

What is Missing in Mathematics and Theoretical Physics

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Abstract:- In particular, probability and statistics are a missing part of mathematics and classical theoretical physics.

The introduction of the stochastic transition matrix B proposed by the Cairo Physical Technique suggests that probability and statistics should belong to physics rather than mathematics.

We first study the comparison between chains of stochastic Markov transitions (mathematics) and chains of B transition matrices (physics) as a clear approach to elucidate this subject and better understand both from the point of view of probability by definition and limitation of application.

The in-depth study shows that the B-matrix chains originally introduced to solve the general case of the time-dependent diffusion equation can also find numerical integration formulas, numerically derive the normal/Gaussian distribution, and find the numerical values of the Gamma function in the positive x domain.

The numerical results presented are surprisingly accurate and validate the B transition matrix hypothesis.

I. INTRODUCTION

Many people think that mathematics already contains everything, but the opposite is also true. In particular statistical integration, statistical differentiation and statistical solution of time-dependent partial differential equations is a missing part of mathematics.

In general, probability and statistics are a missing part of mathematics and should rather belong to physics than to mathematics.

In this article, we study the comparison between stochastic Markov transition chains (mathematics) and matrix B chains (physics) imposed by the Cairo technique as a clear approach to elucidate this topic.

Markov was part of the great tradition of mathematics in Russia and never missed his goal.

He designed and passed on a non-physical stochastic chain and never claimed it could solve physical problems.

We present the following brief comparison between classical stochastic Markov chains and recent Cairo

technique B-matrix chains originally introduced to solve the general case of the time-dependent diffusion equation.

Markov defined his original $n \times n$ square transition matrix $M(i,j)$ via two assumptions[1,2]:

- 1) The transition probabilities $M_{i,j}$ are equal to or greater than zero.
- 2) The sum of the transition probabilities

$\sum_{i=1}^n M_{i,j} = 1$, for all columns $j=1$ to n , (called vertical left matrix)

OR

The sum of the transition probabilities $\sum_{j=1}^n M_{i,j} = 1$, for all the rows $i=1$ to n , (called horizontal or right matrix)

Obviously, stochastic Markov chains can be converted to B-matrix chains and vice versa.

On the other hand, the well-founded B-matrix and B-transition matrix chains rely on four assumptions [3,4].

The first two assumptions of matrix B are Markov assumptions with a slight modification of the second condition to allow the introduction of boundary conditions BC.

Namely, for lines adjacent to the Dirichlet boundary, the sum of transition probabilities $M_{i,j}$ for all lines may be equal to or less than 1.

The other two hypotheses 3 and 4 are physical hypotheses,

- 3) The matrix B is symmetric in the geometry of 3D space like nature itself,

$$i.e. B_{ij} = B_{ji}$$

condition (3) represents two of the physical facts,

- Detailed balance rule.
- Reciprocity rule.

The reciprocity rule was highly respected by A. Einstein and led to his discoveries in special relativity and the atomic and molecular gas laser.

Note that while $M_{i,j}$ comes as a set of individual entries, $B_{i,j}$ comes as a set of pairs.

We recall the following remarks:

- The Boltzmann statistics referring to "Stosszahlansatz" gas molecules or the molecular chaos hypothesis rely on binary or pairwise collisions.
- The classical derivation of the classic time-independent Boltzmann equilibrium distribution for discrete or continuous energy levels via the cell-box method relies on the mutual transition between two cells or two energy levels.

In short, it seems that nature's response resembles some sort of pairwise symmetry.

4) The input elements of the main diagonal (RHS) are all equal to a constant input RO.

Where RO is an element of [0,1].

Note that condition (4) is independent and not implied by condition (3).

Physically speaking, RO represents the residual or remanence of energy density at a free node or point in spacetime.

It is clear that RO must be zero when solving Poisson and Laplace PDEs in vacuum and somewhere between zero and one when solving heat diffusion/conduction PDEs in a material mass medium.

The question arises as to why stochastic Markov chains cannot solve physical problems (heat diffusion/conduction PDEs, Poisson and Laplace PDEs, derivation of integration and differentiation formulas, etc.) that are simply solved by the transition matrix B (B i , j) of the Cairo technique.

What is missing from mathematics and theoretical physics in general and Markov transition chains in particular?

The answer is that the physical conditions (3) and (4) are missing.

