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Bending Analysis of Thick Anisotropic Rectangular Plate using Modified First Shear Deformation Theory

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Abstract:- In this work, displacement functions obtained by direct integration of governing equation were used to analyse anisotropic rectangular plates that are simply supported on its four edges (SSSS) and clamped on two adjacent edges and simply supported on the other two (CCSS). A modified first order shear deformation theory was used to develop the kinematic and constitutive relations of the plate. The total potential energy functional was formulated from which the governing and two compatibility equations were developed and solved to generate the polynomial displacement functions. By satisfying the boundary conditions of the plates, their peculiar displacement functions were determined. With these functions the anisotropic stiffness coefficients were obtained. By differentiating the total potential energy functional with respect to the displacement coefficients, the formulae for the coefficients were obtained. For each boundary condition analysis in this study, the displacement parameter values, in-plane stresses parameter values and out-of-plane stresses parameter values at various span to depth ratios (7.142857, 10 and 20), aspect ratios (0.5 to 2) and angle of fibre orientation of 0^0 were calculated. The solutions of this work were compared with those from various researchers and their results were close.

Keywords:- *Polynomial Displacement Function; Total Potential Energy Functional; Thick Plate; Anisotropic; Displacement; Stress, Governing Equation.*

I. INTRODUCTION

Plates are plane surface structural elements bounded by two parallel planes called faces, which are separated from one another by thickness whose dimension is much smaller than the other dimensions (Ibearugbulem, Ezeh, and Ettu 2014). Rectangle plates are frequently used as structural elements in many branches of modern technology namely aeronautics, electronics, marine, optical, nuclear and structural engineering. Plates are classified according to their material organization as isotropic, orthotropic and anisotropic plates (Ventsel and Krauthammer, 2001). Such plates are often subjected to static and dynamic loads, which are predominantly perpendicular to the plate's surface. Thus, the understanding of bending behaviour of isotropic, orthotropic and anisotropic thick plates are very important to the engineering structural designers. Various plate theories with different assumptions have been developed over the years to accurately describe the static and dynamic behaviour of plates. The earliest theory suggested for the plates was the Kirchhoff's plate theory or classical (CPT) plate theory (Gorman, 1977). This theory has been used by various researchers to analyze bending problems of isotropic, orthotropic and anisotropic rectangular plates (Zhao, Wei and Xiang, 2000). In CPT, the straight line normal to the plane (in-plane) is assumed to be straight and normal in the deformed configuration of the plate. These assumptions led to CPT expression of in-plane displacements (u and v) and out of plane displacement (w) as expressed in Equations (1), (2) and (3) respectively.

$$u_{(x,y,z)} = -z\frac{\partial w_{(x,y)}}{\partial x} \qquad (1)$$

$$v_{(x,y,z)} = -z\frac{\partial w_{(x,y)}}{\partial y} \qquad (2)$$

$$w_{(x,y,z)} = w_{(x,y)} \qquad (3)$$

This assumption neglects the transverse shear effects, which have significant impact on the behaviour of thick plates meaning that CPT idealizes all plates as thin plates thereby limiting the usage of the theory to only thin plates (Ghugal and Sayyad 2011a). This limitation was taken care of by the first order shear deformation (FSDT) plate theories. Reissner was the first to develop a stress-based approach, which incorporated the effect of transverse shear strains. On the other hand, Mindlin employed a displacement-based approach for first order shear deformation, which assumed transverse shear stress as constant throughout the thickness of the plate (Shimpi and Patel, 2005). Reissner`s and Mindlin`s theories are called first order theories because the shear deformation profile line, F(z), (the vertical line that is initially normal to mid surface) remains straight though no longer normal to the middle surface after bending but this violates the shear stresses free surface conditions. This means that F(z) is equal to z {F(z) = z}. Mindlin`s theory satisfies constitutive relations for transverse shear strains and shear stresses in an approximate manner (because their theory violates the shear stress free surface conditions) by way of using shear correction factor. This correction factor is to correct the discrepancy between the real distribution of the transverse shear stress from FSDT and the one resulting from the utilization of the kinematic relations (Mihai and Seriu, 2013; Shahrokh and Arsaniani, 2005). The assumptions of FSDT`s that the normal to the mid-surface remains straight but not necessarily perpendicular to the mid-surface after deformations led to Mindlin's FSDT expression of in-plane and out of plane displacements as:

$$u_{(x,y,z)} = -z\emptyset_{x(x,y)} \quad (4)$$

$$v_{(x,y,z)} = -z\emptyset_{y(x,y)} \quad (5)$$

$$w_{(x,y,z)} = w_{(x,y)} \quad (6)$$

Where $\emptyset_{x(x,y)}$ and $\emptyset_{y(x,y)}$ are the rotations at x and y axis. The limitations of the FSDT`s led to the evolution of the second order shear deformation theory (SSDT), Third order shear deformation (TSDT), Hyperbolic shear deformation theories (HPSDT), Trigonometric shear deformation theories (TST) and Exponential shear deformation theories (ESDT) etc (Dawe and Roufaeil, 1980; Liew, Hung and Lim, 1995; Cheung and Zhou, 2000). These theories assumed that the shear deformation line, that is F(z) is not straight and not normal to the middle surface after bending resulting to most of the higher order theories expression of in-plane and out of plane displacements as:

$$u_{(x,y,z)} = -z\frac{dw}{dy} + F(z)\Theta_{zx} \quad (7)$$

$$v_{(x,y,z)} = -z\frac{dw}{dy} + F(z)\Theta_{zy} \quad (8)$$

$$w_{(x,y,z)} = w_{(x,y)} \quad (9)$$

Where Θ_{zx} and Θ_{zy} are rotation in x and y axes respectively (Sadrnejad, Daryan and Ziaei, 2009; Punit and Hiren, 2013; Yuwaraj and Meghraj, 2011;Ghugal and Sayyad, 2011b; Shahrjerdi and Mustapha, 2011; Sayyad and Ghugal, 2012; Chikalthnkar, Sayyad and Nandedkar, 2013; Sayyad and Ghugal, 2014; Ivo, Nikola, Dae-Seung and Tae-muk, 2014). These theories (higher order theories) have common relationship for shear deformation line, F(z) and shear stress profile, G(z). That is:

$$\tau_{max} = \tau . G(z) = \tau . \frac{\partial F}{\partial z} \quad (10)$$

where F from Equation (10) is usually denoted as F(z) and is a model in cubic or higher polynomial function, trigonometric function, exponential function, hyperbolic function etc (Chikalthankar et al. 2013). Hence, this use of model as the shear deformation profile of the vertical section after bending satisfies the specious assumption for the expression of in-plane displacements and vertical rotation brought about by the assumption that a vertical section that is initially straight and normal to the middle surface before bending no longer remains straight after bending.

All these higher order theories have limitations. In order to solve the problems associated with the use of higher order shear deformation theories, Ibearugbulem (2016) introduced two new theories called Alternative 1 theory and Alternative 2 theory, which are modifications of existing first shear deformation theory. Alternative 2 theory assumed that a section that is initially normal and straight to the middle plane of the plate before deformation will remain straight but no longer normal to the middle surface after deformation. Alternative 2 theory expressed in-plane and out of plane displacements as:

$$u_{(x,y,z)} = u_c + u_s \quad (11)$$

$$v_{(x,y,z)} = v_c + v_s \quad (12)$$

$$w_{(x,y,z)} = w_{(x,y)} \quad (13)$$

Equation (11) and Equation (12) show that Alternative 2 theory expressed in-plane displacements as two components. Where u_c is the classical component of in-plane displacement in x direction and u_s is the shear component of in-plane displacement in x direction while v_c is the classical component of in-plane displacement in y direction and v_s is the shear component of in-plane displacement in y direction.

Alternative 1 theory assumed that a vertical section that is initially straight and normal to the middle surface before bending will remain straight but no longer normal to the middle surface after bending this assumption makes it a first shear deformation theory which led to the Alternative 1 theory expression of in-plane and out of plane displacements as:

$$u_{(x,y,z)} = z\emptyset_{x(x,y)} \quad (14)$$

$$v_{(x,y,z)} = z\emptyset_{y(x,y)} \quad (15)$$

$$w_{(x,y,z)} = w_{(x,y)} \quad (16)$$

From Equations (14) and (15), it can be seen that the negative signs used in expressing in-plane displacements both in FSDT and HSDT have been eliminated in Alternative 1 theory. Also, Alternative 1 theory made the correct assumption for vertical rotations and in-plane displacements and used a linear function, z for the expression of shear deformation line unlike the FSDT that assumed constant shear stress across the thickness of the plate.

Due to the desirable high strength to weight and other excellent properties of anisotropic plates, anisotropic materials are the most important structural materials used in structural and civil engineering fields (sarangan and singh, 2017). Researchers have carried out bending analysis of anisotropic plates using various theories and methods. Ezeh, Ibearugbulem, Anya and Ozioko (2020) analyzed thick anisotropic plates that are simply supported and clamped on

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all edges using a third order shear deformation model in Ritz energy method that employed exact approach. Sayyad (2013) carried out flexural analysis of orthotropic thick plates using a variationally consistent exponential shear deformation theory that considered transverse shear deformation. Gholami, Hassani, Mousavi and Alashti (2019) introduced a three-dimensional simple solution for the analysis of anisotropic functionally graded plates using a method known as differential quadrature method. Lisboa and Marczak (2018) applied adomian decomposition method to moderately thick anisotropic plates under linear bending using a first order shear theory. Aghdam and Mohammadi (2008) presented bending analysis of orthotropic moderately thick sector plates under various loading conditions. Ibearugbulem, Ebirim, Anya and Ettu (2020) employed alternative II theory that is based on polynomial shape function to analyze stability and free vibration of thick orthotropic and isotropic plates having simply and free support conditions. The methods and theories employed by the various researches are either built upon the classical plate theory or used a shear correction factor or adopted a displacement field that consists of classical and shear deformation parts or are based on the erroneous assumption that a vertical section that is initially straight and normal to the middle surface of the plate before bending is no longer straight after bending. The motivation behind this present study is to use a modified first shear deformation theory which is alternative 1 theory that have addressed the limitations of the various theories and methods as outlined in this paper to analyzed bending of thick anisotropic rectangular plates having simply support conditions on all edges (SSSS) and plate clamped on two adjacent edges and simply supported on the other two edges (CCSS).

