# Improving First-Year Pre-Service Teacher's Concept of Fractions using "Cuisenaire Rods" 

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#### Abstract

This study explored the efficacy of using "Cuisenaire rods" as a pedagogical tool for enhancing the instruction of fractions among first-year pre-service educators at Presbyterian Women's College of Education - Aburi located in Ghana's Eastern Region. A unit comprising thirty-six (36) pre-service teachers in their third year of study at the Presbyterian Women's College of Education was selected as the sample for this study. The instruments utilized for collecting data in this study were the pre-test, intervention, post-test, and questionnaire. Following the pre-test, a six-week intervention period was undertaken to enhance the teaching of fractions among pre-service teachers through the use of Cuisenaire rods. A post-intervention assessment was administered subsequent to the implementation of the intervention. The methodology employed in this study involved the analysis of the scores derived from both the pre-test and post-test through the utilization of a paired sample $t$-test. Additionally, the questionnaire was analyzed. The research results evinced that the application of educational devices and resources, particularly the Cuisenaire rod method, remarkably augmented the instruction of fractions in the domain of mathematics. It was recognized that a comprehensive comprehension of equivalent fractions was a prerequisite for First-year pre-service teachers to acquire proficiency in the addition and subtraction of fractions. It has been observed that First-year pre-service teachers exhibit an improved understanding of a concept when it is presented to them in a systematic progression starting from the concrete level and progressing to the semiconcrete and ultimately to the abstract level. The implications derived from the study indicate that it would be beneficial to motivate First-year pre-service educators to incorporate the utilization of Cuisenaire rods as a pedagogical tool during the initial lessons on a new concept. It is recommended to instruct students on the concept of Equivalent fractions before addressing the topics of addition and subtraction of fractions.


Keywords:- Fractions, Concept, Cuisenaire Rods, PreService Teachers, Addition, Subtraction.

## I. INTRODUCTION

Mathematics is considered a crucial discipline among the foundational subjects comprising of the fundamental curriculum for primary education across numerous nations globally. The subject holds a distinguished and influential status within the educational curriculum, as possessing proficiency in it has the potential to enhance one's societal
mobility. This elevated position was achieved following its adoption as a substitute for traditional languages such as Latin or Greek. Prior to the first half of the 20th century, these languages were employed as filters for admission into advanced education and particular vocations. According to Carmi, \& Tamir, (2023), the significance of Mathematics is exemplified by its extensive utilization in our diurnal routines, as well as its pivotal contribution to the advancement of technology. Mathematics is widely regarded as a cohesive element that unifies diverse fields of science. The acquisition of a superficial understanding of scientific disciplines may be attributed to the absence of mathematics (He, Qin, Gou, \& Wu, 2023). The aforementioned suggests that without a comprehensive understanding of the fundamental principles of mathematics, students would be unable to attain and implement the requisite competencies and concepts in the realm of Science and Technology. The development of Ghana as a nation may be impeded in the absence of sustainable measures aimed at enhancing the pedagogical aspects of mathematics teaching and learning.

This is attributed to the fact that said pupils or students serve as prospective leaders of the nation. If the current educational framework fails to provide students with a strong mathematical foundation, they will lack the necessary resources and technical skills required to effectively contribute to the progression of Ghana. According to Fasinu, \& Alant, (2023), mathematics possesses the capability to facilitate students in attaining a more comprehensive comprehension of scientific concepts by furnishing the means to measure and elucidate scientific connections. Despite its significance, many students tend to avoid the subject of mathematics. There exists a cohort of students who harbour a desire to be exempted from participating in mathematics instruction. Despite an awareness of the utility of the subject matter, the individuals in question encounter challenges in their acquisition and comprehension of said subject. This phenomenon results in the development of an adverse outlook towards the matter at hand. The discipline of mathematics has been widely acknowledged as a challenging realm in various spheres of society. A significant proportion of candidates are unable to secure admission to tertiary institutions, partly as a result of poor performance in the subject of mathematics. According to Gnanasagaran, Rahim, \& DeWitt, (2023), it is a requirement for every student entering the education system to study mathematics up until the matriculation level.

According to Cui, Hu, \& Rahmani, (2023), the conventional way of expressing fractions offers the benefit of being readily comprehensible when discussing divisions
of a complete fraction. The manipulation of fractions presents significant challenges to many individuals, particularly those pursuing diplomas in education. The candidates encountered challenges in successfully solving mathematical problems that incorporated fractions. According to the Chief Examiner's Report from the Institute of Education at the University of Cape Coast in 2007, candidates frequently omit crucial steps in their work (Sie, \& Agyei, 2023). Despite numerous targeted initiatives aimed at enhancing Mathematics instruction, there has been a marked escalation in research deficiency in this subject over several decades (Lemos, 2023). It is reasonable to posit that the underlying reasons for educational shortcomings are rooted in the pedagogical methods employed, the manner in which children acquire knowledge, and the contextual aspects of instruction (Vieluf, \& Klieme, 2023) Active participation is widely regarded as the most efficacious approach to acquiring a concept or skill.

## > Statement of the Problem

Etsey, posits that the Chief Examiner's Report of the West African Examination Council for the years 2002, 2004, and 2008 identified some weaknesses in the Basic Education Certificate Examination (BECE). Specifically, the report indicated that candidates displayed limited proficiency in the comprehension and resolution of questions related to fractions. Furthermore, it was observed that First-year pre-service educators displayed deficiencies in solving mathematical queries that encompass the fundamental operations of addition, subtraction, multiplication, and division with regard to fractions. The researchers was not surprised by the inadequacy of some candidates in answering fraction-related questions in the BECE. This is attributed to the limited grasp of the concept of fractions by many service teachers, thereby hindering their ability to teach it effectively.

Moreover, the pedagogical proficiency exhibited by First-year pre-service educators during their On-Campus Teaching Practice (OCTP) while imparting knowledge on the subject of addition and subtraction of fractions, was disappointingly inadequate. There appears to be a challenge for educators to present practical lessons on fractions. There is a common practice of utilizing the Least Common Multiple (LCM) as a means of addition or subtraction; however, the underlying practical significance of this methodology is generally overlooked in educational contexts. Individuals who utilize Teaching Learning Materials (TLMs) often encounter challenges when demonstrating arithmetic operations on dissimilar fractions, such as addition or subtraction of fractions with different denominators (+/-). Pre-service educators are envisioned to possess proficiency in utilizing teaching and learning materials (TLMs) such as the Cuisenaire rods to visually explicate fractions; however, they themselves demonstrate an inability to execute this task. A subset of individuals exhibit difficulty in correlating the colors with the dimensions of the rods. The mention of Cuisenaire rods alone evokes apprehension among a significant portion of the population. There exists a limited cohort who possess proficiency in the manipulation of Cuisenaire rods, whereby
they are adept at employing them as an instructional tool for imparting knowledge on the operations of integers. However, they are unable to apply the same principles to fractions. The present study aimed to confront the challenges encountered by First-year pre-service teachers while instructing the concept of fractions. The quality of mathematics instruction is contingent upon the extent of mathematics pedagogical content knowledge (PCK)) possessed by educators. Given that mathematics serves as the foundation for developing students' mathematical abilities, its content merits attention across all educational levels. It is possible that curriculum developers may overlook this aspect, as their focus is primarily geared towards designing and implementing instructional materials that are believed to facilitate optimal educational outcomes. the fact that it is imperative to maintain a clear understanding that the importance of the issue at hand cannot be overstated.

The premise that the practice of teaching is integral to achieving meaningful advancements is well-established in academic discourse. It is not unexpected that a significant number of Ghanaian educators possess inadequate comprehension and proficiency in mathematics, as noted by (Quansah, Tsotovor, Titty, Ochour, Oppong, Opoku, \& Acheampong, 2022). Concurrently, the degree of significance attributed to mathematics renders it increasingly necessary that every child is instructed in obtaining a proficient comprehension and innate aptitude for the subject matter.

