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The Effect of Coordinate and Boundary Conditions on Displacement and Strain of Thin Rectangular Plate with Large Deflection

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Abstract:- The objective of this research is to investigate the impact of coordinate and boundary conditions on the displacement and strain properties of a thin rectangular plate subjected to substantial deflection. The formulas for nonlinear displacement and nonlinear strain were found by utilising the Von-Karman strain-displacement equation. The Von-Karman equations were mathematically integrated with regard to the variables x and y, resulting in the determination of the nonlinear displacement in both the x and y directions. The nonlinear displacements were further differentiated with respect to both the x and y coordinates, leading to the derivation of the nonlinear strain-displacement equations. The researchers in the study conducted by Ibearugbulem et al. (2020) employed the total potential energy functional of a thin rectangular plate in their investigation of pure bending. The functional was minimised with respect to displacement, resulting in the derivation of a governing equation and two compatibility equations. The aforementioned equations were subsequently solved in order to obtain the in-plane displacements as a function of deflection. The energy functional was further minimised to determine the coefficient of deflection and produce the various formulas employed in the analysis of plates exhibiting considerable bending. The utilisation of polynomial displacement functions was employed in the analysis of pure bending. The load characteristics that were established were compared to those obtained by Levy and Ibearugbulem, revealing a maximum discrepancy of 21.53% and 18.9% respectively. This supports the current methodology. The nonlinear displacement and strain values for thin rectangular plates of SSSS and CCCC were obtained in two distinct coordinate systems. The initial set of coordinates is characterised by the values (0.5, 0.5, 0.5), whereas the subsequent set of coordinates is defined by the values (0.25, 0.25, 0.5). A comparison was made between the findings obtained from the SSSS and CCCC plates.

Keywords:- *Von-Karman; Nonlinear Kinematic; Coordinate and Boundary Conditions.*

I. INTRODUCTION

The link between engineering strain and displacement is referred to as kinematics in the context of structural mechanics. Therefore, the engineering strains must be defined as displacements. The relationship between displacement and strain can be established with the right definitions for both terms. The three cardinal coordinates—*x*, *y*, and *z*—are included in the displacement field. The letters *u*, *v*, and *w* stand for the three dimensions' *x*, *y*, and *z* coordinates, respectively [7].

Plates, which are initially flat structural components, have significantly smaller thicknesses than their other dimensions. Popular examples of plates include tabletops, street manhole coverings, side panels and roofs of structures, turbine discs, bulkheads, and tank bottoms. Plates are employed extensively in machine components, aircraft, bridges, missiles, submarines, ships, and architectural structures. Many engineering problems in the actual world can be categorised as "plates in bending" or "shells in bending" [21].

In structural plates, deflections and deformations are frequently considered, and for convenience, their deflections are investigated under loading conditions [19]. In practice, many plate structures are subjected to heavy loads that can result in significant deflections. By producing membrane pressure, this significant deflection stretches the plate's central plane [9].

[16] states that Von Karman nonlinear equations are not amenable to analytic solutions. [10] and [17] also support this position. [4] the Von-Karman model of nonlinear strain-displacement relation is utilised in the majority of studies examining the strength of highly deformed rectangular plates. This Von-Karman type strain-displacement relation has two components: a linear component (Kirchhoff's strain-displacement, here assumed to be bending strain-displacement) and a non-linear component (membrane strain-displacement). According to [4] and [7], the following describes the plane Von-Karman strain-displacement relations:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} + \left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial u_0}{\partial x}\right] \quad (1)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} + \left[\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\partial v_0}{\partial y}\right] \quad (2)$$

One of the challenges encountered in handling the membrane component of the Von-Karman strain displacement pertains to the examination of plates exhibiting large deflection. Finding mathematical formulas for the u_0 and v_0 in-plane displacements of the membrane is the primary challenge in this situation. Earlier analyses of rectangular plates with significant deflection assumed the existence of the formulations for u_0 and v_0. In light of this, their conclusive findings should not be considered exact ([12], [13], [1], [2], [20], [5] and [10]).

For solving the Von-Karman equations, a stress function, also known as Airy's stress function, is employed. The results are approximations because previous researchers frequently assumed expression for Airy's stress function ([13], [20] and, [18]). This is comparable to membrane displacement in the same plane. It seems impossible to obtain the exact expression for Airy's stress function. In their doctoral research, [14], [15] and [3] discovered polynomial formulations for Airy's stress function. By integrating the governing equation and the plate compatibility equation, they were able to attain this fit. However, the learners' formulations for Airy's stress functions are quite exhaustive and lengthy. A solitary exposure to these assertions can demotivate an analyst.

