ISSN No:-2456-2165

Decision-Making Approach using Similarity Measures under Neutrosophic Cubic Sets and its Applications in Pattern Recognition Scenarios

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Abstract:- By extending the idea of the cubic set, this research study introduces a challenge associated with Neutrosophic Cubic Sets (NCS). We investigate both the ENCS (External Neutrosophic Cubic Set) and INCS (Internal Neutrosophic Cubic Set) properties. Our objective is to show that these sets may successfully deal with problems involving uncertainties. To do this, we suggest a decision-making method that can be adjusted using NCS and a similarity measure. To demonstrate how our strategy works in practice, we offer an example situation. The suggested methodology is specifically made to handle uncertain and ambiguous knowledge that is a common problem in many different real-world scenarios. It uses Neutrosophic cubic sets to describe the levels of informational accuracy, falsehood, and uncertainty. To estimate the similarities between Neutrosophic cubic sets, similarity metrics are also used. This method is used in the context of image segmentation to separate a picture into distinct sections based on its texture features. We show the usefulness of our suggested technique in managing unclear and ambiguous information by the outcomes of our experiments. A technique with use a different method in simulation of decision-making that entail uncertainty and ambiguity.

Keywords:- *Cubic Set · Neutrosophic Cubic Set · Internal Neutrosophic Cubic Set · External Neutrosophic Cubic Set · Decision Making.*

I. INTRODUCTION

Fuzzy-set theory was presented by Lotfi Zadeh[1] in 1965 as a tool in mathematics for representing the grade of certainty or uncertainty in diverse declarations. Zadeh too proposed the notion of linguistic variables, which enabled subjective human language to be expressed mathematically. Later, Peng et al.[2] demonstrated an application practical of fuzzy set theory under multi-criteria decision-making difficulties. Building on Zadeh's work, Türkşen I et al.[3], [4] and Ashtiani B et al.[5] prolonged fuzzy set theory to create interval-valued fuzzy sets. These have found numerous real-world applications, including in fields such as medicine and interval-valued logic. For example, Huiobro P et al.[6] has applied interval valued fuzzy sets to pact with uncertainty in decision-making processes, while Abdel-Basset M et al.[7] has used them in medical applications. Today, fuzzy set theory related with wide range area of mathematical simulation for practical problems.

Cubic-sets were introduced by Jun et al.[8] as a mathematical tool that combines the ideas of fuzzy sets and interval-valued fuzzy sets. In their research, Jun et al. investigated the properties of cubic sets, including internal and external cubic sets. Jun et al.[9], describe many more ideas on sub-algebras/ideals in different BCK/BCI. Later on Aslam et al.[10] mention the generalized formula for cubic formation on the other hand Jun et al. [11] generalized many results on cubic q-ideals with BCI-algebras formation. Many years ago the different scientist work with phenomenon with Neutrosophic sets and related notion with generalized properties related with different applications such as like application in decision making, image processing, data analysis and medical treatment for more see refe. [2], [9], [11]–[23]. INS also suggest different way to solve incomplete problem , where the boundaries of the interval can be used to represent the unknown information. In addition Smarandache explain our original work in the given references [24]–[26], and most probably contributed authors such as Ye and Liu, who made the different concept of definite novel in interval-neutrosophic soft sets, and Wang et al. [27], who applied INS to finite multi-criteria and decision-making difficulties.

In the given paper, we proposes the notion of neutrosophic-cubic sets, along with internal and external variations. Neutrosophic cubic sets extend the different notion of cubic sets, and various properties of such new concept are explored. Using these notions, a decision-making technique, called the neutrosophic-cubic technique, is constructed. The efficacy of this method is demonstrated through an application that highlights its ability to effectively solve problems that contain uncertainties.

II. PRELIMINARIES

In this process, to establish important definition for several types of sets that have been widely studied in the literature. The definitions provided here include fuzzy-sets[1], interval-valued fuzzy sets[4], neutrosophic-sets[24], interval-valued neutrosophic-sets, and cubic sets. These set types have been extensively explored in numerous in areas such as artificial intelligence, decision-making, and data-analysis. For further reading, interested readers can refer to works by Zadeh[1] , Atanassov [28], Smarandache[24].