The implementation of robust standards and highcalibre curricula is of paramount significance in the education system. However, it is imperative to acknowledge that curriculum is a static construct and that its instruction necessitates active facilitation. The effective implementation of standards and curricula within school systems is reliant upon the expertise of proficient educators who possess a comprehensive understanding of the subject matter. Given the particular challenge that pupils and their teachers encounter when confronted with the addition and subtraction of fractions, it is imperative for mathematics educators to embrace sound pedagogical practices that are grounded in strong subject matter expertise. Specifically, this entails the development of robust mathematics knowledge of teaching (MKT), which enables educators to effectively select and employ instructional materials, accurately assess the progress of their students, and make judicious decisions regarding the delivery, emphasis, and ordering of instructional content.
$>$ Purpose of the Study
The study is to find out;

- The challenges First-year pre-service teachers encounter in using the Cuisenaire rods in teaching adding, subtracting, dividing and multiplying fractions.
- Whether the use of manipulatives (Cuisenaire rods) will improve the teaching of fractions.
- Whether the use of Cuisenaire rods will facilitate the use in solving addition, subtraction, division and multiplication of fractions.
> Research Questions
The following research questions will guide the conduct of the study;
- What challenges do First-year pre-service teachers face while utilizing Cuisenaire rods in teaching addition and subtraction of fractions?
- To what degree will the incorporation of Cuisenaire rods enhance the pedagogical approach to the concept of fractions?
- To what extent will Cuisenaire rods facilitate the resolution of addition and subtraction of equivalent fractions?


## $>$ Significance of the Study

The findings of the study will serve as valuable guidance to First-year pre-service educators in the Colleges of Education with respect to enhancing their instructional techniques regarding fractions by implementing the Cuisenaire rod as a teaching approach. This statement proposes that policymakers and relevant stakeholders involved in teacher education prioritize the provision of teaching and learning resources, particularly Cuisenaire rods, to the Colleges of Education in Ghana.

## II. REVIEW OF RELEVANT LITERATURE

The aforementioned proposition alludes to the notion that a fraction can be depicted as a constituent component of a larger entity, whereby said entity may either be a singular unit or a collection of multiple objects.

According to Sempere-Valverde, Guerra-García, García-Gómez, Espinosa, (2023), children at the primary and secondary levels of education dedicate a significant amount of time annually to fraction-related assignments. Students acquire the ability to perform addition and subtraction operations on two fractions, multiplication of one fraction with another, and division of one fraction by another through the process of learning. Subsequently, they are acquainted with expeditious techniques and idiomatic expressions such as "invert and multiply. " After completing their calculations involving fractions, teaching practice students are frequently tasked with redoing previously assigned work which had been completed by students but was not comprehended. It is essential for children enrolled in secondary institutions or other forms of advanced education to possess the ability to perform mathematical operations such as addition, subtraction, multiplication, and division with common fractions. The use of alphabetic symbols to denote numerical values, commonly observed in mathematics and science, warrants specific consideration with regard to their computational and analytical operations (Sempere-Valverde, et al, 2023),.

In various mathematical contexts, such as solving equations and altering formulae, algebraic fractions frequently emerge as a recurrent theme. Therefore, it is of crucial importance to possess knowledge and comprehension of the methods employed in performing fundamental mathematical operations, including addition, subtraction, multiplication, and division.

Knight, (2023), posits that observations, with the exception of whole-number computation, the elementary mathematics curriculum entails no area that necessitates more extensive study than that of fractions. Notwithstanding the extensive duration of academic pursuit, a considerable proportion of high school students exhibit inadequacies in terms of comprehensive grasp of fractions, compounded by a deficient comprehension in fundamental operations involving fractions. When queried regarding their recollections of fractions, grown-ups frequently retort with the following phrase: "One's duty is not to question why; rather, invert and proceed with multiplication. "According to Knight, (2023). the fraction may be regarded as the resolution of the problem as the issue surrounding the allocation of resources or distribution of shares is a prevalent concern. The distribution of 3 dollars among four individuals can be mathematically represented as 3 divided by 4 . In order to facilitate students' comprehension of the relationship between whole numbers and fractions, it is imperative that instruction on the subject of fractions and their corresponding operations is founded upon tangible models. It is imperative to establish a strong foundational comprehension of number sense in relation to fractions, as well as a more profound understanding of the algorithms governing operations with fractions, prior to engaging in formal work with fractions.

According to Radick, (2023), making reference to Brunner's work observed that during the acquisition of a new concept by either a child or an adult, there are three distinct stages that must be traversed Throughout human life, these stages are implemented concomitantly to facilitate the acquisition of new skills. The authors divulged that it is the responsibility of educators to initiate instruction with tangible materials, followed by transitioning towards illustrations and diagrams, progressing through increasingly abstract stages from the semi-concrete to ultimately the purely theoretical. The concept of "fraction" is explicated by Moldavan, Franks, \& Richardson, (2022), in their scholarly discourse who were of the view that Vulgar fractions can be considered equivalent to simple fractions. As an illustration, it has been explicated that. $\frac{1}{4}$ they explained that the lower number that is the 4 is the denominator and 1 is the numerator. The golden rule sets that before fractions could be added or subtracted, they must have the same denominators that is $\frac{1}{5}$ and $\frac{2}{5}$ could be added to get $\frac{3}{5}$ however $\frac{1}{2}$ and $\frac{1}{3}$ cannot be added until they changed into fractions of the same denominators. They also explained that if a fraction is multiplied by a whole number, the denominator remains the same. For instance $3 x$ $\frac{4}{12}=\frac{12}{12}=1$, moreover when a fraction is multiplied
by a fraction, multiply the numerator together and the denominators also together, for example, the rule also says that divisions of fractional numbers averts the divisor and multiply the numerator and the denominator for instance $\frac{8}{15} \div \frac{2}{5}=\frac{8}{15} \times \frac{5}{2}=\frac{40}{30}=\frac{4}{3}$. Kalt, (2022). stated that addition or subtraction with different denominators can be solved using fractions as illustrated in the examples below: $\frac{4}{7}+\frac{3}{5}$

Equivalent fractions such as $\frac{4}{7}$ are

$$
\frac{8}{14}=\frac{12}{21}=\frac{16}{28}=\frac{20}{35}=\frac{24}{42}
$$

And those of are $\frac{3}{5}$

$$
\begin{aligned}
& \qquad \begin{array}{l}
\frac{6}{10}=\frac{9}{15}=\frac{12}{20}=\frac{15}{25}=\frac{18}{30}=\frac{21}{35} \\
\frac{4}{7}+\frac{3}{5}= \\
\frac{20}{35}+\frac{21}{35}=\frac{41}{35} \\
\text { In the same way; }
\end{array} \frac{\frac{2}{3}-\frac{1}{4}}{}
\end{aligned}
$$

$$
\text { Equivalent fractions of } \frac{2}{3} \text { are } \frac{4}{6}=\frac{6}{9}=\frac{8}{12}=\frac{10}{15}
$$

$$
\frac{2}{3}-\frac{1}{4}=\frac{8}{12}-\frac{3}{12}=\frac{5}{12}
$$

$$
\text { And of } \frac{1}{4} \text { are } \frac{2}{8}=\frac{3}{12}=\frac{4}{16}
$$

According to Hurst, Butts, \& Levine, (2022), diagrams have the ability to aid in the organization of certain classifications that children have made, thus promoting the development of their knowledge. Hurst, Butts, \& Levine, (2022), expounded upon the utility of diagrams as didactic and pedagogical aids, particularly in the realm of mathematical problem-solving. The aforementioned proposition conclusively indicates that diagrams have the capability to furnish the problem-solver with a comprehensive understanding of the problem under consideration, facilitate the development of diverse strategies which may aid in its resolution, and assist in the construction of proofs of numerous theorems.

Vale, \& Barbosa, (2023), asserts that the utilization of drawings by children to assess their endeavors to convey the concept of fractions results in a subsequent utilization of spatial arrangements to aid their production of solutions for computational examples via a process of problem-solving. Vale, \& Barbosa emphasizes that visual aids such as pictures, concrete objects, and models possess a unique ability to convey mathematical concepts that are beyond linguistic and symbolic representation. It can be inferred that comprehension of concepts is contingent upon their
elucidation through the use of object diagrams or visual depictions. This is supported by Zhang, Shang, Pelton, \& Pelton, (2020), who also asserts that a beneficial approach to commencing a fraction operation is through the sharing of quantities. The idea of sharing is used to help explain division. When a quantity is shared equally between people, what you're calculating is how much or how large each person gets (Zhang, et al, 2020),

## > Inadequacy of Teaching and Learning Materials.

According to Ayanwale, Mosia, Molefi, \& Shata, (2023), in Ghana, the majority of educational institutions lack sufficient resources for effective teaching and learning. Hence the need to emphasize the use of "No-cost or Lowcost teaching materials". "No-cost" means that the material to be used in teaching does not cost at all and "Low cost" means that the materials to be used in the teaching-learning process have as minimal cost. According to Kaldarova, Omarov, Zhaidakbayeva, Tursynbayev, Beissenova, Kurmanbayev, \& Anarbayev, (2023). appropriate utilization of instructional aids and games can effectively facilitate the learning process, resulting in a simultaneous, effortless, and enjoyable experience. Regrettably, there has been a decline in the utilization of these materials. Cuisenaire rods may serve as an effective tool for imparting knowledge related to fractions (Kaldarova, et al, 2023).

