Varying Order Sizes for One Time Discount Offer – A Net Present Value Analysis

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Abstract:- One of the methods of taking optimal decisions under uncertainty is a probabilistic approach. Under the situation of one-time discount offer with uncertain ending duration, the optimal order sizes need not remain same. The cash flow that occurs at different time point depends on order sizes very much. The optimal order sizes will also depends on probability of discount closes before next replacement. The objective of the paper is to determine an optimal ordering policy when a discounted price is offered over a temporary period and it is for a random duration. The paper discusses the method of finding optimal order quantities and reorder levels to maximize the net present value of the profit. A comparison of policies based on net present value (NPV) concept and with that of the absence of NPV is also studied. A sensitivity analysis based on inventory parameters is carried out.

Keywords: Dynamic inventory policies, temporary price discount, special order, net present value, profit function, price increase.

I. INTRODUCTION

An inventory problem is concerned with the making of decisions that minimize the total cost or maximize the total profit of an inventory system. Decisions always affect the costs or profits. One of the main decisions is time of placing order, and most often this decides the quantity to be ordered. The time element and the quantity element are the variables that are subject to control in an inventory system. The inventory problem is to find the specific values of these variables that minimize the total cost or maximize the total profit. It is more appropriate to discuss maximizing the total profit rather than minimizing the total cost, in the situation discussed here.

Many researchers have proposed models to deal with a variety of inventory problems. One of them is limited-time price incentives such as a one-time-only or temporary price reduction. Limited time price reduction is frequently used in retail. Nowadays it is a common practice for industrial suppliers to give a reduction in price for various reasons to their retailers. For example to reduce excess inventories or for promoting sale means, to take up the slack in their production facilities thereby encouraging consumers to order larger than usual quantities at a discounted price. The main purpose of price decrease is to order additional units to take advantage of the lower (present) price. The vendor offers a discount for a short duration, for instance, one or two months. Whenever the close of the sale is nearer, it may be profitable to place a replenishment to get the opportunity of a lower price. Also while placing an order, if there is a higher chance that price reduction may close before the next replenishment then placing a higher order may be profitable. Such orders that are placed under reduced price are called as special orders. Placing special orders is such that it can balance between lower purchasing cost and higher holding cost due to storing larger quantities. Inventory models have been proposed by many researchers to deal with price reduction problems. Pakkala and Babitha (2013) developed an inventory model to determine optimal order quantities and reorder points for one time discount offer by the vendor with the assumption that inventory parameters like the cost of the item, demand rate, and ordering cost remain same. Since the close of price discount is sure but the exact date is not known, the problem of determining optimal decision variables became complicated.

Another situation where this model is applicable is that the present purchase price will surely increase very soon but the exact date of increase will not be known in advance. As the increase in price occurs quite often, facing this problem is quite possible in business. For discussion purpose, we mention in this paper only about the price discount.

Number of research work has been carried out in price change problems. Initially, Naddor (1966) discussed single order in case of price change but the model needed further improvement. Later on, Lev and Weiss (1990) correctly point out that only two policies must be considered for determining the optimal policy under the deterministic duration of the price change and given expressions for both finite and infinite planning horizon. Goyal (1990) gave a procedure for determining the economic ordering policy when the supplier offers price reduction for a specified period by assuming constant demand over an intimate time horizon. Goyal and Gupta (1990) gave the simpler procedure requiring few EOQ calculations for determining the lot size. Datta and Pal (1991 adopted a price reduction special sale policy to increase the vendor's profit. Arcelus and Srinivasan (1995, 1998) also presented models on one time only discount sale. Arcelus et al. (2001, 2006 and 2008) developed a model for maximizing the profit when a vendor offers temporary sale at a discounted price. Pakkala and Babitha (2013) considered a common practical situation of the unknown ending date of the discount in their paper.