II. ACADEMIC FORMULATION

Two in-plane displacements (u and v) and one out-of-plane displacement called deflection (w) constitute the displacement field. The spatial dimensions (lengths) of the plate are "a, b, and t" along x, y, and z directions respectively. The domain of the plate along the x direction is $0 \leq x \leq a$. Along y and z directions, the domains are $0 \leq y \leq b$ and $-t/2 \leq z \leq t/2$ respectively. These are shown in Figure 1.

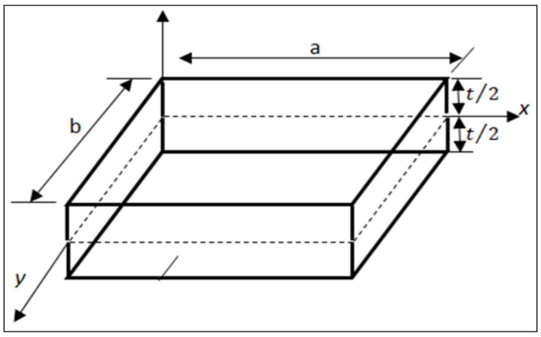

Fig 1 Three-Dimensional Coordinates of a Rectangular Plate

Consider a fibre of the plate indicated as DF that is oriented in the z direction as shown in Figure 2, which shows the plate's section before and after deformation. This fibre takes the positions $D''F''$ and $D'F'$ as a result of shear and bending deformations respectively in the x-z plane. Let ϕ_x be the rotation in the x-z plane of a line formally normal to the middle plane before deformation. The movement (displacement) of a point E that is at a distance $+z\phi_x$ from the mid-plane in the line of action of the x axis is.+z Similarly, the displacement of point E along y axis is $+z\phi_y$. Were ϕ_y is the rotation in the y-z plane of a line that is formerly normal to the middle plane before the deformation of the plate.

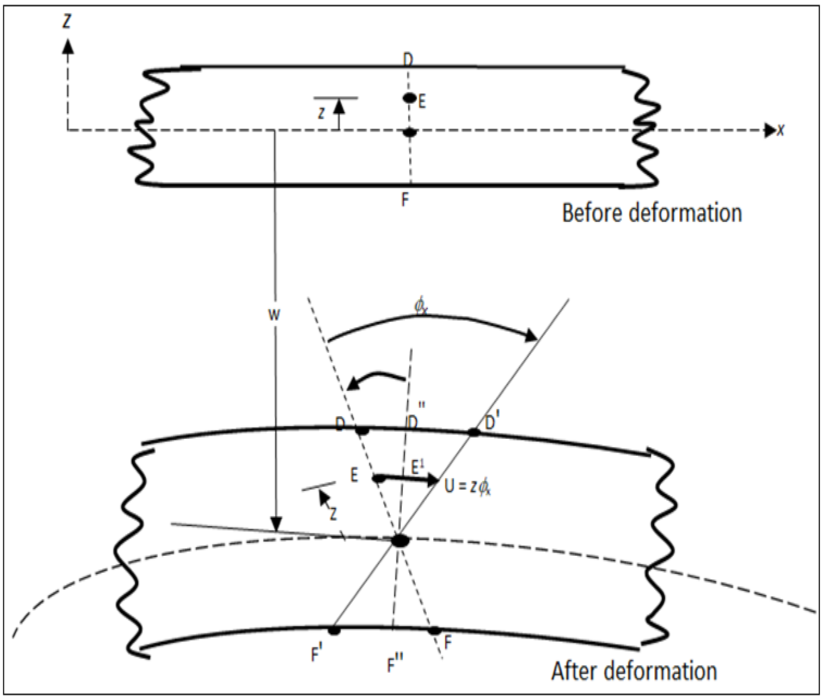

Fig 2 Deformation of a Section of a thick Anisotropic Plate

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III. KINEMATIC RELATIONSHIPS

From Figure 2, it can be stated that the in-plane displacement of any point (like E) in the plate is given by Equations (17) and (18) respectively.

$$u = z\phi_x \qquad (17)$$

$$v = z\phi_y \qquad (18)$$

The five engineering strain components are given in Equations (19) to (23).

$$\varepsilon_x = \frac{\partial u}{\partial x} = z\frac{\partial \phi_x}{\partial x} \qquad (19)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = z\frac{\partial \phi_y}{\partial y} \qquad (20)$$

$$\gamma_{xy} = 2\varepsilon_{xy} = 2\varepsilon_{yx} = 2z\frac{\partial \phi_x}{\partial y} = 2\,z\frac{\partial \phi_y}{\partial x} \qquad (21)$$

$$\gamma_{xz} = \varepsilon_{xz} + \varepsilon_{zx} = \phi_x + \frac{\partial w}{\partial x} \qquad (22)$$

$$\gamma_{yz} = \varepsilon_{yz} + \varepsilon_{zy} = \phi_y + \frac{\partial w}{\partial y} \qquad (23)$$

The vertical rotations in the x-direction (ϕ_x) and the vertical rotation in the y-direction (ϕ_y) are given in Equations (24) to (25)

$$\phi_x = \gamma_{xz} - \frac{\partial w}{\partial x} = c_x\frac{\partial w}{\partial x} \qquad (24)$$

$$\phi_y = \gamma_{yz} - \frac{\partial w}{\partial y} = c_y\frac{\partial w}{\partial y} \qquad (25)$$

IV. CONSTITUTIVE RELATIONSHIPS

Following Hooke's law, the engineering strains of anisotropic material are defined in terms of stress, Poisson's ratios and Young's modulus of elasticity as given in Equations (26) to (30).

$$\varepsilon_{11} = \frac{\sigma_{11}}{E_{11}} - \mu_{21}\frac{\sigma_{22}}{E_{22}} \qquad (26)$$

$$\varepsilon_{22} = -\mu_{12}\frac{\sigma_{11}}{E_{11}} + \frac{\sigma_{22}}{E_{22}} \qquad (27)$$

$$\gamma_{12} = \frac{1}{G_{12}}.\tau_{12} \qquad (28)$$

$$\gamma_{13} = \frac{1}{G_{13}}.\tau_{13} \qquad (29)$$

$$\gamma_{23} = \frac{1}{G_{23}}.\tau_{23} \qquad (30)$$

Where: E_{11} and E_{22} are Young's moduli in the 1 and 2 material directions of an anisotropic plate. μ_{12} and μ_{21} are Poisson's ratios in the 1-2 and 2-1 local planes of an anisotropic plate respectively. G_{12}, G_{13} and G_{23} are the shear moduli in the 1-2, 1-3 and 2-3 planes, respectively. Also σ_{11} and σ_{22} are normal stresses in the 1 and 2 local directions of the plate.

By solving Equations (26) and (27) simultaneously and rearranging Equations (28), (29) and (30) respectively and putting them in matrix form gave Equation (31a)

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\\ \tau_{13}\\ \tau_{23}\end{bmatrix} = E_{00}\begin{bmatrix}e_{11} & e_{12} & 0 & 0 & 0\\ e_{12} & e_{22} & 0 & 0 & 0\\ 0 & 0 & e_{33} & 0 & 0\\ 0 & 0 & 0 & e_{44} & 0\\ 0 & 0 & 0 & 0 & e_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\\ \gamma_{13}\\ \gamma_{23}\end{bmatrix} \qquad (31a)$$

Where:

$$E_{00} = E_0/(1-\mu_{12}\mu_{21}) \qquad (31b)$$

$$e_{11} = \frac{E_{11}}{E_0} \qquad (31c)$$

$$e_{12} = \frac{\mu_{21}.E_{11}}{E_0} = \frac{\mu_{12}.E_{22}}{E_0} \qquad (31d)$$

$$e_{22} = \frac{E_{22}}{E_0} \qquad (31e)$$

$$e_{33} = \frac{G_{12}}{E_0}.(1-\mu_{12}\mu_{21}) \qquad (31f)$$

$$e_{44} = \frac{G_{13}}{E_0}.(1-\mu_{12}\mu_{21}) \qquad (31i)$$

$$e_{55} = \frac{G_{23}}{E_0}.(1-\mu_{12}\mu_{21}) \qquad (31j)$$

E_0 *is the reference Elastic modulus. It can be* E_{11} *or* E_{22}

Putting Equations (19), (20), (21), (22) and (23) in matrix form gives Equation (32).

$$\varepsilon = \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix} = \begin{bmatrix} z\frac{\partial \phi_x}{\partial x}\\ z\frac{\partial \phi_y}{\partial y}\\ z\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\right)\\ \left(\phi_x+\frac{\partial w}{\partial x}\right)\\ \left(\phi_y+\frac{\partial w}{\partial y}\right)\end{bmatrix} \qquad (32)$$

Transforming Equation (31a) from local coordinate (1-2 coordinate) to global coordinate (x-y coordinate) using the transformation matrix resulted to Equation (33a)

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$$\sigma = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = E_{00} \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{12} & a_{22} & a_{23} & 0 & 0 \\ a_{13} & a_{23} & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{45} & a_{55} \end{bmatrix} \begin{bmatrix} z\dfrac{\partial \phi_x}{\partial x} \\ z\dfrac{\partial \phi_y}{\partial y} \\ z\left(\dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x}\right) \\ \left(\phi_x + \dfrac{\partial w}{\partial x}\right) \\ \left(\phi_y + \dfrac{\partial w}{\partial y}\right) \end{bmatrix} \quad (33a)$$

Where:

$$a_{11} = m^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + n^4 e_{22} \quad (33b)$$

$$a_{12} = e_{12}(n^4 + m^4) + m^2 n^2 (e_{11} + e_{22} - 4e_{33}) \quad (33c)$$

$$a_{13} = m^3 n(e_{11} - e_{12} - 2e_{33}) + mn^3(e_{12} - e_{22} + 2e_{33}) \quad (33d)$$

$$a_{22} = n^4 e_{11} + 2m^2 n^2 (e_{12} + 2e_{33}) + m^4 e_{22} \quad (33e)$$

$$a_{23} = mn^3 e_{11} - m^3 n e_{22} + (m^3 n - mn^3)(e_{12} + 2e_{33}) \quad (33f)$$

$$a_{33} = m^2 n^2 (e_{11} - 2e_{12} + e_{22} - 2e_{33}) + e_{33}(m^4 + n^4) \quad (33g)$$

$$a_{44} = m^2 e_{44} - 2mn\, e_{45} + n^2 e_{55} \quad (33h)$$

$$a_{45} = mn\,(e_{44} - e_{55}) + (m^2 - n^2) e_{45} \quad (33i)$$

$$a_{55} = n^2 e_{44} + 2mn\, e_{45} + m^2 e_{55} \quad (33j)$$

Where: $m = \cos\theta$ and $n = \sin\theta$. θ is the angle of orientation of the fibers

V. DETERMINATION OF TOTAL POTENTIAL ENERGY FUNCTIONAL

The total potential energy is defined as Equation (34).

$$\Pi = U + V \quad (34)$$

Where: U is the strain energy of thick anisotropic rectangular plate and V is the external work on the thick anisotropic rectangular plate.