The decisive consequence is that in Markov chains which ignore x-space, the time interval dt is chosen arbitrarily and is not necessarily space-relevant whereas in B-matrix chains time is merged or weaved in a 4D x-t unit space, which means that the time interval dt is imperial.

➤ *In B-Matrix Strings, dt is Imperial and Real Time is Completely Lost.*

It should be mentioned that the B-matrix strings are relativistic invariant as they should be.

Moreover, what are the limits of Markov analysis?

If the actual time interval is too short, Markov models are inappropriate because the individual moves are not random, but rather deterministically related in time. This case suggests that Markov models are generally inappropriate over sufficiently short real time intervals.

II. THEORY

The core of the proposed stochastic B-transition chains is the transition probability matrix B which is well defined through the four conditions 1-4 mentioned earlier in the introduction.

The B-matrix chains were originally introduced to solve the general case of the time-dependent diffusion equation,

$$d U / d t \text{ partial} = D \cdot \text{Nabla}^2 U + S(x, t) \dots \dots (1)$$

Subjected to Dirichlet boundary conditions BC and initial condition $U(x,0)$.

Where,

$U(x,t)$ is the energy density function.

D is the assumed positive diffusion coefficient.

Nabla^2 is the well-known Laplacian operator.

$S(x,t)$ is the source/sink term at the considered node or point in space.

The unconventional statistical method proposed to solve Equation 1 is based on a rigorous physical statistical hypothesis.

This hypothesis assumes the existence of a transition matrix B such that it verifies the following recurrence relation in a time step dt, [3,4]

$$U_{i,j,k}^{N+1} = B \cdot (b + S) + U_{i,j,k}^N \dots \dots (2)$$

Where b is the Dirichlet vector of the boundary conditions BC and S is the source/sink term expressed in the appropriate units.

Surprisingly enough, the same matrix B, originally introduced to solve the general case of the time-dependent diffusion equation, can be applied to find Simpson-like formulas for numerical integration, the formula for the normal/Gaussian distribution, the numeric value of the Gamma Function in positive x space, etc.

This is explained in more detail in section iv (Examples of realizations of physical transition chains).

If the main numerical statistical hypothesis of the existence of a physical statistical transition matrix B such that the recurrence formula 2 is true for each jump or time interval dt is valid,

Then, it follows that the time-dependent solution $U(N)$ of the diffusion Eq 1 at time $t = N$ iterations will be given by,

$$U(N) = D(N) \cdot (b+S) + B^N \cdot U(x,0) \dots \dots \dots (3)$$

Where $D(N)$ is expressed by the following matrix power sum,

$$D(N) = B + B^2 + B^3 + \dots \dots \dots + B^N \dots \dots (4)$$

Therefore, the starting point or the key to numerical calculations is to find the base transition matrix B .

However, the steps of the solution procedure itself are not complicated and the matrix B is well defined by the conditions (1)-(4) explained above in the introduction.

Note that in addition to the transfer matrix $D(N)$ defined via the finite series (4), there is also another important matrix called the transfer matrix E defined by the infinite series of power matrices:

$$E = B^0 + B + B^2 + \dots \dots \dots + B^N \dots \dots (5)$$

For N large enough.

It is obvious that $B^0 = I$ the unit matrix.

And we can prove that,

$$E = (I - B)^{-1} \dots \dots \dots (6)$$

Equation (6) can be used to find the transfer matrix $E(N)$ and hence the transfer matrix $D(N) = E(N) - I$ for a large number of iterations N .

That is, at the equilibrium steady state solution.

It remains to explain the whole procedure of the proposal numerical statistical method which should be done in 3 specific consecutive steps:

➤ **First Step*

Discretize the 1D, 2D or 3D geometric spatial domain into n equidistant free nodes and find the appropriate stochastic transition matrix $B(n \times n)$.

The inputs $B_{i,j}$ are subject to the following numerical conditions:

i- $B_{i,j} = 1/2$ for 1D, $1/4$ for 2D and $1/6$ for 3D for i adjacent to j and $B_{i,j} = 0$ otherwise.

This means that there is no spatial preference direction.

ii- $B_{i,i} = RO$, i.e.the main diagonal consists of constant

• *RO Inputs.*

The transition of the system is an identical collective process of its participants.