The optimal order sizes and reorder sizes are determined for a general probability distribution for the discount duration. Determination of optimal orders is based on combination of the first order is embedded with remaining orders through a dynamic programming solution to the model. Further to this model, Babitha and Pakkala (2014) developed an optimal inventory policy under temporary discount. A functional relationship between order levels and hazard rate function and also the relationship between order levels and mean residual life function is established. There are some other situations where retailer does not pass all the quantities purchased under discount during the sale period to the customer at a reduced price (see Abad 2003). Mark Armstrong and Yongmin Chen in 2019 discussed two reasons why a discounted price rather than a merely low price can make a consumer more willing to purchase. They suggested a behavioral explanation that is consumers with reference-dependence preferences are more likely to buy if they perceive the price as a bargain relative to the earlier price. They also showed discount pricing is an effective marketing technique, and a seller may wish to deceive potential customers by offering a false discount. A review of the papers regarding price discount can be seen in Shah and Dixit (2005).

The most frequently used method for making financial decisions is net present value and also is the standard methodology in theoretical analysis. But because of the complexity of the formulae and the robustness of the EOQ model, net present value is rarely used in production and inventory decisions.

In this study we considered large planning horizon, when the planning horizon is sufficiently long the cash flow at different time points cannot be absolutely compared because of change in the money value. Both revenue and payment occur at different time points throughout the planning horizon. In such cases, these cash flow need to be discounted or compounded with respect to a single reference point.

In particular, the time value of money represents the interest one might earn on a payment received today until that future date. Hadley (1964) compares the optimal order quantities of a single product based on two different objective functions, one of them is the net present value. Trippi and Lewin (1974) gave the discounted cash flow approach for the analysis of the basic EOQ model. Buzacott (1975) investigated the effects of inflation rate on the EOQ formula and the pricing policies. Chandra and Bahner (1985) examined the discounting effects of inflation on the optimal inventory policies of the order-level system and economic lot-size system.

Kim et al. (1986) extended the above approach to various inventory systems. Aucamp and Kuzdrall (1986) gave an expression for the order quantity that minimizes the discounted cash flows for a one-time discount. Also, the work contains derived expressions for the order quantity, cost savings, minimum acceptable percentage price discount and minimum vendor quantity requirements.

Daning Sun and Maurice Queyranne (2000) shows that average cost is a good approximation to the net present value when the demands are deterministic. They investigated the general multiproduct, multistage production and inventory model using the net present value of its total cost as the objective function. Grubbstrom and Kingsman (2004) applied the net present value principle to consider the problem of determining the optimal ordering quantities of a purchased item where there are step changes in price. Smith and Martinez-Flores(2007) compared the Economic Order Quantity formula and similar models against the net present value (NPV) derived by them and proved that the difference in discounted cost is small. For the finite planning horizon, Cheng-Kang Chen et al. (2007) developed a model to obtain the optimal number and the corresponding optimal time points of inventory replenishments using the net present value approach. Mohini and Pakkala (2012) discussed the optimum order quantity of the EOQ model and developed a model for both the situations where payment is made before and after trade credit limit.

While discount duration is random and at the time of placing order if discount is on then the residual probability distribution of discount durations differ except when discount duration is negative exponential distributed. Hence the optimal order quantities cannot be equal order size as discussed in the earlier work by Arcelus et al. (2001, 2006, and 2008). As the optimal order quantities depend on probability distribution of residual duration of discount, hence it is optimal to place varying order sizes. Since the number of possible orders during discount duration being random and depends on several varying order sizes complication of the solution process increases. Hence, in this paper, optimal decision analysis is carried out by taking probability aspect of discount being closed before the next replenishment and a discounted cash flow approach is applied to determine optimal policies under price change problem when the duration of discount or duration is uncertain. We develop an inventory model to determine optimal order quantities and reorder points for one time discount offer by the vendor with the assumption that inventory parameters like the cost of the item, demand rate, and ordering cost remain the same.

II. MODEL ASSUMPTIONS AND ANALYSIS

The objective of the study includes various approaches to find the solution in case of purchase price changes once in the planning horizon. At the time of evaluation point, the announcement of a temporary price discount is made or it has been already announced and surely it will continue for some duration of time. The model is applicable for both of these situations. But at what time the discount closes is not known in advance, that is why the discount duration is considered as a random variable. In order to capture the advantage of lower price, special orders are placed. The special orders different from regular orders and are chosen so as to get higher profit of the discounted purchase price. As mentioned earlier, order sizes optimally differ depending on the probability of discount getting over before the next order, unlike many of the work carried out earlier. Once if

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discount is off then regular orders are placed whenever inventory depletes. There are two types of orders, one under price discount and other under regular price. The optimal order levels are expected to be different when the discount price is present and they are same for the rest of the period.

