Quadratic Rank Transmutation of the Nwikpe Distribution

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Abstract:- In this paper, we proposed the generalization of the Nwikpe distribution using quadratic rank transmutation. Thus, a new two parameter continuous probability distribution, named the transmuted Nwikpe (TND) distribution was obtained. Some important statistical properties such as the shape of its density for different values of the parameter, the moment generating function, the crude moment and the distribution of its order statistics were derived. The parameter of the new distribution was estimated using maximum likelihood method. The flexibility of the new distribution was shown using a real-life data set. The goodness of fit shows that the TND performed better than the base distribution and other competing distributions for the data sets used in this work.

Keywords:- *Quadratic Rank Transmutation, Nwikpe Distribution, Maximum Likelihood, Moment, Crude Moment.*

I. INTRODUCTION

Science and technology improvements have resulted in a rapid increase in the amount of data generated from various disciplines and aspects of life. The application of statistical tools for the modelling of these data sets depends on an underlying probability distribution, especially in parametric statistics. Most of the classical distributions have been found not to give good fit to some of these data sets. With most classical probability functions failing to provide a satisfactory fit to some data sets, developing new flexible probability distributions that will improve the quality of statistical analysis is required. As a result, in recent years, the extension and generalization of existing probability distributions have attracted several statisticians. Consequently, a huge number of probability distributions have being developed by statisticians lately. A simple approach for generalizing a probability distribution is proposed by Guptaet al. (1998) and is known as the exponentiated family of distributions. The exponentiated family of distributions provides flexibility by adding one more parameter to the baseline distribution and has attracted several authors(Md. Mahabubur et al., 2018). Thus, the concept of generalizing a probability distribution is hinged on adding new parameter(s) to a standard distribution to enhance its modelling flexibility. Several generalized (or G) classes are currently available in Statistical literature. For instance, generalized families (G-families)of distributions such as Marshall-Olkin-G-families by Marshall and Olkin (1997), Beta Generalised (Beta-G) family of distributions by Eugene *et al*., (2002) amongst others, have been used to modify most standard distributions, increasing the number of probability distributions available in distribution theory. Furthermore, Shaw and Buckley (2007), have also introduced another interesting method of adding new parameter to an existing distribution for solving problems related to financial mathematics and named the family as quadratic transmuted family (QTF) of distributions. Subsequently, Aryal and Tsokos (2011) have modified the Weibull distribution using the quadratic transmuted family of distribution, Merovci and Puka (2014) proposed the transmuted Pareto distribution, Bourguignon et al. (2017) derived the transmuted Birnbaum-Saunders distribution, Abdullahi and Ieren (2018) derived the transmuted exponential Lomax distribution, Samuel (2019) derived the transmuted Logistic distribution amongst others. However, there still remains a large number of practical problems that does not follow any of the standard probability distributions. Therefore, developing new probability distributions is very pertinentin statistical theory.

The Nwikpe distribution is a one parameter continuous probability function derived by Nwikpe et al. (2021). The distribution in is a mixed model derived from gamma and exponential distributions. Though it has been demonstrated that the Nwikpe distribution outperforms the baseline distribution (exponential distribution) and some probability distributions, its main drawback is its reliance on only one parameter. This shortcoming has inhibited the modelling performance (flexibility) of the model. It is this gap that the present study seeks to fill. The main focus of this study is to enhance the flexibility of the one parameter Nwikpe distribution by introducing an additional parameter to the distribution using the quadratic rank transmutation approach.

A continuous random variable X is said to follow the Nwikpe distribution if its probability density function (PDF) is given as:

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$$f\ \ (x) = \frac{\vartheta^3}{12(\vartheta^2+10)}(\vartheta^3x^5+12)e^{-\vartheta x} \qquad x>0,\vartheta>0 \qquad (1)$$

The corresponding cumulative distribution of the Nwikpe distribution in (1) is given as

$$F(x) = \left[1-\left(1+\frac{\vartheta^5x^5+5\vartheta x^3(4+x)+60x\vartheta(\vartheta x+2)}{12(\vartheta^2+10)}\right)e^{-\vartheta x}\right] \qquad (2)$$