RO can take any value in the closed interval $[0,1]$ and plays a crucial role in the heat diffusion equation. for Laplace and Poisson PDE, $RO = 0$

In other words, solving Poisson and Laplace PDE via B-chains and a zero-input principal diagonal matrix corresponds to the assumption of zero residue or no energy storage in vacuum after each time step dt.

➤ ***Second Step*

Define b vector which is the vector of the Dirichlet boundary conditions by arranging BC in the correct order.

Calculate the source / sink term vector in appropriate units.

➤ ****Third Step*

Compute the transfer matrix E and D via equations 4 and 5 and hence find the steady state equilibrium solution for sufficiently large number of iterations N or the time dependent transient solution for small number of iterations,

$$U(N) = D(N) \cdot (b + S) + B^N \cdot U(x,0) \dots \dots (7)$$

Where $N = 1, 2, \dots N$. That is to say the solution $U(x,t)$ at iteration N or at time $= N dt$ is given by Equation 7.

Note that for N large enough, B^N tends to zero and the initial condition vector for the energy density is absorbed by the Dirichlet boundaries.

Practically all entries in the B^N matrix quickly converge to zero as N tends to a large enough number, since the sum of one or more rows is less than unity when connected to Dirichlet BC. It is a necessary condition for the convergence of the matrices E and D .

More or less, it is not complicated to calculate the matrix E or D . Finite and infinite series (4&5) can be evaluated in a simple way via separate calculation algorithm.

➤ *Double Precision Algorithms are Indispensable in Such a Case.*

However, in order not to worry too much about the details of the theory, let's go straight to the numerical results and the realization of the chains of the matrix B .

III. NUMERICAL RESULTS FOR THE B-MATRIX

For each $n \times n$ square B matrix, there are two main kinds, namely the open B matrix denoted B and the closed B matrix denoted B_c .

Each of the two kinds mentioned above can be 1D, 2D or 3D, which makes the possible number of matrix B six.

In all the six types mentioned above, the matrix element $B_{i,j}$ is defined as:

$B_{i,j} = 1/6 \cdot RO/6$ for i adjacent to j and zero elsewhere.
(3D matrix)

$B_{i,j} = 1/4 \cdot RO/4$ for i adjacent to j and zero elsewhere.
(2D matrix)

$B_{i,j} = 1/2 \cdot RO/2$ for i adjacent to j and zero elsewhere.
(1D matrix)

In addition to the main diagonal constraint $B_{i,i} = RO$.

Which means that there is no direction of spatial preference.

Again, for all the matrices mentioned above, there are an infinite number of principal diagonal constant (RHS) input choices RO where RO is an element of the closed interval $[0,1]$.

Note that the closed matrix B_c will be a stochastic matrix with double symmetry in the sense that the sum of any row is equal to 1 and the sum of any column = 1 for all numerical values of RO .

B_c is called the double stochastic matrix.

Here are some concrete examples.

➤ *6x6 One D Bc Matrix with $RO=0$*

1/2	0	0	0	0	1/2
1/2	0	1/2	0	0	0
0	1/2	0	1/2	0	0
0	0	1/2	0	1/2	0
0	0	0	1/2	0	1/2
1/2	0	0	0	1/2	0

Let's move on to the more complicated case of a rectangleoid in 3D geometric space.