➤ *Notations*

Following are the notations discussed in the model:

- *r_0 -initial inventory at time $t_0=0$*
- *X -random discount period from t_0.*
- *Q_0- order quantity under regular price.*
- *R -constant demand rate per year per units.*
- *T ($=Q_0/R$) - replenishment time under regular price.*
- *τ -starting time point of EOQ when discount is off.*
- *D_0- present value at τ of the profit starting from τ.*
- *Q_i- i^{th} order quantity under discount price*
- *S_i- i^{th} order level under discount price*
- *r_{i+1} be the i^{th} reorder level.*
- *t_i– time at which $(i+1)^{st}$ replenishment is placed, i=0, 1, 2, ….*
- *G(x) - cumulative distribution function of X.*
- *g(x) - density function of X.*
- *c -purchasing price per unit under regular price.*
- *d -rate of discount.*
- *P -selling price per unit.*
- *K -setup cost per order.*
- *F - fraction of holding charge per unit price per year.*
- *r-rate of interest.*
- *L-planning horizon.*

➤ *Ordering Strategy*

At the starting point of time $t_0=0$, and it is evaluation point also,

- *Place a Special Order of Size Q_i, at Different Reorder Points r_{i-1} i=1,2,…, if Discount is on*
  Otherwise (when discount is off)

- *Wait till Inventory Reaches Zero and from that Point Place a Regular Order of size Q_0if Discount is off.*

The retailer's ordering strategy is given in the following figure. Since the probability of closure of discount is random, dynamic orders are considered.

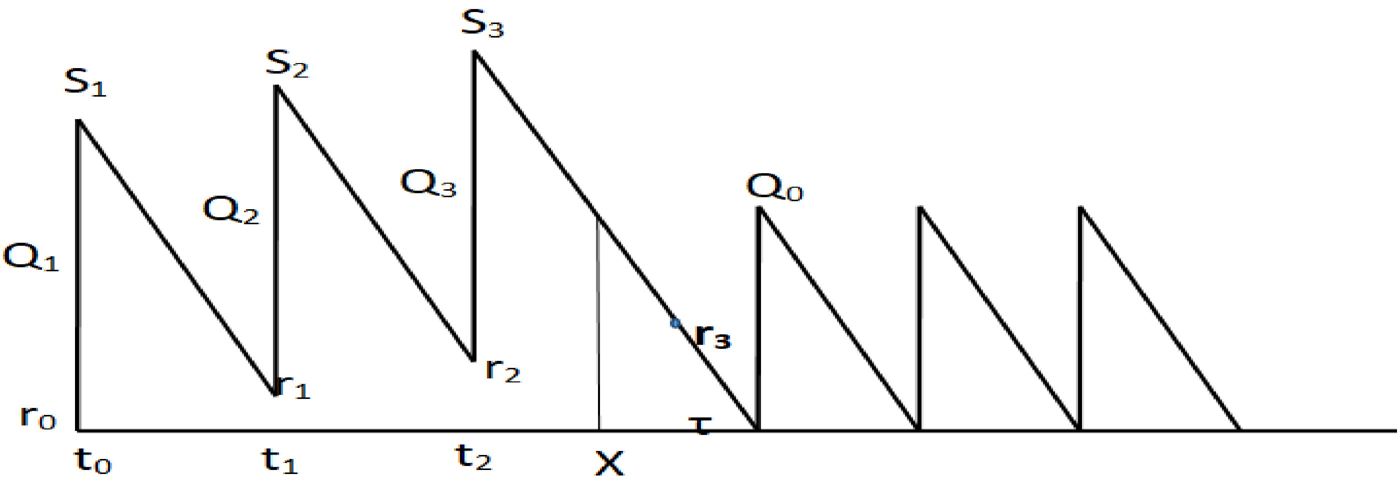

Fig 1 Ordering Strategy

The present value corresponding to the purchasing cost variable over the remaining planning horizon from τ evaluated at τ is

$$cQ_o + cQ_o e^{-\frac{rQ_o}{R}} + cQ_o e^{-2\frac{rQ_o}{R}} + \dots \quad = cQ_o\left(1 + e^{-\frac{rQ_o}{R}} + e^{-2\frac{rQ_o}{R}} + \dots\dots\right)$$
$$= \frac{cQ_o}{1 - e^{-\frac{rQ_o}{R}}} \qquad (1)$$