These issues serve as the impetus for the current investigation. Can Von-Karman strain-displacement relations be used to analyse rectangular plates with significant deflection without utilising or introducing Airy's stress function? is the fundamental research query. It is necessary to avoid using Airy's stress function in the analysis of rectangular plates with large deflection. However, [8] addressed the issue by developing a general mathematical model for the nonlinear displacement and engineering strain of isotropic thin rectangular plates. They did so in accordance with equations (1) and (2). The general equations were applied to plates that were supported on all sides and clamped on all sides, using a polynomial form function. Utilising a polynomial shape function, the present work aims to evaluate the effects of coordinates and boundary conditions on the displacement and strain of the SSSS and CCCC thin rectangular plates.

II. METHODOLOGY

A. The Nonlinear Kinematic Equation

[8] derived a general mathematical model for displacement and strain and also formulated the total potential energy functional of an isotropic thin plate under pure bending analysis, as shown

➤ *Middle Surface Displacement of the Plate:*

$$u_0 = -\frac{1}{3}\frac{\partial w^2}{\partial x} \quad (3)$$

$$v_0 = -\frac{1}{3}\frac{\partial w^2}{\partial y} \quad (4)$$

Where u_0 and v_0 are the middle surface displacement in the x and y axes of the plate.

➤ *Middle Surface Strain of the Plate:*

Differentiating (3) and (4) with respect to x and y respectively gives:

$$\varepsilon_{x_0} = \frac{\partial u_0}{\partial x} = -\frac{1}{6}\left(\frac{\partial w}{\partial x}\right)^2 \quad (5)$$

$$\varepsilon_{y_0} = \frac{\partial v_0}{\partial y} = -\frac{1}{6}\left(\frac{\partial w}{\partial y}\right)^2 \quad (6)$$

Where ε_{x0} and ε_{y0} are the middle surface strain in the x and y axes of the plate.

➤ *In-Plane Displacement of the Plate:*

Simplifying (1) and making ∂u_0 subject formula gives (7)

$$\partial u = \left(-z\frac{\partial^2 w}{\partial x^2} + \left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial u_0}{\partial x}\right]\right)\partial x$$

$$\partial u = -z\frac{\partial^2 w}{\partial x^2}\partial x + \left[\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial x}\right)\partial x\right] + \frac{\partial u_0}{\partial x}\partial x$$

$$\partial u = -z\frac{\partial^2 w}{\partial x} + \left(\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)\partial w + \partial u_0\right) \quad (7)$$

Integrating (7) with respect to x gives (7*a*)

$$\int \partial u = \int -z\frac{\partial^2 w}{\partial x} + \int \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)\partial w + \int \partial u_0$$

$$u = -z\frac{\partial w}{\partial x} + \left[\frac{w}{2}\left(\frac{\partial w}{\partial x}\right) + u_0\right] \quad (7a)$$

Simplifying (2) and making ∂v_0 subject formula gives (7b)

$$\partial v = \left(-z\frac{\partial^2 w}{\partial y^2} + \left[\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\partial v_0}{\partial y}\right]\right)\partial y \quad (7b)$$

$$\partial v = -z\frac{\partial^2 w}{\partial y^2}\partial y + \left[\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)\left(\frac{\partial w}{\partial y}\right)\partial y\right] + \frac{\partial v_0}{\partial x}\partial x$$

$$\partial v = -z\frac{\partial^2 w}{\partial y} + \left(\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)\partial w + \partial v_0\right)$$

Integrating Equations (7b) with respect to y gives Equations (7c)

$$\int \partial v = \int -z\frac{\partial^2 w}{\partial y} + \int \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)\partial w + \int \partial v_0$$

$$v = -z\frac{\partial w}{\partial y} + \left[\frac{w}{2}\left(\frac{\partial w}{\partial y}\right) + v_0\right] \quad (7c)$$

Substituting Equation (3) into Equation (7a) gives:

$$u = -z\frac{\partial w}{\partial x} + \left[\frac{w}{2}\left(\frac{\partial w}{\partial x}\right) - \frac{w}{3}\left(\frac{\partial w}{\partial x}\right)\right]$$

Further simplification gives:

$$u = -z\frac{\partial w}{\partial x} + \left[\frac{w}{6}\left(\frac{\partial w}{\partial x}\right)\right] \quad (8)$$

Similarly, substituting Equation (4) into Equation (7c) gives:

$$v = -z\frac{\partial w}{\partial y} + \left[\frac{w}{6}\left(\frac{\partial w}{\partial y}\right)\right] \quad (9)$$

Where u and v are the in-plane displacement in the x and y axes of the plate.

