Optimization of the Compressive Strength Characteristics of Four-Component Concrete Mixes Made with Unwashed Local Gravel using Second-Degree Polynomial

[1]Arogo Eyaramuuonan Charles
[1]Dept. of Civil Engineering, Federal University Otuoke,
P.M.B. 26 Yenagoa, Nigeria

[2]Adinna Boniface Okafor
[2]Dept of Civil Engineering, Nnamdi Azikiwe University
Akwa, Nigeria

Abstract:- Being able to predict the strength characteristics of concrete can be of great advantage in the design and construction of structural members. This research work set out to develop a model for the compressive strength characteristics of concrete made with unwashed local grave based on Prof. Osadebe's optimization theory or second-degree polynomial. The unwashed aggregate was from Abagana and the river sand from Amansea, both in Anambra State. These aggregates were tested for their physical and mechanical properties based on BS 812: Part 2:1975 and BS 812: Part 3: 1975. Sixty concrete cubes of dimensions 150 mm X 150mm X 150mm —three cubes for each experimental point were made, cured and tested according to BS 1881:1983. The model equation developed was Ŷ= -2006.1Z1+ 1401.02Z2 -90.3Z3 -105.47Z4 +91.79Z1Z2 --9086Z1Z3 + 3268.26Z1Z4 +7888.31Z2Z3 -1636.57Z2Z4 +427.5Z3Z4. The student's t-test and the Fisher's test were used to prove the adequacy of the model. The strengths predicted by the model were in complete agreement with the experimentally obtained values and the null hypothesis was satisfied.

Keywords:- *Characteristics, Compressive, Concrete, Component, Local Gravel, Mixes, Optimization, Polynomial, Second-Degree, Strength, Unwashed.*

I. INTRODUCTION

- *Osadebe's Concrete Optimisation Theory*

Concrete is a four-component material of mixing water, cement, fine and coarse aggregates. These ingredients are mixed in rational proportions to achieve desired strength of the hardened concrete [1]. Let us consider an arbitrary amount, S, of a given concrete mixture and S_i, the portion of the i^{th} component of the four constituent materials of the concrete where i = 1,2,3,4, then in keeping with the principle of absolute volume or mass [2]:

$$\sum Si = S \text{--} 1$$

Dividing through by S and substituting Z_i for S_i/S gives:

$$\sum Zi = 1 \text{--} 2$$

Then, the compressive strength of concrete can be expressed as equation 3:

$$Y = f\,(Zi) \text{ --} 3$$

Using Taylor's theorem and the assumption that Y is continuous, equation 3 becomes:

$$f\,(Z) = f(Z^{(0)}) + \sum \partial f / \partial Z_i(Z^{(0)})(Z_i - Z_i^{(0)}) + ½\,!\sum \partial^2 f(Z^{(0)}) / \partial Z_i \partial Z_j(Z^{(0)})(Z_i - Z_i^{(0)})(Z_j - Z_j^{(0)}) + ½\,!\sum \partial^2 f(Z^{(0)}) / \partial Z_i^2 \partial Z_j(Z_i - Z_i^{(0)})^2 + \text{--------------------} 4$$

$(1 \le i \le 4,\ 1 \le i \le 3,\ 1 \le i \le 4,\ 1 \le i \le 4)$ respectively and $Z^{(0)} = 0$.

If $b_0 = f(0)$, $b_i = \partial f(0)/ \partial Z_i$, $b_{ij} = \partial f^2(0)/ \partial Z_i \partial Z_j$ and $b_{ii} = \partial^2 f(0)/\partial Z^2_i$, then eqn. 4 can be written as follows:

$$f\,(Z) = b_0 + \sum b_i Z_i + \sum b_{ij} Z_i Z_j + \sum b_{ii} Z_i + \text{--------------------} 5$$

Multiplying eqn.2 by b_0 we have

$$b_0 Zi = b_0 \text{--} 6$$

Also, multiplying eqn. 2 by Z_1, Z_2, Z_3 and Z_4 in succession, making Z_1^2, Z_2^2, Z_3^2 and Z_4^2 the subject of the formula, substituting into eqn. 5 and factorizing gives:

$$Y = \sum \beta_i Z_i + \sum \beta_{ij} Z_i Z_j \text{--------------------------------------} 7$$

$(1 \le i \le j \le 4)$

Where $\beta i = b_0 + b_i + b_{ii}$ and $\beta_{ij} = b_{ij} - b_{ii} - b_{jj}$ (i,j = 1, 2, 3, 4)