- *Definition 1.*[1] Let U be a universe of discourse. Then, a fuzzy set F over U is defined by

$$F=\left\{\left\langle u:\mu_F(u)\right\rangle \mid \mathrm{u}\in U\right\}$$

Where $\mu_F(u)$ is called membership grade function of u and defined by $\mu_F(u):U\to[0,1]$. For each $u\in U$, the value $\mu_F(u)$ denotes the grade of u belonging to the fuzzy set F .

- *Definition 2.*[4] Let U be a universe of discourse. Then, an interval valued fuzzy set G over U is defined by

$$G=\left\{\left\langle u:[M^-(u),\ \mathrm{M}^+(u)]\right\rangle \mid u\in U\right\}$$

Where $M^-(u)$ and $\mathrm{M}^+(u)$ are referred as lower degree of membership and upper degree of membership over $u\in U$, respectively. Such that $0\le M^-(u)+M^+(u)\le 1$ and for convenient we write $M(u)=\left[M^-(u),\ \mathrm{M}^+(u)\right]$.

- *Definition 3.* [8]Let U be a non-empty universal set. Then, a cubic set Λ over U is a structure

$$\Lambda=\left\{\left\langle u:M(u),\mu(u)\right\rangle \mid u\in U\right\}$$

In which $M(u)=\left[M^-(u),\ \mathrm{M}^+(u)\right]$ is an interval valued fuzzy membership grade function and $\mu(u)$ is a fuzzy membership grade function. It is denoted by $\Lambda=\langle M,\mu\rangle$.

- *Definition- 4.* [8] Let U is a non-empty universal set. A Cubic-set $\Lambda=\langle M,\mu\rangle$ in U is named an internal-cubic set (ICS) if

$$M^-(u)\le\mu(u)\le M^+(u) \text{ for all } u\in U$$

$M^-(u)$ and $\mathrm{M}^+(u)$ are referred as lower grade of relationship and upper degree of relationship over $u\in U$, respectively.

- *Definition 5.* [8]Let U is a non-empty universal set. A cubic-set $\Lambda=\langle M,\mu\rangle$ in U is named an external cubic set (ECS) if

$$\mu(u)\notin\left(M^-(u),M^+(u)\right) \text{ for all } u\in U .$$

- *Definition 6.* [8]Let $\Lambda_1=\langle M_1,\mu_1\rangle$ and $\Lambda_2=\langle M_2,\mu_2\rangle$ be cubic sets in U . Then we define

- (Equality) $\Lambda_1=\Lambda_2$ if and only if $M_1=M_2$ and $\mu_1=\mu_2$.
- (P--order) $\Lambda_1\subseteq_P\Lambda_2$ if and only if $M_1\subseteq M_2$ and $\mu_1\le\mu_2$.
- (R--order) $\Lambda_1\subseteq_R\Lambda_2$ if and only if $M_1\subseteq M_2$ and $\mu_1\ge\mu_2$.

- *Definition7.*[8] For any

$$\Lambda_i=.\left\{\left\langle u:M_i(u),\ \mu_i(u)\right\rangle \mid\ u\in U\right\},$$

Where $i\in\tau$, we define

- $\underset{i\in\tau}{\cup_p}\Lambda_i=\left\{\left\langle u:\left(\underset{i\in\tau}{\cup}M_i\right)(u),\ \left(\underset{i\in\tau}{\vee}\mu_i\right)(u)\right\rangle \mid u\in U\right\}$
- $\underset{i\in\tau}{\cap_p}\Lambda_i=\left\{\left\langle u:\left(\underset{i\in\tau}{\cap}M_i\right)(u),\ \left(\underset{i\in\tau}{\wedge}\mu_i\right)(u)\right\rangle \mid u\in U\right\}$
- $\underset{i\in\tau}{\cup_R}\Lambda_i=\left\{\left\langle u:\left(\underset{i\in\tau}{\cup}M_i\right)(u),\ \left(\underset{i\in\tau}{\wedge}\mu_i\right)(u)\right\rangle \mid u\in U\right\}$
- $\underset{i\in\tau}{\cap_R}\Lambda_i=\left\{\left\langle u:\left(\underset{i\in\tau}{\cap}M_i\right)(u),\ \left(\underset{i\in\tau}{\vee}\mu_i\right)(u)\right\rangle \mid u\in U\right\}$