➢ *Nonlinear Strain of the Plate:*

Differentiating (8) and (9) with respect to x and y respectively gives:

$$\frac{du}{dx} = \varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2} + \frac{1}{6}\left(\frac{\partial w}{\partial x}\right)^2 \quad (10)$$

$$\frac{dv}{dy} = \varepsilon_{yy} = -z\frac{\partial^2 w}{\partial y^2} + \frac{1}{6}\left(\frac{\partial w}{\partial y}\right)^2 \quad (10a)$$

The in-plane shear strain within x-y plane is given as:

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} \quad (10b)$$

Substituting (10) and (10a) into the in-plane shear strain equation as given in (10b):

$$\gamma_{xy} = -z\frac{\partial^2 w}{\partial x\partial y} + \frac{1}{6}\frac{\partial w}{\partial y}\left(\frac{\partial w}{\partial x}\right) + \left[-z\frac{\partial^2 w}{\partial y\partial x} + \frac{1}{6}\frac{\partial w}{\partial x}\left(\frac{\partial w}{\partial y}\right)\right]$$

$$\gamma_{xy} = 2\left[-z\frac{\partial^2 w}{\partial x\partial y} + \frac{1}{6}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\right] \quad (11)$$

Where γ_{xy} is the in-plane shear strain in the xy axes of the plate.

➢ *Nonlinear Stress of the Plate*

The basic constitutive relations of a rectangular plate with plane stress are

$$\sigma_{xx} = \frac{E}{(1-\mu^2)}(\varepsilon_{xx} + \mu\varepsilon_{yy}) \quad (12)$$

$$\sigma_{yy} = \frac{E}{(1-\mu^2)}(\mu\varepsilon_{xx} + \varepsilon_{yy}) \quad (13)$$

$$\tau_{xy} = \frac{E}{2(1+\mu)}\gamma_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xy} \quad (14)$$

The nonlinear strain in (10), (10b) and (11) were substituted into the basic constitutive relations of a rectangular plate with plane stress in the (12), (13) and (14) to give nonlinear stress of the plate.

$$\sigma_{xx} = \frac{E}{(1-\mu^2)}\left(-z\left[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right] + \frac{1}{6}\left[\left(\frac{\partial w}{\partial x}\right)^2 + \mu\left(\frac{\partial w}{\partial y}\right)^2\right]\right) \quad (15)$$

$$\sigma_{yy} = \frac{E}{(1-\mu^2)}\left(-z\left[\mu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] + \frac{1}{6}\left[\mu\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]\right) \quad (16)$$

$$\tau_{xy} = \frac{E(1-\mu)}{(1-\mu^2)} * \left[-z\frac{\partial^2 w}{\partial x\partial y} + \frac{1}{6}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\right] \quad (17)$$

Where (15), (16), and (17) are the equations for nonlinear stress on the x, y, and xy axes of the plate.

B. Total Potential Energy Functional of a Rectangular Plate

The nonlinear stress and nonlinear strain relations were substituted into the total potential energy functional of a classical rectangular plate in pure bending and carrying out the closed domain integration with respect to z coordinate gives:

$$\Pi = \frac{D}{2}\iint\left\{\left[\left(\frac{d^2 w}{dx^2}\right)^2 + 2\left(\frac{d^2 w}{dxdy}\right)^2 + \left(\frac{d^2 w}{dy^2}\right)^2\right] + \frac{g}{36}\left[\left(\frac{\partial w}{\partial x}\right)^4 + 2\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^4\right] - \frac{2}{D}qw\right\}dx.dy \quad (18)$$

$$Where: D = \frac{Et^3}{12(1-\mu^2)};\ g = \frac{12}{t^2} \quad (19)$$

Expressing the variable (18) in relation to the non-dimensional coordinates ($R = x/a$, $Q = y/b$, $S = z/t$) within the enclosed domain can be achieved by utilising the plate lengths along the x and y axes denoted as a and b, respectively.