Average strain energy mobilized by the plate when it is subjected to load is given as Equation (35)

$$U = \frac{1}{2}\iiint \varepsilon^T \sigma \; dx.dy.dz \quad (35)$$

By substituting Equations (32) and (33a) into Equation (35) and simplifying the resulting equation gave Equation (36a) which is the strain energy equation of thick anisotropic rectangular plate based on alternative 1 theory

$$U = \frac{D_{00}}{2}\iint \{\left(\frac{\partial \phi_x}{\partial x}\right)^2 .a_{11} + 2a_{xy}\frac{\partial \phi_x}{\partial x}.\frac{\partial \phi_y}{\partial y} + \left(\frac{\partial \phi_y}{\partial y}\right)^2 a_{22}$$

$$+2\left[\frac{\partial\phi_x}{\partial y}\frac{\partial\phi_x}{\partial x}+\frac{\partial\phi_y}{\partial x}\frac{\partial\phi_x}{\partial x}\right]a_{13}+2\left[\frac{\partial\phi_x}{\partial y}\frac{\partial\phi_y}{\partial y}+\frac{\partial\phi_y}{\partial x}\frac{\partial\phi_y}{\partial y}\right]a_{23}$$

$$+\frac{12}{t^2}\left(\phi_x{}^2+2\frac{\partial w}{\partial x}\phi_x+\left(\frac{\partial w}{\partial x}\right)^2\right)a_{44}+\frac{24}{t^2}\left(\frac{\partial w}{\partial x}.\phi_y+\frac{\partial w}{\partial x}.\frac{\partial w}{\partial y}+\phi_y\phi_x+\frac{\partial w}{\partial y}\phi_x\right)a_{45}$$

$$+\frac{12}{t^2}\left(2\frac{\partial w}{\partial y}\phi_y+\left(\frac{\partial w}{\partial y}\right)^2+\phi_y{}^2\right)a_{55}\}\,dxdy \qquad (36a)$$

Where:

$$D_{00}=\frac{E_{00}t^3}{12} \qquad (36b)$$

$$a_{xy}=(a_{12}+2a_{33}) \qquad (36c)$$

Lateral load external work (V_q) employed is as given in Equation (37).

$$V_q = q\int_0^a\int_0^b w\,dxdy \qquad (37)$$

By substituting Equations (36a) and (37) into Equation (34), and putting the resulting equation in non-dimensional forms Equation (38a) is obtained.

$$\Pi=\frac{\beta D_{00}}{2}\iint\{\left(\frac{\partial\phi_R}{\partial R}\right)^2.a_{11}+2\frac{a_{xy}}{\beta}\frac{\partial\phi_R}{\partial R}.\frac{\partial\phi_Q}{\partial Q}+\frac{1}{\beta^2}\left(\frac{\partial\phi_Q}{\partial Q}\right)^2a_{22}+2\left[\frac{1}{\beta}\frac{\partial\phi_R}{\partial Q}\frac{\partial\phi_R}{\partial R}+\frac{\partial\phi_Q}{\partial R}\frac{\partial\phi_R}{\partial R}\right]a_{13}$$

$$+2\left[\frac{1}{\beta^2}\frac{\partial\phi_R}{\partial Q}\frac{\partial\phi_Q}{\partial Q}+\frac{1}{\beta}\frac{\partial\phi_Q}{\partial R}\frac{\partial\phi_Q}{\partial Q}\right]a_{23}+\frac{12}{t^2}\left(a^2\phi_R{}^2+2a\frac{\partial w}{\partial R}\phi_R+\left(\frac{\partial w}{\partial R}\right)^2\right)a_{44}$$

$$+\frac{24}{t^2}\left(a\frac{\partial w}{\partial R}.\phi_Q+\frac{1}{\beta}\frac{\partial w}{\partial R}.\frac{\partial w}{\partial Q}+a^2\phi_Q\phi_R+\frac{a}{\beta}\frac{\partial w}{\partial Q}\phi_R\right)a_{45}+\frac{12}{t^2}\left(\frac{2a}{\beta}\frac{\partial w}{\partial Q}\phi_Q+\frac{1}{\beta^2}\left(\frac{\partial w}{\partial Q}\right)^2+a^2\phi_Q{}^2\right)a_{55}$$

$$-2\frac{qwa^2}{D_{00}}\}\,dRdQ \qquad (38a)$$

Where:

$$\beta=\frac{b}{a};\ \ \partial x=a\partial R\ and\ \partial y=b\partial Q \qquad (38b)$$

VI. DETERMINATION OF THE GOVERNING EQUATION AND TWO COMPATIBILITY

By minimizing Equation ($38a$) with respect to the deflection (w) the governing equation is obtained and presented in a Simplified form as Equation (39).

$$\frac{12}{t^2}\left(a\frac{\partial\phi_R}{\partial R}+\frac{\partial^2w}{\partial R^2}\right)a_{44}+\frac{12}{t^2}\left(a\frac{\partial\phi_Q}{\partial R}+\frac{a}{\beta}\frac{\partial\phi_R}{\partial Q}+\frac{2}{\beta}\frac{\partial^2w}{\partial R\partial Q}\right)a_{45}+\frac{12}{t^2}\left(\frac{a}{\beta}\frac{\partial\phi_Q}{\partial Q}+\frac{1}{\beta^2}\frac{\partial^2w}{\partial Q^2}\right)a_{55}-\frac{qa^2}{D_{00}}=0 \qquad (39)$$

Minimizing Equation ($38a$) with respect to ϕ_R gives compatibility equation in x-z plane as Equation (40).

$$\frac{\partial^2 \phi_R}{\partial R^2}.a_{11} + \frac{a_{xy}}{\beta}\frac{\partial^2 \phi_Q}{\partial R \partial Q} + \left[\frac{2}{\beta}\frac{\partial^2 \phi_R}{\partial R \partial Q} + \frac{\partial^2 \phi_Q}{\partial R^2}\right]a_{13} + \left[\frac{1}{\beta^2}\frac{\partial^2 \phi_Q}{\partial Q^2}\right]a_{23} + \frac{12}{t^2}\left(a^2\phi_R + a\frac{\partial w}{\partial R}\right)a_{44} + \frac{12}{t^2}\left(a^2\phi_Q + \frac{a}{\beta}\frac{\partial w}{\partial Q}\right)a_{45} = 0 \quad (40)$$

Minimizing Equation ($38a$) with respect to ϕ_Q gives compatibility equation in y-z plane as Equation (41).

$$\frac{a_{xy}}{\beta}\frac{\partial^2 \phi_R}{\partial R \partial Q} + \frac{1}{\beta^2}\frac{\partial^2 \phi_Q}{\partial Q^2}a_{22} + \frac{\partial^2 \phi_R}{\partial R^2}a_{13} + \left[\frac{1}{\beta^2}\frac{\partial^2 \phi_R}{\partial Q^2} + \frac{2}{\beta}\frac{\partial^2 \phi_Q}{\partial R \partial Q}\right]a_{23} + \frac{12}{t^2}\left(a^2\phi_R + a\frac{\partial w}{\partial R}\right)a_{45} + \frac{12}{t^2}\left(a^2\phi_Q + \frac{a}{\beta}\frac{\partial w}{\partial Q}\right)a_{55} = 0 \quad (41)$$

VII. DETERMINATION OF THE GENERAL DISPLACEMENT EQUATIONS

By solving equations (39), (40) and (41) and simplifying the resulting equation, Equation (42) is obtained.

$$w = (a_{00} + a_{01}R + a_{02}R^2 + a_{03}R^3 + a_{04}R^4)(b_{00} + b_{01}Q + b_{02}Q^2 + b_{03}Q^3 + b_{04}Q^4) \quad (42a)$$

Equation $(42a)$ can be written as Equation $(42b)$

$$w = A_1 h \quad (42b)$$

Where:

$$h = [1 \quad R \quad R^2 \quad R^4 \quad R^4].\ [1 \quad Q \quad Q^2 \quad Q^4 \quad Q^4] \quad (42c)$$

A_1 is the coefficient of deflection; h is the shape function

By substituting Equation (42b) into the non-dimensional form of Equations (24) and (25) respectively gives equations (43) and (44).

$$\phi_R = \frac{c_x}{a}.A_1.\frac{\partial h}{\partial R} = \frac{A_2}{a}.\frac{\partial h}{\partial R} \quad (43)$$

$$\phi_Q = \frac{A_3}{a\beta}.\frac{\partial h}{\partial Q} \quad (44)$$

Where:

$$A_2 = A_1.\ c_x\,;\, A_3 = A_1.\ c_y$$

Equations ($42a$) can be written in split deflection form as Equation (45).

$$w = w_R \times w_Q \quad (45)$$

Where: w_R and w_Q are expressed respectively as Equations ($46a$) and ($46b$).

$$w_R = (a_{00} + a_{01}R + a_{02}R^2 + a_{03}R^3 + a_{04}R^4) \quad (46a)$$

$$w_Q = (b_{00} + b_{01}Q + b_{02}Q^2 + b_{03}Q^3 + b_{04}Q^4) \qquad (46b)$$

Equation ($46a$) is the deflection equation of a strip of the rectangular plate along x axis while Equation ($46b$) is the deflection equation of a strip of the rectangular plate along y axis.