- *Definition 8.* [29]Let U be a non-empty set of genere. An interval- neutrosophic set (INS) E over U is defined as

$$E=\left\{\left\langle u:\left[\Phi^-(u),\Phi^+(u)\right],\ \left[\Psi^-(u),\Psi^+(u)\right],\left[\Upsilon^-(u),\Upsilon^+(u)\right]\right\rangle \mid\ u\in U\right\}$$

ISSN No:-2456-2165

Where $\Phi(u)=\left[\Phi^{-}(u),\Phi^{+}(u)\right]$ is the truth-membership function, $\Psi(u)=\left[\Psi^{-}(u),\Psi^{+}(u)\right]$ is the indeterminacy-membership function and $\Upsilon(u)=\left[\Upsilon^{-}(u),\Upsilon^{+}(u)\right]$ is the falsity-membership function. For each point $u\in U$, $\Phi(u),\Psi(u),\Upsilon(u)\subseteq[0,1]$. Let any two INS

$$E_1=\left\{\left\langle u:\left[\Phi_1^{-}(u),\Phi_1^{+}(u)\right],\ \left[\Psi_1^{-}(u),\Psi_1^{+}(u)\right],\ \left[\Upsilon_1^{-}(u),\Upsilon_1^{+}(u)\right]\right\rangle \mid u\in U\right\}$$

And

$$E_2=\left\{\left\langle u:\left[\Phi_2^{-}(u),\Phi_2^{+}(u)\right],\ \left[\Psi_2^{-}(u),\Psi_2^{+}(u)\right],\ \left[\Upsilon_2^{-}(u),\Upsilon_2^{+}(u)\right]\right\rangle \mid u\in U\right\}$$

Then

- $E_1 \mathrel{\tilde{\subseteq}} E_2$ if and only if $\Phi_1^{-}(u)\le\Phi_2^{-}(u)$, $\Phi_1^{+}(u)\le\Phi_2^{+}(u)$, $\Psi_1^{-}(u)\ge\Psi_2^{-}(u)$, $\Psi_1^{+}(u)\ge\Psi_2^{+}(u)$, $\Upsilon_1^{-}(u)\ge\Upsilon_2^{-}(u)$ and $\Upsilon_1^{+}(u)\ge\Upsilon_2^{+}(u)$ for all $u\in U$.
- $E_1=E_2$ if and only if $\Phi_1^{-}(u)=\Phi_2^{-}(u)$, $\Phi_1^{+}(u)=\Phi_2^{+}(u)$, $\Psi_1^{-}(u)=\Psi_2^{-}(u)$, $\Psi_1^{+}(u)=\Psi_2^{+}(u)$, $\Upsilon_1^{-}(u)=\Upsilon_2^{-}(u)$ and $\Upsilon_1^{+}(u)=\Upsilon_2^{+}(u)$ for all $u\in U$..
- $E_1^{\tilde{c}}=\left\{\left\langle u:\left[\Upsilon_1^{-}(u),\Upsilon_1^{+}(u)\right],\left[\Psi_1^{-}(u),\Psi_1^{+}(u)\right],\left[\Phi_1^{-}(u),\Phi_1^{+}(u)\right]\right\rangle \mid u\in U\right\}$
- $$E_1\tilde{\cup}E_2=\left\{\left\langle \begin{array}{l} u,\left[\max\left\{\Phi^{-}(u),\Phi^{-}(u)\right\},\max\left\{\Phi^{+}(u),\Phi^{+}(u)\right\}\right],\\ \left[\min\left\{\Psi^{-}(u),\Psi^{-}(u)\right\},\min\left\{\Psi^{+}(u),\Psi^{+}(u)\right\}\right],\\ \left[\min\left\{\Upsilon^{-}(u),\Upsilon^{-}(u)\right\},\min\left\{\Upsilon^{+}(u),\Upsilon^{+}(u)\right\}\right]\end{array}\right\rangle : u\in U\right\}$$
- $$E_1\tilde{\cap}E_2=\left\{\left\langle \begin{array}{l} u,\left[\min\left\{\Phi^{-}(u),\Phi^{-}(u)\right\},\min\left\{\Phi^{+}(u),\Phi^{+}(u)\right\}\right],\\ \left[\max\left\{\Psi^{-}(u),\Psi^{-}(u)\right\},\max\left\{\Psi^{+}(u),\Psi^{+}(u)\right\}\right],\\ \left[\max\left\{\Upsilon^{-}(u),\Upsilon^{-}(u)\right\},\max\left\{\Upsilon^{+}(u),\Upsilon^{+}(u)\right\}\right]\end{array}\right\rangle : u\in U\right\}$$

