ISSN No:-2456-2165

Theory and Design of Audio Rooms: A Statistical View

Dr. Ismail Abbas
Senior Lecture at MTC, Cairo

Abstract:- In this paper, the numerical solution of the statistical chains of matrix B is successfully used to calculate the sound intensity field in audio rooms.

Here we use B-chain techniques as a real breakthrough with the time-dependent sound field problem in 3D geometric space. We offer the appropriate design of audio rooms via an example of a cuboid pieces.

We also show that B-chain techniques can produce rigorous statistical proof of Sabines' imperial formula for reverberation time in audio rooms.

In addition, the definition of so-called statistical weights of geometric shapes is introduced and found to be effective in solving double and triple integration as well as sound diffusion transfer equation in audio rooms.

I. INTRODUCTION

During the last century, the classical mathematical statistical methods sublimated by the precise calculation of probabilities succeeded in solving different physical situations and consequently entered one branch of science after another.

In statistical methods, logic and common sense replace heavy mathematics.

Nowadays, a new theory of probability and statistics has appeared.

This theory[1,2,3]called the Cairo technique is expected to be able to solve an ever widening field of complex mathematical and physical situations as a natural extension of its surprising success in solving the partial differential equations of Laplace and Posson as well as the heat diffusion/conduction equation (in its most general form).

Later, the same theory was successfully applied to solve sound scattering, double and triple integrals, and Gamma function integration[1,2,3].

A new weapon in numerical methods has emerged just from nature's own probabilities and statistics.

We hypothesize that the inability of current computational methods[4,5] to handle large physical situations is mainly due to the lack of a correct definition of probability in 4D x-t space which should itself be the cornerstone of all statistical operations as in the case of B matrix strings and the whole Cairo technique.

However, chains of transition matrices B work thanks to a new theoretical device: when the probability statistic tends towards certainty for a large number of trials.

The transition probability statistical matrix Bi,j conforms to the linearity, symmetry and binarity imposed by nature itself.

The application of the B-matrix statistical chains in the theory and design of audio rooms is the subject of this article.

In more detail, sound quality in audio rooms is determined by four main factors,

- an appropriate reverberation time T_R
- an appropriate sound level or sound intensity I_s
- uniform sound distribution.
- low noise / signal ratio.

Sabine's semi-imperial formula sometimes referred to as Sabine's theory, proposed a century ago, remains the main formula for calculating T_R reverberation time in audio rooms.

Moreover, it is also an approximate basis for calculation of sound intensity I_s W/m^2 in audio rooms.

I_s is assumed to be uniform and is practically expressed in decibels.

The reverberation time T_R seconds for an empty room, as given by Sabines formula, can be expressed as follows:

T_R = 53.46 V/ C A S. . second. . . . . . (1)

Assuming the speed of sound in air C, at NPT is 330 m/s, then,

T_R = 0.162 V/A S,. . second. . . . . (2)

For empty rooms.

where V is the volume of the room in m^3, A is its total interior

area in m^2 and S is the average sound absorption coefficient S(av) defined as.

S (av) = (A1 S1 + A2 S2 +.. An Sn) / (A1 + A2 + ... An). . . (3)

The sound absorption of an average human individual is between 0.2 and 0.4 Sabine, so for soundproofing rooms populated by N humans, we propose that the denominator in equation 2 be simply changed to AS+ N (humans) * 0.3 Sabine units.

In other words, equation 2 becomes,

TR = 0.162 V / (A S+Nh *0.3) . second. . . . . (4)

Note that :

- The appropriate or recommended T_R for large cathedrals and mosques is between 2 and 2.5 seconds.
- T_R =2 seconds is an optimal reverberation time for a concert entrance,
- T_R =One second is an optimal reverberation time for a amphitheater conference room.
- T_R =0.3 to 0.5 seconds is standard for recording studios.

T_R Below 0.3 seconds is an acoustically dead room while

T_R above 2.5 seconds is an annoying echogenic piece.

On the other hand, Sabinestheorem predicts the intensity of sound in audio rooms I_s is given by, [1,2]

I_s = sum of sound power sources (P1+P2+..+Pn)in watts / (A S + N (humans

* 0.25) . . . Watts/m^2. ………………………... . . (5),

Eq 5 can be expressed in terms of reverbration timeTR as,

Is = sum of sound power sources P in watts * T_R / 0.161 V .