II. MATERIALS AND METHODS

A. Quadratic Rank Transmutation

Let Y be a non-negative random variable from an arbitrary distribution with probability density function (PDF) and cumulative distribution function (CDF) $f(y)$ and $F(y)$ respectively. According to Shaw and Buckley (2007), the CDFof transmuted distributions is given as:

$$F(y) = (1+\gamma)F(y)-\gamma[F(y)]^2\ ,y>0\ |\gamma|\le 1 \qquad (3)$$

Where $F(y)$ is the CDF of the baseline distribution, if $\gamma = 0$the baseline distribution is obtained.As given by Shaw and Buckley (2007), the corresponding PDF is the derivative of equation (1) with respect to the random variable y given as follows:

$$f(y) = f(y)[1+\gamma-2\gamma F(y)] \qquad y>0\ |\gamma|\le 1 \quad (4)$$

B. The Transmuted Nwikpe Distribution

The transmuted Nwikpe distribution is derived using equations (3) and (4)

A random variable X, is said to have a transmuted Nwikpe distribution if its CDF and PDF are given respectively by:

$$F_{TND}(x;\vartheta,a) = (1+a)\left[1-\left(1+\frac{\vartheta^2x^2+\vartheta x(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right]-a\left[1-\left(1+\frac{\vartheta x^2+\vartheta x(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right]^2$$

$$=\left[1-\left(1+\frac{\vartheta^2x^2+\vartheta x(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right]\times\left\{(1+a)-a\left[1-\left(1+\frac{\vartheta^2x^2+\vartheta x(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right]\ \right\}x>0 \quad (5)$$

and

$$f_{TND}(x;\vartheta,a) = \left(\frac{\vartheta^3\left(x^2\ \ +x\vartheta\right)}{(\vartheta^2+2)}e^{-\vartheta x}\right)\left\{(1+a)-2a\left[1-\left(1+\frac{\vartheta^2x^2+\vartheta x(\vartheta^2+2)}{(\theta^2+2)}\right)e^{-\vartheta x}\right]\right\}$$

$$=\left(\frac{\vartheta^3\left(x^2\ \ +x\vartheta\right)}{(\vartheta^2+2)}e^{-\vartheta x}\right)\left\{(1-a)+2a\left(1+\frac{\vartheta^2x^2+\vartheta x(\theta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right\} \qquad (6)$$

- *Theorem 1: The transmuted Nwikpe distribution is a valid probability distribution. This suffices that:*

$$\lim_{x\to\infty} F_{TND}(x;\vartheta,a) = 1$$

- Proof:

  Let $\lim_{x\to\omega} F_{TND}(x;\vartheta,a) = 1$

  Where $\omega\to\infty$

$$\lim_{x\to\omega} F_{TND}(x;\vartheta,a) = \lim_{x\to\omega}\left[1-\left(1+\frac{\vartheta^2x^2+\vartheta x(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right]\left\{(1+a)-\alpha\left[1-\left(1+\frac{\vartheta^2x^2+\vartheta x(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right]\ \right\}$$

$$=\left[1-\left(1+\frac{\vartheta\omega^2+\vartheta\omega(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta\omega}\right]\left\{(1+a)-\alpha\left[1-\left(1+\frac{\vartheta^2\omega^2+\vartheta\omega(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta\omega}\right]\ \right\}$$

Recall that $e^{-\vartheta\omega} = 0$, Where $\omega\to\infty$

$$\lim_{x\to\infty} F\ \ _{TND}(x;\vartheta,a) = (1-0)\ \ \big((1+a)-a(1-0)\big)\ = 1+a-a \qquad = 1$$

Thus,

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$$F_{TND}(x;\vartheta,a)=(1+a)\left[1-\left(1+\frac{\vartheta^2x^2+\vartheta x(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right]-a\left[1-\left(1+\frac{\vartheta x^2+\vartheta x(\vartheta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right]^2$$

Is a CDF and

$$f_{TND}(x;\vartheta,a)=\left(\frac{\vartheta^3\left(x^2\quad+x\vartheta\right)}{(\vartheta^2+2)}e^{-\vartheta x}\right)\left\{(1-a)+2a\left(1+\frac{\vartheta^2x^2+\vartheta x(\theta^2+2)}{(\vartheta^2+2)}\right)e^{-\vartheta x}\right\}$$