III. NEUTROSOPHIC CUBIC SET

In this section, we extend many more concept of the cubic set concept to neutrosophic sets, resulting in the introduction of neutrosophic cubic sets. Many more interested readers could refer to works by Smarandache and Ali (2020) and Farhadinia and Pedrycz (2022) for further details on neutrosophic sets and their extensions.

- *Definition 10.*[30] Let U be set. Then, a NCS set Đ is an entity taking the formula

$$Đ=\left\{\left\langle u,E(u),N(u)\right\rangle \mid u\in U\right\},$$

Where $E(u)$ is an INS in U and $N(u)$ is a NS in U. We basically represent a NCS as $Đ=\langle E,N\rangle$.

It should be noted that $C(Đ)$ is the notation used to represent the assortment of all neutrosophic cubic sets that are defined over U.

- *Example- 1.* Let $U=\{u_1,u_2,u_3\}$ be a universal set. Then an INS E over U well-defined by

$$E=\left\{\begin{array}{l}\left\langle u_1:[0.11,0.19],[0.22,0.3],[0.5,0.6]\right\rangle,\\ \left\langle u_2:[0.2,0.3],[0.4,0.8],[0.3,0.8]\right\rangle,\\ \left\langle u_3:[0.5,0.6],[0.2,0.4],[0.3,0.6]\right\rangle\end{array}\right\}$$

And a NS N is a set of U well-defined by

$$N=\left\{\left\langle u_1:0.02,0.29,0.39\right\rangle,\left\langle u_2:0.09,0.08,012\right\rangle,\left\langle u_3:0.28,0.26,0.80\right\rangle\right\}$$

Then $Đ=\langle E,N\rangle$ is a neutrosophic cubic set in U.

- *Definition 10.* [30] Let $Đ=\langle E,N\rangle\in C(Đ)$. If $\Phi^{-}(u)\le\phi(u)\le\Phi^{+}(u)$; $\Psi^{-}(u)\le\varphi(u)\le\Psi^{+}(u)$ and $\Upsilon^{-}(u)\le\gamma(u)\le\Upsilon^{+}(u)$, for all $u\in U$, then Đ is termed an INCS.

- *Example: 2.* Let $Đ=\langle E,N\rangle\in C(Đ)$.

If

$$E(u)=\left\langle\left\langle[0.4,0.6],[0.5,0.7],[0.5,0.7]\right\rangle\right\rangle$$

And

$$N(x)=\left\langle 0.5,0.6,0.6\right\rangle \text{ for all } u\in U,$$

Then

$Đ=\langle E,N\rangle$ is an INCS.

- *Definition 11.* [30] Let $Đ=\langle E,N\rangle\in C(Đ)$ .If $\phi(u)\notin[\Phi^-(u),\ \Phi^+(u)]$, $\varphi(u)\notin[\Psi^-(u),\Psi^+(u)]$ and $\gamma(u)\notin[\Upsilon^-(u),\ \Upsilon^+(u)]$ for all $u\in U$ then $Đ=\langle E,N\rangle$ is termed an external neutrosophic cubic set (ENCS).

- *Example:- 3.* Let assume $Đ=\langle E,N\rangle\in C(Đ)$. If $E(u)=\langle\langle[0.4,0.6],[0.5,0.7],[0.5,0.7]\rangle\rangle$ and $N(u)=\langle 0.7,0.4,0.9\rangle$ for all $u\in U$, then $Đ=\langle E,N\rangle$ is an ENCS.