➤ *27x27 3D Matrix with 54 Dirichlet Boundary Conditions (Fig.1)*

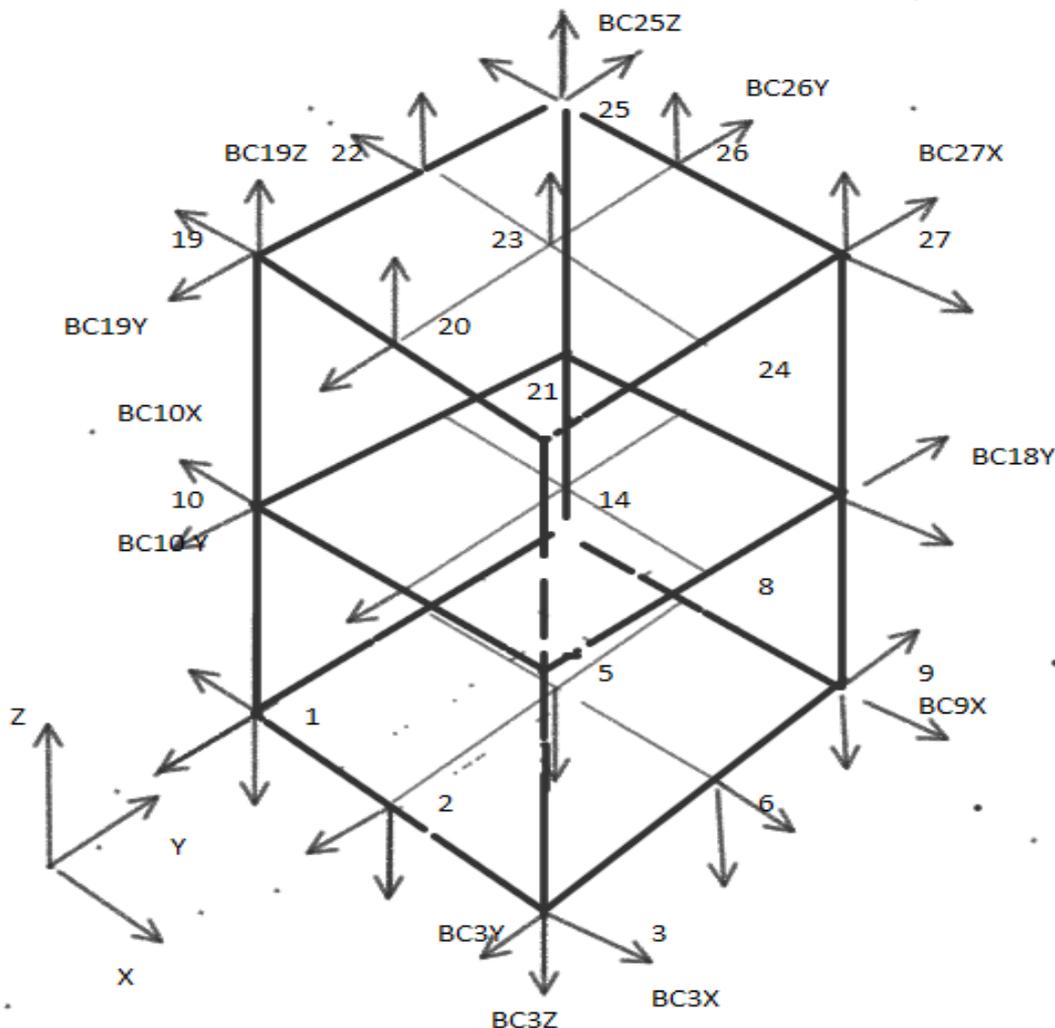


Fig 1 A Rectangleoid with 27 Equidistant Free Nodes and 54 Dirichlet Boundary Conditions.

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Note how the modified probability condition (2) is applied to the 27 free nodes.

The sum of $B_{i,j}$ for all $j=1$ to 27 is <1 except for the free node 14 which is the only node far from the borders and therefore the sum $B_{14,j}$ for $j=1$ to 27 is 1.00000000.

IV. EXAMPLES OF REALIZATIONS OF PHYSICAL TRANSITION CHAINS

In the following we present some examples of realizations of physical transition chains (B-matrix chains).

Note that these realizations are made possible via an adequate definition of a pairwise transition probability.

➤ *Numerical Statistical Integration Formulas*

The derivation of the numerical statistical integration formulas via the chains of the matrix B is well explained in reference 5 so we content ourselves here with presenting the significant result.

We arrive at the following statistical integration formula for 7 nodes via the closed matrix 1D - 7x7 [5],

$$I = \frac{h}{77}(6.Y_1 + 11.Y_2 + 14.Y_3 + 15.Y_4 + 14.Y_5 + 11.Y_6 + 6.Y_7) \dots (7)$$

$I = \int Y(x) dx$ or area under the curve,

$Y(1)$ corresponds to $F(x_1)$, $Y(2)$ corresponds to $F(x_2)$, and so on.

h is the space interval between two nodes $= x_2 - x_1 = x_3 - x_2$ etc.

Equation (7) is surprisingly accurate and is the statistical equivalence of Simpson's rule for 7 nodes.

Note that numerical statistical integration formulas for Simpson-like rules such as Equation 7 can be extended to any odd or even number (n) beyond 7 knots, and accuracy increases as n increases.