$$\Pi = \frac{bD}{2a^3}\int_0^1\int_0^1\left\{\left[\left(\frac{d^2 w}{dR^2}\right)^2 + \frac{2}{\beta^2}\left(\frac{d^2 w}{dRdQ}\right)^2 + \frac{1}{\beta^4}\left(\frac{d^2 w}{dQ^2}\right)^2\right] + \frac{g}{36}\left[\left(\frac{\partial w}{\partial R}\right)^4 + \frac{2}{\beta^2}\left(\frac{\partial w}{\partial R}\right)^2\left(\frac{\partial w}{\partial Q}\right)^2 + \frac{1}{\beta^4}\left(\frac{\partial w}{\partial Q}\right)^4\right] - 2\frac{qa^4}{D}w\right\}dR\,dQ \quad (20)$$

The solution of Equation (20) is in the polynomial form and is given as:

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$$w = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 \end{bmatrix} \begin{bmatrix} 1 \\ R \\ R^2 \\ R^3 \\ R^4 \end{bmatrix} \times \begin{bmatrix} b_0 & b_1 & b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} 1 \\ Q \\ Q^2 \\ Q^3 \\ Q^4 \end{bmatrix} \quad (21)$$

$$w = a_i h_x \times b_i h_y = Ah \quad (22)$$

Substituting Equation (22) into Equation (20) gives:

$$\Pi = \frac{A^2 bD}{2a^3} \int_0^1 \int_0^1 \left[\left(\frac{d^2 h}{dR^2} \right)^2 + \frac{2}{\beta^2} \left(\frac{d^2 h}{dRdQ} \right)^2 + \frac{1}{\beta^4} \left(\frac{d^2 h}{dQ^2} \right)^2 \right] dR\, dQ +$$

$$\frac{A^4 \text{bgD}}{36 \times 2a^3} \int_0^1 \int_0^1 \left[\left(\frac{\partial h}{\partial R} \right)^4 + \frac{2}{\beta^2} \left(\frac{\partial h}{\partial R} \right)^2 \left(\frac{\partial h}{\partial Q} \right)^2 + \frac{1}{\beta^4} \left(\frac{\partial h}{\partial Q} \right)^4 \right] dR\, dQ - Aabq \iint h\, dR\, dQ \quad (23)$$

Where Π and w are the total potential energy and deflection of the plate.

a) Formulas for Analysing the Rectangular Plate:

In a denotational form, Equation (23) becomes:

$$\Pi = \frac{A^2 bD}{2a^3} \left[k_{bx} + \frac{2k_{bxy}}{\beta^2} + \frac{k_{by}}{\beta^4} \right] + \frac{A^4 \text{bgD}}{72a^3} \left[k_{mx} + \frac{2k_{mxy}}{\beta^2} + \frac{k_{my}}{\beta^4} \right] - Aabq\, k_q \quad (24)$$

Where:

$$k_{bx} = \int_0^1 \int_0^1 \left(\frac{d^2 h}{dR^2} \right)^2 dR\, dQ;\ k_{bxy} = \int_0^1 \int_0^1 \left(\frac{d^2 h}{dR\, dQ} \right)^2 dR\, dQ;\ k_{by} = \int_0^1 \int_0^1 \left(\frac{d^2 h}{dQ^2} \right)^2 dR\, dQ$$

$$k_{mx} = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial R} \right)^4 dR\, dQ;\ k_{mxy} = \int_0^1 \int_0^1 \left(\frac{\partial h}{\partial R} \right)^2 \left(\frac{\partial h}{\partial Q} \right)^2 dR\, dQ;\ k_q = \iint h\, dR\, dQ$$

Minimising (24) with respect to the amplitude of deflection, A gives:

$$\frac{\partial \Pi}{\partial \text{A}} = \frac{AbD}{a^3} \left[k_{bx} + \frac{2k_{bxy}}{\beta^2} + \frac{k_{by}}{\beta^4} \right] + \frac{A^3 \text{bgD}}{18a^3} \left[k_{mx} + \frac{2k_{mxy}}{\beta^2} + \frac{k_{my}}{\beta^4} \right] - abq\, k_q = 0 \quad (25)$$

Simplifying and writing (25) in denotational form gives:

$$A\, k_{bT} + \frac{A^3 \text{g}}{18} (k_{mT}) - \frac{qa^4}{D} k_q = 0 \quad (26)$$

$$k_{bT} = k_{bx} + \frac{2}{\beta^2} k_{bxy} + \frac{1}{\beta^4} k_{by};\quad k_{mT} = k_{mx} + \frac{2}{\beta^2} k_{mxy} + \frac{1}{\beta^4} k_{my} \quad (27)$$