- *Theorem 1.*[30] Let $Đ=\langle E,N\rangle\in C(Đ)$ which is not ENCS. Then, $\exists$ $u\in U$, such that $\Phi^-(u)\ \le\ \phi(u)\ \le\ \Phi^+(u)$, $\Psi^-(u)\ \le\ \varphi(u)\ \le\ \Psi^+(u)$ and $\Upsilon^-(u)\ \le\ \gamma(u)\ \le\ \Upsilon^+(u)$.

- *Theorem 2.*[30] Let $Đ=\langle E,N\rangle\in C(Đ)$. If $Đ=\langle E,N\rangle$ is both INCS and ENCS. Then $\phi(u)\in.\{U(\Phi),L(\Phi)\}$, $\varphi(u)\in.\{U(\Psi),L(\Psi)\}$ and $\gamma(u)\in.\{U(\Upsilon),L(\Upsilon)\}$ for all $u\in U$,

Where

$$U(\Phi)=\{\Phi^+(u):u\in U\},$$

$$L(\Phi)=\{\Phi^-(u):u\in U\},U(\Psi)=.\{\Psi^+(u):u\in U\},$$

$$L(\Psi)=.\{\Psi^-(u):u\in U\},$$

$$U(\Upsilon)=.\{\Upsilon^+(u):u\in U\},L(\Upsilon)=\{\Upsilon^-(u):u\in U\}$$

- *Proof:* Assume that $Đ=\langle E,N\rangle$ is together INCS & ENCS. Then through definition (11) & (12), we take

$$\Phi^-(u)\le\phi(u)\le\Phi^+(u)$$

$$\Psi^-(u)\le\varphi(u)\le\Psi^+(u)$$

$$\Upsilon^-(u)\le\gamma(u)\le\Upsilon^+(u)$$

And

$$\phi(u)\notin[\Phi^-(u),\Phi^+(u)]$$

$$\varphi(u)\notin[\Psi^-(u),\Psi^+(u)]$$

$$\gamma(u)\notin[\Upsilon^-(u),\Upsilon^+(u)]$$

For all $u\in U$.

Thus

$$\phi(u)=U(\Phi)\text{ or }L(\Phi)=\phi(u),$$

$$\varphi(u)=U(\Psi)\text{ or }L(\Psi)=\varphi(u)$$

And

$$\gamma(u)=U(\Upsilon)\text{ or }L(\Upsilon)=\gamma(u).$$

Hence

$$\phi(u)\in\{U(\Phi),L(\Phi)\}$$

$$\varphi(u)\in\{U(\Psi),L(\Psi)\}$$

$$\gamma(u)\in\{U(\Upsilon),L(\Upsilon)\}$$

- *Example 4.* Let $Đ=\langle E,N\rangle\in C(Đ)$ where $E(u)=\langle\langle[0.3,0.6],[0.4,0.7],[0.5,0.8]\rangle\rangle$ and $N(u)=\langle 0.5,0.9,0.6\rangle$ for all $u\in U$, then $Đ=\langle E,N\rangle$ is $\frac{2}{3}$-INCS.

- *Example 5.* Let $Đ=\langle E,N\rangle\in C(Đ)$ where $E(u)=\langle\langle[0.3,0.5],[0.4,0.6],[0.6,0.8]\rangle\rangle$ and $N(u)=\langle 0.4,0.9,0.9\rangle$ for all $u\in U$, then $Đ=\langle E,N\rangle$ is $\frac{1}{3}$-INCS.

- *Theorem 3.* Let $Đ=\langle E,N\rangle\in C(Đ)$. Then

  All ICSs can be seen as specific instances of INCSs.

  All ECSs can be seen as specific instances of ENCSs.

All Cubic Sets can be seen as specific instances of NCSs.

- *Proof:* The aforementioned definitions directly lead to the proofs.