Is a valid PDF

III. RESULTS

A. Graphs of the CDF of Transmuted Nwikpe Distribution

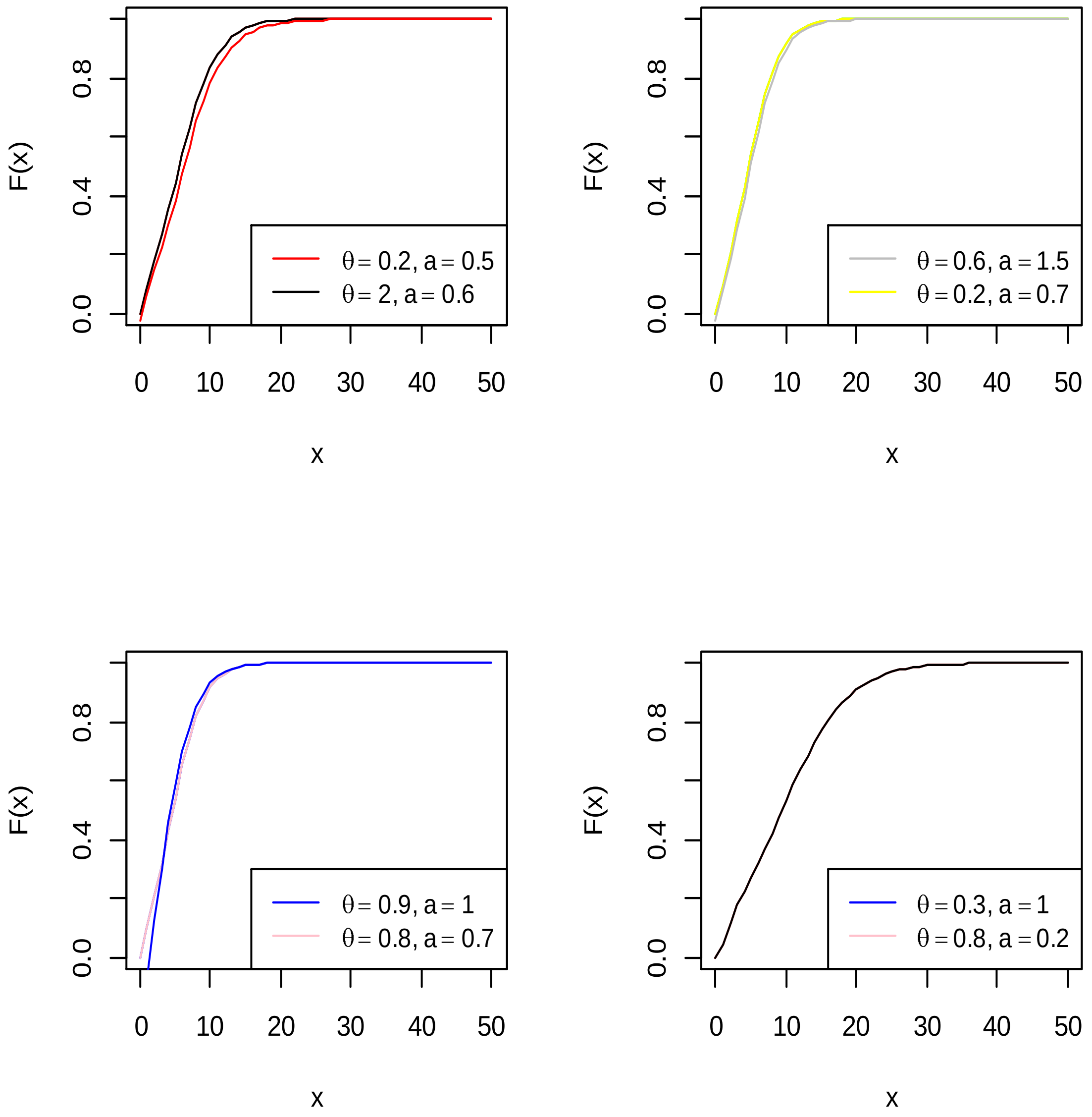

Fig. 1: Graph of the CDF of the Transmuted N-E Type1 Distribution

$a=$ *the transmuted parameter and* $\theta=\vartheta$ *is the shape parameter*

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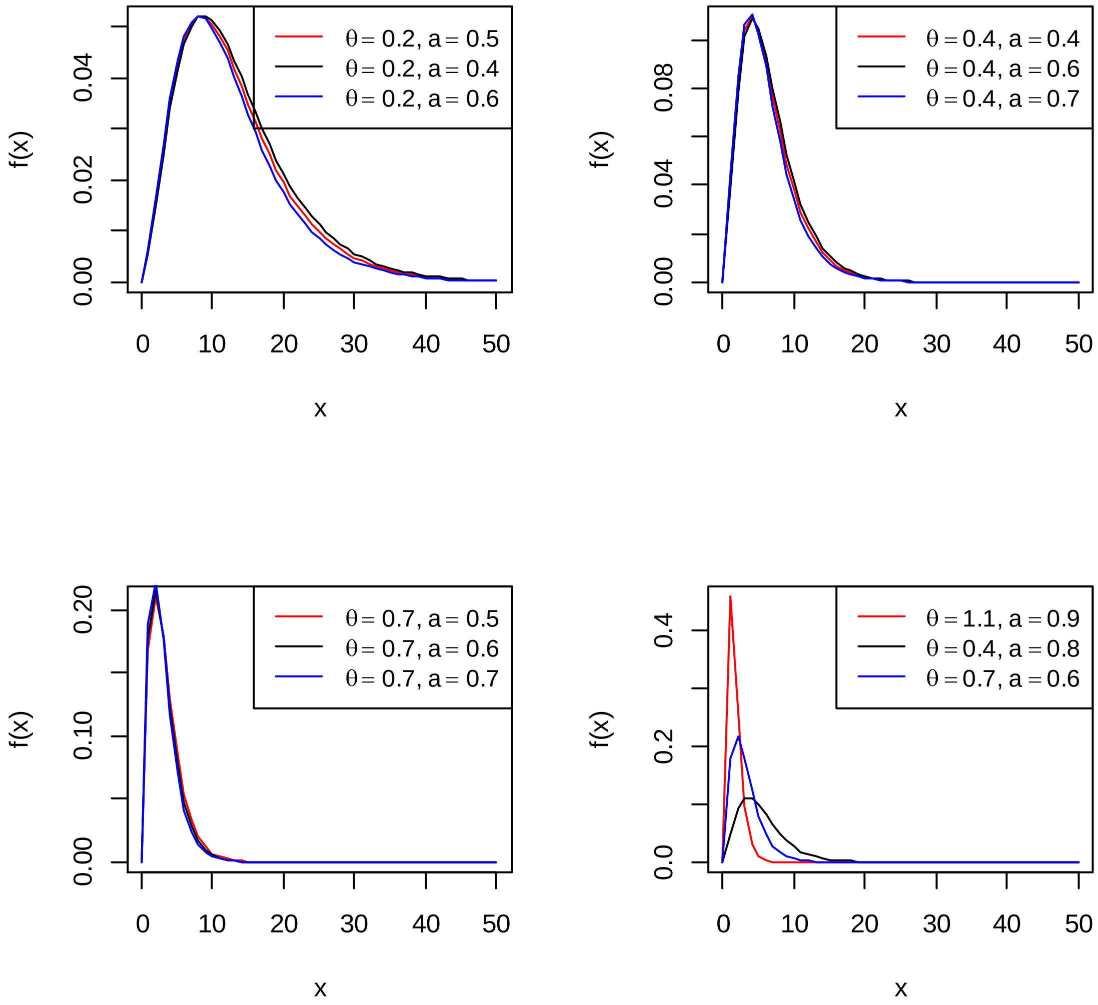
B. Graph of the PDF of the Transmuted Nwikpe Distribution



a = *the transmuted parameter and* $\theta = \vartheta$ *is the shape parameter*

Fig. 2: Graph of the PDF of the Transmuted N-E Type 1 Distribution

C. Survival Function of the Transmuted Nwikpe Distribution

The survival function of a random variable X is given as;

$$S(x) = 1 - F(x) \tag{7}$$

If the random variable X has the transmuted Nwikpe distribution with parameter θ, then

$$S_{TND}(x) = 1 - F_{TND}(x)$$

$$= 1 - (1+a)\left[1 - \left(1 + \frac{\theta^2 x^2 + \theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right] - a\left[1 - \left(1 + \frac{\theta^2 x^2 + \theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]^2 \tag{8}$$