## > Models for Fraction

Three concepts of fractions illustrated by Yeo, \& Webel, (2022), are stated below;

## - Fraction as Sharing (Part - Whole Model)

This is the case where some children share a number of items like oranges or sweets. In a situation where the number of items being shared is not enough for the children, it becomes necessary for them to cut or break the items up into bits and share. This is the use of a fraction to denote part of a whole. In the fraction $\frac{a}{b}$ the bottom number " $b$ " indicates the number of equal parts in a whole and the top number "a" also indicates the number of parts taken out from the whole.

## - Part Group Model

In the day-to-day activities of children, it often becomes necessary for them to consider part of a set in relation to the major set. For instance, in a primary school set up, Basic School 1 (B. S. 1) is part of the school which implies that it is a subset of the school.

## - Ratio Model

This shows the relationship between objects of the same kind. It is a way of comparing the objects and this ends up in the form of a fraction. For instance, in using the Cuisenaire rods to compare the lengths of two rods, it takes 3 red rods to equal the length of 1 dark green rod in Figure (1) below. Hence the length of the red rod is of the length of dark green rod.

The ratio of the length of the red rod to the dark green $\operatorname{rod}$ is $1: 3=\frac{\frac{1}{3}}{3}$


## Dark green

## > Common Errors in Fractions

There are several common errors that students commit when solving questions on fractions. These include common misconceptions in fractions, language errors and Conceptual errors.

## - Language Errors

The manner in which an educator communicates with students holds substantial significance in their ability to create a beneficial effect on their learners. According to Daayeng and Asuo-Baffour, effective use of mathematical language is of great significance and must be approached with a meticulous and precise methodology right from the outset. The researchers of the study discovered that the instruction of mathematical vocabulary is necessary and should be imparted in association with concrete experiences. As an illustration, the fractional notation of ought to be expressed verbatim as "two-seventh," with a focus on the numeral two: Malliakas (2022), contends that in Mathematics, a fractional notation means that a number can be written in the form of $\mathrm{a} / \mathrm{b}$. (i.e.,) numerator/denominator. The top number is called the numerator and the bottom number is called the denominator. The fraction can be classified as proper fraction, improper fraction or mixed fraction Lots of students can make common errors during the manipulation of fractions. The author enumerates prevalent errors, which encompass fractional notations

$$
2 \frac{1}{2} \times 1 \frac{2}{5}=\frac{5}{2} \times \frac{7}{2}=\frac{35}{2}
$$

According to Makhubele (2021), the aforementioned outcome emerges from the utilization of the principle of augmenting fractions that possess equating denominators. Shure, Rösken-Winter, and Lehmann (2022) noted that fractions pose a significant challenge for both educators and students. The writer delineated the subsequent tactics for instructing the notion of fractions;
$\checkmark$ Instructing students to perceive fractions as constituent components of complete entities or assemblies of entities may facilitate their understanding and utilization of these mathematical concepts.
$\checkmark$ It is pertinent to note that the fundamental principle underlying the concept of fractions is that of equitable partitioning.
$\checkmark \quad$ It is suggested to gradually construct the understanding of fractions by first introducing the concept of onefourth, one-third and other similar units.
$\checkmark$ Utilizing manipulatives such as paper folding, sharing, a chart, or a fractional board can be employed to compare different fractions and determine their equivalent forms.

Other common errors identified in fractions are as follows; dividing the given pieces to the value of the denominator of the fractional statement and multiplying it to the value of the numerator, adding the fractional statement to the number that is stated as part by multiplying the numerator and denominator of the fractional statement. Students may also have difficulty simplifying fractions because of a shaky understanding of division. Students often order fractions incorrectly and cannot place them on a number line. They cannot easily count fractions the way they count whole numbers ( $1,2,3,4 \ldots$ ).

## - Conceptual Error in Fractions

Conceptual mistakes occur when students have misconceptions or misunderstandings about the concepts related to the problem, such as the concept of how to add two fractions

- Common Misconception in Fractions

Some common misconceptions in fractions have to do with
$\checkmark$ Missing the importance of equal parts: The denominator in a fraction not just represents the parts into which the whole has been divided, but it also implies that the whole has been divided into as many equal parts.
$\checkmark$ Focus only on the number of parts, and not at their being equal. Most students misunderstand "counting up" as involving changing the denominator rather than the numerator.
$\checkmark$ Confuses the unit fraction with a whole number when counting. Confuses the unit fraction with a whole number when counting. " $1,2,3,4,5$, etc."

Students believe that the numerator and denominator are the same. Let's start with the fractions basics to help address this misconception. 'Denominator' means 'that which names' in Latin. This translation identifies the denominator as a name the same way 'one', 'two' and 'three' are names Students are wrong
$\checkmark$ In rewriting the known components of the problem.
$\checkmark$ To apply the concept of fractional counting operations.
$\checkmark$ To convert mixed fractions into ordinary fractions and vice versa.
$\checkmark$ To change integers to fractions.

## > Challenges Encountered by Students

Fraction notation and terminologies can be challenging
Words like "numerator" and "denominator" are used sometimes forgetting that students may not have internalized those terms and have most likely never heard them. Words like "equivalence" "improper" and "reduce" can also add to the confusion. Many students encounter challenges in effectively managing fractions. The underlying cause for this phenomenon could stem from a lack of adequate comprehension of the aforementioned concept. The comprehension of subject matter is contingent upon the capacity of students to establish connections between symbols and linguistic expressions. The process of
multiplying fractions by an integer was previously perceived as a challenging task. However, the task of adding fractions can be considered a more significant predicament. As an example, the value of $2 \times \frac{1}{3}$ was presented as $\frac{2}{6}$. The following displays are instances of students' output that were presented to the researchers.

$$
-7 \frac{3}{4}+12 \frac{2}{5}=-31+62=31
$$

The students provided a rationale whereby they performed arithmetic operations of multiplication and addition to obtain the values of -31 and 62 , respectively, by multiplying 4 by 7 and adding 3 , as well as by multiplying 5 by 12 and adding 2 . These values were then combined to yield a sum of 31 .

$$
\frac{2}{3}+\frac{4}{5}=\frac{6}{8}
$$

The student elucidated that the summation of the numerators and denominators $\frac{6}{8}$ resulted in the computation of $(2+4)$ and $(3+5)$. Upon analyzing the computational errors that arise in the context of fraction exercises, it becomes clear that a considerable amount of difficulty can be attributed to a deficient comprehension of the underlying concepts. This notion is reinforced by the observations made by Liu and Jacobson (2022). Based on the existing empirical data, a plausible inference can be made that exposing children to a wide variety of grouping encounters before introducing them to the abstract notion of fractions can prove advantageous.

## - Challenges Encountered by First-year Pre-service Teachers

The present study further delved into the challenges that are faced by pre-service teachers in their third year of teacher education. The researchers identified a formidable challenge for pre-service teachers in their third year of training to impart fractional concepts in an effective manner. Certain First-year preservice teachers employed oranges as a tool for introducing the concept of fractions to their students. It must be noted, however, that oranges are not didactic materials suitable for imparting such domainspecific knowledge. The following excerpts constitute some examples of First-year pre-service teachers, elicited through a quiz administered by the researchers in order to gauge their comprehension of the concept of fractions.

$$
\frac{5}{4}-\frac{1}{4}=\frac{4}{0}=4
$$

The majority of First-year pre-service teachers opted to perform subtraction operations on both the numerators (specifically, 5-1) and the denominators (specifically, 4 4). The resulting values obtained were 4 and 0 , respectively. The numerical operation of subtraction was performed between the values of zero and four, yielding an output value of four.

$$
\frac{2}{6}+\frac{1}{6}=\frac{3}{12}=3+12=15
$$

The majority of First-year pre-service teachers employed a method of summing the numerators and denominators, as well as summing the resultant values, to derive a final solution of 15 .

$$
\frac{2}{9} \div \frac{2}{3}=\frac{1}{3}
$$

The majority of pre-service teachers adopted a method of division whereby both the numerators and denominators were divided, resulting in an outcome of 1 for $2 \div 2$ and 3 for $9 \div 3$. The desired outcome was ultimately achieved. In multiplication, the product of both the numerator and denominator were determined to obtain the resultant value. The present study affirms that First-year pre-service teachers are deficient in their comprehension of fractions and are thus unable to effectively impart this knowledge to their pupils. Given that both are greater than, it can be inferred that the solution exceeds 8 . Fostering a robust comprehension of numerical quantities and mathematical operations would facilitate the identification of logical solutions to various problems.