➤ *Numerical Derivation of Normal/Gaussian Distribution*

Again, The Numerical derivation of Normal/Gaussian distribution formulas via the chains of the matrix B is well explained in reference 6 so we content ourselves here with presenting the significant result.

If we use the closed B matrix (Bc) 15X15, we arrive at the following distribution explained in ref.[6],

- 0.0000000
- 0.0000000
- 9.765625E-4
- 9.7656250E-3
- 4.3945313E-2
- 0.11718750
- 0.20507813
- 0.246093750
- 0.20507813
- 0.11718750
- 4.3945313E-2
- 9.7656250E-3
- 9.765625E-4
- 0.0000000
- 0.0000000

Note that,

*The above distribution is the proposed unconventional numerical Gaussian distribution law that corresponds to the classical Gaussian distribution curve.

** the total sum of the 15 elements of proposed distribution D, which represents the area under the normal/Gaussian distribution curve is equal to 1.0000000, which is surprisingly accurate.

***The presented value of the normalization constant A in the unconventional Gaussian law given by the proposed Distribution is still 0.2461 which corresponds precisely to the value of $1/\sqrt{2\pi}$ in the classic Gaussian law.

➤ *Calculate the Gamma Function without Numerical Integration.*

Again, the numerical computation of the Gamma function via the B matrix strings is well explained in ref. 7.

The derivation process itself relies on the matrix B and the emergent results for the Gamma function are in agreement with those given by the specialist tables in refs 8,9,10 - We therefore present here only the significant result as Curve 1 and Curve 2 .

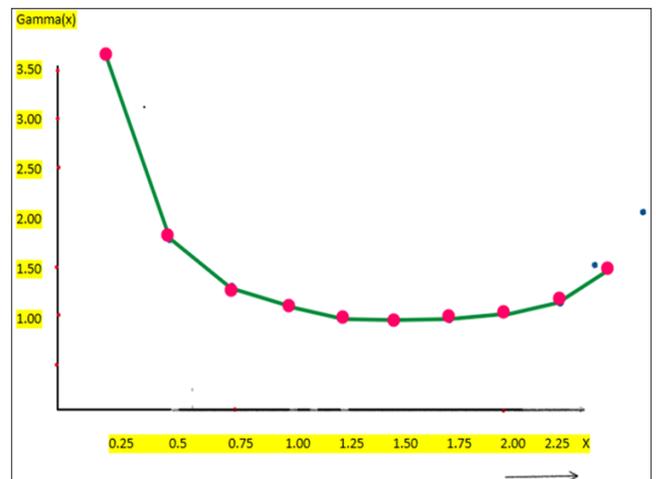


Fig 1 Part of the Results of the Gamma Function Obtained in Ref.7

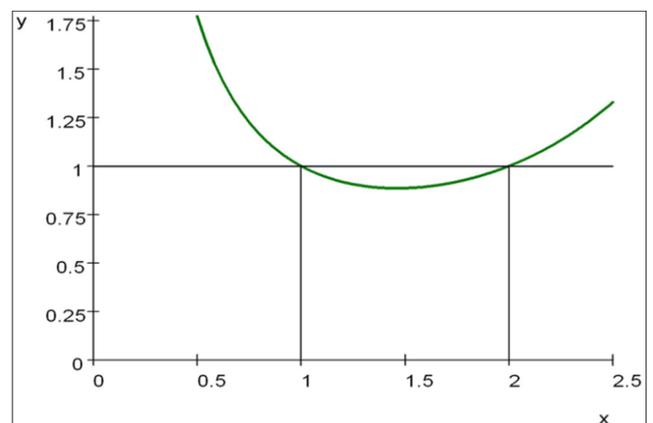


Fig 2 Part of the Results of the Gamma Function Obtained in Ref.7

V. CONCLUSION

Probability and statistics is a missing part in mathematics and the case of Markov stochastic transition matrix is an example.

The in-depth study shows that the B-matrix chains originally introduced to solve the general case of the time-dependent diffusion equation can also find numerical integration formulas, numerically derive the normal/Gaussian distribution, and find the numerical values of the Gamma function in the positive x domain.

The numerical results presented are surprisingly accurate and validate the B transition matrix hypothesis.

We also assume that laws based on physical probability and statistics must satisfy the rules of symmetry and reciprocity as the matrix B itself.

NB. All calculations in this article were produced using the author's double-precision algorithm to ensure maximum accuracy, as follows by ref. 11 for example

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