ISSN No:-2456-2165

- *The Coefficients of the Regression Equation*

If the K^{th} response (compressive strength for the serial number k) is $y^{(k)}$, substituting the vector of the corresponding set of variables, i.e., $Z^{(K)} = [Z_1^{(K)}, Z_2^{(K)}, Z_3^{(K)}, Z_4^{(K)}]^T$ (see table 1) into eqn.7 generates the explicit matrix of equation 8:

$$[y^{(k)}] = [B]\,[Z] \qquad (8)$$

Re-arranging eqn.8 yields:

$$[Z]^T\,[B]^T = [y^{(k)}] \qquad (9)$$

Solution of eqn.9 gives the values of the unknown coefficients of the regression equation (eqn 7).

- *The Student's T-Test*

The unbiased estimate of the unknown variance S^2 is given by [3],

$$S_Y^2 = \frac{\sum\left(y_i - \breve{y}\right)^2}{n-1} \qquad (10)$$

If $a_i = z_i\,(2z_i - 1)$, $a_{ij} = 4\,z_i\,z_j$; for ($1 \le i \le q$) and ($1 \le i \le j \le q$) respectively.

Then, $\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$ (11)

Where ε is the error of the predicted values of the response.

The t-test statistic is given by [3]

$$t = (\Delta y\sqrt{n}/s_Y)\sqrt{(1+\varepsilon)} \qquad (12)$$

Where $\Delta y = y_0 - y_t$; y_0 = observed value, y_t = theoretical value; n = number of replicate observations at every point; ε = as defined in eqn.11.

- *The Fisher's Test*

The Fishers-test statistic is given by

$$F = \mathbf{S}_1^2/\mathbf{S}_2^2 \qquad (13)$$

The values of $\mathbf{S_1}$ (lower value) and $\mathbf{S_2}$ (upper value) are calculated from equation 10.

II. MATERIALS AND METHOD

- *Preparation, Curing and Testing of Cube Samples*

The aggregates were sampled in accordance with the methods prescribed in BS 812: Part 1:1975 [4]. The test sieves were selected according to BS 410:1986 [5]. The water absorption, the apparent specific gravity and the bulk density of the coarse aggregates were determined following the procedures prescribed in BS 812: Part 2: 1975 [6]. The Los Angeles abrasion test was carried out in accordance with ASTM. Standard C131: 1976 [7]. The sieve analyses of the fine and coarse aggregate samples were done in accordance with BS 812: Part 1: 1975 [4] and satisfied BS 882:1992[8]. The sieving was performed by a sieve shaker. The water used in preparing the experimental samples satisfied the conditions prescribed in BS 3148:1980 [9]. The required concrete specimens were made in threes in accordance with the method specified in BS 1881: 108:1983 [10]. These specimens were cured for 28 days in accordance with BS 1881: Part 111: 1983 [11]. The testing was done in accordance with BS 1881: Part 116:1983 [12] using compressive testing machine.

Table 1 Selected Mix Ratios and Component's Fraction based on Osadebe's Second Degree Polynomial

S/NO	MIX RATIOS				COMPONENT'S FRACTION			
	S_1	S_2	S_3	S_4	Z_1	Z_2	Z_3	Z_4
1	0.88	1	2.5	4	0.105	0.119	0.298	0.477
2	0.86	1	2	4	0.109	0.127	0.254	0.509
3	0.855	1	2	3.5	0.116	0.136	0.272	0.476
4	0.86	1	2	3	0.125	0.146	0.292	0.437
5	0.855	1	2.5	3.5	0.109	0.127	0.318	0.446
6	0.865	1	3	4	0.098	0.113	0.338	0.451
7	0.87	1	3	4.5	0.093	0.107	0.320	0.480
8	0.86	1	1.5	3	0.135	0.157	0.236	0.472
9	0.86	1	2.75	3.4	0.107	0.125	0.343	0.424
10	0.865	1	2	4.25	0.107	0.123	0.246	0.524
			CONTROL					
11	0.858	1	2.43	4	0.104	0.121	0.293	0.483
12	0.86	1	1.75	3	0.130	0.151	0.265	0.454
13	0.855	1	2.4	3.5	0.110	0.129	0.309	0.451
14	0.86	1	2	4.33	0.105	0.122	0.244	0.529
15	0.862	1	2.25	3.13	0.119	0.138	0.311	0.432
16	0.858	1	2	2.83	0.128	0.150	0.299	0.423
17	0.858	1	2.67	3.29	0.110	0.128	0.342	0.421
18	0.86	1	3	4.13	0.096	0.111	0.334	0.459