## III. METHODOLOGY

This research utilized a combination of qualitative and quantitative approaches. The purpose of this research was to enhance the pedagogical proficiency of pre-service educators in the instruction of fractions through the application of Cuisenaire rods.

## A. Population

A population can be defined as a collection of various elements or variables that share a common characteristic or feature. Regardless of whether the subject is a human, an object or an event, it must meet specific criteria in order to be of interest to researchers who intend to generalize their findings. The current study's population comprises of Firstyear students enrolled in the Presbyterian Women's College of Education situated in Aburi. A total of two hundred students comprised five classes. The researcher, due to teaching constraints during the semester in question, made the decision to select solely one specific class for the purposes of the study. The selection of this option was solely determined by its convenience.

## B. Sampling Size and Sampling Procedure

Utilizing the probability sampling technique, a specific First-year class was deliberately selected from a pool of five First-year classes of students enrolled at Presbyterian Women's College of Education, Aburi. This selection was a result of the researchers' appointment as Mathematics tutors responsible for instructing the mathematics in that particular class. Therefore, the researchers employed a convenience sampling technique to select a cohort comprised of 36 firstyear pre-service educators. The convenience sampling method is a non-probability sampling technique employed to determine a sample population by selecting units readily accessible to the researchers. There are various factors that could account for this phenomenon, which include geographical proximity, temporal availability, and inclination to participate in the study. The curriculum
comprises of the following modules;

- Learning, Teaching and Applying Number and Algebra
- Teaching and Assessing Numeracy
- Teaching and Assessing Mathematics I (Intermediate)
- Teaching and Assessing Mathematics II (Advanced)

As at the time of the study, the researchers were teaching the first two courses listed above.

## C. Demonstration (Paper Folding Method)

The concept of fractions was introduced by the researchers through the implementation of a pedagogical strategy involving the folding of a sheet of paper in equal portions, which was subsequently distributed among a pair of students. The researchers engaged in a discourse with the student cohort, elucidating that a fraction commonly designated as one-half refers to a single portion of a whole, the latter of which has been partitioned into two identical parts, hence the appellation one-half. When a unitary object is partitioned into seven congruent parts and three of these parts are removed, the resulting fraction of the whole that has been removed is three-sevenths, while the fraction of the object that remains is four-sevenths. The First-year preservice teachers were segregated into cohorts of three and equipped with Cuisenaire rods. The subjects were directed by the researchers to select the orange rod in its entirety. It is recommended that they endeavour to construct a maximum number of rows utilizing rods of a singular colour. It is imperative that every row maintains uniformity in length, matching that of the initially selected rod or the orange rod, as depicted in the following illustration.

$$
\mathrm{W}=\text { white }
$$

| Orange |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Red |  | Red |  | Red |  | Red |  | Red |  |
| W | W | W | W | W | W | W | W | W |  |

From the diagram, five red rods make an orange rod. In a fraction statement, a red ione-fifth of the orange whole which is written as $\frac{1}{10}$. Also, ten white rods make an orange.

In fraction statement, a white is one-tenth of the orange which is written as $\frac{1}{10}$

## > Equivalent Fractions

The concept of equivalence is prominent and therefore, it is advisable to utilize every conceivable opportunity to engage in discourse with a teacher for pre-service educators. Teachers position posits that the genesis of the idea ought to stem from the First-year pre-service teachers' experiential learning, rather than being imparted to them as an isolated study theme. "Compiling the diverse concepts acquired is a beneficial approach. "

For example long strips

| Whole |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\frac{1}{2}$ | $\frac{1}{2}$ |  |  |  |
| $\frac{1}{4}$ |  | $\frac{1}{4}$ | $\frac{1}{4}$ |  |  |

Using Cuisenaire rods to teach equivalent fractions
The study directed pre-service educators to select any rod or assortment of rods to serve as the "whole. " Subsequently, they were asked to generate as many rows as possible utilizing rods of the identical colour, such as exclusively red or solely brown. In order to maintain consistency with the selected original entity, it is necessary that each row is of equivalent length.

| Orange and Dark green |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Brown |  |  |  |  |  |  |  | Brown |  |  |  |  |  |  |  |
| Purple |  |  |  | Purple |  |  |  | Purple |  |  |  | Purple |  |  |  |
| Red |  | Red |  | Red |  | Red |  | Red |  | Red |  | Red |  | Red |  |
| W | W | W | W | W | W | W | W | W | W | W | W | W | W | W | W |

> Pre-service teachers were guided to write down their observations in words. Their observations included:

- Two brown rods make the orange and dark green whole.
- Two purple rods make one brown rod.
- Four purple rods make two brown rods.
- Two red rods make one purple rod.
- Four red rods make one brown rod.
- Eight red rods make the orange and dark green whole.
- Two red rods make one purple rod.
- Four white rods make one purple rod.
- Eight white rods make one brown rod.
- Sixteen white rods make the orange and dark green whole.
- These colour observations are then turned into fractional statement such as:
- A brown is one half of the orange and dark green whole.
- A purple is one-fourth of the orange and dark green whole.
- A red is one-eighth of the orange and dark green whole
- A white is one-sixteenth of the orange and dark green a hole.

From Table 3, we have the pattern as $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{8}{16}$ and also $\frac{1}{4}=\frac{2}{8}=\frac{4}{16} \quad$ Equivalent fractions are fractions of the same value but have different names or structure for instance $\frac{1}{2}$ and $\frac{2}{4}$.

## D. Teaching Addition and Subtraction of Like Fractions

 (Same Denominators)
## > Example 1.

Using the Cuisenaire rods to teach the addition of like fraction such as $\frac{1}{5}$ and $\frac{2}{5}$ to develop the algorithm for the addition of like fractions. Chose the yellow rod as a whole and the white rods each represent one-fifth. The reason why the yellow rod was chosen is that the yellow rod can be split into five equal rods (yellow).

respectively. Putting these white rods together, we will have 3 whites that are three-fifths.

$>$ Example II: Using the Cuisenaire rods we want to solve; $\frac{2}{3}+\frac{1}{3}$
Taking the dark green as the whole, then the red rods are each one-third of the whole. $\frac{2}{3}$ is taken as 2 reds. $\frac{1}{3}$ is also taken as 1 red.

| Dark green |  |  |
| :---: | :---: | :---: |
| Red | Red | Red |

We now have 2 red rods and 1 red rod which are 3 red rods. This is the same as the whole dark green. Hence $\frac{2}{3}+\frac{1}{3}=\frac{3}{3}=1$.

## > Example III:

To solve $\frac{3}{5}-\frac{1}{5}$ using the Cuisenaire rods. Choose a rod or a train of rods to be your whole. Use the orange rod as a whole, and then the red rods are each one-fifth.


$$
\text { Hence } \frac{3}{5}-\frac{1}{5}=\frac{2}{5}
$$

## E. Teaching Addition and Subtraction of unlike Fractions and Mixed Numbers

## > Example 1:

Using the Cuisenaire rods solve the following fractions $\frac{3}{5}+\frac{2}{10}$ and develop the algorithm for addition of unlike fraction. 10 is a multiple of 5 and as such we need to choose a whole rod which can be split into ten. Therefore, the orange is the appropriate rod needed.

| Orange |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Red |  | Red |  | Red |  | Red |  | Red |  |
| W | W | W | W | W | W | W | W | W | W |

1 red rod represents $\frac{1}{5}$ and 1 white rod also represents $\frac{1}{10}$ of the whole rod (orange) 2 white rods represent


Change 3 red rods for 6 white rods, that is $\frac{3}{5}$ has changed to $\frac{6}{10}$ (equivalent fractions).

| W | W | W | W | W | W |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\frac{6}{10}$ |  |  |  |  |  |

Now putting all the white rods together


Therefore $\frac{3}{5}+\frac{2}{10}$ becomes $\frac{6}{10}+\frac{2}{10}=\frac{8}{10}$
Example II: Solve the following fraction $\frac{1}{2}+\frac{1}{3} .6$ is a multiple of 2 and 3. You need to choose a rod which can be split into six. Therefore the dark green is the appropriate rod.

| Dark Green |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Green |  | Green |  |  |  |
| Red |  | Red |  | Red |  |
| W | W | W | W | W | W |

1 green rod represents $\frac{1}{2}, 1$ red rod represents $\frac{1}{3}$ and 1 white rod also represent $\frac{1}{6}$ of the whole that is dark green.

