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Investigating the Limitations of Some of the Most Popular Lines and Points Based Camera Calibration Techniques in Photogrammetry and Computer Vision

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Abstract:- The need for camera calibration has been a fundamental requirement since the foundation of photogrammetry. As the number of photogrammetry applications grows and the technology advances, camera calibration became more complex to undertake. However, when measurements derived from the imagery is used for scene modelling purposes, the consequences of small imaging errors can be significant on the accuracy of derived models. Thus, the development of cheaper lenses such as those of consumer grade cameras and their integration in the Photogrammetry process requires from camera calibration approaches to accurately model the projection process from the 3D scene onto the 2D image plan and also offer robust solutions to derive with high accuracy the various camera parameters. Several line based and points based camera calibration methods have been proposed in literature and reported producing promising results but the majority of such approaches were found either numerically instable or suffer from serious limitations when it comes to removing distortions at the edges of imagery. The fact that these techniques rely of the traditional brown's model which assumes symmetric radial distortions make them no suitable for consumer grade digital cameras which are known for their instable internal geometry. This study found undisputable that new analytical camera calibration techniques more adapted to the internal geometry of consumer grade cameras are needed.

I. INTRODUCTION

The need for camera calibration has been a fundamental requirement since the foundation of photogrammetry. With the increasing number of close range photogrammetry applications and advances in imaging technologies, camera calibration methods are becoming more complex to perform. Indeed with the introduction of consumer grade cameras which possess off the shelves lenses into the photogrammetry processes, the need for high quality metrics from photographs has grown over the past decades, making camera calibration a very crucial task.

The aim of camera calibration is to determine camera internal and external parameters that enabled the mapping of a 3D object space onto the 2D image space. Without accurate modelling of these camera internal and external parameters it is impossible to achieve a good geometric description of the projection from the 3D world scene onto the 2D image plan. The accuracy of the modeled transformation depends on a certain number of parameters including the mathematical components of the distortion model, the number of parameters considered by the model and the robustness of the mathematical solution used to solve for the camera parameters. This study will try to investigate some of the mostly employed lines and points based camera calibration approaches in order to identify their short-comings of the current camera calibration methods when dealing consumer grade digital cameras. The study will also investigate the suitability of Brown's radial distortion model when it comes to modelling pincushion, barrel profiles that are not always symmetric with reference to the distortion center. Finally, the study will analyze the effects of the number of additional parameters in lens distortion models, on the accuracy of the calibration procedure.

II. CAMERA CALIBRATION APPROACHES

A. Ahmed and Farag Calibration Method

In Ahmed and Farag (2005), a new camera calibration approach based on the slope of a distorted line was proposed. The camera model is based on the perspective projection of a straight line in which every point on a line satisfies the linear equation expressed by the following:

$$ax + by + c = 0 \qquad [1]$$

Where *a*, *b* and *c* are constants for a specific line *l*, and $s = \frac{a}{b}$ the slope of the line. Considering the origin of the image coordinate system $O(x_0, y_0)$, the proposed model relates a point $P_u(x_u, y_u)$ on the undistorted line *l* and its distorted corresponding $P_d(x_d, y_d)$ on the distorted line by using the traditional radial and decentring distortions models originally proposed by Brown (1971) as follows:

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$$f(x_u, y_u) = a\left\lfloor x_d + (x_d - x_0)\left(k_1 r_d^{\ 2} + k_2 r^4 + k_3 r^6\right) + \left\lfloor p_1\left(r_d^{\ 2} + 2(x_d - x_0)^2\right) + 2p_2(x_d - x_0)(y_d - y_0)\right\rfloor\left\lfloor 1 + p_3 r_d^{\ 2}\right\rfloor\right\rfloor$$
$$+ b\left[y_b + (y_d - y_0)\left(k_1 r_d^{\ 2} + k_2 r_d^{\ 4} + k_3 r_d^{\ 6}\right) + \left[p_2\left(r_d^{\ 2} + 2(y_d + y_0)^2 + 2p_1(x_d - x_0)(y_d - y_0)\right)\right]\left[1 + p_3 r_d^{\ 2}\right]\right] + c \quad [2]$$