D. Hazard Function or Failure Rate of the Transmuted Nwikpe Distribution

If the random variable X has the transmuted Nwikpe distribution, then the hazard function of X is;

$$H_{TND}(x) = \frac{f_{TND}(x;\vartheta,a)}{S_{TND}(x)}$$

$$= \frac{\left(\frac{\theta^3\left(x^2 \quad + x\theta\right)}{(\theta^2+2)} e^{-\theta x}\right)\left\{(1-a) + 2a\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right\}}{1-\left\{(1+a)\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right] - a\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]^2\right\}} \quad (9)$$

E. The crude Moments of the Transmuted Nwikpe Distribution

By definition the kth crude moments of a random variable $X$is defined as:

$$\mu_k^{'} = E(x^k) = \int_{-\infty}^{\infty} x^k f(x) dx \quad (10)$$

We obtain the kth crude moment of the transmuted Nwikpe distribution $\mu_{TND}^{'}$by substituting equation 4 in 10and setting $\theta = \vartheta$

$$\mu_{TND}^{'} = \int_0^{\infty} x^k \left[\left(\frac{\theta^3\left(x^2 \quad + x\theta\right)}{(\theta^2+2)} e^{-\theta x}\right)\left\{(1-a) + 2a\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right\}\right]$$

$$= \int_0^{\infty} \left(\frac{\theta^3 x^{k+2}e^{-\theta x} + \theta^4 x^{k+1} e^{-\theta x}}{(\theta^2+2)}\right)\left\{(1-a) + 2a\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right\}$$

$$= \frac{(1-\alpha)\theta^3}{(\theta^2+2)}\left(\frac{\Gamma(k+3)}{\theta^{k+3}}\right) + \frac{2\alpha\theta^3}{(\theta^2+2)}\left\{\left(\frac{\Gamma(k+3)}{2\theta^{(k+3)}}\right) + \int_0^{\infty} \frac{x^{k+2}e^{-2\theta x}\left(\theta^2x^2+\theta x(\theta^2+2)\right)}{(\theta^2+2)}\right\} +$$

$$\frac{(1-\alpha)\theta^4}{(\theta^2+2)}\left(\frac{\Gamma(k+3)}{\theta^{k+3}}\right) + \frac{2\alpha\theta^3}{(\theta^2+2)}\left\{\left(\frac{\Gamma(k+2)}{2\theta^{(k+2)}}\right) + \int_0^{\infty} \frac{x^{k+1}e^{-2\theta x}\left(\theta^2x^2+\theta x(\theta^2+2)\right)}{(\theta^2+2)}\right\}$$

$$= \frac{(1-\alpha)\theta^3}{(\theta^2+2)}\frac{(k+2)!}{\theta^{k+3}} + \frac{2\alpha\theta^3}{(\theta^2+2)}\left\{\left(\frac{(k+2)!}{2\theta^{(k+3)}}\right) + \frac{\theta^2}{(\theta^2+2)}\frac{\Gamma(k+5)}{2\theta^{(k+5)}} + \frac{\theta^3}{(\theta^2+2)}\frac{\Gamma(k+4)}{2\theta^{(k+4)}} + \frac{2\theta}{(\theta^2+2)}\frac{\Gamma(k+4)}{2\theta^{(k+4)}}\right\} +$$