The present value of ordering cost for placing all the regular orders from τ evaluated at τ is,

$$K + Ke^{-\frac{rQ_o}{R}} + Ke^{-2\frac{rQ_o}{R}} + \dots \quad = K\left(1 + e^{-\frac{rQ_o}{R}} + e^{-2\frac{rQ_o}{R}} + \dots\dots\right)$$

$$= \frac{K}{1 - e^{-\frac{rQ_o}{R}}} \qquad (2)$$

Similarly, the present value of holding cost during the planning horizon after tow is

$$\frac{cFQ_0}{r\left(1-e^{-\frac{rQ_o}{R}}\right)}-\frac{cFR}{r^2} \tag{3}$$

The present value of revenue calculated from τ evaluated at τ is,

$$P\int_{\tau}^{\infty} e^{-r(t-\tau)}dt \quad = Pe^{r\tau}\left[\frac{e^{-rt}}{-r}\right]_{\tau}^{\infty} \quad = Pe^{r\tau}\left(\frac{0-e^{-r\tau}}{-r}\right)$$

$$=\frac{P}{r} \tag{4}$$

Subtracting (1),(2), and (3) from (4) will give the present value of profit from τ. That is,

$$PV(T)=\frac{P}{r}-\left[\frac{K+cQ_0}{1-e^{-\frac{rQ_o}{R}}}+\frac{cFQ_0}{r\left(1-e^{-\frac{rQ_o}{R}}\right)}-\frac{cFR}{r^2}\right]$$

$$=\frac{\mathrm{P}}{\mathrm{r}}-\left[\frac{\mathrm{K}+\mathrm{cRT}}{1-\mathrm{e}^{-\mathrm{rT}}}+\frac{\mathrm{cFRT}}{\mathrm{r}\left(1-\mathrm{e}^{-\mathrm{rT}}\right)}-\frac{\mathrm{cFR}}{\mathrm{r}^2}\right] \tag{5}$$

Differentiate (5) and equate to zero to obtain optimal T, say T^*, that implies,

$$\frac{\partial PV(T)}{\partial T}=\frac{\partial}{\partial T}\left\{\frac{\mathrm{P}}{\mathrm{r}}-\left[\frac{\mathrm{K}+\mathrm{cRT}}{1-\mathrm{e}^{-\mathrm{rT}}}+\frac{\mathrm{cFRT}}{\mathrm{r}\left(1-\mathrm{e}^{-\mathrm{rT}}\right)}-\frac{\mathrm{cFR}}{\mathrm{r}^2}\right]\right\}=0$$

$$\Rightarrow cR\left(1+\frac{F}{r}\right)\left[e^{rT^*}-1-rT^*\right]-Kr=0$$

$$or \quad Kr=cR\left(1+\frac{F}{r}\right)\left[e^{rT^*}-1-rT^*\right] \tag{6}$$

One can get T* by solving the above expression using any numerical method and there by the optimal order quantity Q_0*(=RT*).

- *Profit Function*

At $t_0=0$, it is known that discount is on, place a special quantity of size (S_1- r_0). At any time point $t_i i=1, 2...$ inventory will be non-negative, that is, whenever inventory level reaches $r_i \geq 0$, $i=0,1....$the next order (S_i-r_{i-1}) is placed. The net present value of revenue for sales of (S_1- r_0) items over the period (t_0, t_1) is equal to $PR\int_{t_0}^{t_1} e^{-rt}dt$ where $t_1=\frac{S_1-r_1}{R}$, and the net present value of the holding cost and setup cost or ordering cost is, $(c-d)F\int_{t_0}^{t_1} I(t)e^{-rt}dt$ and $K\,e^{-rt_0}$.