By calculating the elemental change of f (δf) from equation [2] one obtains the following equation:

$$\delta f = a\left[\frac{\partial x_u}{\partial x_d}\delta x_d + \frac{\partial x_u}{\partial y_d}\partial y_d\right] + b\left[\frac{\partial y_u}{\partial x_d}\partial x_d + \frac{\partial y_u}{\partial y_d}\partial y_d\right] \quad [3]$$

This equation represents the tangent to the distorted curve and its slopes can be estimated by the equation [4] as follows:

$$s(x_d, y_d) = \frac{\dfrac{\partial y_u}{\partial x_d} + \dfrac{\partial y_u}{\partial y_d}\dfrac{\delta y_d}{\delta x_d}}{\dfrac{\partial x_u}{\partial x_d} + \dfrac{\partial x_u}{\partial y_d}\dfrac{\delta y_d}{\delta x_d}} \quad [4]$$

Given a chain of edge points on the distorted line $\left(x_d^{\ i}, y_d^{\ i}, i = 1,2,....N\right)$, the error E between points positions on the distorted line can be estimated through the squared difference between slopes of the line. This can be expressed as follows:

$$E = S\left(x_d^{\ i}, y_d^{\ i}\right)^2 - S\left(x_d^{\ i-1}, y_d^{\ i-1}\right)^2 \quad [5]$$

In the case where several lines are considered on the distorted image the error in [8] would be estimated as the sum of errors as follows:

$$E = \sum_{i=2}^{N}\left(S\left(x_d^{\ i} - x_d^{\ i}\right) - S\left(x_d^{\ i-1} - x_d^{\ i-1}\right)\right)^2 \quad [6]$$

Under the correct values of the distortion parameters, the error estimated in (6) should be zero. This error can be minimized using non-linear optimization algorithm starting with initial guess values of distortion parameters. Although the proposed projection model was reported robust, the distortion model used in the complete model assumes a symmetry of radial distortion and produces strong correlation between distortion parameters and the model only models distortions at image corners (Jacobsen, 2003). A variation of the technique was earlier proposed in 1997 by Prescott and McLean. The two stages techniques starts by establishing an undistorted line model by joining two end points representing a line using an edge detector program. The detected edge points' coordinates are then used to build the equation of the undistorted line given by:

$$x\cos\phi + y\sin\phi = \rho \quad [7]$$

With ρ the distance from the origin of the line to the end point and ϕ is the angle this line makes with the horizontal axis. Once the equation of the undistorted line is established, a search process to identify distorted points in the neighborhood of the undistorted line is performed. This process groups pixels based on their grey level and spatial connectedness on Line Support Regions (LSRs). The set of identified pixels are then used as input in a line fitting process which leads to the estimation of radial distortion parameters using a Least Squares technique. The obtained distortion parameters are then employed in a wrap function which removes distortions from the imagery by mapping distorted points onto their ideal locations. The wrap parameters estimated by the technique are only limited to the two first coefficients of radial distortion k_1, k_2 and the coordinates of the distortion center x_c, y_c. However, a limitation of this approach is the fact that it ignores other types of distortion such as decentering or film deformation distortions. Moreover, the radial distortion model employed by the technique is not adequate for consumer grade cameras as it relies on consistent radial distance which assumes symmetric radial distortions. The other limitation of the technique is the influence of image noise that can be mistaken as a distorted point during the point search in the neighborhoods of the undistorted line model. Wang et al., (2009) proposed another line-based technique which employs rational models model derived from the traditional Brown's radial distortion model and described in Fitzgibbon (2001). The model relates the distorted and undistorted points on an image by the following functions:

$$x_u = \frac{x_d}{1 + \lambda r_d^{\ 2}} \text{ and } y_u = \frac{y_d}{1 + \lambda r_d^{\ 2}}, \quad [8]$$

Where λ is the first coefficient of radial distortion. These produce the equation of the undistorted line as follows:

$$\frac{y_d}{1 + \lambda r_d^{\ 2}} = a\frac{x_d}{1 + \lambda r_d^{\ 2}} + b \quad [9]$$

After reformulation the relation in [8] gives:

$$y_d = ax_d + b + b\lambda\left(x_d^{\ 2} + y_d^{\ 2}\right) \quad [10]$$

Which after development produces the following:

$$x_d^{\ 2} + y_d^{\ 2} + \frac{a}{b\lambda}x_d - \frac{1}{b\lambda}y_d + \frac{1}{\lambda} = 0 \quad [11]$$

The equation [11] is the equation of a circle showing the graphics of a distorted straight line (Wang et al., 2009). By extracting three straight lines from the image and by determining parameters that satisfy their equations then substituting those parameters into [10] the technique can recover the parameters of radial distortion. Although the technique was reported suitable to deal with severe distortions, it still relies on a consistent radial distance which describes symmetric radial distortions. Moreover, other distortions such decentring distortions have not been considered by the technique.