➢ *Definition 15.*[30] Let $Đ_1 = \langle E_1, N_1 \rangle$ and $Đ_2 = \langle E_2, N_2 \rangle$ be neutrosophic cubic sets in U. Then

- (Equality) $Đ_1 = Đ_2$ if and only if $E_1 = E_2$ and $N_1 = N_2$.
- (P-order) $Đ_1 \subseteq_P Đ_2$ if and only if $E_1 \tilde{\subseteq} E_2$ and $N_1 \tilde{\subseteq} N_2$.
- (R-order) $Đ_1 \subseteq_R Đ_2$ if and only if $E_1 \tilde{\subseteq} E_2$ and $N_1 \tilde{\supseteq} N_2$.

➢ *Definition 13.* [30] Let $Đ = \langle E, N \rangle \in C(Đ)$, where $i \in \kappa = \{1, 2, ..., n\}$, we define

- $\cup_P Đ_i = \left\{ \left\langle u : \left(\overleftarrow{\bigcup_{i \in \kappa}} E_i \right)(u), \left(\overleftarrow{\bigcup_{i \in \kappa}} N_i \right)(u) \right\rangle \mid u \in U \right\}$ (P--union).
- $\cap_P Đ_i = \left\{ \left\langle u : \left(\overleftarrow{\bigcap_{i \in \kappa}} E_i \right)(u), \left(\overleftarrow{\bigcap_{i \in \kappa}} N_i \right)(u) \right\rangle \mid u \in U \right\}$ (P--intersection).
- $\cup_R Đ_i = \left\{ \left\langle u : \left(\overleftarrow{\bigcup_{i \in \kappa}} E_i \right)(u), \left(\overleftarrow{\bigcap_{i \in \kappa}} N_i \right)(u) \right\rangle \mid u \in U \right\}$ (R--union).
- $\cap_R Đ_i = \left\{ \left\langle u : \left(\overleftarrow{\bigcap_{i \in \kappa}} E_i \right)(u), \left(\overleftarrow{\bigcup_{i \in \kappa}} N_i \right)(u) \right\rangle \mid u \in U \right\}$ (R--intersection).

➢ *Definition 14.* [30] Let $Đ = \langle E, N \rangle \in C(Đ)$. Then, the complement of $Đ = \langle E, N \rangle$ is defined as

$$Đ^c = \left\{ \left\langle u : \overleftarrow{E^c}(u), \overleftarrow{N^c}(u) \right\rangle : u \in U \right\}$$

- *Theorem 4.* Let $Đ = \langle E, N \rangle \in C(Đ)$, where $i \in \tau = \{1, 2, 3, ..., n\}$, the subsequent holds.

✓ $\left(\cup_{P} Đ_i \right)^c = \cap_P (Đ_i)^c$ and $\left(\cap_P Đ_i \right)^c = \cup_P (Đ_i)^c$ ($i \in \tau$).
✓ $\left(\cup_R Đ_i \right)^c = \cap_R (Đ_i)^c$ and $\left(\cap_R Đ_i \right)^c = \cup_R (Đ_i)^c$ ($i \in \tau$)

- *Theorem 5.* Let $Đ_1 = \langle E_1, N_1 \rangle$ and $Đ_2 = \langle E_2, N_2 \rangle$ be neutrosophic cubic sets in U. The function $d(Đ_1, Đ_2) = C(Đ) \to \mathbb{R}^+$ given by Definition 18 is system of measurement, where $\mathbb{R}^+$ is the set of all non-negative real numbers.

- *Proof.* The proof is easy.

- *Theorem 6.* Let $Đ_1 = \langle E_1, N_1 \rangle$ and $Đ_2 = \langle E_2, N_2 \rangle$ be neutrosophic cubic sets in U. Then,

✓ $d(Đ_1, Đ_2) \geq 0$;
✓ $d(Đ_1, Đ_2) = d(Đ_2, Đ_1)$.
✓ $d(Đ_1, Đ_2) = 0$ iff $Đ_2 = Đ_1$.
✓ $d(Đ_1, Đ_2) + d(Đ_2, Đ_3) \geq d(Đ_1, Đ_3)$

IV. A PROBLEM ON PATTERN RECOGNITION

The decision-making process for pattern recognition is presented in a novel way in this part, with a focus on comparing two Neutrosophic Cubic Sets (NCS) using similarity metrics. The technique, which is based on earlier research, entails determining whether a sample pattern fits into a group of ideal patterns. The method presupposes that the sample pattern belongs to the ideal pattern family if the similarity score between it and the ideal pattern is 0.5 or greater. The NCS of the ideal pattern is computed and stored to put this strategy into practice. The NCS of each sample pattern is then calculated and put into comparison with the ideal pattern's stored NCS using a similarity metric.