$$\frac{(1-\alpha)\theta^4}{(\theta^2+2)}\frac{(k+1)!}{\theta^{k+2}} + \frac{2\alpha\theta^3}{(\theta^2+2)}\left\{\left(\frac{(k+1)!}{2\theta^{(k+2)}}\right) + \frac{\theta^2}{(\theta^2+2)}\frac{\Gamma(k+4)}{2\theta^{(k+4)}} + \frac{\theta^3}{(\theta^2+2)}\frac{\Gamma(k+3)}{2\theta^{(k+3)}} + \frac{2\theta}{(\theta^2+2)}\frac{\Gamma(k+3)}{2\theta^{(k+3)}}\right\}$$

$$= \frac{(1-\alpha)\theta^3}{(\theta^2+2)}\frac{(k+2)!}{\theta^{k+3}} + \frac{2\alpha\theta^3}{(\theta^2+2)}\left(\frac{(k+2)!}{2\theta^{(k+3)}}\right) + \frac{2\alpha\theta^5}{(\theta^2+2)^2}\frac{(k+4)!}{2\theta^{(k+5)}} + \frac{2\alpha\theta^6}{(\theta^2+2)^2}\frac{(k+3)!}{2\theta^{(k+4)}} + \frac{4\alpha\theta^4}{(\theta^2+2)^2}\frac{(k+3)!}{2\theta^{(k+4)}}$$

$$+ \frac{(1-\alpha)\theta^3}{(\theta^2+2)}\frac{(k+1)!}{\theta^{k+2}} + \frac{2\alpha\theta^4}{(\theta^2+2)}\left(\frac{(k+1)!}{2\theta^{(k+2)}}\right) + \frac{2\alpha\theta^6}{(\theta^2+2)^2}\frac{(k+3)!}{2\theta^{(k+4)}} + \frac{2\alpha\theta^7}{(\theta^2+2)^2}\frac{(k+2)!}{2\theta^{(k+3)}} + \frac{4\alpha\theta^5}{(\theta^2+2)^2}\frac{(k+2)!}{2\theta^{(k+3)}}$$

By collecting like terms and simplifying we get

$$\mu_{TNk}^{'} = \frac{(1-\alpha)[(k+2)! + \theta^2(k+1)!]}{\theta^k(\theta^2+2)} + \frac{\alpha[(k+2)! + 2\theta^2(k+1)!]}{4(2\theta)^k(\theta^2+2)} + \frac{\alpha[(k+4)! + 2\theta^2(k+3)!]}{16(2\theta)^k(\theta^2+2)^2} + \frac{\theta^2\alpha[(k+3)! + \theta(k+2)!]}{8(2\theta \quad)^k(\theta^2+2)^2} + \frac{\alpha[(k+4)! + 2\theta^2(k+3)!]}{4(2\theta \quad)^k(\theta^2+2)^2} \quad (11)$$

F. The Mean of the Transmuted Nwikpe Distribution

The mean of the transmuted N-E type1distribution is obtained by setting $\boldsymbol{k = 1}$in equation (11) above, thus, we have:

$$E(x) = \frac{(1-\alpha)[2\theta^2+6]}{\theta \quad (\theta^2+2)} + \frac{\alpha[4\theta^2+6]}{8\theta(\theta^2+2)} + \frac{\alpha[48\theta^2+120]}{32\theta(\theta^2+2)^2} + \frac{\theta^2\alpha[6\theta+24]}{16\theta(\theta^2+2)^2} + \frac{\alpha[12\theta^2+12]}{8\theta(\theta^2+2)^2} \quad (12)$$

G. Distribution of Order Statistics for the Transmuted Nwikpe Distribution

Let $y_{1,}y_2, y_3, \dots, y_{n,}$ be a random sample a distribution with PDF$f(y)$, suppose the corresponding order statistics obtained from the sample is$Y_{1:n} > Y_{2:n} > Y_{3:} > \cdots > Y_{n:n}$.By definition, the PDF of the kthorder statistics is given by:

$$f_{X:n}(y) = \frac{n!}{(k-1)!\,(n-k)!}\left(F(y)\right)^{k-1}[1-F(y)]^{n-k}f(y)$$

$$= \frac{n!}{(k-1)!\,(n-k)!}\sum_{i=o}^{n}\binom{n-k}{i}(-1)^{i}\left(F(y)\right)^{k-1+i}f(y)$$