Change 1 green $\operatorname{rod} \frac{1}{2}$ for 3 white rods, that is $\frac{1}{2}$ has change to $\frac{3}{6}$ (equivalent fraction).
Change 1 red $\operatorname{rod} \frac{1}{3}$ for 2 white rods, that is $\frac{1}{3}$ has change to $\frac{2}{6}$ (equivalent fraction).

Now putting all the white rods together.


Hence $\frac{1}{2}+\frac{1}{3}$ becomes $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

Example III: Solve the following fraction $\frac{1}{2}-\frac{1}{8}$. Now, 8 is a multiple of 2 and as such you need to choose a whole rod which can be split into eight equal parts. The appropriate rod is the brown.

| Brown |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Purple | Wurple |  |  |  |  |  |  |
| W W W W | W | W | W | W | W | W | W |

1 purple rod represents $\frac{1}{2}$ and 1 white rod also represents $\frac{1}{8}$ of the whole rod (brown).

1 purple rod represents $\frac{4}{8} \quad w$

Change 1 purple rod for 4 white rods i.e. $\quad \frac{1}{2}$ has changed to $\frac{4}{8}$ (equivalent fractions).


Therefore $\frac{1}{2}-\frac{1}{8}$ has now become $\frac{4}{8}-\frac{1}{8}$


Out of the 4 white rods take one from it. You will realize that there will be three (3) white rods left. Hence $\frac{1}{2}$ $-\frac{1}{8}=\frac{4}{8}-\frac{1}{8}=\frac{3}{8}$

Example IV: Solve the following fraction $\frac{3}{5}-\frac{1}{4}$ using Cuisenaire rods. 20 is a multiple of 5 and 4. So you need to choose rod(s) which can be split into twenty. The appropriate rods are two oranges.

| Orange and Orange |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Yellow |  |  |  |  | Yellow |  |  |  |  | Yellow |  |  |  |  | Yellow |  |  |  |  |
| Purple |  |  |  | Purple |  |  |  | Purple |  |  |  | Purple |  |  |  | Purple |  |  |  |
| W | W | W | W | W | W | W | W | W | W | W | W | W | W | W | W | W | W | W | W |

1 purple rod represents $\frac{1}{5}$ and 1 yellow rod also represents $\frac{1}{4}$ of the whole that is orange and orange.
Change 3 purple rods which represents $\frac{3}{5}$ for white rods and that will be $\frac{12}{20}$ (equivalent fractions).

| $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ | $\mathbf{w}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Change 1 yellow rod which represents $\frac{1}{4}$ for white rods and that will be $\frac{5}{20}$ (equivalent
fractions)


Therefore $\frac{3}{5}-\frac{1}{4}$ has now become $\frac{12}{20}-\frac{5}{20}$ out of the 12 white rods take away 5 white rods from it and 7 white rods would be left.


Hence $\frac{3}{5}-\frac{1}{4}=\frac{12}{20}-\frac{5}{20}=\frac{7}{20}$

Example V: Solve the following mixed fraction $1 \frac{2}{5}+2 \frac{1}{10}$
This can be done either by converting each mixed fraction into an improper fraction as $\frac{7}{5}+\frac{21}{10}$ or by adding the whole numbers together and the fractions together as
$1 \frac{2}{5}+\frac{1}{10}=3+\frac{2}{5}+\frac{1}{10}$

Using the Cuisenaire rods to choose a whole which can be split into fifths and tenths. The orange rod is appropriate.

| Orange |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Red |  | Red |  | Red |  | Red |  | Red |  |
| $\mathbf{W}$ | $\mathbf{W}$ | $\mathbf{W}$ | $\mathbf{W}$ | $\mathbf{W}$ | $\mathbf{w}$ | $\mathbf{W}$ | $\mathbf{W}$ | $\mathbf{W}$ | $\mathbf{W}$ |

For $\frac{2}{5}$ take 2 red and 1 white rods for $\frac{1}{10}$


Change each red for 2 whites.


Joining together we have 5 white rods.

| $W$ | $W$ | $W$ | $W$ | $W$ | $\frac{5}{105} \quad$ And exchanging for 1 yellow |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Yellow  |  |  |  |  |  | | 10 |
| :--- |

Comparing the yellow to the original whole we have

Hence $1 \frac{2}{5}+2 \frac{1}{10}=3+\frac{2}{5}+\frac{1}{10}$

$$
=3+\frac{4}{10}+\frac{1}{10}
$$

$$
=3+\frac{5}{10}=3 \frac{5}{10}=3 \frac{1}{2}
$$

## F. Teaching Multiplications of Common Fractions and Mixed Fractions and Developing the Algorithm

## > Example 1:

Multiplication of a whole number by a fraction such as $5 \times \frac{2}{3}$ multiplication is thought of as a repeated addition. Therefore, 5 $x \frac{2}{3}$ can be interpreted as 5 lots or 5 groups of $\frac{2}{3}$. Using the rods, we need to identify a whole which can be divided into thirds. The light green is the appropriate rod. Take two-thirds five times.


For two thirds we take 2 white. Therefore for $5 \frac{2}{3}$ we take two whites 5 times. This gives 10 white rods. Comparing this to our original whole of light green we have $\frac{10}{3}=3$

| Light Green |  |  | Light Green |  |  | Light Green |  |  | Light Green |  |  | Light Green |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$\frac{2}{3} \quad \frac{2}{3}$
$\frac{2}{3}$
$\frac{2}{3} \quad$ that is $5 \times \frac{2}{3}=\frac{10}{3}=3 \frac{1}{3}$

Example II: Multiplication of a fraction by another fraction such as $\frac{2}{3} x \frac{3}{5}$ is thought as $\frac{2}{3}$ of
$\frac{3}{5}$
Choosing a suitable whole rod which divides exactly fifths. An orange rod is proper. We then find which colour of rods represents one-fifth. An orange is the same as 5 reds.

We take three of these rods to represent $\frac{3}{5}$

| Orange |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| Red | Red | Red | Red | Red |  |

Our whole now is $\frac{3}{5}$ of which we need to find two thirds. The new whole is three reds

which divides the original whole.

| Red | Red | $\frac{3}{5}$ |
| :--- | :--- | :--- |
|  | $\frac{2}{3}$ | $\frac{2}{6}$ |

Hence $\frac{2}{3} \times \frac{3}{5}=\frac{2}{5}$

## G. Teaching Division of Common Fractions and Mixed Numbers

By means of the Cuisenaire rods to develop the algorithms, the operation of division applied to fractions is tremendously challenging for students to understand for instance $4 \div 1 \frac{1}{3}$ means how many one-thirds are in four.

Example 1: Dividing a whole number by a fraction such as $4 \div \frac{1}{\mathbf{3}}$
Choose two rods one of which is a third of the other. Dark green and red rods are the appropriate. 4 wholes are taken as 4 dark green.

| Dark Green |  |  | Dark Green |  |  | Dark Green |  |  | Dark Green |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R | R | R |  |  |  |  |  |  |  |  |  |
| R | R | R | R | R | R | R | R | R | R | R | R |

4 Wholes

One red indicates $\frac{1}{3}$ we have 12 reds and this gives twelve-thirds that is $4 \div \frac{1}{3}=12$ Example II: Dividing a

## fraction by another fraction

Solve $\frac{1}{3} \div \frac{1}{2}$
Take a rod that can be divided into three and two. The dark green is the most appropriate rod. One light greed rod will be equal to $\frac{1}{2}$ of the dark green whole and two red rods equal
to $\frac{2}{3}$ of the dark green whole.


Comparing the light green rod along with the two red rods gives one light green plus one white rod. 1 white rod is equivalent to $\frac{1}{3}$ of light green rod. This indicates thathalve-in $\frac{2}{3}$ will be
$1 \frac{1}{3}$
Hence $\frac{2}{3} \div \frac{1}{3}=\frac{4}{3}=1 \quad \frac{1}{3}$
Example III: Dividing a mixed fraction by a fraction using the Cuisenaire rods such as
$2 \frac{1}{2} \div \frac{1}{3}$. Use a rod that can be divided into 2 and 3. Dark green rod be ok for that. Taking three dark green rods to represent three wholes, one light green rod will be equivalent to $\frac{1}{2}$ of a dark green whole and 1 red rod will be equal to $\frac{1}{3}$ of a dark green whole.