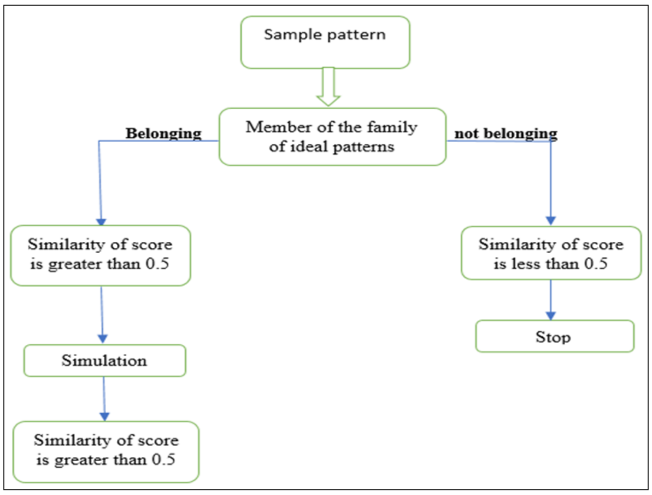

Fig 1 Decision Making Graph in Pattern Recognition Problem.

➢ *Procedure:*

- *Step 1.* Create a model of an optimal NCS $Ð=\langle E,N\rangle$ on U.
- *Step 2.* Create NCSs $Ð_i=\langle E_i,N_i\rangle,\ i=1,2,\ldots,n$, on U for the model patterns that must be identified.
- *Step 3.* Compute its distances $d(Ð,Ð_i),\ i=1,2,\ldots,n$.
- *Step 4.* If $d(Ð,Ð_i)\le 0.5$ then the pattern $Ð_i$ is to be acknowledged as being a part of the ideal-Pattern $Ð$ and if $d(Ð,Ð_i)\le 0.5$ then pattern $Ð_i$ remains aware about not fitting the ideal-Pattern $Ð$.

We now present a modified instance as of [25].

➢ *Example 6.* In order to demonstrate how similarity measures between two Neutrosophic Cubic Sets (NCSs) can be used in pattern recognition problems, a fictitious numerical example is provided in this section. The example focuses on three sample patterns that need to be recognized using the proposed method. Let $U=\{u_1,u_2,u_3\}$ be the universe. Also let $Ð$ be NCS set of the ideal pattern and $d(Ð,Ð_i),\ i=1,2,\ldots,n$ be the NCSs of three sample patterns.

- *Step 1.* Create an ideal NCS $Ð=\langle E,N\rangle$ on U as;

$$Ð=\left\langle \begin{array}{l} E(u)=\left\{\begin{array}{l}\langle u_1:[0.3,0.5],[0.4,0.6],[0.4,0.6]\rangle,\\ \langle u_2:[0.6,0.8],[0.1,0.6],[0.3,0.4]\rangle,\\ \langle u_3:[0.7,0.9],[0.1,0.2],[0.4,0.5]\rangle\end{array}\right\}\\ N(u)=\left\{\begin{array}{l}\langle 0.2,0.3,0.5\rangle/x_1,\\ \langle 0.2,0.3,0.3\rangle/x_2,\\ \langle 0.4,0.2,0.8\rangle/x_3\end{array}\right\}\end{array}\right\rangle$$

- *Step 4.* Meanwhile $d(Ð,Ð_1)\le .0.5$, $d(Ð,Ð_2)\le 0.5$ and $d(Ð,Ð_3)\ge .0.5$, The patterns that have NCS sets represented by $Ð_1$ and $Ð_2$ are considered to be similar to the ideal patterns represented by NCS set $Ð$. On the other hand, the pattern with NCS set represented by $Ð_3$ is not a part of the intimate of ideal patterns represented by NCSs $Ð$.