B. Tsai Calibration Method

In 1987, Tsai proposed a calibration technique based on a single image. The two stages calibration technique estimates the intrinsic camera parameters using a perspective projection before refining them in a second stage by applying a non-linear optimisation technique. The first stage of the camera model proposed by Tsai requires coordinates of 3D points to estimate the ideal coordinates of projected 2D points. With known ideal 2D coordinates and the observed coordinates, the second phase of the model estimates the radial distortion parameters (Horn, 2000). Tsai model start by estimating the relationship between camera coordinates and ideal image coordinates (undistorted) by the following expression:

$$\begin{bmatrix} X_u \\ Y_u \end{bmatrix} = f \begin{bmatrix} \frac{x_c}{z_c} \\ \frac{y_c}{z_c} \end{bmatrix} \quad [12]$$

Where f, the principal distance is measured from the centre of projection to the image plane and X_u, Y_u representing the undistorted image coordinates. The quantities x_c, y_c, z_c represent the coordinates measured in the camera coordinate system. Tsai model only considers radial distortions and the relationships between the distorted (X_d, Y_d) and undistorted image coordinates are expressed using the traditional Brown's radial distortion model as follow:

$$\begin{bmatrix} X_u \\ Y_u \end{bmatrix} = \begin{bmatrix} X_d + X_d\left(k_1 r^2 + k_2 r^4 + \ldots\right) \\ Y_d + Y_d\left(k_1 r^2 + k_2 r^4 + \ldots\right) \end{bmatrix} \quad [13]$$

The proposed technique requires the position and altitude of the calibration targets to be recovered with respect to the camera coordinate system. For instance, if (x_s, y_s) are the coordinates of a point measured in the scene coordinate system and (x_c, y_c) the coordinates measured in the camera coordinate system, the perspective projection model expressing the relationship between the two coordinates is given as follows:

$$(x_c, y_c) = R(x_s, y_s) + t \quad [14]$$

Where t is the translation and $R(....)$ the rotation matrix. This can be reformulated by the following expression:

$$\begin{bmatrix} X_d + X_d\left(k_1 r^2 + k_2 r^4 + \ldots\right) \\ Y_d + Y_d\left(k_1 r^2 + k_2 r^4 + \ldots\right) \end{bmatrix} = f \begin{bmatrix} \frac{r_{11}x_s + r_{12}y_s + r_{13}z_s + t_x}{r_{31}x_s + r_{32}y_s + r_{33}z_s + t_z} \\ \frac{r_{21}x_s + r_{22}y_s + r_{23}z_s + t_y}{r_{31}x_s + r_{32}y_s + r_{33}z_s + t_z} \end{bmatrix} \quad [15]$$

The conversion from distorted coordinates to image pixel is done by the following model:

$$u = \frac{s_x x_d}{d_x} + c_x \qquad v = \frac{y_d}{d_x} + c_y \quad [16]$$

With s_x the scale factor introduced by the image capture hardware, d_x, d_y the horizontal and vertical distances between centres of adjacent cells in the CCD array. The expression in [15] can be substituted in [14] to estimate the coordinates in pixels. Although the model proposed by Tsai was reported viable for 3D machine vision measurements and produced acceptable calibration results (Horn, 2010), a certain number of limitations were identified. Firstly, the technique is very laborious to implement as it requires very large amounts of points to perform well. Moreover, the proposed model assumes the image centre to be the centre of projection, which does not hold true for consumer grade cameras (Li et al., 2014). The other limitation is the pinhole perspective projection model which does not characterise the physical and optical behaviour of consumer grade cameras with unstable internal geometry. Furthermore, Tsai technique only works perfectly for symmetric radial distortions as it assumes the radial distance constant. The model also ignores the decentring component of distortions.