Therefore the profit for placing the first special order of size (S_1- r_0) is,

$$\varphi_1 = PR\int_{t_0}^{t_1} e^{-rt}\,dt - (c-d)(S_1 - r_0) - (c-d)F\int_{t_0}^{t_1} I(t)\,e^{-rt}\,dt - Ke^{-rt_0} \tag{7}$$

$$where\ \ I(t) = Q_1 - tR$$

The conditional profit over the planning horizon if discount closes before the second special order

$$\psi_1 = \left(\varphi_1 + \xi_1 + PV(T)e^{-r\frac{S_1 - r_1}{R}}\right)$$

$$where\quad \xi_1 = PR\int_{t_1}^{t_1 + r_1/R} e^{-rt}dt - (c-d)F\int_{t_1}^{t_1 + r_1/R} \left(r_1 - R(t - t_1)\right)e^{-rt}dt \tag{8}$$

The last expression in equation (8) is the profit from (t_1, $t_1 + \frac{r_1}{R}$) which is not included in equation (7).

If the discount is on when inventory reaches r_{i-1} then place an order of size (S_i-r_{i-1}). The present value of various costs i.e. purchase cost, holding cost, setup cost or ordering cost and revenue for placing an i[th] order under discount is,

- $(c-d)e^{-rt_{i-1}}(S_i - r_{i-1})$
- $(c-d)F\int_0^{t_i - t_{i-1}} I(t)e^{-r(t_{i-1}+t)}dt$
- $where\quad I(t) = Q_i - \left(t - \frac{S_{i-1} - r_{i-1}}{R}\right)R$
- $Ke^{-rt_{i-1}}$
- $PR\int_0^{t_i - t_{i-1}} e^{-r(t_{i-1}+t)}dt$

Combining all these cost and revenue will give expected profit for this part is,

$$\varphi_i = PR\int_0^{t_i - t_{i-1}} e^{-r(t_{i-1}+t)}\,dt - (c-d)e^{-rt_{i-1}}(S_i - r_{i-1}) - (c-d)\,F\int_0^{t_i - t_{i-1}} I(t)e^{-r(t_{i-1}+t)}dt - K\,e^{-rt_{i-1}}$$

The above expression calculates the profit from the depletion of an order of size (Q_i-r_{i-1}) till r_i and not necessarily to zero.

The expression (5) gives the net present value of profit which is evaluated from τ. The profit is evaluated for the entire planning horizon. So the net present value of the profit should be calculated from the beginning. Therefore the net present profit at the starting point for the transactions beyond τ if n orders are placed under discount price is,

$$D(n) = PV(T)\,e^{-r\left(\sum_{j=1}^{n} t_j\right)}$$

Finally, the actual profit is,

$$\psi = \sum_{n=1}^{J} P(N=n)\left[\sum_{j=1}^{n}\varphi_j + \xi_n + D(n)\right] \tag{9}$$

where $$P(N=n) = \int_{t_{n-1}}^{t_n} g(x)\,dx$$

Where J is large enough such that the probability of ordering beyond J is negligibly small. An extra computation is required for the last order discount. If the discount duration is closed before inventory reaches r_n, then the profit from the remaining inventory r_n which is not included in the profit function is separately calculated using the following formula. That is,

$$\xi_n = PR\int_{t_n}^{t_n + r_n/R} e^{-rt}dt - (c-d)F\int_{t_n}^{t_n + r_n/R}\left(r_n - R(t-t_n)\right)e^{-rt}dt$$

Here the objective is to determine optimal order quantity and reorder levels over an infinite planning horizon. The profit is considered as a function of the present value of setup cost, of purchasing cost, holding cost, and revenue. The profit function has a similar property as discussed earlier.