ISSN No:-2456-2165

19	0.855	1	2	3	0.125	0.146	0.292	0.438
20	0.8595	1	2.75	4	0.100	0.116	0.319	0.465

LEGEND: S_1= water/cement ratio; S_2=Cement; S_3=Fine aggregate; S_4=Coarse aggregate, $Z_i = S_i/S$

Table 2 Z^T Matrix

Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
0.105	0.119	0.298	0.477	0.013	0.031	0.050	0.036	0.057	0.142
0.109	0.127	0.254	0.509	0.014	0.028	0.056	0.032	0.065	0.129
0.116	0.136	0.272	0.476	0.016	0.032	0.055	0.037	0.065	0.129
0.125	0.146	0.292	0.437	0.018	0.037	0.055	0.042	0.064	0.127
0.109	0.127	0.318	0.446	0.014	0.035	0.049	0.041	0.057	0.142
0.098	0.113	0.338	0.451	0.011	0.033	0.044	0.038	0.051	0.153
0.093	0.107	0.320	0.480	0.010	0.030	0.045	0.034	0.051	0.154
0.135	0.157	0.236	0.472	0.021	0.032	0.064	0.037	0.074	0.111
0.107	0.125	0.343	0.424	0.013	0.037	0.046	0.043	0.053	0.146
0.107	0.123	0.246	0.524	0.013	0.026	0.056	0.030	0.065	0.129

Table 3 Responses of the Mix Ratios

S/NO	S_1	S_2	S_3	S_4	AVERAGE COMPRESSIVE STRENGTH[N/mm2]
1	0.88	1	2.5	4	13.2
2	0.86	1	2	4	13.8
3	0.855	1	2	3.5	14.6
4	0.86	1	2	3	15.3
5	0.855	1	2.5	3.5	14.5
6	0.865	1	3	4	10.3
7	0.87	1	3	4.5	9.8
8	0.86	1	1.5	3	16.6
9	0.86	1	2.75	3.4	14.9
10	0.865	1	2	4.25	13.5

LEGEND: S_1= water/cement ratio; S_2=Cement; S_3=Fine aggregate; S_4=Coarse aggregate

➢ *Testing the Fit of the Quadratic Polynomials*

The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. The null hypothesis was denoted by H_0 and the alternative by H_1.