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C. Weng Calibration Method

The camera model proposed by Tsai (1987) only models radial distortion. The accuracy of this method was reported sufficient for most of photogrammetry applications (Horn, 2000). However, in some cases where the camera lens needs to be accurately modelled, a simple radial approximation is not sufficient. Following this, Weng (1992) proposed a two steps calibration approach in which the transformation from the 3D space coordinate to the 2D image coordinates is firstly modelled by a perspective projection composed of a rotation and a translation as follows:

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = R \begin{bmatrix} x \\ y \\ z \end{bmatrix} + T \qquad [17]$$

Where R is a 3×3 rotation matrix defining the camera orientation and T is a translation vector defining the camera position. Considering the image plane coordinate system $(0', u, v)$ and the camera coordinate system (o_c, x_c, y_c, z_c) with $0'$ representing the principal point and u, v axes respectively parallel to x_c and y_c, the transformation between the image coordinates and the camera coordinates is given by the following:

$$\begin{bmatrix} u \\ v \end{bmatrix} = f \begin{bmatrix} \frac{x_c}{z_c} \\ \frac{y_c}{z_c} \end{bmatrix} \qquad [18]$$

As geometrical distortions concern the positions of image points in the image plane, the relations in [16] do not hold true and should be replaced by the relations in [19] as follow:

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} u + \delta_u(u,v) \\ v + \delta_v(u,v) \end{bmatrix} \qquad [19]$$

Where u, v are the image undistorted coordinates and u', v' their respective distorted coordinates. The distortion expressions $\delta_u(u,v)$ and $\delta_v(u,v)$ comprise radial distortion functions, decentring distortion and thin prism distortions along the u and v axes. The radial distortion functions are given by the following:

$$\begin{bmatrix} \delta_u \\ \delta_v \end{bmatrix} = \begin{bmatrix} k_1 u(u^2 + v^2) \\ k_1 v(u^2 + v^2) \end{bmatrix} \qquad [20]$$

In addition, the decentring distortion functions are given by:

$$\begin{bmatrix} \delta_{ud} \\ \delta_{vd} \end{bmatrix} = \begin{bmatrix} p_1(3u^2 + v^2) + 2p_2 uv \\ 2p_1 uv + p_2(u^2 + 3v^2) \end{bmatrix} \qquad [21]$$

Finally, the thin prism distortions along u and v directions that arises from the imperfection in lens design and manufacturing as well as camera assembly are given by:

$$\begin{bmatrix} \delta_{up} \\ \delta_{vp} \end{bmatrix} = \begin{bmatrix} s_1(u^2 + v^2) \\ s_2(u^2 + v^2) \end{bmatrix} \qquad [22]$$

With s_1 and s_2 the coefficients of thin prism distortions. The total amount of distortions along u and v axes is then expressed as the sum of radial, decentring and thin prism distortions as follows:

$$f \begin{bmatrix} \dfrac{r_{11}x + r_{12}y + r_{13}z + t_y}{r_{31}x + r_{32}y + r_{33}z + t_z} \\ \dfrac{r_{21}x + r_{22}y + r_{23}z + t_y}{r_{31}x + r_{32}y + r_{33}z + t_z} \end{bmatrix} = \begin{bmatrix} u' \\ v' \end{bmatrix} - \begin{bmatrix} [k_1 u(u^2 + v^2) + p_1(3u^2 + v) + 2p_2 uv + s_1(u^2 + v^2)] \\ k_1 v(u^2 + v^2) + p_2(u^2 + 3v^2) + 2p_1 uv + s_2(u^2 - v^2) \end{bmatrix} \qquad [23]$$

Although the model is reported to perform better than Tsai's model it relies on the perspective projection which does not characterise the physical behaviour of consumer grades cameras. Moreover, the technique uses the traditional Brown's model which assume consistent radial distance.

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D. The Tommaseli Calibration Method

Tommaseli et al., (2012) proposed a line-based camera model based on the equivalence between the vector normal to the projection plane in the image space and the vector normal to the projection plane in the object space. In order to build that equivalence the authors considered a perspective centre point PC, two objects points P_1 and P_2 in the 3D object space and their images p_1 and p_2 in the image space. In the figure1 below, the authors showed the vector $\vec{n}$ normal to the projection plane in the image space, the vector $\vec{N}$ also normal to the projection plane in the object space and the two vectors $\vec{p_1 p_2}$ and $\vec{PCp_1}$.