. . Watt/meter ^ 2 .... (6)

However, since the practical unit of Is is the decibel (db),

where,

I in decibel = 10 Log (base10) Is / Is (0). . ………. . (7)

I(0) is conveniently chosen as the hearing threshold for a normal human ear of a healthy person.

I(0)= 10^-12 watt/m^2 or zero decibel.

And the pain threshold that causes damage to the human ear is 1 watt/m^2, which is 120 db.

The range of 40 to 70 db is the recommended loudness in audio rooms to be quite audible and more comfortable for the human ear.

In conclusion :

Sabine's semi-imperial incomplete formula, sometimes called Sabine's theory, proposed a century ago is still a widely accepted formula for calculating reverberation time in audio rooms (RT), and in roughly estimate the volume of the sound.

Clearly, a reconsideration or further investigation is needed, which is the subject of this article.

II. THEORY

The claim that mathematics is the language of physics has always been a given and widely accepted fact, but the idea that the reverse might be true is quite unexpected.

Through this work, we apply the statistical chains of the matrix B to calculate the sound intensity inside the audio rooms as well as their reverberation time.

Note that the B-matrix chains do not use any mathematical law or formula, but they fit in experimentally (via the simulation algorithm) like nature itself.

However, the B-matrix itself is well defined via the following physical conditions:

For Cartesian coordinates in 1,2 and 3D space, the inputs $B_{i,j}$ respect or are subject to the following conditions:

- $B_{i,j}$ = 1/2-RO/2,1/4-RO/4 and 1/6-RO/6 in 1D, 2D and 3D respectively for i adjacent to j and B i, j = 0 otherwise . RO=$B_{i,i}$ ie the main diagonal input elements. Condition (i) translates an equal a priori probability of all directions in space, ie no preferred direction.

Note that i condition is not a strict law but rather a sort of statistical thermodynamic regularity.

Also, condition i had been used by Maxwell in his derivation of the Maxwell-Boltzmann velocity distribution formula.

- $B_{i,j} = B_{j,i}$ for all i, j.
  Matrix B is symmetrical to conform to nature's symmetry and physical principles of reciprocity and detailed balance.
- $B_{i,i}$ = RO, i.e. the main diagonal consists of equal or constant entries RO .
  RO can take any value in the interval [0,1].

Condition (ii) corresponds to the assumption of equal and similar residue after each jump or time step dt for all the free elementary nodes.

- The sum of $B_{i,j}$ = 1 for all rows (or columns) away from the borders and the sum $B_{i,j}$ < 1 for all the rows connected to the borders.

- The condition iv means that the probability of the whole space = 1. Obviously, the statistical matrix B is very different from the mathematical Laplacian matrix and the mathematical statistical matrix of the Markov transition probability.

The physical nature of B is clear and briefly explained above through its four conditions i-iv which support the hypothesis of being an accurate model of how nature works.

It is worth mentioning that the transition matrix B mentioned above is eligible for studying canonical thermodynamic systems where the exchange of energy between the system and its environment is allowed.

Another closed matrix B (called Bc) for closed or isolated systems (such as soundproof audio rooms) where there is no energy exchange between the system and its environment is similar to matrix B at except for condition iv which should be replaced by:

The sum of $B_{i,j} = 1$ for all rows (or columns) away from borders and similarly the sum of $B_{i,j} = 1$ for all rows connected to borders.

A thorough study of the acoustic properties of rooms requires the introduction and calculation of the so-called statistical weights of geometric shapes having a certain number of nodes as shown below.

The statistical weights themselves are best explained with numerical examples as follows,

A. 1D geometric shape (straight line),

Numerical integration I=∫ y dx from x=a to x=b can be reached by chains of matrices Bc where one arrives at [6],

I=6h/77(6.Y1 +11.Y2 + 14.Y3+15.Y4 +14.Y5 + 11.Y6 + 6.Y7)

h is the space interval between two nodes.

The statistical integration formula for the area under the curve, written above for 7 nodes, which is an extension of Sympson's rule to 3 nodes, can be expressed as follows:

I=SW1.Y1+SW2.Y2+SW3.Y3+SW4.Y4+SW5.Y5+SW6.Y6+SW7.Y7

It is obvious that SW1=6/77 . . etc

It is obvious that SW1=6/77 . . etc

B. 2D rectangular shape

The finite double integral I=∬ f(x,y) dx dy... for the domain D1

a<=x<=b and c<=y<=d

Similarly, the process of double numerical integration ,

I=∬ f(x,y) dxdy

on the D1 domain divided into 9 equidistant nodes can be realized via the Bc-Matrix chains like,

IBc = 9h^3/29.5(2.75Z(1.1)+3.5Z(1.2)+2.75.Z(1.3)+3.5Z(2.1)+4.5Z(2.2)+ 3 .5Z(2.3)+2.75Z(3.1)+3.5Z(3.2)+2.75Z(3.3))

where h is the equidistant interval on the x and y axes.