➢ *Solution Procedure*

Since the number of orders under the discount price is a random variable, finding the optimal solutions for the above discussed mathematical model is not simple. We need to use the optimizing algorithm by fixing the number of decision variables. Observing the expected profit (9), it is noted that the decision variables differ depending on the number of orders placed within the random duration of discount, which itself is varying. Hence any fixed choice of decision variables (Q_i, r_{i-1}) selected will not be valid when *N*, the number of orders under entire discount duration, varies. Hence we need to fix an N and for that N=n value we have 2n decision variables and it can be determined using an optimization toolbox available. In order to determine a specific n, among the possible values of N, an n that is most probable can be taken as a representative. Suppose the random variable representing the discount duration is unimodal and ranges over a finite duration. Treating that discount is available until the last point, the number of orders *m* according to Schwarz's formula is determined. Now divide the range into *m* equal parts and find the range which possesses maximum probability. Let that be nth part. That is, in the nth part of the range the discount is most likely to close. Hence corresponding n is taken for the computation of (Q_i, r_{i-1}). However actual computation is done exactly without considering the equal order sizes or equal length for the replenishment period. When the range of the discount duration random variable is infinite, instead of the entire range we take only up to 0.95 quantile, which is up to 95th percentile. Then apply the above procedure with the assumption that discount remains until the 95th percentile point. With 2n decision variables, the optimal decision variables corresponding to the maximum profit is finally taken. So obtained decision variables are taken for implementation of the inventory policy,

➢ *Numerical illustration*

In this section behavior of the model is studied. Certain insights are observed through this study, which otherwise is not observed directly.

- *When Discount Duration follows Weibull Distribution:*

Let the random discount duration X be assumed to follow the Weibull density function with a shape parameter β and a scale parameter λ, that is,

$$g(x) = \lambda\beta x^{\beta-1}e^{-\lambda x^{\beta}} \text{ for } x > 0;\ \beta > 0 \text{ and } \lambda > 0$$

The Basic parameters are (K=50, c=9, F=0. 2, d=3, R=1890, P=13, L=100, λ=2, β =1)

Optimal order quantity and profit for basic parameters are, Q_1=1482.44, Q_2=1702.52, Q_3=4789.77, and Profit=67783.33

Table 1 Showing the Most Likely Number of Orders, Order Quantity, and Profit for a Different set of Parameters

Parameters (λ, β)	Most likely number of orders	Order quantity	Profit
(0.4,1)	3	966.55 1434.30 3064.18	46266.77
(0.8,1)	2	2634.43 5201.25	54717.83
(1.2,1)	3	1211.46 1771.17 3651.04	59718.29

(1.6,1)	2	3503.74 4578.25	61234.56
(2,1.4)	3	949.31 1039.53 4556.26	61372.22
(2,1.5)	3	984.71 3000.08 4812.12	61419.82
(2,1.6)	2	2632.29 5228.87	61952.17

It is observed from the above table that there is a significant variation in order quantities (in the case of (2, 1.4)) in the third column of the table, this is because the chances of discount getting over is more. As the conditional probability of closure of discount increases the order sizes also increases. Also one can see that when the parameters of the distribution increases the profit also increases.

Having determined optimal policies based on NPV we can compare the policies without using NPV. Suppose optimal policies are determined without discounting for money value then we get different optimal policies, which are considered as policy without NPV. In the study carried out, we have determined optimal policies (i) using NPV (ii) without NPV. Under the framework of NPV, the optimal solution obtained without NPV will not be considered optimal because it does not take into consideration the money value. Under the NPV framework profit can be computed for the solutions obtained under without NPV, which is denoted by P*. That is, P* is a profit under the NPV model but for a suboptimal solution corresponding to the approach of without NPV. In the table below the first column gives the various choices of parameters taken. The second column gives the profit for the corresponding optimal policies for the solutions obtained under NPV. The third column gives the profit for the optimal solutions obtained without considering NPV. These profits are inflated because discounting for money value is not considered. For the same solutions if the profit is seen through the concept of NPV we get P* which are given in column four. It can be seen that the profit given in column four is always lower than the optimal policy given under column two. Hence establishing the sub-optimality of the solutions that give profit under columns three and four.