Substituting (19) for g and D into (26) and dividing through by t gives:

$$\left(\frac{A}{t} \right) k_{bT} + \frac{2}{3} \left(\frac{A}{t} \right)^3 k_{mT} - 12(1 - \mu^2) \frac{q}{E} \left(\frac{a}{t} \right)^4 k_q = 0 \quad (28)$$

Rearranging (28) gives:

$$\left(\frac{A}{t} \right)^3 + \frac{3}{2} \left(\frac{A}{t} \right) \frac{k_{bT}}{k_{mT}} - 18(1 - \mu^2) \frac{q}{E} \left(\frac{a}{t} \right)^4 \frac{k_q}{k_{mT}} = 0 \quad (29)$$

$$\text{let } X = \frac{3k_{bT}}{2k_{mT}} \tag{30}$$

$$Y = -18(1-\mu^2).\frac{k_q}{k_{mT}}.\frac{qa^4}{Et^4} \tag{31}$$

Solving the cubic polynomial, we have:

$$\left(\frac{A}{t}\right) = \left(-\frac{Y}{2} + \sqrt{\Delta_0}\right)^{1/3} + \left(-\frac{Y}{2} - \sqrt{\Delta_0}\right)^{1/3} \tag{32}$$

Where;

$$\Delta_0 = \frac{Y^2}{4} + \frac{X^3}{27} \tag{33}$$

$$\text{Let; } \Delta = \frac{A}{t} \tag{34}$$

So therefore;

$$\Delta = \left(-\frac{Y}{2} + \sqrt{\Delta_0}\right)^{1/3} + \left(-\frac{Y}{2} - \sqrt{\Delta_0}\right)^{1/3} \tag{35}$$

Hence, the amplitude of the out of plan displacement of a plate experiencing large deflection is:

$$A = \Delta t \tag{36}$$

Substituting (36) into (22) gives:

$$w = \Delta th \tag{37}$$

Where t, h, D, and Δ are thickness of the plate, shape function, flexural rigidity of the plate and coefficient of deflection. Substituting (37) into (8) and (9) and writing them in terms of non-dimensional coordinates gives:

$$u = \frac{\Delta t^2}{a}\left[-S\frac{\partial h}{\partial R} + \frac{\Delta}{6}\frac{\partial h^2}{\partial R}\right] \tag{38}$$

$$v = \frac{\Delta t^2}{a\beta}\left[-S\frac{\partial h}{\partial Q} + \frac{\Delta}{6}\frac{\partial h^2}{\partial Q}\right] \tag{39}$$

Writing (10), (10a) and (10b) in terms of the non-dimensional coordinates gives:

$$\varepsilon_{xx} = \frac{\Delta t^2}{a^2}\left[-S\frac{\partial^2 h}{\partial R^2} + \frac{\Delta}{6}\frac{\partial^2 h}{\partial R^2}\right] \tag{40}$$

$$\varepsilon_{yy} = \frac{\Delta t^2}{a^2\beta^2}\left[-S\frac{\partial^2 h}{\partial Q^2} + \frac{\Delta}{6}\frac{\partial^2 h}{\partial Q^2}\right] \tag{41}$$

$$\gamma_{xy} = \frac{2\Delta t^2}{a^2\beta^2}\left[-S\frac{\partial^2 h}{\partial R\partial Q} + \frac{\Delta}{6}\frac{\partial^2 h}{\partial R\partial Q}\right] \tag{42}$$

C. Polynomial Deflected Shape Functions

According to [6], deflection equation for SSSS and CCCC are shown in Equation (43) and (44) respectively.

$$w = a_4b_4(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \tag{43}$$

$$w = a_4b_4(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \tag{44}$$

Where

$$A = a_4 b_4 \text{ and } h = \text{product of } R_{strip} \text{ and } Q_{strip}$$

D. Numerical Example

This analysis focuses on the behaviour of SSSS and CCCC plates subjected to a uniformly distributed force, resulting in large deflection. The plate material exhibits a Poisson's ratio of 0.316 and a Young's elastic modulus of 200 kN/mm2. The dimensions of the plate are as follows: the span is denoted as "a" and measures 500mm, the thickness is represented by "t" and measures 5mm, and the surface roughness is indicated by "S" and measures 0.5mm.

Note that for the numerical values in Table 4–6, non-dimensional values were used for the computation of the parameters.