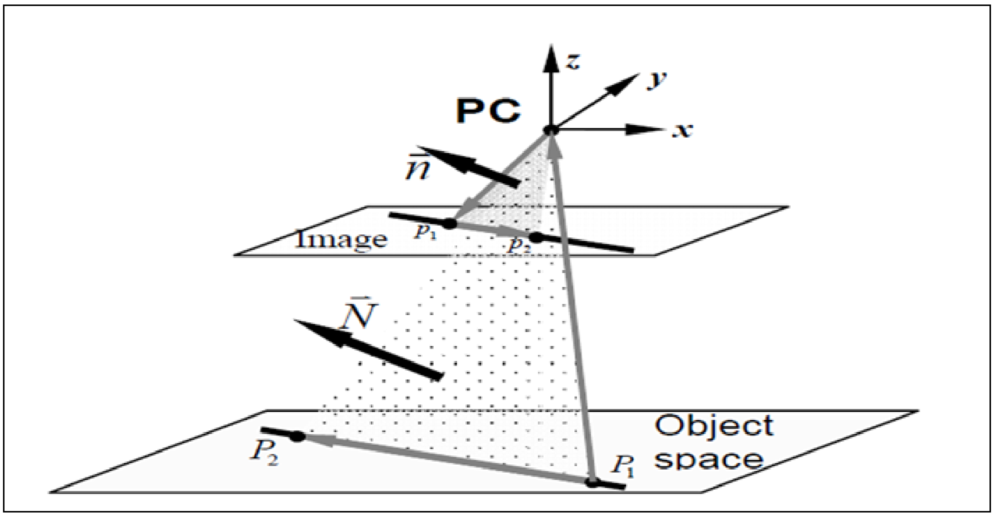

Fig 1 Projection Plane and Normal Vectors

This vector is expressed by the equation (22) below:

$$\vec{n} = \begin{bmatrix} x_{p2} - x_{p_1} \\ y_{p_2} - y_{p_2} \\ 0 \end{bmatrix} \wedge \begin{bmatrix} x_1 \\ y_1 \\ -f \end{bmatrix} = \begin{bmatrix} -f(y_{p_2} - y_{p_1}) \\ f(x_{p_2} - x_{p_1}) \\ x_2 y_1 - x_1 y_2 \end{bmatrix} \quad [24]$$

The projection plane in the object space is composed of the vector $\vec{P_1P_2}$ and $\vec{P_1PC}$, the vector normal $\vec{N}$ to the object projection plane is expressed by the equation [25] below:

$$\vec{N} = \begin{bmatrix} X_{P_1} - X_{P_1} \\ Y_{P_2} - Y_{P_1} \\ Z_{P_2} - Z_{P_1} \end{bmatrix} \wedge \begin{bmatrix} X_{P_1} - X_{PC} \\ Y_{P_1} - Y_{PC} \\ Z_{P_1} - Z_{PC} \end{bmatrix} = \begin{bmatrix} (Y_{P_2} - Y_{P_1})(Z_{P_1} - Z_{PC}) - (Y_{P_1} - Y_{PC})(Z_{P_2} - Z_{P_1}) \\ (X_{P_1}\ X_{PC})(Z_{P_2} - Z_{P_1}) - (X_{P_2} - X_{P_1})(Z_{P_1} - Z_{PC}) \\ (Z_{P_2} - Z_{P_{P_1}})(Y_{P_1} - Y_{PC}) - (X_{P_1} - X_{PC})(Y_{P_2} - Y_{P_1}) \end{bmatrix} \quad [25]$$

The equivalence between the two vectors can be obtained by equalling equation [24] and [25] and gives equation [26] as follows:

$$\begin{bmatrix} -f(y_{p_2} - y_{p_1}) \\ f(x_{p_2} - x_{p_1}) \\ x_2 y_1 - x_1 y_2 \end{bmatrix} = \begin{bmatrix} (Y_{P_2} - Y_{P_1})(Z_{P_1} - Z_{PC}) - (Y_{P_1} - P_{PC})(Z_{P_2} - Z_{P_1}) \\ (x_{P_1} - X_{PC})(Z_{P_2} - Z_{P_1}) - (X_{P_2} - X_{P_1})(Z_{P_1} - Z_{PC}) \\ (Z_{P_1} - Z_{PC})(Y_{P_1} - Y_{PC}) - (X_{P_1} - X_{PC})(Y_{P_2} - Y_{P_1}) \end{bmatrix} \quad [26]$$