Obviously the 9 statistical weights in the 2D example above are as follows,

2.75/29.5,3.5/29.5,2.75/29.5,3.5/29.5,4.5/29.5,3.5/29.5,2.75/29.5,3.5/29.5,2.75Z/29.5

which is an extension of Sympson's 9-node double integration rule.

C. 3D cuboid shape

The finite double integral I=∭ f(x,y,z) dx dy dz for the domain D1

a<=x<=b, c<=y<=d and e<=y<=f

subdivided into 27 equidistant nodes as shown in Figure 1.

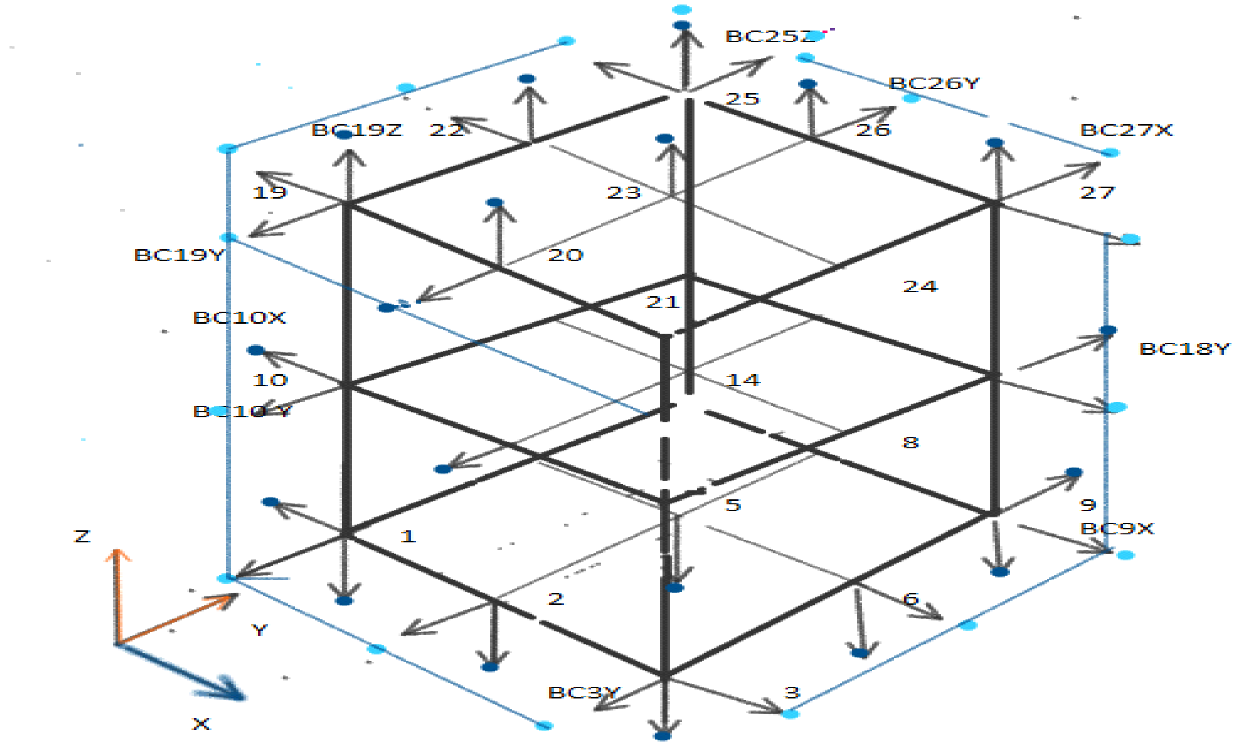

Fig. 1: 3D cuboid subdivided into 27 equidistant nodes

Again, the finite triple statistical numerical integration process,

$I=\iiint f(x,y,z)\, dx\, dy\, dz$

on the D1 domain divided into 27 equidistant nodes can be realized via the Bc-Matrix chains like cases i and ii.

$I=\iiint W(x,y,z)\, dxdydz$

on the domain of the cube.

