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Is it Time to Reformulate the Partial Differential Equations of Poisson and Laplace?

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Abstract:-

The question of whether it is time to reformulate the time-independent partial differential equations of Poisson and Laplace is no longer a matter of debate but rather an urgency and a principle of decision.

The Poisson partial differential equation PPDE subject to Dirichlet boundary conditions which is currently expressed as,

L.U=s

Should it be rephrased as:
dU/dt)partial = D. LU +S

And the Laplace partial differential equation LPDE subject to Dirichlet boundary conditions which is currently expressed as,
LU=0
Should it be rephrased as:
dU/dt)partial = D. LU

Where L is the Laplacian operator and S is the source term.

It is quite surprising that the solution of time-dependent PPDEs and LPDEs is more accessible than that of time-independent solutions.

We apply chains of B matrices in a revolutionary step to solve Laplace and Poisson PDEs numerically.

We present two applications of solving PPDE and LPDE via the time-dependent model using the B-matrix chain statistical technique.

The numerical results of the proposed model are accurate and fast and superior to classical solutions using the finite difference technique.

We have shown theoretically via the time-dependent statistical model and experimentally by a rigorous experimental technique that the supposed mathematical singularity at the center of the solid sphere of the energy density field has no physical existence.

I. INTRODUCTION

The question of whether it is time to reformulate the time-independent partial differential equations of Poisson and Laplace is no longer a matter of debate but rather an urgency and a principle of decision.

Many mathematicians and physicists still believe that these PDE equations are currently valid and do not need to be changed.

Moreover, they argue that despite constant criticism, the Poisson and Laplace equations remain widely used in physics and mathematics.

These equations are relatively simple to understand and can be used to solve a wide variety of problems. Moreover, the equations were found to be correct in many cases, suggesting that these equations are not outdated and can accurately describe the physical world.

But the question arises; how important is the time-dependent PPDE and Laplace's time-dependent PDE when we live happily without it?

First, but not last, the moment we reformulate the LPDE and PPDE equations into time-dependent equations, the world is turned upside down and life becomes more comfortable.

The Poisson partial differential equation PPDE which is currently expressed as,

L.U=s . . . .(1)

Should it be rephrased as:

dU/dt)partial = D. LU +S . . . . . .(1")

And the Laplace partial differential equation LPDE which is currently expressed as,

LU=0 ....(2)

Should it be rephrased as:

dU/dt)partial = D. LU . . . . . .(2")

Where L is the Laplacian operator and S is the source term.

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Obviously, PPDEs and LPDEs should be subject to Dirichlet boundary conditions or another suitable BC.

In a way, the classical time-independent equations only hold because the speed of their energy transfer is close to that of light and therefore equilibrium is reached almost instantaneously.

All physical laws should contain time and therefore we assume the claims of keeping PDE independent of time as it is neither physically nor mathematically correct.

On the other hand, there are a large number of reasons why the Poisson-Laplace PDE should be reformulated as a function of time, the sooner the better because:

The two equations become similar to the heat equation and all three are self-solving nature equations i.e. they do not need much math to solve them like the need FDM techniques or even the PDE itself. Life becomes more comfortable.

Additionally, time-dependent PPDEs and LPDEs can reveal many interesting facts and absence rules such as:

- Many mathematicians introduce "a time step dt" as the only way to get the time in the image only as a tool where dt is the successive overrelaxation iteration step. Unfortunately they mistakenly think that dt is not real time and time is not a variable in the equation itself.
- Proposed time-dependent PPDEs and LPDEs can be successfully used to solve the energy density distribution of the electric potential field as well as the thermal energy in its most general time-dependent three-dimensional situation [3,4].
- The proposed time-dependent PPDEs and LPDEs have already been successfully used to solve finite double and triple integrations and produce Sympson-like formulas for any number of nodes never found before[5].
- The proposed time-dependent PPDEs have already been successfully used to solve the most general case of time-dependent PPDEs in 3D geometry with arbitrary initial conditions, arbitrary boundary conditions and arbitrary source term[6].
- It allows the description of the so-called Non-Local Poisson equation in the sense that there is a type of interaction at a distance.: The non-local Poisson equation is a generalization of the Poisson equation which uses a non-local kernel. The nonlocal Poisson equation can be used to describe systems that exhibit nonlocal interactions [7,8].