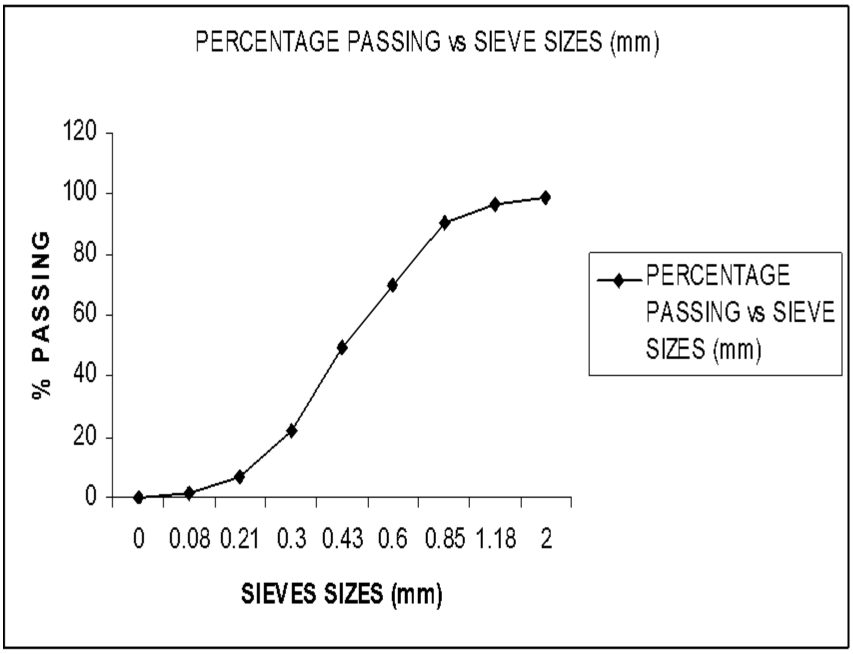

Fig 1 Grading Curve for the Fine Aggregate

ISSN No:-2456-2165

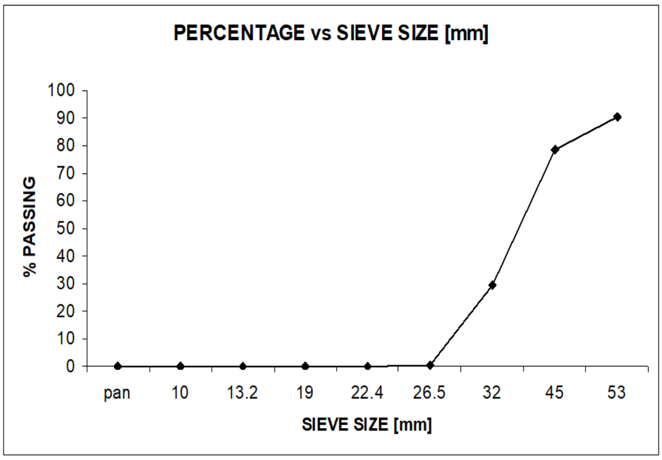

Fig 2 Grading Curve for the Local Gravel

III. RESULTS AND DISCUSSION

- *Physical and Mechanical Properties of Aggregates*

Sieve analyses of both the fine and coarse aggregates were performed and the grading curves shown in figures 1 and 2. These grading curves showed the particle size distribution of the aggregates. The maximum aggregate size for the unwashed gravel was 53 mm and 2mm for the fine sand. The local gravel had water absorption of 4.55%, moisture content of 53.25 %, apparent specific gravity of 1.88, Los Angeles abrasion value of 60% and bulk density of 1302.7 kg/m^3.

- *The Regression Equation for the Compressive Strength Tests Results*

Solution of eqn.9, given Z^T values of table 2 and the responses (average compressive strengths) in table 3 gave the values of the unknown coefficients of the regression equation (eqn.7) as follows:

β_1= -2006.1, β_2 = 1401.02, β_3 = -90.3, β_4 = -105.47, β_{12}= 91.79, β_{13}= -9086, β_{14}= 3268.26, β_{23}= 7888.31, β_{24}= -1636.57, β_{34} = 427.5. Thus, from eqn.7, the model equation based on Osadebe's second-degree polynomial was given by:

$$\hat{Y}= -2006.1Z_1 + 1401.02Z_2 -90.3Z_3 - 105.47Z_4 +91.79Z_1Z_2 -- 9086Z_1Z_3 + 3268.26Z_1Z_4 + 7888.31Z_2Z_3 - 1636.57Z_2Z_4 + 427.5Z_3Z_4.$$

- *Fit of the Polynomial*

Selected mix ratios and component's fraction based on Osadebe's second degree polynomial was shown in table 1. The polynomial regression equation developed i.e., $\hat{Y}$= -2006.1Z_1+ 1401.02Z_2 -90.3Z_3 -105.47Z_4 +91.79Z_1Z_2 --9086Z_1Z_3 + 3268.26Z_1Z_4 +7888.31Z_2Z_3 -1636.57Z_2Z_4 +427.5Z_3Z_4, was tested to see if the model agreed with the actual experimental results. There was no significant difference between the experimental and the theoretically expected results. The null hypothesis, H_0 was satisfied.

Table 4 T –Statistic for the Controlled Points, Unwashed-Gravel Concrete Compressive Test, based on Osadebe's Second –Degree Polynomial Polynomial

RESPONSE	i	j	a_i	a_{ij}	a_i^2	a_{ij}^2	ε	y̌	Ŷ	t
C_1	1	2	-0.082	0.050	0.007	0.003	0.4835	16.0	16.9	-1.58008
	1	3	-0.082	0.121	0.007	0.015				
	1	4	-0.082	0.120	0.007	0.0399				
	2	3	-0.092	0.142	0.01	0.020				
	2	4	-0.092	0.233	0.013	0.0543				
	3	4	-0.121	0.566	0.016	0.3202				
	4	—	-0.017	—	0.001	—				
				Σ	0.052	0.4316				
	Similarly									
C_2	—	—	—	—	—	—	0.4809	16.3	15.9	0.617625
C_3	—	—	—	—	—	—	0.9234	16.1	17.1	-1.9825
C_4	—	—	—	—	—	—	0.4642	16.2	17.0	-1.26789
C_5	—	—	—	—	—	—	0.5053	13.5	12.8	1.086064
C_6	—	—	—	—	—	—	0.4966	15.7	14.4	2.377246
C_7	—	—	—	—	—	—	0.5707	14.6	13.4	2.026749
C_8	—	—	—	—	—	—	0.5624	13.1	14.6	-2.72798
C_9	—	—	—	—	—	—	0.4949	15.7	16.9	-2.20202
C_{10}	—	—	—	—	—	—	0.5236	14.8	15.3	-0.91441

LEGEND: c_i =response; $a_i = z_i (2z_i - 1)$; $a_{ij} = 4 z_i z_j$; $\varepsilon = \Sigma a^2_i + \Sigma a^2_{ij}$; y̌ = experimentally-observed value; Ŷ= theoretical value; t = t-test statistic.

