103	527	36	563	472.068
		+10	.033	3.952	5.051	5.452	102	518	36	554	473.466
		+20	.036	3.916	5.012	5.416	101	514	36	550	474.155
d_c	0.02	-20	0.016	3.991	5.093	5.491	103	522	36	559	472.727
		-10	0.018	3.990	5.091	5.49	103	522	36	559	472.727
		+10	0.022	3.988	5.089	5.488	103	522	36	559	472.727
		+20	0.024	3.987	5.088	5.487	103	522	36	559	472.727
c_s	3	-20	2.4	4.339	5.045	5.839	65	517	45	562	471.790
		-10	2.7	4.161	5.070	5.661	85	520	40	560	472.330
		+10	3.3	3.820	5.107	5.32	121	524	33	557	473.131
		+20	3.6	3.655	5.121	5.155	138	526	30	556	473.434
c_j	4	-20	3.2	4.478	5.613	5.978	106	583	33	615	390.901
		-10	3.6	4.216	5.334	5.716	105	550	35	585	431.960
		+10	4.4	3.788	4.875	5.288	102	498	37	535	513.360
		+20	4.8	3.620	4.682	5.12	100	476	39	515	553.755

The following observations are concluded from Table 1:

- The company's ideal procurement quantity (Q^*) experiences a notable rise with increasing values of A and a. Conversely, the optimal Q^* decreases as the remaining parameters $b, p, \theta, L, h, d_c, c_s$ and c_j increase. When the cost associated with each order placement rises, it's advisable for the company to augment the optimal procurement quantity to minimize the average ordering cost per unit. Similarly, as the initial market demand escalates, the total demand increases, prompting the company to enlarge the order size accordingly. Notably, the optimal Q^* remains unchanged despite variations in the deterioration cost d_c per unit.

- The maximum backordering level (R^*) increases with the rise in values of parameters A, a, θ and c_j ,while R^* shrinks concerning the positive variations of the parameters b, p, L and c_s . It is worth mentioning that the parameter h_c and d_c has no influence on the idealbackordering level.
- The company's optimal initial positive inventory level (S^*), increases with positive variations in the values of A, a, L and c_s . Conversely, it decreases with positive changes in the parameters b, p, θ, h_c and c_j . Additionally, it's noteworthy that the optimal value of the company's initial positive inventory level S^* remains unaffected by changes in the parameter d_c .
- The company's best reorder point (S_1^*) upsurges for the increasing values of a, L and c_s while S_1^* reduces due to the change in the parameters A, b, p, θ, h_c and c_j in positive way. The optimal value of the company's bestreorder point level remain unchanged for the changes in the parameter d_c .
- The optimal reordering time (t_1^*) for the inventory rises for the increasingvalues of A, b and p , while t_1^* shrinkages with respect to $a, \theta, L, h, d_c, c_s$ and c_j .
- The ideal duration for available inventory in stock (T^*) increases for the positive changes inthe parameters A, b, p, L and c_s , on the other hand T^* declineswith thepositive changes in a, θ, h_c, d_c and c_j .
- The company's optimal cycle length (T_1^*) for each business cycle significantly decreases with increases of parameters $a, \theta, L, h, d_c, c_s$ and c_j , whereas it increases with increases in parameters A, b and p .
- The total minimum cost (TC^*) incurred by the company increases with the rise in the value of all parameters except for parameters b and p . Parameters a and c_j exert significant influences on the company's minimum cost.

VIII. MANAGERIAL INSIGHTS

On the basis of the performed sensitivity analyses conducted for the proposed model, the following recommendations can be offered to decision-makers or managers:

- In cases where the cost associated with replenishing per order is substantial, managers should consider negotiating with suppliers or manufacturers to lower this cost. Alternatively, they can mitigate this expense by increasing order sizes significantly.
- The cost per unit of purchase has the second most detrimental impact on the overall cost per unit of time. Managers are advised to make efforts to diminish the per unit acquisition cost, potentially by assuring suppliers or manufacturers that a lowerunit acquisition cost will lead to larger order sizes.
- Supplier lead times introduce a delay between order placement and order receipt. Thus setting an accurate reorder point is a crucial need for a manager to prevent stockouts during the lead time. Since in the proposed model the shortages cost per unit is directly proportional to the supplier's lead time, a shorter lead time can substantially decrease the shortages cost and, consequently, the overall cost per unit time.
- Due to the product's deterioration property, some items incur constant damage, resulting in lost revenue. To mitigate costs, managers should focus on reducing the number of damaged products. Consequently, it is recommended for managers to enhance preservation facilities to decrease the deterioration rate of the product.

IX. CONCLUSION

This research investigates into the impact of lead time induced shortages on optimal inventory preparationfor a company dealing instantaneous deterioration items. The customer's consumption behaviour is considered asprice dependent.The study aims to develop an optimal inventory planning with a perfect reorder point, order quantity and replenishment time to minimize the total inventory cost under stockout situation.

To provide guidelines for minimizing the total inventory cost, sensitivity analyses are conducted by examining the evolving pattern of the company's optimal inventory approachbased on variations in each system parameter. For example, in situations where the expense associated with each order is significant, the suggestion is to raise the quantity purchased to diminish the average ordering cost per unit.Additionally, suggestions include reducing the rate of item deterioration through proper storage and guaranteeing the provision of top-notch products from the supplier. The company manager is advised to select suppliers with shorter lead time to reduce shortages cost.

The study mathematically proves the existence and uniqueness of optimal solutions for total costs and verifies them graphically using MATLAB software with anumerical illustration. Sensitivity analyses highlight the importance of focusing on ordering cost, lead time and demand parameters to reduce the total inventory cost.

The suggested inventory model could be expanded by incorporating further practical aspects like stock-dependent demand, time varying holding costs, partial backlogging , inflation, trade credit policies.Notably, the selling price is not considered as a decision variable in the current model, leaving room for an interesting extension by incorporating nonlinear demand patterned model or a profit-maximizing inventory model. Another potential extension involves introducing imprecise inventory parameters, such as fuzzy-valued or interval-valued inventory cost.

However, the study acknowledges certain limitations, such as not considering the influence of unit selling price on customers' consumption behaviour during model formulations. Therefore, a future research direction could involve exploring the impact of stock and time on demand. Additionally, addressing the uncertainty in demand is identified as an important immediate extension of the proposed inventory procedures.

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