This is because the equation involves integral operators from the boundary conditions.

In conclusion,

- It is important to understand that mathematics is only a tool for quantitatively describing physical phenomena, it cannot replace physical understanding.
- It is claimed that mathematics is the language of physics, but the reverse is also true, physics can be the language of mathematics as in the case of numerical integration and the derivation of the normal/ Gaussian distribution law via the statistical B-matrix chains.

The statement 2 that mathematics is the language of physics always was a given and widely accepted, but the idea that the reverse could be true is quite unexpected.

However, in a revolutionary technique, chains of B matrices are used to numerically solve PDEs, double and triple integrals as well as the general case of time-dependent 3D partial differential equations with arbitrary Dirichlet boundary conditions and conditions arbitrary initials.

II. THEORY

The basis of the present numerical statistical theory is the following recurrence relation:

$$U(x,y,z,t+dt)= B.\ U(x,y,z,t) \quad \ldots\ldots (3)$$

or equivalently in matrix notation,

$$U^{l,m,n,k+1}=B.\ U^{l,m,n,k} \quad \ldots\ldots (3'')$$

Obviously,

$$x=l\ dx\ ,\ y= m\ dy,\ z= n\ dz \text{ and } t= k\ dt.$$

The recurrence relation in equation 3 is the cornerstone of the matrix chain model B introduced to show how nature works in the 4D x-t unit space [9].

The nature itself is known to be linear, binary and symmetrical.

Consequently a proper statistical transition matrix that satisfy linearity, binarity and symmetry such as the B-matrix or any other adequate statistical transition matrix would be able to model the nature behavioue through time chains.

the B-transition matrix itself is well defined through the following four conditions:

For Cartesian coordinates in 1,2 and 3D space, the entries of the transition matrix B (B i , j corresponds to the binary transition probability) respect or are subject to the following conditions:

- B i , j = 1/2-RO/2,1/4-RO/4 and 1/6-RO/6 in 1D, 2D and 3D for i adjacent to j and B i, j = 0 otherwise. Condition (i) translates an equal a priori probability of all directions in space, ie no preferred direction.

- B i, i = RO, i.e. the main diagonal consists of equal or constant entries RO .

RO can take any value in the interval [0,1].

Condition (ii) corresponds to the assumption of an equal residue after each jump or time step dt for all the free elementary nodes.

For emw in free space RO=0 and for a perfect insulator in the heat diffusion/conduction equation RO=1.

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For heat diffusion/conduction PDE RO is equal to a specified value between 0 and 1 depending on the thermal diffusivity and the size of the body concerned.

- B i, j = B j, i, for all i, j.

Matrix B is symmetrical to conform to nature's symmetry and physical principles of reciprocity and detailed balance.

- The sum of B i, j = 1 for all rows (or columns) away from borders and the sum B i, j < 1 for all rows connected to borders.

The condition iv means that the probability of the whole space = 1.

Obviously, the statistical matrix B is very different from the mathematical Laplacian matrix and the mathematical statistical matrix of the Markov transition probability.

The physical nature of B is clear and briefly explained above through its four conditions **i-iv** which support the hypothesis of being an accurate model of nature itself.

Note that n which represents the number of rows or columns in the nxn square matrix B should not be confused with N which is the number of iterations or the number of time steps dt.

It can be shown that the B-statistical transition matrix which satisfies conditions **i-iv** above would generate the solution of PPDE with Dirichlet boundary conditions as a simple map [3,4,6],

$$U(x,y,z,t)= D(N).(b+S). \ldots\ldots (4)$$

where b is the Dirichlet boundary condition vector and S is the source term vector.

The boundary conditions b are no longer problematic but rather a source of simplified solutions.

The transfer matrix D(N) is defined as the matrix power series,

$$D(N)= B + B^2 +B^3+.....+B^N \ldots\ldots (5)$$

Which gives the transitory solution dependent on time at time t=N dt.