Comparing the red rods along with the 2 new 1 whole, we have 7 red rods plus one white rod. One white rod is equal to $\frac{1}{2}$ of the red rod. This indicates that one thirds in 2 will be $7 \frac{1}{2}$. Hence $2 \frac{1}{2} \div \frac{1}{3}$ the mixed fraction could be first change to improper fraction that is $2 \frac{1}{2} \div \frac{1}{3}$ becomes $\frac{5}{2} \div \frac{1}{3}=\frac{15}{2}=7$

## H. Method of Data Collection

The researchers used six weeks for the collection of the data. The first week was used to conduct the pre-test. Four weeks used for the intervention and the last week used for the post-test.

## I. Validity and Reliability of Instruments

The statistical analysis process involved the deliberate use of data in order to extract essential information. The collection of data is a crucial prerequisite for the successful completion of a thriving statistical analysis. As posited by Kavzoglu and Teke (2022), the reliability and validity derived from a statistical analysis primarily depend on the quality, relevance, and accuracy of the data employed. The concept of validity for a research instrument pertains to the extent to which the items comprising it are able to accurately assess the intended constructs. Reliability refers
to the extent to which an assessment instrument produces dependable results when utilized to measure a particular construct. Specifically, the measuring instrument is capable of generating similar or identical outcomes upon repeated administration to a given participant operating under similar environmental or contextual conditions. To guarantee the dependability of the measuring tool, the investigators opted to dispense uniform interrogations and intervention tactics to all students enrolled in the investigation on numerous instances, with a minimum frequency of two instances. The aforementioned process was executed with the objective of establishing coherence in both the responses and the resulting scores. The research utilized a triangulation methodology for the gathering of data, which entailed the utilization of diverse practices of inquiry. The scholars employed pre-test, post-test, and survey methodologies in order to attain a thorough understanding of the
circumstance. During the administration of both the pre-test and post-test, the researchers afforded supervision to the participants.

## J. Data Collection Instruments

Two forms of instruments were used in this study, namely a questionnaire and standardized tests, specifically. The objective of this study was to assess the proficiency of First-year pre-service teachers in utilizing the Cuisenaire rods approach for the instruction of fractions through the implementation of a pre-test and post-test evaluation design. The purpose of the questionnaire was to examine the rationale behind the perceptions held by teacher-trainees towards the fundamental properties of fractions as well as the efficacy of the Cuisenaire rods approach in teaching fractions. The pre-test was developed with the intention of evaluating the fractional comprehension and manipulation skills of the teacher-trainees. The test's contents pertained to the computation of fractions through addition and subtraction. The interrogatives were derived from the material presented in the Basic Education Diploma textbooks and the syllabus of the educational institution. Please refer to Appendices B to E for detailed information
regarding the pre-test and recommended response strategies for both the pre-test and post-test assessments.

## K. Method of Data Analysis

The researchers based on the pre-test and post-test scores for the analysis. The Paired Sample t-test was used to analyze the data.

## IV. RESULTS / FINDINGS AND DISCUSSION

The data collected from the First-year pre-service teachers of Presbyterian Women's College of EducationAburi were analyzed and interpreted by the researchers. The data information includes those obtained from the responses to the questionnaire, pre-test and post-test of the pre-service teachers.
> Analysis and Data Interpretation of Questionnaire by Pre-service Teachers
Research question 1: What are the challenges that First-year pre-service teachers encountered when using Cuisenaire rods in teaching addition and subtraction of fractions?

Table 1 Responses on the Challenges that First-year pre-service teachers exhibits when using Cuisenaire rods in teaching addition and subtraction of fractions

| S No. | Statement | SA | A | SD | D |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Students can add fractions without much difficulty <br> when taken through the Cuisenaire rod approach. | (T) $\%$ | (T) $\%$ | (T) $\%$ | (T) $\%$ |
| 2. | Trainees can subtract fractions without much <br> difficulty when taken through the Cuisenaire rod <br> approach. | $(0) 0$ | $(22) 61.1$ | (7) 19.4 | (7) 19.4 |

Table 1 depicts Item 1, which pertains to the proposition that pre-service teachers have the ability to add fractions when instructed using the Cuisenaire rods method. The results indicate that a majority of respondents, specifically twenty-two (22) pre-service teachers, representing $(61.1 \%)$ per cent of the sample under investigation, have expressed their agreement towards the aforementioned notion. A total of fourteen (14) respondents, accounting for $19.4 \%$ in each instance, expressed strong disagreement and disagreement towards the notion under discussion, with each subgroup comprising seven (7) individuals.

This discovery aligns with Miller, (2023), assertion that gaining an understanding of a fraction through problemsolving facilitates the development of multiple strategies to address the issue and further aids in constructing proof for various theorems. The results suggest that a majority of the students, namely $61.1 \%$, demonstrated proficiency in adding fractions using the Cuisenaire rods methodology.

According to the data presented in Table 3, the second item revealed that 22 pre-service teachers, accounting for $61.1 \%$ of the total respondents, reported their ability to perform fraction subtraction with ease using the Cuisenaire rods methodology. A total of 8 participants, comprising
$22.2 \%$ of the sample, strongly expressed disagreement while 6 participants, constituting $16.7 \%$ of the sample, indicated disagreement with the proposition.

This research result substantiates the proposition put forward by Ellerton, \& Clements, (2023), regarding the effectiveness of using instructional aids such as Cuisenaire rods in the context of subtracting fractions. Specifically, such instructional materials offer an opportunity to construct certain classifications that young children may have already accomplished, thus facilitating the acquisition of mathematical knowledge and enhancing the development of diverse problem-solving strategies. Additionally, the use of these tools may enable learners to construct proofs for various theorems. In academic writing, it is preferred to use numerical values instead of written numbers. Therefore, the sentence would be rewritten as: " $61.1 \%$ of the sample population demonstrated. "The pre-service teachers reported that the utilization of the Cuisenaire rods approach facilitated their ability to perform subtraction operations involving fractions with ease.

## $>$ Research Question 2: To what extent will the use of Cuisenaire rods improve the teaching of fractions?

Table 2 Responses to the use of Cuisenaire rods to improve the teaching of fractions

| No. | Statement | SA (T) \% | A (T) \% | SD (T)\% | D (T)\% |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | The use of Cuisenaire rods as a teaching and <br> learning material greatly improves the teaching of <br> fractions in Mathematics. | $(26) 72.2$ | $(7) 19.4$ | $(2) 5.6$ | $(1) 2.8$ |
| 2. | Concepts of fractions were better understood when <br> Cuisenaire rods were used as <br> teaching and learning materials. | $(23) 63.9$ | $(7) 19.4$ | $(0) 0$ | (6) 16.7 |

> Note: SA represents Strongly Agreed, A represents Agreed, SD represents Strongly Disagreed, D represents Disagreed, T represents the number of First-year preservice teachers who responded to the questionnaire.

According to the data presented in Table 1, it was observed that the use of Cuisenaire rods as a teaching aid was strongly favoured by the majority of the sample, comprising of twenty-six (26) students, representing 72. $2 \%$ of the surveyed individuals. Additionally, seven (7) students, representing $19.4 \%$ of the sample, agreed that the implementation of Cuisenaire rods would positively impact the teaching of fractions in the domain of mathematics.

The aforementioned discovery corroborates the study conducted by Patsiomitou, (2022). wherein the ratio model was expounded and the efficacy of Cuisenaire rods in enhancing comprehension of mathematical fractions was underscored. The present study demonstrates that the utilization of Cuisenaire rods as pedagogical and instructional tools markedly enhances comprehension of the fundamental principles related to fractions.

Table 2 presents item 2, which pertains to the preservice teacher's comprehension of fractions as facilitated by the use of Cuisenaire rods as instructional aids. According to the results of the study, a majority of the participants, comprising twenty-three individuals or $63.9 \%$, strongly agreed that pre-service teachers' comprehension of fractions was significantly improved through the utilization of Cuisenaire rods as an instructional tool. An additional seven individuals, or $19.4 \%$, concurred with this notion, indicating their agreement. Approximately $167 \%$ of the respondents expressed dissent towards the proposition, which was represented by six (6) individuals. The current discovery concurs with the assertion made by Delfin, \& Wang, (2022). The perspective that the acquisition of a novel concept necessitates exposure to concrete teaching and learning materials, exemplified by Cuisenaire rods, which is correlated with an enhanced understanding of the concept of fractions among learners. The utilization of Cuisenaire rods exhibits a considerable impact on students' comprehension.
$>$ Research question 3: To what extent would Cuisenaire rods be used in solving addition and subtraction of equivalent fractions?