Equations ($46a$) and ($46b$) is written in generalized form as Equation (47).

$$w_\propto = (\Delta_{00} + \Delta_{01}\propto + \Delta_{02}\propto^2 + \Delta_{03}\propto^3 + \Delta_{04}\propto^4) \qquad (47)$$

Where $\propto$ can be *R* or *Q* as the case may be and Δ can be a or b as the case may be.

Equation (47) is the generalized split deflection equation.

VIII. PECULIAR DISPLACEMENT EQUATIONS

Peculiar displacement equations are obtained by substituting respectively the boundary conditions of a particular plate with specified edge conditions into the general deflection polynomial equation of the thick anisotropic plate. Carrying out this procedure using the generalized split deflection equation gave the following deflection equations of the plate along simply supported (S-S) strip and clamped at one end and simply supported at the other end (C-S) strip as presented on Equations (48) and (49) respectively.

$$\mathrm{S-S} = \Delta_{04}(\propto - 2\propto^3 + \propto^4) \qquad (48)$$

$$\mathrm{S-C} = \Delta_{04}(1.5\propto^2 - 2.5\propto^3 + \propto^4) \qquad (49)$$

By combining the peculiar deflections along various strips, the peculiar deflection equations for plate of various support conditions are obtained. This are summarized on Table 1

IX. DETERMINATION OF THE FORMULAS FOR CALCULATING DEFLECTION COEFFICIENTS, DISPLACEMENTS AND STRESSES

Substituting Equations (42*a*), (43) and (44) into Equation (38*b*) and minimizing the resulting equation with respect to the coefficient of deflection (A_1) and coefficient of x-z shear rotation along x-direction (A_2) and coefficient of y-z shear rotation along y-direction (A_3) respectively gave Equations (50), (51) and (52) respectively.

$$\frac{d\Pi}{dA_1} = \frac{\beta D_{00}}{2}\{24a_{44}\left(\frac{a}{t}\right)^2(A_1 + A_2)k_{N_R} + \frac{24a_{45}}{\beta}\left(\frac{a}{t}\right)^2(2A_1 + A_2 + A_3)k_{N_{RQ}} + \frac{24a_{55}}{\beta^2}\left(\frac{a}{t}\right)^2(A_1 + A_3)k_{N_Q} - 2\frac{qa^4}{D_{00}}k_q\} = 0 \qquad (50)$$

$$\frac{d\Pi}{dA_2} = \frac{\beta D_{00}}{2}\{2A_2a_{11}k_R + 2A_3\frac{a_{xy}}{\beta^2}k_{RQ} + \frac{2a_{13}}{\beta}[2A_2 + A_3]k_{RRQ} + \frac{2a_{23}}{\beta^3}[A_3]k_{RQQ} + 12a_{44}\left(\frac{a}{t}\right)^2(2A_1 + 2A_2)k_{N_R} + \frac{24a_{45}}{\beta}\left(\frac{a}{t}\right)^2(A_1 + A_3)k_{N_{RQ}}\} = 0 \qquad (51)$$

$$\frac{d\Pi}{dA_3} = \frac{\beta D_{00}}{2}\{2A_2\frac{a_{xy}}{\beta^2}k_{RQ} + 2A_3\frac{a_{22}}{\beta^4}k_Q + \frac{2a_{13}}{\beta}[A_2]k_{RRQ} + \frac{2a_{23}}{\beta^3}[A_2 + 2A_3]k_{RQQ} + \frac{24a_{45}}{\beta}\left(\frac{a}{t}\right)^2(A_1 + A_2)k_{N_{RQ}} + \frac{12a_{55}}{\beta^2}\left(\frac{a}{t}\right)^2(2A_1 + 2A_3)k_{N_Q}\} = 0 \qquad (52)$$

Note:

$$k_R = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial R^2}\right)^2 dRdQ; \quad k_{RQ} = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial R \partial Q}\right)^2 dRdQ \quad k_Q = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial Q^2}\right)^2 dRdQ;$$

$$k_{RRQ} = \int_0^1 \int_0^1 \frac{\partial^2 h}{\partial R \partial Q} \frac{\partial^2 h}{\partial R^2} dRdQ; \quad k_{RQQ} = \int_0^1 \int_0^1 \frac{\partial^2 h}{\partial R \partial Q} \frac{\partial^2 h}{\partial Q^2} dRdQ; k_{N_R} = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial R}\right)^2 dRdQ;$$

$$k_{N_Q} = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial Q}\right)^2 dRdQ; \quad k_{N_{RQ}} = \int_0^1 \int_0^1 \frac{\partial h}{\partial R} \cdot \frac{\partial h}{\partial Q} dRdQ; \quad k_q = \int_0^1 \int_0^1 h \, dRdQ$$

By solving simultaneously, the simplified form of Equations (51) and (52) gave Equations (53) and (54)

Table 1 The Deflection of the Plate for SSSS and SSCC Boundary Conditions

Plate	Strip R	Strip Q	Deflection along R strip	Deflection along Q strip	Peculiar Deflection equations
SSSS	S-S	S-S	$a_{04}(R - 2R^3 + R^4)$	$b_{04}(Q - 2Q^3 + Q^4)$	$A_1(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$
CCSS	C-S	C-S	$a_{04}(1.5R^2 - 2.5R^3 + R^4)$	$b_{04}(1.5Q^2 - 2.5Q^3 + Q^4)$	$A_1(1.5R^2 - 2.5R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)$

$$A_2 = T_2 A_1 = A_1 \frac{(d_{12} \cdot d_{23} - d_{13} \cdot d_{22})}{({d_{12}}^2 - d_{11} d_{22})} \qquad (53)$$

$$A_3 = T_3 A_1 = A_1 \frac{(d_{12} \cdot d_{13} - d_{11} d_{23})}{({d_{12}}^2 - d_{11} d_{22})} \qquad (54)$$

By substituting equations (53) and (54) into the simplified form of Equation (50) and making deflection coefficient (A_1) the subject gives equation (55).

$$A_1 = \frac{q a^4}{D_{00}} \cdot \frac{k_q}{k_T} \qquad (55)$$

Where:

$$d_{11} = a_{11} k_R + 2\frac{a_{13}}{\beta} k_{RRQ} + 12 a_{44} \left(\frac{a}{t}\right)^2 k_{N_R};$$

$$d_{12} = \frac{a_{xy}}{\beta^2} k_{RQ} + \frac{a_{13}}{\beta} k_{RRQ} + \frac{a_{23}}{\beta^3} k_{RQQ} + \frac{12 a_{45}}{\beta} \left(\frac{a}{t}\right)^2 k_{NRQ};$$

$$d_{13} = -12\left(\frac{a}{t}\right)^2 \left[a_{44} k_{N_R} + \frac{a_{45}}{\beta} k_{N_{RQ}}\right]; \quad d_{22} = \frac{a_{22}}{\beta^4} k_Q + 2\frac{a_{23}}{\beta^3} k_{RQQ} + \frac{12 a_{55}}{\beta^2} \left(\frac{a}{t}\right)^2 k_{N_Q};$$

$$d_{23} = -12\left(\frac{a}{t}\right)^2 \left[\frac{a_{45}}{\beta} k_{N_{RQ}} + \frac{a_{55}}{\beta^2} k_{N_Q}\right];$$

$$k_T = 12 a_{44} \left(\frac{a}{t}\right)^2 (1 + T_2) k_{N_R} + \frac{12 a_{45}}{\beta} \left(\frac{a}{t}\right)^2 (2 + T_2 + T_3) k_{N_{RQ}} + \frac{12 a_{55}}{\beta^2} \left(\frac{a}{t}\right)^2 (1 + T_3) k_{N_Q}$$

Substituting Equation (55) into Equation (42b) and simplifying gives Equation (56).

$$\frac{w E_0 t^3}{q a^4} = 12(1 - \mu_{12}\mu_{21}) \cdot \frac{k_q}{k_T} \cdot h \qquad (56)$$

Substituting Equation (53) and (54) into Equations (43) and (44) respectively gave Equations (57) and (58) respectively

$$\phi_R = \frac{T_2 A_1}{a} \cdot \frac{\partial h}{\partial R} = 12(1-\mu_{12}\mu_{21}) T_2 \cdot \frac{q}{E_0}\left(\frac{a}{t}\right)^3 \cdot \frac{k_q}{k_T} \cdot \frac{\partial h}{\partial R} \qquad (57)$$

$$\phi_Q = \frac{T_3 A_1}{a\beta} \cdot \frac{\partial h}{\partial Q} = 12(1-\mu_{12}\mu_{21}) \frac{T_3}{\beta} \cdot \frac{q}{E_0}\left(\frac{a}{t}\right)^3 \cdot \frac{k_q}{k_T} \cdot \frac{\partial h}{\partial Q} \qquad (58)$$

Substituting Equations (57) and (58) into Equations (17) and (18) respectively and simplifying the resulting equations gives Equations (59) and (60)

$$\frac{uE_0 t^2}{qa^3} = 12S(1-\mu_{12}\mu_{21}) T_2 \cdot \frac{k_q}{k_T} \cdot \frac{\partial h}{\partial R} \qquad (59)$$

$$\frac{vE_0 t^2}{qa^3} = 12S(1-\mu_{12}\mu_{21}) \frac{T_3}{\beta} \cdot \frac{k_q}{k_T} \cdot \frac{\partial h}{\partial Q} \qquad (60)$$