For sufficiently large values of N, it can be shown that,

$$D(N)=E(N)-I \ldots\ldots (6)$$

which gives the time-independent stationary solution.

Notice that,

$$E(N)=(I-B)^{-1} \ldots\ldots (7)$$

Two questions arise:

- Why are statistical numerical integration solutions faster, more stable and more accurate than mathematical formulas?
- ii-Why the numerical solution of the B matrix chain is unbeatable and unstoppable even in the "seemingly" most complicated situations like double and triple integration, with or without singularities, as well as solving 3D geometry PPDE, LPDE in as a function of time as well as the heat diffusion/conduction equations

The answer to these questions is explained via illustrative applications and their numerical results section II

III. APPLICATIONS AND NUMERICAL RESULTS

From the moment we reformulate the LPDE and PPDE equation into time-dependent equations, the world is turned upside down.

The two equations become similar to the heat equation and all three are self-solving nature equations i.e. they do not need mathematics to solve them.

It is obvious that the solutions via the B-Matrix chain technique, called Cairo technique, do not need to implement the FDM finite difference methods or even the PDE itself as it was followed in the previous conventional methods. [3,4] . .

The numerical solution of this type of PDE belongs to the type of statistical numerical solution of chains of B-transition matrices where nature works in the 4D unit space according to the recurrence relation,

$$U(x,y,z,t+dt)= B.U(x,y,z,t) \ldots\ldots (3)$$

The transition matrix B is well defined via the four conditions **i-iv.**

Below, we provide illustrative numerical solutions for PPDEs and heat conduction/diffusion PDEs in 2D and 3D geometries that validate the accuracy of the B-matrix statistical chains and its superiority over classical mathematical solutions.

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A. CASE A

Two-dimensional energy density field subject to Dirichlet boundary conditions Fig1.

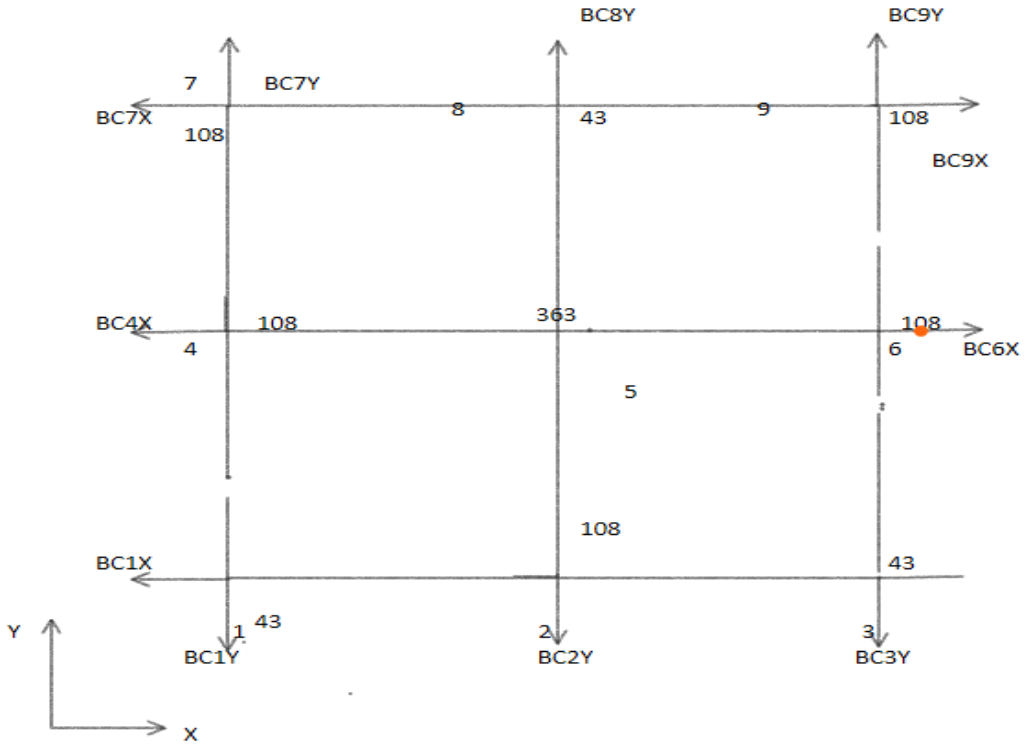

Fig. 1: A 2D rectangular domain with 9 equidistant free nodes

Consider the simple case of a rectangular domain with 9 equidistant free nodes, u1, u2, u3, ... u9 subject to 12 Dirichlet boundary conditions BC1 to BC12 as shown in Figure 1.