- *T -Value from Table*

The t-student's test had a significance level, α = 0.05 and $t_{\alpha/l(ve)} = t_{0.005(9)}$=3.69. This was greater than any of the t values calculated in table 4. Therefore, the regression equation for the unwashed gravel concrete was adequate.

- *F-Statistic Analysis*

The sample variances $\boldsymbol{S}_1^2$ and $\boldsymbol{S}_2^2$ for the two sets of data were not significantly different (table 5). It implied that the error(s) from experimental procedure were similar and that the sample variances being tested are estimates of the same population variance. Based on eqn.10, we had that S_K^2 = 12.319/9 = 1.369, S_E^2 = 22.486/9 = 2.498 & **F =** 1.369/2.498 = 0.548. From Fisher's table, $F_{0.95(9,9)}$ = 3.3, hence the regression equation for the compressive strength of the unwashed gravel concrete was adequate.

Table 5 F –Statistic for the Controlled Points, Granite Concrete Compressive Strength, based on Osadebe's Second –Degree Polynomial

Response Symbol	Y_K	Y_E	Y_K- $\check{Y}_K$	Y_E-$\check{Y}_E$	$(Y_K$- $\check{Y}_K)^2$	$(Y_E$-$\check{Y}_E)^2$
C_1	16.0	16.9	0.778	1.450	0.606	2.102
C_2	16.3	15.9	1.116	0.505	1.245	0.255
C_3	16.1	17.1	0.909	1.674	0.827	2.803
C_4	16.2	17.0	1.047	1.541	1.096	2.374
C_5	13.5	12.8	-1.740	-2.618	3.026	6.855
C_6	15.7	14.4	0.545	-1.085	0.297	1.178
C_7	14.6	13.4	-0.604	-2.003	0.365	4.011
C_8	13.1	14.6	-2.118	-0.818	4.486	0.669
C_9	15.7	16.9	0.462	1.491	0.213	2.223
C_{10}	14.8	15.3	-0.398	-0.122	0.158	0.015
$\sum$	152	154.3			12.319	22.486

Legend: $\check{Y}=\Sigma y/n$ where y is the response and n, the number of observed data (responses) y_k is the experimental value (response) y_E is the expected or theoretically calculated value (response)

IV. CONCLUSION

The strengths (responses) of concrete were a function of the proportions of its ingredients: water, cement, fine aggregate and coarse aggregates. Since the predicted strengths by the model were in total agreement with the corresponding experimentally -observed values, the null hypothesis was satisfied. This meant that the model equation was valid.

REFERENCES

[1]. Neville, A. M., (1995) *Properties of Concrete, Third Edition*, Pitman, London
[2]. Osadebe, N.N. (2003), *Generalised Mathematical Modelling of Compressive Strength of Normal Concrete as a Multivariant function of the properties of its constituent components*, Lecture, University of Nigeria, Nsukka,.
[3]. Biyi, A. (1975), *Introductory Statistics*, Abiprint & Pak Ltd., Ibadan.
[4]. BS 812: Part 1 (1975). *Sampling, shape, size and classification*. Methods for sampling and testing of mineral aggregates, sands and fillers. British Standards Institution Publication, London,
[5]. BS 410 (1986). *Specification for test sieves*. British Standards Institution Publication, London,
[6]. BS 812: Part 2 (1975). Methods for sampling and testing of mineral aggregates, sands and fillers. *Physical properties*. British Standards Institution Publication, London,.
[7]. ASTM. Standard C 131 (1976). *Tests for Resistance to Abrasion of Small Size Coarse Aggregate by Use of the Los Angeles Machine*. American Society for Testing and Materials Publication, New York.
[8]. BS 882 (1992). *Specification for aggregates from natural sources for concrete*. British Standards Institution Publication, London.
[9]. BS 3148 (1980). *Tests for water for making concrete*. British Standards Institution Publication, London,.
[10]. British Standard 1881: Part 108 (1983). *Method for making test cubes from fresh concrete*. British Standards Institution Publication, London.
[11]. British Standard 1881: Part 111 (1983) *Method of normal curing of test specimens (20 °C)*. British Standards Institution Publication, London,.
[12]. British Standard 1881: Part 116 (1983). *Method for determination of compressive strength of concrete cubes*. British Standards Institution Publication, London.