Substituting Equations (56), (57) and (58) into Equation (33*a*) and simplifying the resulting equations, Equations (61), (62), (63), (64), (65) and (66) are obtained.

$$\frac{\sigma_{xx} t^2}{qa^2} = 12\, \frac{k_q}{k_T} \cdot P_{xx} \qquad (61)$$

$$\frac{\sigma_{yy} t^2}{qa^2} = 12\, \frac{k_q}{k_T} \cdot P_{yy} \qquad (62)$$

$$\frac{\tau_{xy} t^2}{qa^2} = 12\, \frac{k_q}{k_T} \cdot P_{xy} \qquad (63)$$

$$\frac{\tau_{xz} t^2}{qa^2} = 12\, \frac{k_q}{k_T} \cdot P_{xz} \qquad (64)$$

$$\frac{\tau_{yz} t^2}{qa^2} = 12\, \frac{k_q}{k_T} \cdot P_{yz} \qquad (65)$$

Where:

$$P_{xx} = a_{11}\frac{\partial^2 h}{\partial R^2} \cdot T_2 S + a_{12}\frac{\partial^2 h}{\partial Q^2} \cdot \frac{T_3}{\beta^2} S + a_{13}(T_2+T_3)\frac{\partial^2 h}{\partial R \partial Q}\frac{S}{\beta};\ P_{yy} = a_{12}\frac{\partial^2 h}{\partial R^2} \cdot T_2 S + a_{22}\frac{\partial^2 h}{\partial Q^2} \cdot \frac{T_3}{\beta^2} S + a_{23}(T_2+T_3)\frac{\partial^2 h}{\partial R \partial Q}\frac{S}{\beta};$$

$$P_{xy} = a_{13}\frac{\partial^2 h}{\partial R^2} \cdot T_2 S + a_{23}\frac{\partial^2 h}{\partial Q^2} \cdot \frac{T_3}{\beta^2} S + a_{33}(T_2+T_3)\frac{\partial^2 h}{\partial R \partial Q}\frac{S}{\beta};\ P_{xz} = a_{44}(T_2+1)\left(\frac{a}{t}\right)\frac{\partial h}{\partial R} + a_{45}(T_3+1)\left(\frac{a}{t}\right)\frac{1}{\beta}\frac{\partial h}{\partial Q};$$

$$P_{yz} = a_{45}(T_2+1)\left(\frac{a}{t}\right)\frac{\partial h}{\partial R} + a_{55}(T_3+1)\left(\frac{a}{t}\right)\frac{1}{\beta}\frac{\partial h}{\partial Q}$$

X. NUMERICAL PROBLEMS

For anisotropic rectangular plate of two different support conditions (SSSS and SSCC) that are subjected to bending, the numerical values of the in-plane displacements in x and y axis (u and v), central deflection (w), in-plane stresses (σ_{xx}, σ_{yy}, and τ_{xy}) and out-plane stresses (τ_{xz} and τ_{yz}) were obtained at span to depth ratios (a/t) of 7.142857, 10 and 20, aspect ratio (β) of 0.5 to 2 and grain fibres orientation angle (θ) of 0^0. For SSSS anisotropic rectangular plate: in-plane displacements were calculated at coordinates: u $(x = 0,\ y = {}^{b}/_{2},\ z = \pm {}^{h}/_{2})$; $v(x = {}^{a}/_{2}, y = 0, z = \pm {}^{h}/_{2})$; out of plane displacement was calculated at coordinates: w $(x = {}^{a}/_{2},\ y = {}^{b}/_{2})$; in-plane stresses were calculated at coordinates: σ_{xx}, σ_{yy} $(x = {}^{a}/_{2},\ y = {}^{b}/_{2},\ z = \pm {}^{h}/_{2})$; in- plane shear stress was calculated at coordinates: τ_{xy} $(x = 0, y = 0, z = \pm {}^{h}/_{2})$; out of plane shear stresses was calculated at coordinates: τ_{xz} $(x = 0,\ y = {}^{b}/_{2})$; τ_{yz} $(x = {}^{a}/_{2},\ y = 0)$. For CCSS anisotropic rectangular plate: in-plane displacements were calculated at coordinates: u $(x = {}^{a}/_{4}, y = {}^{b}/_{2}, z = \pm {}^{h}/_{2})$;$v(x = {}^{a}/_{2}, y = {}^{b}/_{4}, z = \pm {}^{h}/_{2})$; out of plane displacement was calculated at coordinates: w $(x = {}^{a}/_{2},\ y = {}^{b}/_{2})$; in-plane stresses were calculated at coordinates: σ_{xx}, σ_{yy} $(x = {}^{a}/_{2},\ y = {}^{b}/_{2},$ $z = \pm {}^{h}/_{2})$; in- plane shear stress was calculated at coordinates: τ_{xy} $(x = {}^{a}/_{5}, y = {}^{b}/_{5}, z = \pm {}^{h}/_{2})$; out of plane shear stresses was calculated at coordinates: τ_{xz} $(x = {}^{a}/_{5}, y = {}^{b}/_{2})$; τ_{yz} $(x = {}^{a}/_{2}, y = {}^{b}/_{5})$. Both plates are subjected to load that is uniformly distributed on the surface of the plates. The material properties used are as follows: $\frac{E_2}{E_1} = 0.52500, \frac{G_{12}}{E_1} = 0.26293, \frac{G_{13}}{E_1} = 0.15991 \frac{G_{23}}{E_1} = 0.26681, \mu_{12} = 0.44049$ and $\mu_{21} = 0.23124$.

XI. PRESENTATION OF NUMERICAL RESULTS

The numerical problem results on bending analysis of SSSS and CCSS anisotropic rectangular plate with specified material properties as given in the section above and for aspect ratios (β = b/a= 0.5 to 2), span-depth ratios (a/t = 7.142857, 10 and 20) and grain fibre orientation (θ = 0^0) obtained from Equations (56), (59), (60), (61), (62), (63), (64) and (65) are presented on Tables 2 to 7. The present study is validated by compering its solution with those obtained from previous research works of Shimpi and Patel (2006), Reddy (1984) and Srinivas and Roa (1970). The out of plane displacement parameter values ($\overline{w}$), the x-axis in-plane stress parameter values ($\overline{\sigma_{xx}}$) and the y-axis in-plane stress parameter values ($\overline{\sigma_{yy}}$), were calculated at the same coordinates as presented in the section above using the following formula: $\overline{w} = \frac{wE_0}{qt*(1-\mu_{12}\mu_{21})}$; $\overline{\sigma_{xx}} = (\frac{\sigma_x}{q})$; $\overline{\sigma_{yy}} = (\frac{\sigma_x}{q})$. it is importance to note the following: If β = 0.5 for previous authors; for this study $\frac{1}{\beta} = \frac{1}{0.5} = 2.0$. If β = 1.0 for previous authors; for this study $\frac{1}{\beta} = \frac{1}{1} = 1.0$. If β = 2.0 for previous authors; for this study $\frac{1}{\beta} = \frac{1}{2} = 0.5$. If $\frac{t}{a} = 0.05$ for previous authors; for this study $\frac{a}{t} = \frac{1}{0.05} = 20$.

The obtained results compared with the results of previous research works are presented on Tables 8 to 10.

Table 2 Displacement and stress parameter values for SSSS plate at a/t =7.142857 and θ = 0^o

β	$\frac{wE_{22}t^3}{qa^4}$	$\frac{uE_{22}t^2}{qa^3}$	$\frac{vE_{22}t^2}{qa^3}$	$\frac{\sigma_{xx}t^2}{qa^2}$	$\frac{\sigma_{yy}t^2}{qa^2}$	$\frac{\tau_{xy}t^2}{qa^2}$	$\frac{\tau_{xz}t^2}{qa^2}$	$\frac{\tau_{yz}t^2}{qa^2}$
0.5	0.0148	-0.0156	-0.0403	0.1143	0.1533	-0.0601	0.0185	0.0273
1	0.0697	-0.0929	-0.1035	0.3902	0.2532	-0.1652	0.0425	0.0307
1.1	0.0806	-0.1089	-0.1094	0.4406	0.2585	-0.1753	0.0459	0.0300
1.2	0.0908	-0.1239	-0.1134	0.4868	0.2614	-0.1823	0.0489	0.0292
1.3	0.1001	-0.1377	-0.1158	0.5287	0.2625	-0.1865	0.0514	0.0283
1.4	0.1086	-0.1503	-0.1169	0.5664	0.2625	-0.1887	0.0536	0.0274
1.5	0.1162	-0.1617	-0.1171	0.6003	0.2617	-0.1892	0.0556	0.0265
1.6	0.1231	-0.1719	-0.1164	0.6305	0.2604	-0.1884	0.0573	0.0255
1.7	0.1293	-0.1812	-0.1152	0.6576	0.2588	-0.1866	0.0587	0.0246
1.8	0.1349	-0.1895	-0.1137	0.6817	0.2571	-0.1842	0.0600	0.0237
1.9	0.1398	-0.1970	-0.1118	0.7033	0.2553	-0.1813	0.0612	0.0229
2	0.1443	-0.2037	-0.1097	0.7227	0.2535	-0.1780	0.0622	0.0221

Table 3 Displacement and stress parameter values for SSSS plate at a/t =10 and θ = 0^o