If the random variable X follows the transmuted Nwikpedistribution the PDF of its kth order statistics is given by;

$$f_{TND}(x) = \frac{n!}{(k-1)!\,(n-k)!}\sum_{i=o}^{n}\binom{n-k}{i}(-1)^{i}\times$$
$$\left\{(1+\alpha)\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]-\alpha\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]^2\right\}^{k-1+i}\times$$
$$\left(\frac{\theta^3\left(x^2 \quad +x\theta\right)}{(\theta^2+2)}e^{-\theta x}\right)\left\{(1+\alpha)-2\alpha\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]\right\} \quad (12)$$

H. Maximum Likelihood Estimate of the Transmuted Nwikpe Distribution

Given a random sample $x_1, x_2, \ldots, x_n$ of size n from the transmuted Nwikpe distribution with PDF $f(x;\theta,\alpha)$ the likelihood function is defined as

$$L\left(f(x,\theta,\alpha)\right) = \prod_{i=1}^{n} f(x_i;\theta,\alpha)$$

$$= \prod_{i=1}^{n}\left[\left(\frac{\theta^3\left(x^2 \quad +x\theta\right)}{(\theta^2+2)}e^{-\theta x}\right)\left\{(1+\alpha)-2\alpha\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]\right\}\right]$$

$$= \left(\frac{\theta^3}{(\theta^2+2)}\right)^{n}\prod_{i=1}^{n}\left(x_i^2 \quad +x_i\theta\right)e^{-\theta\sum_{i=1}^{n}x_i}\times\prod_{i=1}^{n}\left\{(1+\alpha)-2\alpha\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]\right\}$$

By taking the log of both sides we get

$$LogL(x;\theta,\alpha) = nlog\left(\frac{\theta^3}{(\theta^2+2)}\right)+log\left(x_i^2 \quad +x_i\theta\right)-\theta\sum_{i=1}^{n}x_i+$$
$$log\left\{(1+\alpha)-2\alpha\left[1-\left(1+\frac{\theta^2x^2+\theta x(\theta^2+2)}{(\theta^2+2)}\right)e^{-\theta x}\right]\right\} \quad (13)$$

I. Application of the Transmuted Nwikpe Distribution

To establish the applicability and flexibility of the transmuted Nwikpe distribution, a real data set was used to fit the distribution. The performance of the distribution was compared with TPAN, Nwikpe and Shanker distributions. The Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and -2ln$L$ were used as the goodness of fit criteria for the data set considered. The distribution that is associated with the least AIC, BIC and -2InL is said to be more flexible distribution for the data set.The data set is the waiting time in minutes of 100 bank customers before being attended to. This data has been used previously by Shaker (2015) to fit the Shanker distribution.

0.8,0.8,1.3,1.5,1.8,1.9,1.9,2.1,2.6,2.7,2.9,3.1,3.2,3.3,3.5,3.6,4.0,4.1,4.2,4.2,4.3,4.3,4.4,4.4,4.6,4.7,4.7,4.8,4.9,4.9,5.0,5.3,5.5,5.7,5.7,6.1,6.2,6.2,6.2,6.3,6.7,6.9,7.1,7.1,7.1,7.1,7.4,7.6,7.7,8.0,8.2,8.6,8.6,8.6,8.8,8.8,8.9,8.9,9.5,9.6,9.7,9.8,10.7,10.9,11.0,11.0,11.1,11.2,11.2,11.5,11.9,12.4,12.5,12.9,13.0,13.1,13.3,13.6,13.7,13.9,14.1,15.4,15.4,17.3,17.3,18.1,18.2,18.4,18.9,19.0,19.9,20.6,21.3,21.4,21.9,23.0,27.0,31.6,33.1,38.5

Table 1: Descriptive Statistics of the Data Set

N	Max.	Min	Mean	Median	1st Qu.	3rd Qu.	Kurtosis	Var	Skewness
100	38.500	0.800	9.877	8.100	4.675	13.025	5.540292	52.37411	1.472765