In the next way the step 3 mentioned in the figure 2

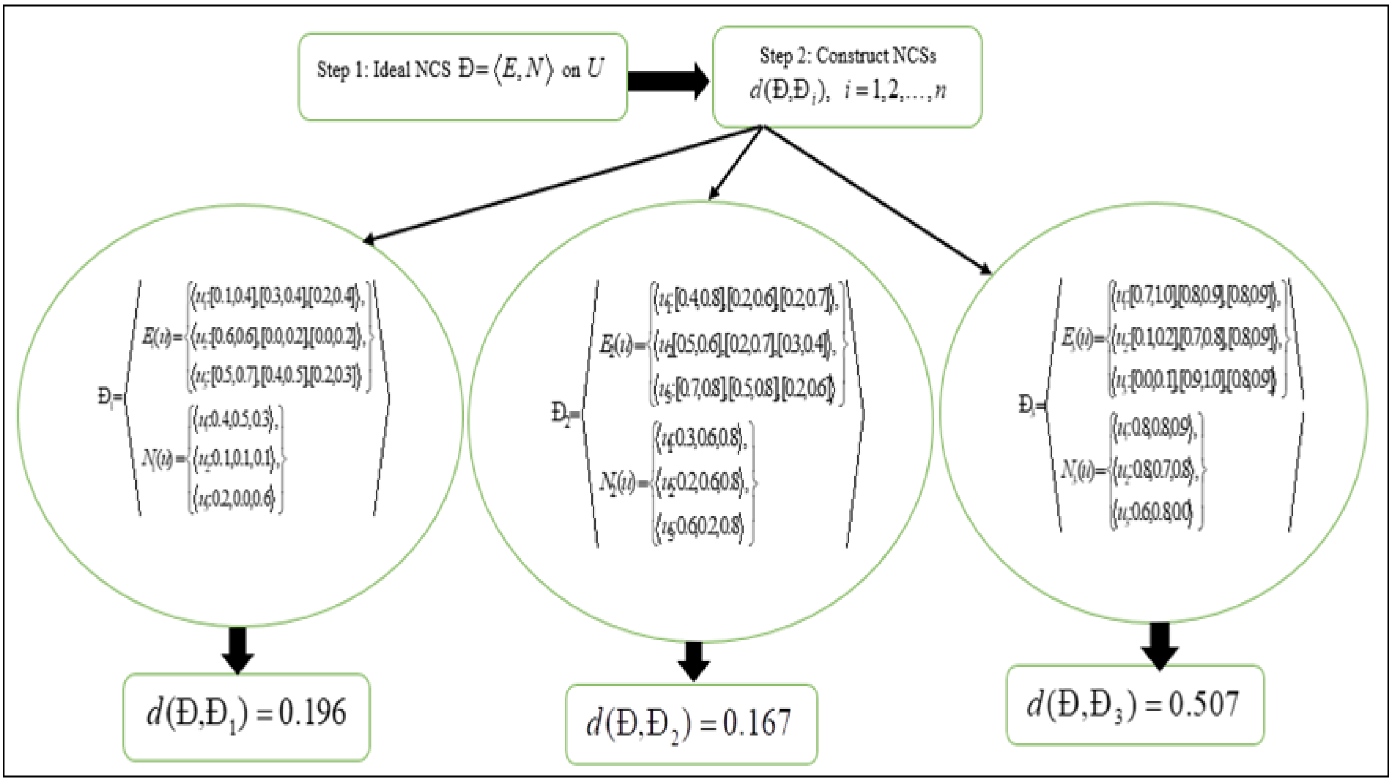

Fig 2 See Example 6 (Pattern Recognition Solutions)

ISSN No:-2456-2165

V. CONCLUSION

NCS is a mathematical concept that combines a NS with an INS. It is an advanced version of the concept with cubic set and has wide uses in numerous fields of mathematics. Here in particular paper, the authors have introduced new notions related to NCS and have provided their basic properties. This research is significant in the sense that it not only contributes to the theoretical foundations of NCS but also has the potential to bring about new insights and developments in related fields of mathematics.

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