The 12 boundary conditions can be reduced to 9 BC for the 9 free nodes as follows,

BC1 = BC1X + BC1Y
BC2 = BC2X + BC2Y
. . . . . .
BC9 = BC9X + BC9Y .

The construction Now of the transition matrix B 9x9 fulfills the **i-iv** conditions for RO = 0 and therefore the corresponding matrix E can be calculated.

Now just multiply the matrix D=E-I (E and D are not a function of BC) by any arbitrary vector BC(b) to get the required solution for the electrostatic voltage distribution V or equivalently the distribution of temperature T c.

J. Mathews [3] classically solved the resulting system of 9 linear algebraic equations using the Gaussian elimination method in a more efficient scheme by extending the tridiagonal algorithm to the more sophisticated pentadiagonal algorithm for its vector BC(b) arbitrarily chosen,

b = [100,20,20,80,0,0,260,180,180] T.. . . . . (8)

and arrived at the solution vector: U=[55,7143,43,2143,27,1429,79,6429,70,0000,45,3571,112 ,357,111,786,84,2857]T.... (9)

Alternatively, the vector BC for the proposed statistical solution of the matrix B corresponding to (1) reduces to

b=[100/4, 20/4, 20/4, 20/4, 80/4, 0, 0.260/4, 180/4, 180/ 4] T. . . (10)

Further, the calculated transfer matrix D=E-I can be multiplied by the vector BC(b) of and we get the statistical solution which is,

U=[55.7132187 43.2126846 27.1417885 79.6412506 69.9978638 45.3555412112.856079 111.784111 84.2846451]T. . . (11)

If we compare the proposed statistical solution (11) with the mathematical solution of Mathews (9), we find a striking simplicity, rapidity and precision.

B. CASE B

Below we present the 3D numerical statistical solution for the transient heating curve (or equivalently the Poisson transient voltage distribution) inside a cube (shown in Figure 2) initially at 0c (or equivalent to zero voltage) suddenly placed in a 76 c container (or equivalent to a 76 volt unit voltage container "Dirichlet BC Equipotential Container")

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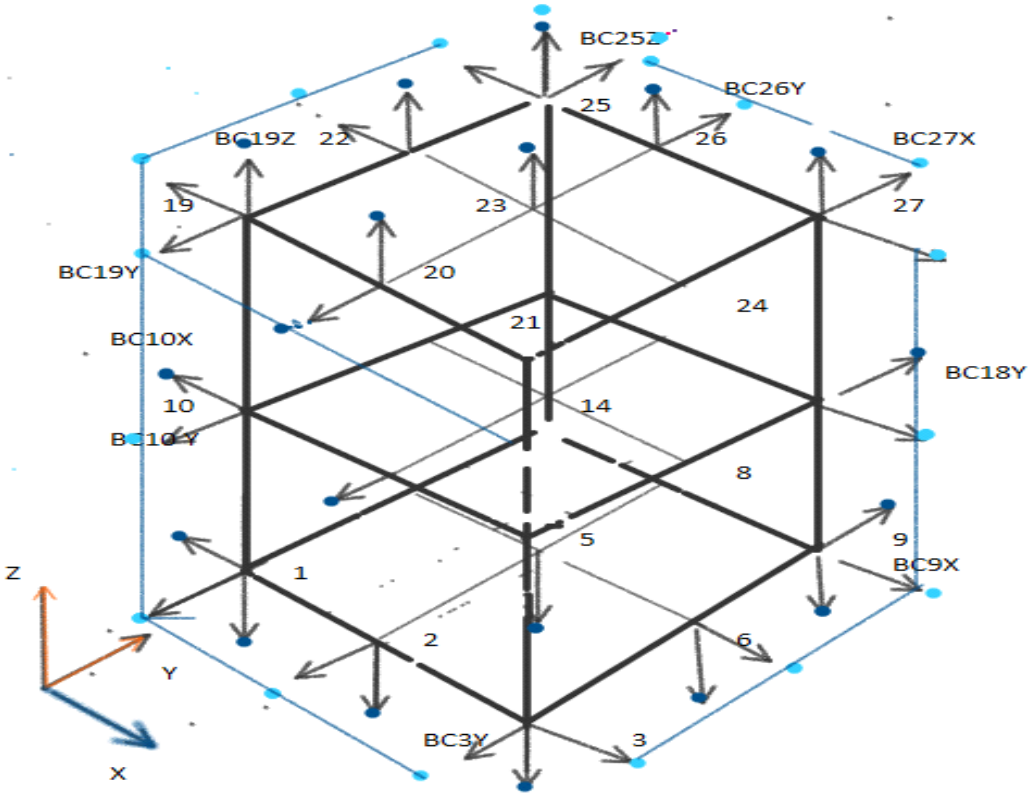