The numerical results are,

I = 27h^4/59(2.555W(1,1,1)+3.13W(1,2,1)+2.555.W(1,3,1)+3.13W(2,1,1)+3.876W(2,2,1)+ 3,13W(2,3,1)+2,555W(3,1,1)+2,555W(3,2,1)+3,13Z(3,3,1) . . . etc.)

This formula is an extension of Sympson's double integration rule to the triple integration rule.

III. APPLICATION TO SOUND ROOMS

This is not absolutely new since a few essays based on sound scattering theory (eg Chiara 2012) have been published. We assume that the main flaw of the long procedure of Chiara [4] and other authors is that they did not use the statistical thermodynamic regularity (condition i) in their approaches.

A. Reverbration time TR

It can be assumed that the temporal evolution of the energy density (sound, heat, energy em..etc) in a 3D system [7,8] is subject to the following relation which is in itself a consequence of the thermodynamic regularity statistic (condition i):

U(t)= U(0). Exp(-const . A/V) . . . (8)

Eq 8 as adjusted to sound rooms becomes,

U(t)= U(0). Exp(-t.C.A.S /V) . . . (9)

The reverberation time TR is defined as the time when U(t) drops to one million (10^-6) from its initial value.

In other words,

Log(10)[U(t=TR)/U(0)]=-6

when substituted in Eq 9 we get,

-6=-TR.C.A.S /V

OR,

TR=6V/CAS . . .time without dimension space of the matrix B (10)

Note that when moving in the unit space of the B 4D x-t matrix, the real time t is completely lost.

Retrieving the real time is not complicated but a bit long.

In short the real time t is

t= n^2. dt . . . condition (4)

where n is the number of dimensions of the geometric object:

Obviously, for the 3D object n=3 and n^2=9

And for the 2D object,

n=2 and n^2=4.

This explains why molecules (or equivalently energy density) take longer to diffuse in 2D than in 3D. We can now rewrite Eq 10 as,

T_R=6x9V/CAS=54V/CAS . . . .sec in real-time space (11)

If we compare equation 11 with Sabines reverberation time formula,

T_R = 53.46 V/c A S. . dry with Eq 11 we find a surprising agreement.

absolute error in Sabine's formula=-.54

relative error=-.54/54=-.01

Showing that Sabine's formula is quite accurate.

Notice that:

- Condition 4 is a serious condition and corresponds to what is called spatial compression. this means it takes more time for molecules or energy density to diffuse in a 2D object than in 3D.

This is a consequence of the statistical regularity condition in the chains of the matrix B which predicts a diffusion coefficient:

$\alpha(3D)=9/4 * \alpha(2D)$

Equations 8 and 9 suggest that:

- The theoretical diffusion coefficient depends on the geometric characteristics of the surrounding room through the A/V ratio sometimes called its characteristic length Lc which is the mean free path between two successive collisions of the sound "ray" with the walls.
- two 3D bodies of different shapes cannot have the same volume to area (V/A) ratio unless both have exactly the same volume and area.

Both are physical rules imposed by the laws of nature itself. However, at the same time, you can find many exceptions to rules i and ii, but only when applied outside their scope.

ISSN No:-2456-2165

B. *The sound energy density field*

The distribution of the sound energy density field in audio rooms[9,10], especially the uniformity, is a serious matter and requires careful attention.

In current literature, uniformity of sound energy density field distribution in audio rooms is achieved by inserting additional audio devices such as reflectors, diffusers, absorbers, etc.

In the following analysis, we describe how to achieve the maximum uniformity possible through proper speaker placement.

This analysis is best explained through the statistical weights SW,s of the cube in Fig.1

The statistical weights are calculated via the closed transition matrix Bc and their numerical values at the 27 nodes are presented in Table I.

Table 1: Numerical values of the statistical weights at the 27 nodes of the cube of Fig.1.

Node	**statistical weight**
1	0.7100
2	0.9728
3	0.710
4	0.9728
5	1.31427
6	0.9728
7	0.7100
8	0.9728
9	0.710
10	0.97281
11	1.31427
12	0.9728
13	1.3142
14	1.7606
15	1.3143
16	0.9728
17	1.3143
18	0.9728
19	0.710
20	0.9728
21	0.710
22	0.9728
23	1.31427
24	0.9728
25	0.710
26	0.9728
27	0.710

Notice that:

- The statistical weights at the 27 nodes add up to 27.
- There is complete symmetry around the center (node 14).