Table 3 Responses of Cuisenaire rods use in solving addition and subtraction of equivalent fractions

| No. | Statement | SA (T) \% | A (T) \% | SD (T) \% | D (T) \% |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | Students were able to solve addition of <br> equivalent fractions using Cuisenaire rods. | $(28) 77.8$ | $(6) 16.7$ | $(1) 2.8$ | $(1) 2.8$ |
| 2. | Students were able to solve subtraction of <br> equivalent fractions using Cuisenaire rods. | $(28) 77.8$ | $(8) 22.2$ | $(0) 0$ | $(0) 0$ |

An examination was conducted on Item 1 in Table 2, which pertains to the utilization of Cuisenaire rods for determining the addition of equivalent fractions. The findings indicated that out of the total number of respondents surveyed, thirty (28) individuals, comprising ( $77.8 \%$ ) of the sample, strongly agreed that Cuisenaire rods can be employed for this purpose. Additionally, six (6) respondents, representing ( $16.7 \%$ ) of the sample, expressed agreement with this assertion. Two respondents, each representing $2.8 \%$ of the total sample, expressed strong disagreement or disagreement with the proposition at hand.

The aforementioned discovery aligns with the perspective put forth by Wilkie, \& Roche, (2022). who posited that Cuisenaire rods are a viable tool for imparting knowledge of the addition of equivalent fractions. According to the results, it was disclosed that the facilitation of instruction concerning the addition of fractions is
rendered more manageable in instances where students possess a firm grasp of commensurate fractions.

The scrutiny of Item 2 featured in Table 2, which pertains to the utilization of Cuisenaire rods for the purpose of solving subtractions of equivalent fractions, was performed. A total of 28 participants, accounted for $77.8 \%$ of the respondents, endorsed a strong agreement with the proposition that Cuisenaire rods possess the potential to facilitate the resolution of subtraction problems involving equivalent fractions. A total of eight (8) participants, amounting to $22.2 \%$, endorsed the proposed concept, with none dissenting. This result is consistent with the statement made by Wilkie, \& Roche, (2022), regarding the educational suitability of Cuisenaire rods for imparting knowledge on equivalent fraction subtraction. The results of the study indicate that prior to the instruction of subtraction involving equivalent fractions, it is imperative for students to have
proficiency in manipulating Cuisenaire rods.
> Analysis of First year Pre-service Teacher's Pre-Test and Post-Test Scores
Table 4 depicts the scores attained by first-year preservice teachers in the pre-test and post-test phases subsequent to their completion of ten questions in each
assessment, with scores being evaluated out of a maximum of 20. Both analyses were conducted incorporating standard protocols ensuring the prevention of any form of collaboration among the students. The tabular representation presented herein reflects the results obtained by the Firstyear pre-service teachers for both the pre-test and post-test assessments.

Table 4 Scores on pre-test and post-test

| Student | Pre-Test | Post-Test | Student | Pre-Test | Post-Test |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 2 | 10 | 19. | 6 | 10 |
| 2. | 4 | 10 | 20. | 8 | 20 |
| 3. | 4 | 14 | 21. | 10 | 20 |
| 4. | 8 | 16 | 22. | 0 | 12 |
| 5. | 10 | 20 | 23. | 12 | 20 |
| 6. | 8 | 16 | 24. | 10 | 20 |
| 7. | 6 | 10 | 25. | 4 | 12 |
| 8. | 0 | 12 | 26. | 6 | 16 |
| 9. | 12 | 16 | 27. | 0 | 12 |
| 10. | 6 | 16 | 28. | 4 | 14 |
| 11. | 8 | 12 | 29. | 6 | 12 |
| 12. | 10 | 20 | 30. | 8 | 16 |
| 13. | 6 | 20 | 31. | 6 | 16 |
| 14. | 6 | 12 | 32. | 0 | 4 |
| 15. | 0 | 16 | 33. | 4 | 12 |
| 16. | 8 | 18 | 34. | 6 | 14 |
| 17. |  |  | 35. | 8 | 16 |
| 18. |  |  | 36. | 4 | 10 |

## > Analysis of the Pre-Test Results

Upon examining the unprocessed scores obtained during the pre-test, it was discerned that no trainee in the teacher education program was able to correctly answer all ten (10) questions. An infinitesimal proportion of the participants, specifically two (2) First-year pre-service teachers, constituting a mere (5.6\%) of the thirty-six (36) trainees, distinguished themselves by obtaining a perfect score of twelve (12) marks. The data revealed that a total of four (4) trainees, which constituted ( $11.1 \%$ ) of the sample,
obtained half of the total marks, while a majority of the participants, specifically twenty-five (25) pre-service teachers or (69.4\%) of the cohort, received between eight (8) to two (2) marks. The remaining (13.9\%) of trainees, comprising five (5) individuals, failed to attain any marks at all. A comprehensive examination of the pre-test scores revealed a disheartening lack of success in the overall performance. The present document presents a frequency distribution table displaying the results obtained in the Pretest.

Table 5 Frequency distribution table for the pre-test results

| Marks | Frequency | Percentages |
| :---: | :---: | :---: |
| Below 1 | 5 | 13.89 |
| $1-3$ | 1 | 2.78 |
| $4-6$ | 17 | 47.22 |
| $7-9$ | 7 | 19.44 |
| $10-12$ | 6 | 16.67 |
| Total | 36 | 100 |

## > Level of Performance of Pre-Service Teachers During Pre-Test



Fig 1 Level of Performance of First-year Pre-service Teachers' during the Pre-Test

## $>$ Analysis of the Post-Test Result

The analysis of post-test scores revealed a promising level of overall performance. Out of the total number of pre-service teachers, seven (7) corresponded to a proportion of $19.4 \%$, and were successful in achieving a perfect score on all of the questions presented to them. Twenty-two (22) achieved a score of twelve (12) and above, constituting $61.1 \%$ of the total number of preservice teachers, while only seven (7) out of thirty-six (36) attained a score of ten (10) or lower, representing $19.4 \%$ The improvement in pre-service teachers' performance as a result of the researchers' intervention is evidenced by their higher scores in the post-test compared to the pre-test. Below is the frequency distribution Table for the Post-test results.

Table 6 Frequency Distribution Table for the Post-Test Results

| Marks | Frequency | Percentages |
| :---: | :---: | :---: |
| Below 1 | 0 | 0 |
| $1-3$ | 0 | 0 |
| $4-6$ | 2 | 5.56 |
| $7-9$ | 0 | 0 |
| $10-12$ | 13 | 36.11 |
| $13-15$ | 4 | 11.11 |
| $16-18$ | 10 | 27.78 |
| $19-21$ | 7 | 19.44 |
| Total | 36 | 100 |

> Figure 2 Shows the Bar chart for the Post-test results.
Bar Chart of Post-Test


Fig 2 Level of Performance of Pre-Service Teachers' after the Intervention Process

Table 7 and Figure 3 shows the Frequency distribution table and the bar chart which was used for comparing the pre-test and post-test results.

Table 7 Frequency distribution table for the pre-test and post-test results

| Marks | Pre-test | Post-test |
| :---: | :---: | :---: |
| Below 1 | 5 | 0 |
| $1-3$ | 1 | 0 |
| $4-6$ | 17 | 2 |
| $7-9$ | 7 | 0 |
| $10-12$ | 6 | 13 |
| $13-15$ | 0 | 4 |
| $16-18$ | 0 | 10 |
| $19-21$ | 0 | 7 |
| Total | 36 | 36 |

> Comparison of Pre- and Post-Test Results of Pre-service Teachers


Fig 3 Comparing the level of Performance of Pre-service teachers after the implementation of the intervention
> Analysis and Interpretation of Post-Observation Assessment
After the researchers intervened by structuring and teaching addition, subtraction, division and multiplication of fractions for the six weeks intervention period, the researchers assessed the pre-service teachers' performance in addition, subtraction, division and multiplication of fractions once again by test to find out if the intervention mechanism has worked to perfection.