β	$\frac{wE_{22}t^3}{qa^4}$	$\frac{uE_{22}t^2}{qa^3}$	$\frac{vE_{22}t^2}{qa^3}$	$\frac{\sigma_{xx}t^2}{qa^2}$	$\frac{\sigma_{yy}t^2}{qa^2}$	$\frac{\tau_{xy}t^2}{qa^2}$	$\frac{\tau_{xz}t^2}{qa^2}$	$\frac{\tau_{yz}t^2}{qa^2}$
0.5	0.0134	-0.0173	-0.0391	0.1181	0.1506	-0.0620	0.0133	0.0195
1	0.0654	-0.0951	-0.1005	0.3951	0.2496	-0.1645	0.0304	0.0219
1.1	0.0758	-0.1110	-0.1062	0.4453	0.2550	-0.1743	0.0328	0.0215
1.2	0.0855	-0.1259	-0.1101	0.4913	0.2581	-0.1809	0.0349	0.0209
1.3	0.0944	-0.1396	-0.1124	0.5329	0.2594	-0.1849	0.0367	0.0202
1.4	0.1025	-0.1520	-0.1135	0.5704	0.2596	-0.1868	0.0383	0.0196
1.5	0.1098	-0.1633	-0.1136	0.6040	0.2590	-0.1872	0.0397	0.0189
1.6	0.1164	-0.1735	-0.1130	0.6340	0.2578	-0.1863	0.0409	0.0182
1.7	0.1223	-0.1826	-0.1118	0.6608	0.2564	-0.1845	0.0420	0.0176
1.8	0.1276	-0.1908	-0.1103	0.6847	0.2548	-0.1820	0.0429	0.0169
1.9	0.1324	-0.1982	-0.1084	0.7061	0.2532	-0.1790	0.0437	0.0163
2	0.1367	-0.2048	-0.1064	0.7253	0.2515	-0.1757	0.0444	0.0158

Table 4 Displacement and stress parameter values for SSSS plate at a/t =20 and θ = 0^o

β	$\frac{wE_{22}t^3}{qa^4}$	$\frac{uE_{22}t^2}{qa^3}$	$\frac{vE_{22}t^2}{qa^3}$	$\frac{\sigma_{xx}t^2}{qa^2}$	$\frac{\sigma_{yy}t^2}{qa^2}$	$\frac{\tau_{xy}t^2}{qa^2}$	$\frac{\tau_{xz}t^2}{qa^2}$	$\frac{\tau_{yz}t^2}{qa^2}$
0.5	0.0122	-0.0186	-0.0383	0.1211	0.1485	-0.0634	0.0066	0.0098
1	0.0620	-0.0968	-0.0981	0.3990	0.2468	-0.1639	0.0152	0.0110
1.1	0.0720	-0.1126	-0.1037	0.4490	0.2523	-0.1734	0.0164	0.0107
1.2	0.0813	-0.1274	-0.1075	0.4948	0.2555	-0.1798	0.0175	0.0104
1.3	0.0899	-0.1410	-0.1097	0.5362	0.2570	-0.1836	0.0184	0.0101
1.4	0.0977	-0.1534	-0.1108	0.5735	0.2573	-0.1854	0.0192	0.0098
1.5	0.1048	-0.1646	-0.1109	0.6069	0.2568	-0.1856	0.0199	0.0094
1.6	0.1112	-0.1747	-0.1103	0.6367	0.2558	-0.1847	0.0205	0.0091
1.7	0.1169	-0.1837	-0.1092	0.6633	0.2545	-0.1828	0.0210	0.0088
1.8	0.1220	-0.1919	-0.1077	0.6871	0.2531	-0.1803	0.0214	0.0085
1.9	0.1266	-0.1992	-0.1059	0.7083	0.2516	-0.1773	0.0219	0.0082
2	0.1308	-0.2057	-0.1039	0.7273	0.2500	-0.1739	0.0222	0.0079

Table 5 Displacement and stress parameter values for CCSS plate at a/t =7.142857 and θ = 0^o

β	$\frac{wE_{22}t^3}{qa^4}$	$\frac{uE_{22}t^2}{qa^3}$	$\frac{vE_{22}t^2}{qa^3}$	$\frac{\sigma_{xx}t^2}{qa^2}$	$\frac{\sigma_{yy}t^2}{qa^2}$	$\frac{\tau_{xy}t^2}{qa^2}$	$\frac{\tau_{xz}t^2}{qa^2}$	$\frac{\tau_{yz}t^2}{qa^2}$
0.5	0.0079	-0.0064	-0.0161	0.0671	0.0893	-0.0195	0.0098	0.0204
1	0.0381	-0.0388	-0.0467	0.2413	0.1629	-0.0577	0.0299	0.0207
1.1	0.0437	-0.0452	-0.0493	0.2701	0.1652	-0.0610	0.0328	0.0195
1.2	0.0487	-0.0509	-0.0508	0.2951	0.1653	-0.0629	0.0353	0.0183
1.3	0.0531	-0.0560	-0.0515	0.3165	0.1639	-0.0638	0.0375	0.0171
1.4	0.0568	-0.0604	-0.0515	0.3347	0.1617	-0.0638	0.0392	0.0159
1.5	0.0601	-0.0642	-0.0511	0.3502	0.1590	-0.0633	0.0407	0.0149
1.6	0.0629	-0.0675	-0.0503	0.3634	0.1560	-0.0624	0.0419	0.0139
1.7	0.0653	-0.0704	-0.0493	0.3746	0.1531	-0.0612	0.0429	0.0130
1.8	0.0674	-0.0729	-0.0482	0.3842	0.1502	-0.0598	0.0438	0.0122
1.9	0.0693	-0.0751	-0.0470	0.3925	0.1474	-0.0583	0.0445	0.0115
2	0.0708	-0.0770	-0.0457	0.3996	0.1448	-0.0568	0.0451	0.0109

Table 6 Displacement and stress parameter values for CCSS plate at a/t =10 and θ = 0^o

β	$\frac{wE_{22}t^3}{qa^4}$	$\frac{uE_{22}t^2}{qa^3}$	$\frac{vE_{22}t^2}{qa^3}$	$\frac{\sigma_{xx}t^2}{qa^2}$	$\frac{\sigma_{yy}t^2}{qa^2}$	$\frac{\tau_{xy}t^2}{qa^2}$	$\frac{\tau_{xz}t^2}{qa^2}$	$\frac{\tau_{yz}t^2}{qa^2}$
0.5	0.0068	-0.0070	-0.0158	0.0694	0.0884	-0.0201	0.0072	0.0145
1	0.0344	-0.0403	-0.0445	0.2459	0.1588	-0.0572	0.0216	0.0146

1.1	0.0395	-0.0467	-0.0467	0.2745	0.1608	-0.0601	0.0237	0.0137
1.2	0.0441	-0.0523	-0.0480	0.2992	0.1608	-0.0618	0.0254	0.0128
1.3	0.0480	-0.0573	-0.0485	0.3203	0.1595	-0.0624	0.0269	0.0120
1.4	0.0514	-0.0616	-0.0483	0.3382	0.1573	-0.0623	0.0281	0.0112
1.5	0.0544	-0.0653	-0.0478	0.3533	0.1548	-0.0616	0.0292	0.0104
1.6	0.0569	-0.0686	-0.0470	0.3662	0.1520	-0.0606	0.0300	0.0098
1.7	0.0591	-0.0714	-0.0460	0.3771	0.1493	-0.0594	0.0307	0.0092
1.8	0.0610	-0.0738	-0.0449	0.3864	0.1466	-0.0580	0.0313	0.0086
1.9	0.0627	-0.0759	-0.0438	0.3945	0.1440	-0.0565	0.0319	0.0081

Table 7 Displacement and stress parameter values for CCSS plate at a/t =20 and θ = 0

β	$\frac{wE_{22}t^3}{qa^4}$	$\frac{uE_{22}t^2}{qa^3}$	$\frac{vE_{22}t^2}{qa^3}$	$\frac{\sigma_{xx}t^2}{qa^2}$	$\frac{\sigma_{yy}t^2}{qa^2}$	$\frac{\tau_{xy}t^2}{qa^2}$	$\frac{\tau_{xz}t^2}{qa^2}$	$\frac{\tau_{yz}t^2}{qa^2}$
0.5	0.0059	-0.0075	-0.0155	0.0714	0.0876	-0.0206	0.0036	0.0072
1	0.0315	-0.0416	-0.0426	0.2498	0.1554	-0.0568	0.0109	0.0072
1.1	0.0362	-0.0479	-0.0446	0.2782	0.1573	-0.0594	0.0119	0.0068
1.2	0.0404	-0.0535	-0.0457	0.3026	0.1572	-0.0609	0.0128	0.0063
1.3	0.0440	-0.0584	-0.0460	0.3233	0.1558	-0.0613	0.0135	0.0059
1.4	0.0472	-0.0626	-0.0458	0.3409	0.1538	-0.0611	0.0141	0.0055
1.5	0.0499	-0.0662	-0.0452	0.3558	0.1514	-0.0603	0.0146	0.0052
1.6	0.0522	-0.0694	-0.0444	0.3684	0.1488	-0.0592	0.0151	0.0048
1.7	0.0543	-0.0721	-0.0434	0.3791	0.1462	-0.0579	0.0154	0.0045
1.8	0.0560	-0.0745	-0.0424	0.3882	0.1437	-0.0565	0.0157	0.0043
1.9	0.0575	-0.0765	-0.0412	0.3961	0.1413	-0.0550	0.0160	0.0040
2	0.0588	-0.0783	-0.0401	0.4028	0.1391	-0.0534	0.0162	0.0038

Table 8 Out of plane displacement parameter results of present study compared with the results of previous research for SSSS thick orthotropic rectangular plate at θ = 0^o.