Fig. 2: A 3D cuboid domain with 27 equidistant free nodes

Consider the more complicated case of a cuboid domain with 27 equidistant free nodes, u1, u2, u3, ... u27 subject to 54 Dirichlet boundary conditions BC1 to BC54 as shown in Figure 2.

The 54 boundary conditions can be reduced to 27 BC for the 27 free nodes in a procedure *RO is set equal to zero which corresponds to the diffusion of heat in the perfect conductor and the distribution of voltage in bounded free space.

The results of the numerical calculation for the 27 free nodes are the following:

time=dt

30.400	20.270	30.400	20.270	10.133	20.270	30.400	20.270	30.400
20.270	10.133	20.270	10.133	0.000	10.133	20.270	10.133	20.270
30.400	20.270	30.400	20.270	10.133	20.270	30.400	20.270	30.400

time= 2 dt

44.587	35.129	44.587	35.129	22.969	35.129	44.587	35.129	44.587
35.129	22.969	35.129	22.969	8.107	22.969	35.129	22.969	35.129
44.587	35.129	44.587	35.129	22.969	35.129	44.587	35.129	44.587

time= 3 dt

53.369	45.307	53.369	45.307	34.543	45.307	53.369	45.307	53.369
45.307	34.543	45.307	34.543	19.996	34.543	45.307	34.543	45.307
53.369	45.307	53.369	45.307	34.543	45.307	53.369	45.307	53.369

time= 4 dt

59.197 52.771 59.197 52.771 43.872 52.771 59.197 52.771 59.197

52.771 43.872 52.771 43.872 31.634 43.872 52.771 43.872 52.771

59.197 52.771 59.197 52.771 43.872 52.771 59.197 52.771 59.197

time= 5 dt

63.348 58.306 63.348 58.306 51.270 58.306 63.348 58.306 63.348

58.306 51.270 58.306 51.270 41.424 51.270 58.306 51.270 58.306

63.348 58.306 63.348 58.306 51.270 58.306 63.348 58.306 63.348.

time= 6 dt

66.392 62.493 66.392 62.493 57.007 62.493 66.392 62.493 66.392

62.493 57.007 62.493 57.007 49.301 57.007 62.493 57.007 62.493

66.392 62.493 66.392 62.493 57.007 62.493 66.392 62.493 66.392 .....

etc..

Another interesting physical situation is the case where the same cube in Figure 2 heats up (OR equivalently the Poisson voltage distribution builds up) when held in a reservoir of 0c and heated by a source of 100 units placed at node 1. (OR equivalently the Poisson voltage distribution builds up when a source of 100 units of voltage is placed at node 1).

The 3D solution as a function of time is represented for the 27 free nodes by the following transient heating/voltage rise curve,

time = dt

120.0 13.3 0.00 13.3 0.00 0.00 0.00 0.00 0.00

13.3 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

0.00 0.00 0.00 0.00 0.00 0.00 0.0 0.00 0.00.