However, due to the opposite directions between the two normal vectors and fact that the 3D object plane and the 2D image are linked by a scale relationship, the equation [27] can be written as :

$$\lambda.\vec{n} = R.\vec{N} \quad [27]$$

With λ is a scale factor and R is the rotation matrix defined by the sequence $.R_x(\omega), R_y(\varphi), R_z(\kappa)$ of the rotation. This relationship is now expressed as:

$$\lambda\begin{bmatrix} -f(y_{P_2}-y_{P_1}) \\ f(x_{P_2}-x_{P_1}) \\ x_{P_2}y_{P_1}-x_{P_1}y_{P_2} \end{bmatrix} = R\begin{bmatrix} (Y_{P_2}-Y_{P_1})(Z_{P_1}-Z_{PC})-(Y_{P_1}-Y_{PC})(Z_{P_2}-Z_{P_1}) \\ (x_{P_1}-X_{PC})(Z_{P_2}-Z_{P_1})-(X_{P_2}-X_{P_1})(Z_{P_1}-Z_{PC}) \\ (Z_{P_1}-Z_{PC})(Y_{P_1}-Y_{PC})-(X_{P_1}-X_{PC})(Y_{P_2}-Y_{P_1}) \end{bmatrix} \quad [28]$$

By introducing the following new variables:

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} (Y_{P_2}-Y_{P_1})(Z_{P_1}-Z_{PC})-(Y_{P_1}-Y_{pc})(Z_{P_2}-Z_{P_1}) \\ (X_{P_1}-X_{PC})(Z_{P_2}-Z_{P_1})-(X_{P_2}-X_{P_1})(Z_{P_1}-Z_{PC}) \\ (Z_{P_1}-Z_{PC})(Y_{P_1}-Y_{PC})-(X_{P_1}-X_{Pc})(Y_{P_2}-Y_{P_1}) \end{bmatrix} \quad [29]$$

And by expending [30] one obtains:

$$\begin{bmatrix} -\lambda f(y_{p_2}-y_{p_1}) \\ \lambda(x_{p_2}-x_{p_1}) \\ \lambda(x_{p_2}y_{p_1}-x_{p_1}y_2) \end{bmatrix} = \begin{bmatrix} \omega_{11}N_1+\omega_{12}N_2+\omega_{13}N_3 \\ \varphi_{21}N_1+\varphi_{22}N_2+\varphi_{23}N_3 \\ \kappa_{31}N_1+\kappa_{32}N_2+\kappa_{33}N_3 \end{bmatrix} \quad [30]$$

In order to eliminate the scale factor λ the first two equations in [28] are divided by the last equation and give:

$$\begin{bmatrix} \dfrac{-\lambda f(y_{p_2}-y_{p_1})}{\lambda(x_{p_2}yp_1-x_{p_1}y_{p_2})} \\ \dfrac{\lambda f(x_{p_2}-x_{p_1})}{\lambda(x_{p_2}y_{p_1}-x_{p_1}y_2)} \end{bmatrix} = \begin{bmatrix} \dfrac{\omega_{11}N_1+\omega_{12}N_2+\omega_{13}N_3}{\kappa_{31}N_1+\kappa_{32}N_2+\kappa_{33}N_3} \\ \dfrac{\varphi_{21}N_1+\varphi_{22}N_2+\varphi_{33}N_3}{\kappa_{31}N_1+\kappa_{32}N_2+\kappa_{33}N_3} \end{bmatrix} \quad [31]$$

And by expending [30] one obtains:

$$\begin{bmatrix} f(y_{p_1}-y_{p_2})(\kappa_{31}N_1+\kappa_{32}N_2+\kappa_{33}N_3) \\ f(x_{p_2}-x_{p_1})(\kappa_{31}N_1+\kappa_{32}N_2+\kappa_{33}N_3) \end{bmatrix} + \begin{bmatrix} (x_{p_1}y_{p_2}-x_{p_2}y_1)(\omega_{11}N_1+\omega_{12}N_2+\omega_{33}N_3) \\ (x_{p_1}y_{p_2}-x_{p_2}y_1)(\varphi_{21}N_1+\varphi_{22}N_2+\varphi_{33}N_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad [32]$$