β	Theory	$\frac{wE_0}{qt*(1-\mu_{12}\mu_{21})}$		
		a/t		
		7.142857	**10**	**20**
0.5	Present study (P)	42.89	149.19	2173.36
	Shimpi and Patel (2006), (SP)	39.26	137.82	2042.74
	Reddy (1984), (R)	40.21	139.8	2051.0
	Srinivas and Roa (1970), (SR)	39.79	139.08	2048.7
	% Difference between P and SP	8.46	7.62	6.01
	% Difference between P and R	6.25	6.29	5.63
	% Difference between P and SR	7.23	6.78	5.74
1	Present Study (P)	202.01	728.17	11044.95
	Shimpi and Patel (2006), (SP)	187.75	681.73	10413.4
	Reddy (1984), (R)	191.60	689.5	10450.0
	Srinivas and Roa (1970), (SR)	191.07	688.57	10443.0
	% Difference between P and SP	7.06	6.38	5.72
	% Difference between P and R	5.15	5.31	5.39
	% Difference between P and SR	5.42	5.44	5.45
2	Present study (P)	418.22	1522.02	23301.28
	Shimpi and Patel (2006), (SP)	384.20	1402.24	21513.5
	Reddy (1984), (R)	387.5	1408.5	21542.0
	Srinivas and Roa (1970), (SR)	387.23	1408.5	21542.0
	% Difference between P and SP	8.13	7.87	7.67
	% Difference between P and R	7.35	7.46	7.55
	% Difference between P and SR	7.35	7.46	7.55

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Table 9 *x*-axis in-plane stress parameter results of present study compared with the results of previous research for SSSS thick orthotropic rectangular plate at θ = 0^o.

β	Theory	$(\frac{\sigma_x}{q})$		
		a/t		
		7.142857	**10**	**20**
0.5	Present study (P)	5.8316	11.81	48.44
	Shimpi and Patel (2006), (SP)	5.32	10.33	40.98
	Reddy (1984), (R)	5.068	10.05	40.67
	Srinivas and Roa (1970), (SR)	5.0364	10.025	40.657
	% Difference between P and SP	8.77	12.53	15.40
	% Difference between P and R	13.09	14.90	16.04
	% Difference between P and SR	13.63	15.11	16.07
1	Present Study (P)	19.9082	39.51	159.6
	Shimpi and Patel (2006), (SP)	18.68	36.36	144.68
	Reddy (1984), (R)	18.34	36.01	144.3
	Srinivas and Roa (1970), (SR)	18.346	36.021	144.31
	% Difference between P and SP	6.17	7.97	9.35
	% Difference between P and R	7.88	8.86	9.59
	% Difference between P and SR	7.85	8.83	9.58
2	Present study (P)	36.872	72.53	290.92
	Shimpi and Patel (2006), (SP)	33.96	66.07	262.78
	Reddy (1984), (R)	33.84	65.95	262.6
	Srinivas and Roa (1970), (SR)	33.862	65.975	262.67
	% Difference between P and SP	7.89	8.91	9.67
	% Difference between P and R	8.22	9.07	9.73
	% Difference between P and SR	8.16	9.04	9.71

Table 10 y-axis in-plane stress parameter results of present study compared with the results of previous research for SSSS thick orthotropic rectangular plate at θ = 0^o.

β	Theory	$(\frac{\sigma_y}{q})$		
		a/t		
		7.142857	**10**	**20**
0.5	Present study (P)	7.821	15.06	59.4
	Shimpi and Patel (2006), (SP)	7.06	13.65	54.04
	Srinivas and Roa (1970), (SR)	7.2794	13.888	54.279
	% Difference between P and SP	9.73	9.36	9.01
	% Difference between P and SR	6.92	7.78	8.62
1	Present Study (P)	12.918	24.96	98.72
	Shimpi and Patel (2006), (SP)	11.21	21.80	86.68
	Srinivas and Roa (1970), (SR)	11.612	22.21	87.08
	% Difference between P and SP	13.22	12.66	12.19
	% Difference between P and SR	10.11	11.02	11.79
2	Present study (P)	12.933	25.15	100
	Shimpi and Patel (2006), (SP)	10.25	19.94	79.30
	Srinivas and Roa (1970), (SR)	10.515	20.204	79.545
	% Difference between P and SP	20.75	20.72	20.7
	% Difference between P and SR	18.69	19.67	20.46

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XII. DISCUSSION ON THE RESULTS OF BENDING ANALYSIS OF SSSS AND CCSS THICK ANISOTROPIC RECTANGULAR PLATE

The displacement parameters and stress parameters numerical results for SSSS and CCSS thick anisotropic rectangular plates are presented on Tables 2 to 7. The span to depth ratios (a/t) used are 7.142857, 10 and 20 while 0^o fibre orientation angle (θ) were considered. Aspect ratio (β) used ranges from 0.5 to 2. Each of these tables presented results for a given value of span to depth ratio and fibre orientation angle over the full range of aspect ratios. From Tables 2, 3, 4, 5,6 and 7, the following observation were made: for SSSS and CCSS anisotropic plates, the out-of-plane displacement parameter ($\frac{wE_{22}t^3}{qa^4}$) values increase as the aspect ratios increase but decrease in value as the span to depth ratios increase. This means that out-of-plane displacement depends on the spatial dimensions of the anisotropic plate and acts more on thin plate than on thick plate. The x-axis in-plane displacement parameter ($\frac{uE_{22}t^2}{qa^3}$) yielded negative values, which increased as the aspect ratios and span to depth ratios increased both for SSSS and CCSS anisotropic plates. The y-axis in-plane displacement parameter ($\frac{vE_{22}t^2}{qa^3}$) yielded negative values, which increased as the aspect ratios increased but decreased as span to depth ratios increased, both for SSSS and CCSS anisotropic plates. The x-axis in-plane stress parameter($\frac{\sigma_{xx}t^2}{qa^2}$) values increase as the aspect ratios and span to depth ratios increase, both for SSSS and CCSS anisotropic plates. For SSSS anisotropic plate, the y-axis in-plane stress parameter ($\frac{\sigma_{yy}t^2}{qa^2}$) values increased for 0.5 to 1.4 aspect ratios but decreased moderately for 1.5 to 2 aspects ratios. The y-axis in-plane stress parameter ($\frac{\sigma_{yy}t^2}{qa^2}$) values decreased gradually as span to depth ratios increased. For CCSS anisotropic plate, the y-axis in-plane stress parameter ($\frac{\sigma_{yy}t^2}{qa^2}$) values increased for 0.5 to 1.2 aspect ratios but decreased moderately for 1.3 to 2 aspects ratios but decreased gradually as span to depth ratios increased. The behaviour of the in-plane displacements and stresses are because of the anisotropic nature of the plate. For SSSS anisotropic plate, the in-plane shear stress parameter ($\frac{\tau_{xy}t^2}{qa^2}$) values, increased for 0.5 to 1.5 aspect ratios but decreased in value for aspects ratios of 1.6 to 2. The in-plane shear stress parameter ($\frac{\tau_{xy}t^2}{qa^2}$) increased and decreased in value as span to depth ratios increased for aspect ratio of 0.5 and aspect ratios of 1 to 2 respectively. For CCSS anisotropic plate, the in-plane shear stress parameter ($\frac{\tau_{xy}t^2}{qa^2}$) values increased for 0.5 to 1.3 aspect ratios but decreased in value for aspects ratios of 1.4 to 2. The in-plane shear stress parameter ($\frac{\tau_{xy}t^2}{qa^2}$) increased and decreased in value as the span to depth ratio increased for aspect ratio of 0.5 and 1 to 2, respectively. The out of plane shear stress parameter ($\frac{\tau_{xz}t^2}{qa^2}$) values increase as the aspect ratios increase but decrease in value as the span to depth ratios increase, both for SSSS and CCSS anisotropic plates. The out of plane shear stress parameter ($\frac{\tau_{yz}t^2}{qa^2}$) values increase for aspect ratios of 0.5 to 1.0 and decreased for aspect ratios of 1.1 to 2 but decrease in value as the span to depth ratios increase both for SSSS and CCSS anisotropic plates. This increase and decline in the value of shear stresses are as a result of the anisotropic characteristics of the rectangular plates.

XIII. COMPARISON OF THE SOLUTIONS OF PRESENT WORK WITH THOSE FROM PREVIOUS AUTHORS

Results of the comparison of the solutions of this present study with the solutions of previous researchers as presented on Tables 8 shows that the out-of-plane displacement parameter values obtained by the present theory has the same trend as those of Shimpi and Patel (2006), Reddy (1984), and Srinivas and Roa (1970). The out-of-plane displacement parameter values for both the present theory and those of previous researchers increase as the span to depth ratio increases. It is also seen from Table 8 that the out-of-plane displacement parameter values of present theory and those of past researchers increase as the aspect ratio increases. The out-of-plane displacement parameter values of present theory and that of Shimpi and Patel (2006), for span to depth ratios of 7.142857, 10 and 20 at aspect ratio of 0.5 differ by 8.45%, 7.62%, and 6.01% respectively and the variance between their results has a maximum value of 8.46% at a span to depth ratio of 7.142857, aspect ratio of 0.5 and a minimum value of 5.72% at span to depth ratio of 20, aspect ratio of 1. The out-of-plane displacement parameter values of present theory and that of Reddy (1984), for span to depth ratios of 7.142857, 10 and 20 at aspect ratio of 0.5 differ by 6.25%, 6.29%, and 5.63% respectively and the variance between their results has a maximum value of 7.55% at a span to depth ratio of 20, aspect ratio of 2 and a minimum value of 5.15% at span to depth ratio of 7.142857, aspect ratio of 1. The out of plane displacement parameter values of present theory and that of Srinivas and Roa (1970), for span to depth ratios of 7.142857, 10 and 20 at aspect ratio of 0.5 differ by 7.23%, 6.78%, and 5.74% respectively and the variance between their results has a maximum value of 7.23% at a span to depth ratio of 7.142857, aspect ratio of 0.5 and a minimum value of 5.42% at span to depth ratio of 7.142857, aspect ratio of 1.