Table 1 Values of Shape Functions and their Derivatives for SSSS Plate

R	**Q**	$\frac{\partial h}{\partial R}$	$\frac{\partial h}{\partial Q}$	$\frac{\partial^2 h}{\partial R^2}$	$\frac{\partial^2 h}{\partial Q^2}$	$\frac{\partial h^2}{\partial R}$	$\frac{\partial h^2}{\partial R \partial Q}$	$\frac{\partial h^2}{\partial Q}$
0.5	0.5	0	0	-0.9375	-0.9375	0	0	0
0.25	0.25	0.153076	0.153076	-0.500977	-0.500977	0.007589	0.472656	0.007589

Table 2 Values of Shape Functions and their Derivatives for CCCC Plate

R	Q	$\frac{\partial h}{\partial R}$	$\frac{\partial h}{\partial Q}$	$\frac{\partial^2 h}{\partial R^2}$	$\frac{\partial^2 h}{\partial Q^2}$	$\frac{\partial h^2}{\partial R}$	$\frac{\partial h^2}{\partial R \partial Q}$	$\frac{\partial h^2}{\partial Q}$
0.5	0.5	0	0	-0.0625	-0.0625	0	0	0
0.25	0.25	0.006592	0.006592	-0.008789	-0.008789	0.000008	0.035156	0.000008

III. RESULTS AND DISCUSSIONS

Table 3 Comparison of Central Deflection for this Present Work and that of Samuel Levy and Ibearugbulem

$\frac{ql^4}{Ebt^4}$	**Coefficient A/t**		**Center deflection**				% Diff	% Diff
	SSSS	**CCCC**	**Present SSSS**	**Present CCCC**	**Samuel Levy SSSS**	**Ibearugbulem et al. SSSS**	**between present. & Levy**	**Between present & Ibearugbulem et al.**
0	0	0	0	0	0	0	0	0
12.1	5.244	44.366	0.512	0.173	0.486	0.498	5.35	2.8
29.4	10.855	106.707	1.060	0.417	0.962	0.974	10.20	8.8
56.9	16.655	200.329	1.626	0.783	1.0424	1.439	14.22	13.0
99.4	22.463	327.148	2.194	1.278	1.87	1.899	17.31	15.5
161	28.187	474.995	2.753	1.855	2.307	2.353	19.32	17.0
247	33.900	632.416	3.311	2.470	2.742	2.807	20.74	18.0
358	39.419	786.389	3.849	3.072	3.174	3.247	21.28	18.5
497	44.800	935.503	4.375	3.654	3.6	3.678	21.53	18.9

Table 4 Effect of Coordinates on Non-Dimensional Displacements and Strain for SSSS Plates (0.5, 0.5, 0.5)

β	$\overline{w/t}$	$\bar{u}$	$\bar{v}$	$\bar{\varepsilon}_{xx}$	$\bar{\varepsilon}_{yy}$	$\bar{\varepsilon}_{xy}$
1.0	0.044668	0	0	0.181717	0.1817172	0
1.1	0.05355	0	0	0.210057	0.1736011	0
1.2	0.062203	0	0	0.235182	0.1633207	0
1.3	0.070471	0	0	0.256896	0.1520093	0
1.4	0.078258	0	0	0.275299	0.1404585	0
1.5	0.085516	0	0	0.290661	0.1291827	0
1.6	0.092229	0	0	0.30332	0.1184845	0
1.7	0.098405	0	0	0.313688	0.1085427	0
1.8	0.104065	0	0	0.322082	0.0994079	0
1.9	0.109242	0	0	0.328838	0.0910908	0
2.0	0.113969	0	0	0.33424	0.0835601	0
2.1	0.118283	0	0	0.338532	0.0767646	0
2.2	0.12222	0	0	0.341916	0.0706438	0
2.3	0.125814	0	0	0.344562	0.0651346	0
2.4	0.129098	0	0	0.34661	0.0601753	0
2.5	0.132101	0	0	0.348173	0.0557077	0

Table 5 Effect of Coordinates on Non-Dimensional Displacements and Strain for SSSS Plate (0.25, 0.25, 0.5)