Table 2 Comparison of Actual Profit and Profit Computed Through Without using NPV

Parameters (λ, β)	**Profit using NPV**	**Profit without using NPV**	**P***
(0.4,1)	46266.77	71522.29	44575.57
(0.8,1)	54717.83	71743.25	50300.85
(1.2,1)	59718.23	71555.95	49886.74
(1.6,1)	61234.56	71504.20	52266.34
(2,1.4)	61372.22	72711.27	55790.68
(2,1.5)	61419.82	72612.26	55827.59
(2,1.6)	61952.17	72500.02	55858.25

The difference could be seen by comparing the second column and the fourth column of the Table given above. Since the difference is significant and NPV is a more appropriate method, the study shows a measure of error due to considering profit without NPV.

- *When Discount Duration follows A Gamma Distribution:*
  Here the discount duration X is assumed to follow the gamma density function.

  The probability density function of gamma distribution with shape parameter α and scale parameter λ is given by,

$$g(x) = \frac{x^{\alpha-1}e^{-\frac{x}{\lambda}}}{\lambda^{\alpha}\gamma(\alpha)} \text{with } \gamma(\alpha) = \int_0^{\infty} t^{\alpha-1}\, e^{(-t)}\text{dt}$$

  Basic parameters are K=50, c=9, F=0. 2, d=3, R=1890, P=13, T=100, α =2, λ =1

  Optimal order quantity and profit for basic parameters are Q_1=1603.93, Q_2=2233.64, Q_3=3109.85, and Profit=56019.96

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Table 3 Table Showing Number of Orders, Order Quantity, and Profit for a Different set of Parameters

Parameters (α, λ)	**Most Likely Number of Orders**	**Order Quantity**	**Profit**
(0.6,1)	2	3276.12 5045.72	60380.12
(0.8,1)	2	1148.13 3826.55	58217.93
(1.2,1)	3	1128.57 1587.69 3275.88	55544.85
(1.4, 1)	3	1261.45 1768.90 2513.88	53896.85
(2,1. 2)	2	2011.87 2435.68	46747.79
(2,1.4)	3	428.70 2618.03 3425.26	44208.35
(2,1.6)	2	639.52 2702.99	42348.38

Here also similar jumps in order quantities could be seen, as in the case of Weibull distribution

Table 4 Comparison of Actual Profit and Profit Computed Through without using NPV

Parameters (α, λ)	**Profit using NPV**	**Profit without using NPV**	**P***
(0.6,1)	60380.12	71732.45	52972.49
(0.8,1)	58217.93	71545.49	49818.24
(1.2,1)	55544.85	71717.12	50350.90
(1.4,1)	53896.85	71834.35	50505.81
(2,1.2)	46747.79	72439.37	45790.68
(2,1.4)	44208.35	72700.36	44114.69
(2,1.6)	42348.38	70813.36	41783.53

A close look at the above table indicates the difference in profits for three different situations. The first where the profit is calculated based on the net present value and it is smaller than the second where the profit is obtained without using net present value also this value is greater than P*.

III. CONCLUSION

In this study, we discuss the situation in which discount in buying price for retailer is announced and will remain for a random duration. A similar situation is faced when price increase is announced but will come into force in near future but exact time is not known. Equal order sizes within the discount duration were suggested in earlier studies. When the discount period is random equal order size is not necessarily optimal. A slightly complicated but better solution is obtained. The model derived here considers variable order sizes over the discount period that prevails for a random duration. Since money transactions take place at various time points the present value of future cash flows is considered. Since total profit is a function of a random number of variables, it cannot be solved in an explicit form. Hence the complexity of getting solutions is increased. A solution procedure is adapted to obtain the optimal policies. The value of money decreases as time passes therefore importance is given for net present value in the current study. Profit is calculated by incorporating net present value and corresponding decision variables. Solutions are also obtained without using net present value and a comparison of profits are made. It is observed that profit is less in the case of net present value which is the true value. The optimal order quantity which is obtained without using the concept of net present value and corresponding profit is recomputed by incorporating the net present value. All the three profits are compared and observed that profit without using net present value shows larger than profit using net present value. The profit obtained for the solutions obtained under NPV is better than the NPV profit for the solutions obtained under without NPV concept. It is observed that all reorder points are negligible, in the numerical situation considered here. The profit function can be derived for any distribution, here Weibull distribution and Gamma distribution are considered for the discount duration as an example for the study.

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