Time = 4 dt

133.3 22.16 2.84 22.16 5.69 0.711 2.84 0.711 0.0

22.16 5.69 0.71 5.69 1.42 0.00 0.71 0.00 0.00000000

2.84 0.71 0.00 0.71 0.00 0.00 0.00 0.00 0.00.

Time = 7 dt

137.5 26.2 4.73 26.17 9.45 2.23 4.73 2.23 0.577

26.17 9.45 2.23 9.45 4.47 1.15 2.23 1.15 0.278

4.73 2.23 0.577 2.23 1.15 0.278 0.577 0.278 5.057E-02

Time = 15 dt

138.9 27.9 5.79 27.9 11.6 3.50 5.79 3.50 1.32

27.9 11.6 3.50 11.6 7.01 2.65 3.50 2.65 1.15

5.79 3.50 1.32 3.50 2.65 1.15 1.32 1.15 0.54

Time = 16 dt

138.95 27.96 5.814 27.96 11.63 3.54 5.81 3.54 1.35

27.96 11.63 3.54 11.63 7.09 2.705 3.54 2.70 1.18

5.81 3.54 1.35 3.54 2.70 1.18 1.35 1.18 0.567

Time = 19 dt

139.0 28.0 5.87 28.0 11.73 3.62 5.87 3.62 1.40

28.0 11.7 3.6 11.7 7.23 2.80 3.617 2.80 1.25

5.87 3.62 1.40 3.62 2.80 1.25 1.40 1.25 0.615

Note that the time-independent stationary equilibrium is almost reached on the 27 free nodes after 19 time steps.

The same model can provide examples of cooling curves for different metals where the RO value is between zero and 1 depending on the size and thermal diffusivity of the object under consideration.

In B-Matrix strings, dt is imperial and real time is completely lost.

It should be mentioned that the B-matrix strings are relativistic invariant as they should be.

C. CASE C

- *Irremovable Singularity*

It is true that the heat conduction/diffusion equation and the voltage energy density distribution as a function of time in the solid sphere have no mathematical solution.

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To our knowledge, the conduction/diffusion of heat as a function of time in the solid sphere has never been solved mathematically, neither analytically nor numerically.

Neither in specific articles nor in respectable reference works suggested by mathematicians and theoretical physicists.

In other words the solution does ont exist, or it exists but mathematically inaccessible due to the existence of an immovable singularity at the center of the sphere r=0 where Nabla^2 r tends towards infinity.

Moreover, it is quite difficult to prove the existence or non-existence of such a solution.

Einstein and some other notable scientists of the time once said that nature does not care about our mathematical difficulties.

It integrates experimentally.

The proposed reformulation of time-dependent PPDEs and LPDEs suggests a simple and elegant form for the time-dependent temperature/voltage at center of mass cooling curve c for a material body of any shape [including solid spheres]:

Tc(t)-T(S)=[Tc(0)-T(S)] . Exp(-C1.t.A/V) . . . (11)

T(S) is the ambient temperature assumed to be uniform in space and constant in time (t).

The pre-imposed condition of T(S) corresponds to the Dirichlet boundary conditions for the considered object.

C1 is a constant function of the body's diffusivity (alpha) where A and V are its area and volume, respectively.

For simplicity, the thermal conductivity and therefore the diffusivity are assumed to be an isotropic scalar quantity.

It is surprising that Eq 11 accurately describes the experimental and theoretical statistical dependence of temperature on time.

Equation 11, which we call the BC equation for distinction, is an elegant time-dependent solution to the heat diffusion/conduction equation for a three-dimensional object of any shape.

The BC equation produces some interesting rules:

- two 3D bodies of different shapes cannot have the same volume to area ratio unless both have exactly the same volume and area,

This physical rule produced by the laws of nature has exceptions, but only when applied outside its scope.

However, these exceptions confirm the rule rather than deny it.

- The minimum value of the V/A ratio of a body of 3D geometry is equal to 1/6 and is satisfied by cubic geometric shapes.
- Two three-dimensional bodies of different shapes and of the same materials can only have the same mass m if they both have exactly the same volume V and the same area A.
- Sabine's imperial formula for the reverberation time Tr in sound rooms:

Tr= constant * V/S. A.

is deduced in a regular way via the equation BC which is a product of the chains of the matrix B

The BC equation itself can be validated through transient cooling curve experiments as well as through statistical techniques.