Table 2: Goodness of Fit of the Transmuted Nwikpe Distribution

Model	Parameter Estimate	-2inL	AIC	BIC	AICC
Nwikpe	0.3001177	643.3718	645.3717	647.372	646.0032
Transmuted	0.1909046	516.3198	520.3198	520.3192	520.44
Nwikpe	0.9863287				
Shanker	0.1983169	635.2598	637.2597	637.2398	637.3
TPAN	0.3052646 195.1176072	645.144	649.1444	649.144	649.2677

IV. DISCUSSION/ CONCLUSION

In this study, we proposed the quadratic rank transmutation of the Nwikpe distribution. The aim was to introduce additional shape parameters to induce skewness to the Nwikpe distribution to enhance its flexibility for modelling heavily skewed data sets. The mathematical expressions for the density function and the cumulative distribution function of the baseline distribution are given in equations (1) and (2) respectively. The mathematical expressions for the survival function, hazard function distribution of order statistics and the crude moment of the new distributions (TND) have also been derived. The PDF plots for the distributions reveal that the distribution could be used to model positively skewed unimodal data sets. The cumulative distribution function of the transmuted Nwikpe distribution tends to one when the value of X approaches infinity as shown in Figure1, this means that the CDF is valid and by extension, its corresponding PDF is a valid probability function.

Table 2 reveals that the transmuted Nwikpe distribution has the smallest AIC, BIC AICC, and -2inL when fitted to the data set used in this study. Thus, the transmuted Nwikpe distribution performs better than the base distribution, Shanker distribution and the TPAN distributions for the data set used. This shows that the transmuted Nwikpe distribution is an improvement over its base distribution.

- **Competing Interests:** The authors have declared no competing interest.

REFERENCES

[1.] Abdullahi, U. K. and Ieren, T. G. (2018). On the Inferences and Applications of Transmuted Exponential Lomax Distribution. *International Journal of Advanced Statistics and Probability*, 6:30–36,doi:10.14419/ijasp.v6i1.8129.

[2.] Aryal, G. R. and Tsokos, C. P. (2009). on The Transmuted Extreme Value Distribution with Application. Nonlinear Analysis: Theory, Methods and Applications, 71:1401–1407, doi:10.1016/j.na.2009.01.168.

[3.] Bourguignon, M., Le˜ao, J., Leiva, V., and Santos-Neto, M. (2017). The transmuted birnbaum-saunders distribution. *Revstat - Statistical Journal*, 15:601–628.

[4.] Eugene, N., Lee, C., and Famoye, F. (2002). Beta-normal distribution and its applications. *Communications in Statistics-Theory and Methods*, 31:497–512.

[5.] Gupta, R. C., Gupta, P., and Gupta, R. D. (1998). Modeling Failure Time Data By Lehmann Alternatives. *Communications in Statistics-Theory and Methods*, 27:887–904, doi:10.1080/03610929808832134.

[6.] Marshall, A. W. and Olkin, I. (1997). A New Method for Adding A Parameter to A Family Of Distributions With Application To The Exponential And Weibull Families. Biometrika, 84:641–652, doi:10.1093/biomet/84.3.641.

[7.] Merovci, F. and Puka, L. (2014). Transmuted Pareto Distribution. *ProbStat Forum*, 7:1–11.

[8.] Md. Mahabubur Rahman, Bander Al-Zahrani, Muhammad Qaiser Shahbaz (2018) A General Transmuted Family of Distribution. *Pakistan Journal of Statistics and Operation Research*, 14(2) 451-469.

[9.] Nwikpe, B. J., Essi, I. D., and Emeka A. (2021) Nwikpe Probability Distribution: Statistical Properties and Goodness of Fit. *Asian Journal of Probability and Statistics* 11(1): 52-61. DOI: 10.9734/AJPAS/2021/v11i130260

[10.] Samuel, A. F. (2019). On the Performance of Transmuted Logistic Distribution: Statistical Properties and Application. *Budapest International Research in Exact Sciences (BirEx) Journal*, 1:26–34.

[11.] Shaw, W. T. and Buckley, I. R. C. (2007). The alchemy of probability distributions: beyond Gram-Charlierexpansions, and a skew-kurtotic-normal distribution from a rank transmutation map. Research report.