β	$\overline{w/t}$	$\bar{u}$	$\bar{v}$	$\bar{\varepsilon}_{xx}$	$\bar{\varepsilon}_{yy}$	$\bar{\varepsilon}_{xy}$
1.0	0.022676	-0.034744	-0.034744	0.097105	0.097105	-0.18323
1.1	0.027185	-0.041589	-0.037809	0.112249	0.092768	-0.17505
1.2	0.031578	-0.048239	-0.040199	0.125675	0.087275	-0.16468
1.3	0.035775	-0.054573	-0.041979	0.137279	0.08123	-0.15328
1.4	0.039729	-0.060523	-0.043231	0.147113	0.075058	-0.14163
1.5	0.043413	-0.066053	-0.044036	0.155322	0.069032	-0.13026
1.6	0.046821	-0.071156	-0.044473	0.162094	0.063318	-0.11948
1.7	0.049956	-0.07584	-0.044612	0.167627	0.058003	-0.10945
1.8	0.05283	-0.080125	-0.044514	0.172112	0.053121	-0.10024
1.9	0.055457	-0.084035	-0.044229	0.175723	0.048677	-0.09185
2.0	0.057857	-0.0876	-0.0438	0.17861	0.044652	-0.08426
2.1	0.060047	-0.090848	-0.043261	0.180903	0.041021	-0.0774
2.2	0.062046	-0.093808	-0.04264	0.182711	0.03775	-0.07123
2.3	0.06387	-0.096507	-0.04196	0.184126	0.034806	-0.06568
2.4	0.065537	-0.09897	-0.041237	0.18522	0.032156	-0.06068
2.5	0.067062	-0.101219	-0.040488	0.186055	0.029769	-0.05617

Table 6 Effect of Coordinates on Non-Dimensional Displacements and Strain for CCCC Plate (0.5, 0.5, 0.5)

β	$\overline{w/t}$	$\bar{u}$	$\bar{v}$	$\bar{\varepsilon}_{xx}$	$\bar{\varepsilon}_{yy}$	$\bar{\varepsilon}_{xy}$
1.0	0.014353	-0.140637	-0.140637	-0.025812	-0.025812	0
1.1	0.017124	-0.200189	-0.18199	-0.063194	-0.052226	0
1.2	0.019641	-0.26336	-0.219467	-0.106229	-0.07377	0
1.3	0.021861	-0.326259	-0.250969	-0.151368	-0.089567	0
1.4	0.023782	-0.38613	-0.275807	-0.195868	-0.099933	0
1.5	0.025427	-0.441367	-0.294245	-0.237951	-0.105756	0
1.6	0.026826	-0.49127	-0.307044	-0.276662	-0.108071	0
1.7	0.028014	-0.535746	-0.315145	-0.311634	-0.107832	0
1.8	0.029024	-0.575058	-0.319477	-0.342869	-0.105824	0
1.9	0.029884	-0.609648	-0.320867	-0.370578	-0.102653	0
2.0	0.030619	-0.640027	-0.320013	-0.395073	-0.098768	0
2.1	0.031251	-0.666705	-0.317478	-0.416697	-0.094489	0
2.2	0.031795	-0.69016	-0.313709	-0.435793	-0.09004	0
2.3	0.032268	-0.710825	-0.309054	-0.452678	-0.085572	0
2.4	0.032680	-0.729077	-0.303782	-0.467637	-0.081187	0
2.5	0.033040	-0.745246	-0.298098	-0.480923	-0.076948	0

Table 7 Effect of Coordinates on Non-Dimensional Displacements and Strain for CCCC Plate (0.25, 0.25, 0.5)

β	$\overline{w/t}$	$\bar{u}$	$\bar{v}$	$\bar{\varepsilon}_{xx}$	$\bar{\varepsilon}_{yy}$	$\bar{\varepsilon}_{xy}$
1.0	0.004542	-0.012093	-0.012093	-0.00363	-0.00363	0.029042
1.1	0.005418	-0.014424	-0.013112	-0.008887	-0.007344	0.058761
1.2	0.006215	-0.016539	-0.013783	-0.014938	-0.010374	0.083001
1.3	0.007525	-0.018404	-0.014157	-0.021286	-0.012595	0.100774
1.4	0.007525	-0.020018	-0.014299	-0.027544	-0.014053	0.112436
1.5	0.008488	-0.021398	-0.014265	-0.033462	-0.014872	0.118988
1.6	0.008488	-0.022572	-0.014108	-0.038905	-0.015197	0.121593
1.7	0.008864	-0.023569	-0.013864	-0.043823	-0.015164	0.121324
1.8	0.009184	-0.024416	-0.013564	-0.048216	-0.014881	0.119064
1.9	0.009456	-0.025137	-0.01323	-0.052112	-0.014436	0.115497
2.0	0.009688	-0.025754	-0.012877	-0.055557	-0.013889	0.111126
2.1	0.009888	-0.026283	-0.012516	-0.058598	-0.013287	0.106312
2.2	0.010061	-0.02674	-0.012155	-0.061283	-0.012662	0.101306
2.3	0.01021	-0.027136	-0.011798	-0.063657	-0.012034	0.096279
2.4	0.01034	-0.027481	-0.011451	-0.065761	-0.011417	0.091345
2.5	0.010455	-0.027783	-0.01111	-0.067629	-0.010821	0.086575