The experimental results validating the BC equation are presented via the following four tables:

These experiments are subjected to the initial temperature T0=76°C and the temperature of the cooling tank Ts is kept constant Ts=4°C [10,11].

Table I, Cooling curve for a 2.5 kg aluminum cube with a side of 10 cm

t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

temp(c) 76 45 33 26.5 23 20 17.2 15.6 13.8 11.7 9.9 8.8 7.9 7.4 6.95

Table II, Cooling curve for a 2.5 kg aluminum cylinder with a diameter of 14.0 cm and a length of 6.5 cm

t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

temp(c) 76 44 32 26.23 20 17.2 15.6 13.8 11.6 9.8 8.6

7.9 7.4 6.

Table III, Cooling curve for a 6.2 kg aluminum pyramid with a 20 cm square base and a height of 19 cm.

t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

temp(c) 76 49 37.5 28 26 22.6 18.3 16.8 15, 13.5 12.6

12 11.3 9

Table IV, Cooling curve for a 2.9 kg solid aluminum sphere, 13 cm in diameter.

t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

temp(c) 76 49 37. 32 28.5 25 22.2 20 18.6 16 14.3 13.3 12.3 11.7 11.2

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It should be noted that;

- The temporal evolution of the temperature in the center of the of 3D geometriy shapes follow precisely the BC equation.
- The temporal evolution of the temperature in the center of the solid sphere is continuous, smooth and regular. It follows precisely the BC equation in a similar way to that of other 3d shapes of similar A/V ratio..

Conclusion is that the mathematical singularity at r=0 does not exist physically.

Table I , Cooling curve for a 2.5 kg aluminum cube with a side of 10 cm

t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

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t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

temp(c) 76 44 32 26.23 20 17.2 15.6 13.8 11.6 9.8 8.6

7.9 7.4 6.

Table 3, Cooling curve for a 6.2 kg aluminum pyramid with a 20 cm square base and a height of 19 cm.

t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

temp(c) 76 49 37.5 28 26 22.6 18.3 16.8 15, 13.5 12.6

--- 12 11.3 9

Table 4, Cooling curve for a 2.9 kg solid aluminum sphere, 13 cm in diameter.

t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

temp(c) 76 49 37. 32 28.5 25 22.2 20 18.6 16 14.3 13.3 12.3 11.7 11.2

It should be noted that;

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Table 4, Cooling curve for a 2.9 kg solid aluminum sphere, 13 cm in diameter.

t(s) 0 30 60 90 120 150 180 210 240 300 360 420 480 540 600

temp(c) 76 49 37. 32 28.5 25 22.2 20 18.6 16 14.3 13.3 12.3 11.7 11.2

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- The temporal evolution of the temperature in the center of the of 3D geometriy shapes follow precisely the BC equation.
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Conclusion is that tha mathematical singularity at r=0 does not exist physically

IV. CONCLUSION

The Poisson partial differential equation PPDE subject to Dirichlet boundary conditions which is currently expressed as,

LU=s

Should be rephrased as follows:

dU/dt)partial = D. LU +S

And the Laplace partial differential equation LPDE subject to Dirichlet boundary conditions which is currently expressed as,

LU=0

Should be rephrased as follows:

dU/dt)partial = D.LU

Where L is the Laplacian operator and S is the source term.

It is quite surprising that the solution of time-dependent PPDEs and LPDEs is more accessible than that of time-independent solutions.

We apply chains of B matrices in a revolutionary step to solve Laplace and Poisson PDEs numerically.

We present two applications of solving PPDE and LPDE via the time-dependent model using the B-matrix chain statistical technique.

The numerical results of the proposed model are accurate and fast and superior to classical solutions using the finite difference technique.

Moreover, we have shown theoretically via the time-dependent statistical model and experimentally by a rigorous experimental technique that the supposed mathematical singularity at the center of the solid sphere of the energy density field has no physical existence.

NB. All calculations in this article were produced using the author's double-precision algorithm to ensure maximum accuracy, as follows by ref. 12 for example.

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