The geometric shape in Fig.1 and its corresponding statistical weights at the 27 nodes suggest that placing 4 loudspeakers at nodes 11, 13, 15, 17 would produce an efficient and uniform sound field.

Table 2: Shows the acoustic energy field at the 27 nodes in Fig.1

23.1387901	40.3952217	23.1387901	40.3952255	46.2775764	40.3952255	23.1387901	40.3952255	23.1387901
58.0422783	52.1599274	58.0422821	52.1599312	116.084564	52.1599312	58.0422821	52.1599350	58.0422859
23.1387901	40.3952217	23.1387901	40.3952255	46.2775764	40.3952255	23.1387901	40.3952255	23.1387901

Table II. The acoustic energy density field at the 27 nodes in Fig.1 due to the placement of 4 loudspeakers each with a power of 100 units at nodes 11, 13, 15, 17.

Alternatively, if the same four loudspeakers are transferred to nodes 2, 4, 6, 8, the numerical results produced are shown in Table III.

Table 3: The numerical results produced when four loudspeakers from Table II are transferred to nodes 2, 4, 6, 8.

50.8281326	40.7318916	50.8281326	40.7318878	101.656258	40.7318878	50.8281326	40.7318878	50.8281288
23.5050011	41.1520615	23.5050011	41.1520615	47.0099983	41.1520615	23.5050011	41.1520615	23.5050011
7.97098923	12.1604652	7.97098923	12.1604652	15.9419785	12.1604662	7.97098923	12.1604662	7.97098923

Notice that:

- The sound energy distribution of Table II is more efficient and more uniform than that of Table III.
- The sum of the energy densities in Table II is 1170 units while the sum of the energy densities in Table III is 870 units.

The ratio of the sums 1170/870(1.345) is equal to the ratio of their statistical weights 1.3143/0.9728(1.3510).

IV. CONCLUSION

In this paper, the numerical solution of the statistical chains of matrix B is successfully used to calculate the sound intensity field in audio rooms.

Also, we show that B-chain techniques can produce rigorous statistical proof of Sabines' imperial formula for reverberation time in audio rooms.

This proves that Sabine's formula for the reverberation time T_R is fairly accurate.

In the current literature the computation of the non-uniform sound energy density field inside audio rooms is abscent whereas the present article we do it with high precision and speed.

In addition, the definition of so-called statistical weights of geometric shapes is introduced and found to be effective in solving double and triple integration as well as sound diffusion transfer equation in audio rooms.

NB. All calculations in this article were produced using the author's double-precision algorithm to ensure maximum accuracy, as follows by ref. 11 for example.

REFERENCES

[1.] I. Abbas ,IJISRT, A Numerical Statistical Solution to Laplace and Poisson Partial Differential Equations, Volume 5, Number 11, November – 2020.

[2.] I. Abbas, IJISRT, A time-dependent numerical statistical solution of the partial differential heat diffusion equation, Volume 6, Issue 1, January - 2021.

[3.] I. Abbas, Researchgate, Effective Unconventional Approach to Statistical Differentiation and Statistical Integration, November 2022.

[4.] Mona Rahmani, UBC, Numerical methods for solving the heat equation, the wave equation and the Laplace equation (finite difference methods, January 2019

[5.] John H. Mathews, Numerical Methods for Mathematics, Science and Engineering, 1994, pp. 520-527.

[6.] I.M.Abbas,Theory and Design of Audio Rooms-Reformulation of Sabine Foroula,IJISRT review-Volume 6, October – 2021, ISSN No:-2456-2

[7.] Uwe M. Stephenson at HafenCity University Hamburg, Hamburg, Germany, Internoise, 2012.

[8.] I. Abbas,IJISRT review,THEORY AND DESIGN OF AUDIO ROOMS-STATISTISTICAL VIEW-July 2023.

[9.] Uwe M. Stephenson at HafenCity University Hamburg, ,Hamburg, Germany, Internoise, 20122-Michael Bannon & Frank Kaputa, The Sabins Formula &The Birth of Modern Architectural Acoustics. Publié le 22 janvier 2015 Différentes hypothèses - différentes formules de réverbération

[10.] THEORY AND DESIGN OF SABINE FOROULA'S AUDIO REFORMULATION ROOMS. Available at: https://www.researchgate.net/publication/35503890

[11.] I. abbas et al, IEEE, June 1996, Pseudospark Discharg spark. Plasma Science Transactions 24(3):1106 - 1119, DOI: 1109/27.533119.