To introduce radial and decentring distortions into the model, the authors considered the Conrady-Brown model of radial and decentring distortions expressed by:

$$\begin{bmatrix} x_u-x_0 \\ y_d \end{bmatrix} = \begin{bmatrix} (x_d-x_0)-[(x_d-x_0)(k_1r^2+k_2r^4+k_3r^6)]+p_1(r^2+2(x_d-x_0)^2)+2p_2(x_d-x_0)(y_d-y_0) \\ (y_d-y_0)-[(y_d-y_0)k_1r^2+k_2r^4+k_3r^6]+p_2(r^2+2(y_u-y_0)^2+2p_1(x_d-x_0)(y_u-y_0)) \end{bmatrix} \quad [33]$$

Where x_u, y_u are the undistorted coordinates and x_d, y_d the observed distorted coordinates in the image reference system, k_1, k_2, k_3 are the coefficients of radial distortions and P_1, P_2 are the coefficients of decentring distortion while x_0, y_0 are the coordinates of the principal point. Although the technique was reported producing good estimates of distortion parameters, it does not guaranty the accuracy of the 3D points coordinates involved in the model as any inaccuracy in points' measurements can be propagated into the estimated distortion parameters. Moreover the Brown-Conrady model used is a model suitable for symmetric lens distortions as it assumes consistent radial distance.

E. Additional Parameters Model.

Additional parameters are the term of a polynomial expression incorporated in the collinearity equations in order to model various systematic errors including lens distortions. Five types of additional parameters models have been proposed in the literature. This includes the Bauer simple model, the Jacobsen model, the Ebner's orthogonal model, the fourteen parameters Brown's Physical model and the sixteen parameters Borwon's model.

The Bauer model has three additional parameters including two parameters that describes the extent of affine deformation on the image and one parameter that describes the symmetric radial distortion (Anguilar et al., 2010; Blazquez and Colomnia, 2010). The distortions applied to the x and y coordinates are given by the following distortion model:

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} a_1 x\left(r^2 - r_0^{\,2}\right) + a_2 x \\ a_1 y\left(r^2 - r_0^{\,2}\right) + a_2 y + a_3 x \end{bmatrix} \quad [34]$$

By combining the Bauer's distortion model into the collinearity equation one obtains the complete distortion model presented as follows:

$$\begin{bmatrix} x_u - x_0 \\ y_u - y_0 \end{bmatrix} = f\left[\frac{r_{11}(X_w - X_0) + r_{12}(Y_w - Y_0) + r_{13}(Z_w - Z_0)}{r_{31}(X_w - X_0) + r_{32}(Y_w - Y_0) + r_{33}(Z_w - Z_0)}\right] - \begin{bmatrix} a_1 x\left(r^2 - r_0^{\,2}\right) + a_2 x \\ a_1 y\left(r^2 - r_0^{\,2}\right) + a_2 y + a_3 x \end{bmatrix} \quad [35]$$

With x_u, y_u the undistorted coordinates, x_0, y_0 the coordinates of the perspective centre and X_0, Y_0, Z_0 the origin of the 3D coordinate system.

The Jacobsen simple model is similar to the Bauer's model but presents one additional parameter and compensate for the first and second order distortion associated with affine deformation and lens distortions (Passini and Jacobsen, 2008). The affine deformations and lens distortion applied to the x and y coordinates are given by the distortion model as follows:

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} a_1 x\left(r^2 - r_0^{\,2}\right) + a_2 x + a_3 y \\ a_1 y\left(r^2 - r_0^{\,2}\right) + a_2 y + a_3 x + a_4 x^2 \end{bmatrix} \quad [36]$$

Similar to Bauer's model, the Jacobsen calibration approach relies on the collinearity equation to form its complete distortion model.

The Ebners's orthogonal model is a twelve additional parameters model which compensates for various types of systematic errors such as scanner errors, affine deformation and film deformation. (Leica Geosystem User Guide, 2015). These additional parameters are orthogonal to one another and to the exterior orientation parameters under circumstances where the ground surface is flat. The model is given by the following:

$$\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}=\begin{bmatrix} a_1x+a_2y-a_3\left(2x^2-\frac{4b^2}{3}\right)+a_4xy+a_5\left(y^2-\frac{2b^2}{3}\right)+a_7x\left(y^2-\frac{2b^2}{3}\right)+a_9y\left(x^2-\frac{2b^2}{3}\right)+ \\ a_{11}\left(x^2-\frac{2b^2}{3}\right)\left(y^2-\frac{2b^2}{3}\right) \\ -a_1y+a_2x-a_3xy-a_4\left(2y^2-\frac{4b^2}{3}\right)+a_6\left(x^2-\frac{2b^2}{3}\right)+a_8y\left(x^2-\frac{2b^2}{3}\right)+a_{10}x\left(y^2-\frac{2b^2}{3}\right)+ \\ a_{12}\left(x^2-\frac{2b^2}{3}\right)\left(y^2-\frac{2b^2}{3}\right)\end{bmatrix} \quad [37]$$