## > Comparing the Pre-Test and Post-Test Results of Pre-

 Service TeachersThe researchers coded the data and inputted in the statistical package SPSS to find out the relationship between the use of Cuisenaire rods and the understanding of the preservice teachers. Table 8 shows the results and analysis of both the Pre- and Post-test of the pre-service teachers’ performance in fractions. Table 8 displays the mean, sample size, standard deviation and standard error for the pre-test and post-test performances of the college students.

Table 8 Paired Samples Statistics of pre-service teachers' Performance in fraction class

| Variable |  | Mean | $\mathbf{N}$ | Std. Deviation | Std. Error Mean |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pair | PRE-TEST | 5.7778 | 36 | 3.33904 | .55651 |
|  | POST-TEST | 14.2222 | 36 | 4.18918 | .69820 |

According to Table 8, it is evident that the preintervention mean score of the students was 5.78 , which is comparatively lesser than the post-test mean score of 14.22 The observed outcome demonstrates a considerable enhancement in the academic achievement of students following the implementation of the intervention. In a similar vein, it is noteworthy that the standard deviation exhibits a disparity of (0.85) as it transitions from (4.19) to (334) The
results indicate a statistically significant improvement in the academic outcomes of students after completing the post-test. The findings imply that the utilization of Cuisenaire rods in the context of instructing fractions resulted in a notable improvement in students' academic outcomes. Table 9 displays the correlation observed between student competency levels in addition, subtraction, division, and multiplication of fractions and the utilization of Cuisenaire rods within a college setting.

## > Paired Sample Correlation of Pre-Test and Post-Test of the use of Cuisenaire Rods By Pre-Service Teachers

Table 9 Paired Samples Correlations

| Variable |  | N | Correlation | Sig. |
| :---: | :---: | :---: | :---: | :---: |
| Pair | PRE-TEST \& POST-TEST | 36 | .796 | .000 |

From Table 9, there is a strong positive correlation between the post-test and pre-test. This also indicates that there is a relationship between the use of Cuisenaire rods and achievement in students' performance.

Table 10 shows the paired differences between the pre-test, where pre-service teachers learnt the use of Cuisenaire rods and the post-test, where college students learnt fractions after mastering the use of Cuisenaire rods.

Table 10 Paired Samples Test

| Variable | Paired Differences |  |  |  |  | t | Df | Sig. (2tailed) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Std. <br> Deviation | Std. Error Mean | 95\% Confidence Interval of the Difference |  |  |  |  |
|  |  |  |  | Lower | Upper |  |  |  |
| PRE-TEST - POSTTEST | 8.4444 | 2.53484 | . 42247 | 9.30211 | 7.58678 | 19.988 | 35 | . 000 |

According to the results presented in Table 10, the $95 \%$ confidence interval provides a mean of estimating the range within which the actual mean difference between pretest and post-test results may be found in $95 \%$ of all potential interventions.

In terms of the pre-test and post-test assessments, the observed scores demonstrate a range extending from a minimum of 9.302 to a maximum of 7.587 The $t$-statistic is acquired through the division of the mean difference by its respective standard error. The t-statistic associated with the pre-test and post-test scores was found to be 19.988. The given column exhibits the likelihood of acquiring a twotailed $t$ statistic that possesses an absolute value equivalent to or exceeding that of the absolute $t$ statistic. According to Table 10, it is evident that the alteration in performance among college students in the areas of addition, subtraction, division, and multiplication of fractions exhibited a statistically significant value of less than 0.005 . Upon analysis of the aforementioned results, the researchers arrived at the conclusion that the suboptimal performance of college students in fractions is not a coincidental occurrence but rather a result of inadequate utilization of suitable instructional and educational tools such as Cuisenaire rods.

## V. SUMMARY, RECOMMENDATIONS AND CONCLUSIONS

The study is about using Cuisenaire rods to improve the teaching of fractions to First-year pre-service teachers in Presbyterian Women's College of Education, Presbyterian Women's in the Eastern Region. The study was guided by the following research questions:

- What challenges do first-year pre-service teachers face while utilizing Cuisenaire rods in teaching the addition and subtraction of fractions?
- To what degree will the incorporation of Cuisenaire rods enhance the pedagogical approach to the concept of fractions?
- To what extent will Cuisenaire rods facilitate the resolution of addition and subtraction of equivalent fractions?

The study population consisted of a group of five firstyear classes enrolled at the Presbyterian Women's College of Education out of which one class was conveniently selected making a sample of thirty-six (36) first-year prospective teachers. The data collection procedure involved the utilization of pre-test, post-test, and questionnaire as the primary research instruments. Following the administration of the pre-test, a six-week intervention was undertaken, targeting First-year pre-service educators with the objective of enhancing their capacity to instruct on fractions, utilizing Cuisenaire rods. A post-intervention assessment was administered subsequent to the implementation of the treatment. The results acquired from the initial and final assessments were subjected to statistical scrutiny using a paired sample test. The questionnaire data were subjected to analysis with the utilization of a Likert-type scale.
> Summary of Main Findings
The present study yields several significant findings.

- The results of the study indicate that the incorporation of Cuisenaire rods is a highly effective method for enhancing the instruction of fractions within the field of mathematics.
- The observations ascertain that pre-service teachers in their first-year exhibit insufficient comprehension of the subject as it pertains to commonplace occurrences.
- In conclusion, it has been observed that the aptitude of first-year pre-service teachers to comprehend the concept of fractions is significantly enhanced by exposure to the Cuisenaire rods approach, followed by gradual progression towards abstract understanding.
> Implications for Practice
Through rigorous analysis and subsequent discourse on the data, the present study has revealed noteworthy educational implications.
- It is recommended that the utilization of the Cuisenaire rods as a teaching method is recommended as the primary approach for instructing the addition and subtraction of fractions. This finding is consistent with Newton's assertion (2023) wherein it was posited that the process of adding or subtracting fractions with different denominators necessitates the conversion of such fractions into equivalent ones, utilizing Cuisenaire rods, in order to facilitate their subsequent resolution.
- The language utilized ought to be easily comprehensible and tailored to the proficiency level of the learners. This assertion is congruent with the perspective of Hiebert and Stigler (2023), who posit that a teacher's use of language is a crucial factor in facilitating a beneficial learning experience for their students. The researchers maintain that the language of mathematics must be employed with meticulous care and precision right from the outset. As an illustration, the numerical representation of the rational number 3/4 should be verbally expressed as "three-fourths" in academic writing. It is advisable to refrain from utilizing expressions such as "three over four" in academic writing.
- This methodological approach involves transitioning from the utilization of concrete materials towards incorporating semi-concrete materials, ultimately culminating in the abstract stage.
- Pre-service educators are facilitated in the utilization of teaching and learning materials for the instruction of mathematical concepts such as fractions.
- The aforementioned implication coincides with the viewpoint expressed by Louka (2023), which asserts that it is imperative for teachers to employ tangible instructional resources in order to initiate the instruction of fractions and gradually progress towards utilizing visuals, such as pictures and diagrams.


## > Conclusion

The concept of fractions should be well explained to students by using the appropriate teaching and learning materials.

- Think, pair and share should be encouraged among Pre-service teachers to present ideas in different ways and solve real-life problems using fractions.
- The language being employed by educators while disseminating information to pupils holds significant weight in determining the degree of impact they have on students. It is advised to refrain from utilizing colloquial expressions like 'one over six' and 'two over ten. ' An academic tone of writing necessitates the use of formal and precise language.

The present analysis culminates in the recognition that an infallible method for ensuring students' acquisition of knowledge remains elusive. Nevertheless, if learners exhibit a robust enthusiasm for learning and are equipped with clearly structured and compelling pedagogical resources that are cognitively appropriate, effective outcomes are more likely to materialize.
> Recommendations
Based on the findings of this study, the following recommendations were proposed for consideration.

- The operations of Cuisenaire rods ought to be utilized to present lessons to upgrade the concept of fractions being taught in schools.
- Pre-service teachers ought to work in sets or bunches when essential. This will empower them to trade thoughts unreservedly in lessons.
- Workshops, classes and conferences must be organized as often as possible to empower First-year pre-service teachers to get them prepared to teach the concept effectively and efficiently.
> Implication for Further Research
In designing future studies, the following suggestions may be considered.
- The sample size should be increased. A much larger size would enhance the validity of the findings.
- Different environmental settings are suggested for future study at the same time.
- The use of Cuisenaire rods as a teaching resource should be supplemented by the use of diagrams and songs accompanied with beats.


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