Results of the comparison of the solutions of this present study with the solutions of previous researchers as presented on Tables 9 shows that the present study predicts higher values of x-axis in-plane stress parameter for all the span to depth ratios and aspect ratios. The x-axis in-plane stress parameter values predicted by the present theory has the same trend as those predicted by the previous researchers since both their values increased as the span to depth ratio increased. It is also seen from Table 9 that the x-axis in-plane stress parameter values of present theory and those of past researchers increased as aspect ratio increased. The x-axis in-plane stress parameter values of present theory and that of Shimpi and Patel (2006), for span to depth ratios of 7.142857, 10 and 20 at aspect ratio of 0.5 differ by 8.77%, 12.53%, and 15.40% respectively. The variance between the results of the present theory and that of Shimpi and Patel (2006), has a maximum value of 15.40% at a span to depth ratio of 20, aspect ratio of 0.5 and a minimum value of 6.17% at span to depth ratio of 7.142857, aspect ratio of 0.5. The x-axis in-plane stress parameter values of present theory and that of Reddy (1984), for span to depth ratios of 7.142857, 10 and 20 at aspect ratio of 0.5 differ by 13.09%, 14.90%, and 16.04% respectively. The variance between their results has a maximum value of 16.04% at a span to depth ratio of 20, aspect ratio of 0.5 and a minimum value of 7.88% at span to depth ratio of 7.142857, aspect ratio of 0.5. The x-axis in-plane stress parameter values of present theory and that of Srinivas and Roa (1970), for span to depth ratios of 7.142857, 10 and 20 at aspect ratio of 0.5 differ by 13.63%, 15.11%, and 16.07% respectively. The variance between their results has a maximum value of 16.07% at a span to depth ratio of 20, aspect ratio of 0.5 and a minimum value of 7.85% at span to depth ratio of 7.142857, aspect ratio of 0.5.

Results of the comparison of the solutions of this present study with the solutions of previous researchers as presented on Tables 10 shows that the present study predicts higher values of y-axis in-plane stress parameter for all the aspect ratios and span to depth ratios. The y-axis in-plane stress parameter values predicted by the present theory has the same trend as those predicted by the previous researchers, owing to the fact that both their values increase as the span to depth ratio increases. Table 10, shows that the y-axis in-plane stress parameter values of present theory and those of past researchers increase as aspect ratio increases. The y-axis in-plane stress parameter values of present theory and that of Shimpi and Patel (2006), for span to depth ratios of 7.142857, 10 and 20 at aspect ratio of 0.5 differ by 9.73%, 9.36%, and 9.01% respectively. The variance between their results has a maximum value of 20.75% at a span to depth ratio of 7.142857, aspect ratio of 2 and a minimum value of 9.01% at span to depth ratio of 20, aspect ratio of 0.5. The y-axis in-plane stress parameter values of present theory and that of Srinivas and Roa (1970), for span to depth ratios of 7.142857, 10 and 20 at aspect ratio of 0.5 differ by 6.92%, 7.78%, and 8.62% respectively. The variance between their results has a maximum value of 20.46% at a span to depth ratio of 20, aspect ratio of 2 and a minimum value of 6.92% at span to depth ratio of 7.142857, aspect ratio of 0.5.

The differences between the present theory results and those of previous researchers are quite acceptable as being close as can be seen from the percentage difference between them which is within acceptable limits in statistics which justifies that the present study provides good solutions to anisotropic plate problems. These differences are as a result of the different approaches employed by the researchers. The present study used alternative 1 theory which is a modified first order shear deformation theory that used an improved Ritz energy method for analysis while Shimpi and Patel (2006) used a two variable refined plate theory, Srinivas and Roa (1970) used a three-dimensional elasticity theory, Reddy (1984) used a higher order shear deformation theory. It is worth to note that the present theory used polynomial series as its displacement function which it obtained by the direct integration of its governing equation while all the previous researchers assumed their displacement function as a double trigonometric series.

XIV. CONCLUSION

In this work, the present theory is applied to bending analysis of thick anisotropic plate of two boundary conditions (SSSS and CCSS). The solutions of this study are compared with those obtained from previous works. Observations show that the results of stresses and displacement predicted by the present theory are in close agreement with those of previous researchers. The present theory is capable of producing reasonably correct solutions to bending problems of thick anisotropic rectangular plate and can be employed by future researchers to solve thick anisotropic rectangular plate problems.

REFERENCES

[1]. Ibearugbulem, O.M, Ezeh, J.C and Ettu, L.O. (2014). Energy Methods in Theory of Rectangular Plates (Use of Polynomial Shape Functions). Owerri: LIU House of Excellence Ventures

[2]. Ventsel, E. and Krauthammer, T. (2001). Thin Plates and Shell (Theory, Analysis and Applications). New York: Marcel Dekker.

[3]. Gorman, D.J. (1977). Free vibration Analysis of the completely free Rectangular plate by the method of superposition. Journal of sound and vibration, 57(3), 437 – 447.

[4]. Zhao, Y.B; Wei, G.W. and Xiang, Y. (2000). Plate Vibration Under Irregular Internal Supports. International Journal of Solids and Structures. 39(5), 1361 – 1363.

[5]. Ghugal, Y.M and Sayyad, A.S. (2011a). Free Vibration of Thick Orthotropic Plate Using Trigonometric Shear Deformation Theory. Latin American Journal of Solids and Structures, 8(3), 229 – 243.

[6]. Shimpi, R.P., and Patel, H.G. (2006). A Two Variable Refined Plate Theory for Orthotropic Plate Analysis. International Journal of Solids and Structures, 43(22), 473 – 490.

[7]. Mihai, V., and Sergiu, B. (2013). Comparative Analysis of the Bending Theories for Isotropic Plates Case Study. Bul. Inst. Polit, 50(58), 134 – 144.

[8]. Shahrokh, H.H. and Arsaniani, M. (2005). Exact Characteristic Equations for Some of Classical Boundary Conditions of Vibrating Moderately Thick Rectangular Plates. International Journal of Solids and Structures, 42(3), 819 – 853.

[9]. Dawe, D.L., and Roufaeil, O.L. (1980). Rayliegh – Ritz Vibration Analysis of Mindlin Plates. Journal of Sound and Vibration, 69(3), 345 – 359.

[10]. Liew, K.M., Hung, K.C., and Lim, M.K. (1995). Vibration of Mindlin Plates Using Boundary Characteristic Orthogonal Polynomials. Journal of Sound and Vibration, 182(1), 77 – 90.
[11]. Cheung, Y.K., and Zhou, D. (2000). Vibrations of Moderately Thick Rectangular Plate in Terms of a Set of Static Timoshenko Beam Functions. Computers and Structures, 78(6), 757 – 768.
[12]. Sadrnejad, S.A., Daryan, A., and Ziaei, M. (2009). Vibration Equations of Thick Rectangular Plates Using Mindlin Plate Theory. Journal of Computer Science, 5(11), 838 – 842.
[13]. Punit, P., and Hiren, P. (2013). Vibration Analysis of Thick Rectangular Plate with Inline Support Using FEA. International Journal of Engineering Research and Technology. 2(6), 1592-1597.
[14]. Yuwaraj, M.G., and Meghraj, D.P. (2011). Buckling and Vibration of Plates by Hyperbolic Shear Deformation Theory. Journal of Aero-Pace Engineering and Technology, 1(1), 1-12.
[15]. Ghugal, Y.M., and Sayyad, A.S. (2011b). Free Vibration of Thick Isotropic Plates Using Trigonometric Shear Deformation Theory Journal of Soil Mechanics, 3(2), 172-182.
[16]. Shahrjerdi, A., and Mustapha, F. (2011). Second Order Shear Deformation Theory (SSDT) For Free Vibration Analysis of a Functionally Graded Quadrangle Plate. Dr. Natalie Baddour (Ed.), In Tech., 1-78.
[17]. Sayyad, A.S., and Ghugal, Y.M. (2012). Bending and Free vibration analysis of thick Isotropic plates by using exponential shear deformation theory. Applied and computational mechanics, 6(1), 65-82.
[18]. Chikalthankar, S.B., Sayyad, I.I., and Nandedkar, V.M. (2013). Analysis of Orthotropic Plate by Refine Plate Theory. International Journal of Engineering and Advanced Technology (IJEAT), 2(6), 310-315.
[19]. Sayyad A.S. and Ghugal, Y.M. (2014). Buckling and Free Vibration Analysis of Orthotropic Plates by Exponential Shear Deformation Theory. Latin American Journal of Solids and Structures, 11(1), 1298 – 1314.
[20]. Ivo, S., Nikola, V., Dae-seung, C., and Tae–milk, C. (2014). Vibration Analysis of Thick Plates: Analytical and Numerical Approaches. ASME 2014 33rd International Conference on Ocean, Offshore and Arctic Engineering, 4(A), 1-10.
[21]. Ibearugbulem, O.M. (2016). Note on Rectangular Plate Analysis. Deutschland: Lambert Academic publishing.
[22]. Sarangan, S., and Singh, B. N. (2017). Evaluation of free vibration and bending analysis of laminated composite and sandwich plates using non-polynomial zigzag models: C0Finite element formulation. Journal of Aerospace science and Technology, 68(6), 496-508
[23]. Ezeh, J.C., Ibearugbulem O.M., Anya U.C., and Ozioko, H.O. (2020). Analysis of SSSS and CCCC Thick Anisotropic Rectangular Plates Using Exact Displacement Function. Saudi Journal of Civil Engineering, 4(9), 150 – 160.
[24]. Gholami, M., Hassani, A., Mousavi, S.S. and Alashti, R.A. (2019). Bending Analysis of Anisotropic Functionally Graded Plates Based on Three–Dimensional Elasticity. European Journal of Computational Mechanics, 10(1), 1 – 25.
[25]. Lisboa, T.V. and Marczak, R.J. (2018). Adomian Decomposition Method Applied to an Isotropic Thick Plates in Bending. European Journal of Mechanics and Solids. 70(1), 95 – 114
[26]. Aghdam, M.M and Mohammadi, M., (2008). Bending Analysis of Thick Orthotropic Sector Plates with Various Loading and Boundary Conditions. Composite Structures, 88(2), 212 – 218.
[27]. Ibearugbulem, O.M., Ebirim, S.I., Anya, U.C., and Ettu, L.O. (2020). Application of Alternative II Theory to Vibration and Stability Analysis of Thick Rectangular Plates (Isotropic and Orthotropic). Nigerian Journal of Technology, 39(1), 52-62.
[28]. Reddy, J.N. (1984). A Refined Nonlinear Theory of Plates with Transverse Shear Deformation, Int. J. Solids Structures, 20(10), 881 – 896.