Table 3 presents the centre deflection values for SSSS and CCCC plates that have been examined under a specific load condition. The centre deflection values for the different plates produced in this study were compared to the values reported by Levy (1942) and Ibearugbulem et al. (2020). The numbers presented in this study are evidently higher than those found in the works of Levy and Ibearugbulem. The observed greatest percentage differences are 21.3% and 18.9%, respectively. The observed discrepancy can potentially be attributed to the utilisation of distinct deflection functions. In the current study, polynomial shape functions were employed, whereas Levy and Ibearugbulem utilised trigonometric shape functions.

Tables 4 and 5 show the effect of coordinates on non-dimensional displacements and strain for a thin rectangular plate of SSSS. Along with being measured at two different coordinates, these strains and displacements were also recorded at R = 0.5, Q = 0.5, and S = 0.5, and R = 0.25, Q = 0.25, and S = 0.5. The aspect ratio for the first coordinate was found to rise when the ratios of w/t increased, although the in-plane displacements in the directions of x and y are both zero. At w/t = 0.132101 and 0.044668, the critical strain for the x and y directions, respectively, occurred. The critical strains are, respectively, 0.348173 and 0.1817172. For all w/t ratios, the shear strain in the xy direction is zero. This suggests that near the centre of the SSSS plate, the shear strain is always zero. As demonstrated in Table 5, the situation is different in the second coordinate (0.25, 0.25, 0.5). When examining the relationship between shear strain and w/t ratios, it was observed that the shear strain had a height value of -0.05617. Furthermore, the critical shear strain was found to occur at a w/t ratio of.067062. When compared to the coordinates (0.5, 0.5, 0.5), the in-plan displacement and the strains in the x and y axes are reduced at the coordinates (0.25, 0.25, 0.5).

Tables 6 and 7 show the effect of coordinates on non-dimensional displacements and strain for a CCCC thin rectangular plate. The first coordinate is at R = 0.5, Q = 0.5, and S = 0.5, and the second coordinate is at R = 0.25, Q = 0.25, and S = 0.5. These displacements and strains were measured at these two different coordinates. A close examination of Table 6 for the first coordinate reveals that the aspect ratio rises as the parabolic profile of the non-dimensional displacements and strains increases. The maximum in-plane displacement in both the x and y directions is the same, at -0.14065, and it happened at an aspect ratio of 1.0. At aspect ratios of 2.5 and 1.9, respectively, the minimal plane displacement measured in the x and y axes is -0.74525 and -0.32087. Although the pattern of the strain in the x and y directions is identical, it is smaller than that of the in-plane displacement. The maximum strain in both the x and y directions is the same, and it was recorded at an aspect ratio of 1.0. Its value is -0.02581. At aspect ratios of 2.5 and 1.6, respectively, the minimum strain measured in the x and y directions is -0.48092 and -0.10807, respectively. For all w/t ratios, the shear strain in the xy direction is zero. This suggests that near the centre of the CCCC plate, the shear strain is always zero. As shown in Table 6, the situation is different at the second coordinate (0.25, 0.25, 0.5). A finding was made indicating that the shear strain exhibited a parabolic distribution in relation to the w/t ratios. The critical shear strain, observed at a deflection of 0.008488, has a value of 0.121593. When compared to the coordinates (0.5, 0.5, 0.5), the in-plan displacement and the stresses in the x and y axes are greater at (0.25, 0.25, 0.5).

IV. CONCLUSION

- The bending behaviour of the SSSS plate at the coordinate (0.5, 0.5, 0.5) exhibits a range of $0.044668 \leq w/t \leq 0.132101$. Similarly, at the coordinate (0.25, 0.25, 0.25), the bending range is $0.022676 \leq w/t \leq 0.067062$. In contrast, the CCCC plates demonstrate bending within the range of $0.014353 \leq w/t \leq 0.033040$ at the coordinate (0.5, 0.5, 0.5), and within the range of $0.004542 \leq w/t \leq 0.010455$ at the coordinate (0.25, 0.25, 0.25). These results indicate that the bending of plates increases with both the increase in span ratio and the coordinates.
- Based on the obtained data, it can be inferred that a thin rectangular plate subjected to clamping at all edges exhibits more stability compared to a plate that is solely supported at all edges. This implies that the CCCC plate exhibited minimal bending in comparison to the SSSSS plate.

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