The Brown's fourteen additional parameters compensate for linear and non-linear forms of plate film deformation and lens distortions. The total distortion applied to x and y image coordinates is given by the following distortion model.

$$\begin{bmatrix}\Delta x\\ \\ \Delta y\end{bmatrix}=\begin{bmatrix} a_1x+a_2y+a_3xy+a_5x^5y+a_6xy^2+a_7x^2y^2+a_{13}\frac{x^3y}{f}+a_{14}x\left(x^2+y^2\right) \\ a_8xy+a_9x^2+a_{10}x^2y+a_{11}xy^2+a_{12}x^2y^2+a_{13}\frac{y^3x^2}{f}+a_{14}y\left(x^2+y^2\right)\end{bmatrix} \quad [38]$$

Like the previous models, Brown's fourteen model relies on the collinearity equation to compose the complete distortion model. Moreover, another Brown's model presenting sixteen additional parameters has been proposed to deal with affine distortion, film and lens distortions.

$$\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}=\begin{bmatrix} [\ a_1xy+a_2y^2+a_3x^2y+a_4xy^2+a_5x^2y^2+\frac{a_{11}x\left(x^2-y^2\right)}{f}+\frac{a_{12}x^3y^2}{f}+\frac{a_{13}x\left(x^4-y^4\right)}{f}+ \\ a_{14}x\left(r-r_0\right)+a_{15}x\left(r-r_0\right)^2+a_{16}x\left(r-r_0\right)^3\ ] \\ \\ [\ a_6xy+a_7x^2+a_8x^2y+a_9xy^2+a_{10}x^2y^2+\frac{a_{11}x\left(x^2-y^2\right)}{f}+\frac{a_{12}x^3y^2}{f}+\frac{a_{13}x\left(x^4-y^2\right)}{f}+ \\ a_{14}y\left(r-r_0\right)+a_{15}\left(r-r_0\right)^2+a_{16}\left(r-r_0\right)^3\ ]\end{bmatrix} \quad [39]$$

The above model does not only differ from the previous one in terms of the number of degrees of freedom but also in the functions composing those degrees of freedom. Although the additional parameter models discussed above deal with variety of distortions, they suffer from numerical instability due to their large number of parameters.

III. CONCLUSION

From the lined-based calibration approaches studied in this investigation, the method proposed by Prescott and McLean (1997) would produce the poorest camera parameter accuracies due to the fact that it is very sensitive to image noise. In fact, during the search process, there is a high chance when dealing with low resolution images that image noise could be mistaken as distorted pixel locations. This is associated to the dependence of the edge detection algorithm to pixel grey levels. Moreover, the radial distortion function employed by the technique is not suitable for consumer grade cameras with instable internal geometry. This poor performance extended to other line based calibration approaches investigated in this study in occurrence of non-symmetric radial lens distortions as the proposed algorithms assume a constant radial displacement measure for all the distorted points within the image. Which does not always hold true with distortion profiles such as barrel and pincushion distortions that create a larger or small towards the edges of the photograph in comparison to the measure around the image centre. Line based camera calibration approaches also rely on optimization algorithms to perfect the numerical estimates of calibrated camera parameters. Although point based calibration method offer better numerical stability due to the fact that they do not need any sophisticated algorithm to measure the coordinates of the projections of the 3D points onto the image plan, they require a large amount of calibration points in order to achieve an optimal calibration results. Moreover, most of point based calibration approaches require some results optimization through an iteration process and iterative camera calibration suffers the limitation of requiring very accurate initial estimates of the calibrated parameters, which are not always available. From the above, it is evident that there is a need to develop new point based camera calibration methods that offer analytical solutions without any intervention of iterative processes and that can achieve satisfactory calibration results with very few calibration points.

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