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Comparison of Results of Variant of Mixed Transportation Problem against Standard Formulation Case Study: (Ghana Bauxite Company Limited)

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Abstract:- The paper models the shipment of bauxite of Ghana Bauxite Company Limited (GBC) from the various sources of production to the various destinations as mixed transportation problem. The paper studies the work done by Mondal et al (2012) and Mondal and Hossain (2012) in which the MVAM technique was used as solution method. The original inequality constraints arrangement was m modified to obtained variants of inequality arrangement for the same data set from Ghana Bauxite Company Limited. Again, the inequality arrangements in Mondal et al (2012) was given different arrangement in each of the data sets with different inequality arrangement. In each case, we obtained the same solution for each variant of the inequality arrangements, first for the data set of Ghana Bauxite Company Limited, second for the data set of Mondal et al (2012). In addition, the standard technique of solving the TP using MODI method with North West Corner method was applied on GBC data set and the solution obtained previously also repeated. It is then concluded that the formulation of the mixed transportation problem does not d differentiate between order of arrangement of the inequalities on a data set and therefore the MVAM solution method is not unique for d different arrangement of inequalities on the data set. In addition, the standard method of solution was also found to have the same solution. However, the advantage of the MVAM is that, it is simple to use, it is also faster due to less number of iteration. It does not involve the problem of degeneracy as compared to the standard method.

Keywords:- *Transportation problem, Mixed inequality constraint, Modified Vogel's Approximation Method (MVAM), North-west Corner Method, MODI Method.*

I. INTRODUCTION

One of the crucial and successful mathematical techniques used in decision making, solving of problem and also in the physical distribution of products by the management of an organization is the transportation problem (TP) (Obinna and Nwosu, 2016). Basically, TP aims to minimize the cost of distributing a commodity from various sources to multiple destinations and it is leveraged by business such as Planning, Communication Networks, Scheduling, Transportation and Allocation (Adlakha et al., 2006). According to Nikky (2020) TPs in most cases have mixed constraints yet we use equality constraints for the optimal solution due to the reason that, TPs with mixed constraints addressed in the literature require rigor to solve them to optimality and therefore literature search revealed no systematic method for finding an optimal solution for TPs with mixed constraints. The More-Fore-Less (MFL) concept in a TP has been analysed to show that, in the distribution (Transportation Model) of commodities from multiple sources to multiple destinations, it is sometimes possible to increase the total shipping quantity at a lower (optimal) total cost for some problems or increase the total production volume for a lower cost, but we must ensure that individual supplies and demands are met (Pandian and Natarajan, 2010).Mathematically, transportation problem was first formulated by Hitchcock (1941) in which the objective was to minimize the cost of shipping commodities from supply points to the demand points. Dantzig (1953) formulated linear programming model and used the simplex method to solve TP. Shafaat and Goyal (1988) developed the degeneracy strategy resolution of transportation problem to obtained initial basic feasible solution. Singh (2015) wrote notes on transportation problem with a new method for resolution of degeneracy. Korukoglu and Balli (2011) proposed an improved Vogel's approximation method for solving TP by finding the initial basic feasible solution. Adlakha et al (2006) provided a heuristic algorithm for solving TPs with mixed constraints and extended the algorithm to finding More - For – Less (MFL) solution. Pandian and Natarajan (2010a) described a new method for find an optimal MFL solution of TPs with mixed constraints. Pandian and Natarajan(2010b) provided a method for solving transportation Problems with Mixed Constraints. Mondal and Hossain (2012) studied a new method for solving transportation problem with mixed constraints and described it as MVAM algorithm to find an optimal MFL solution. Mondal et al (2012) studied a new data set with different inequality arrangement for solving TPs with mixed constraints and described the MVAM algorithm to find an optimal MFL solution. Nikky (2020) proposed zero accomplishment method for find an optimal solution of TP with mixed constraints. Agarwal and Sharma (2020) developed an open loop method for time minimizing TP with mixed constraints. This paper considers the work of Mondal et al (2012) and Mondal and Hossain (2012) to discuss the solution method for the mixed TP. The shipment of bauxite of Ghana Bauxite Company Limited (GBC) from various sources of production to the various destinations was used as a case study model for a mixed TP. Various variant apart from the original inequalities' arrangement provided by the company (GBC) were generated and the resulting tableaux solved by the MVAM method. Consequently, the data was also formulated as the standard TP and solved using the MODI method with North West Corner method.

II. METHODS

A. Mathematical Formulation of Transportation Problem with Mixed Constraints

The Table 1 below consists of all the orders of inequality arrangements of both the demand points and supply points as well as the costs of a mixed TP.

	1	2	3	……	n	*Supply*
1	c_{11}	c_{12}	c_{13}	……	c_{1n}	$\leq/=/\geq a_1$
2	c_{21}	c_{22}	c_{23}	……	c_{2n}	$\leq/=/\geq a_2$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	c_{m1}	c_{m2}	c_{m3}	……	c_{mn}	$\leq/=/\geq a_m$
Demand	$\leq/=/\geq b_1$	$\leq/=/\geq b_2$	$\leq/=/\geq b_3$	……	$\leq/=/\geq b_n$	

Table 1: General tableau of transportation problem with mixed constraints

Therefore, the formulation of the mixed TP is given below:

$$\text{Minimize (z)} = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Subject to

$$\sum_{j=1}^{n} X_{ij} \geq a_i, i\epsilon U \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\sum_{j=1}^{n} X_{ij} \leq a_i, i\epsilon V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\sum_{j=1}^{n} X_{ij} = a_i, i\epsilon W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$\sum_{i=1}^{m} X_{ij} \geq b_j, j\epsilon Q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$\sum_{i=1}^{m} X_{ij} \leq b_j, j\epsilon T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$\sum_{i=1}^{m} X_{ij} = b_j, j\epsilon S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$Xij \geq 0$$

where $ai > 0, \forall i\epsilon I : b_j > 0, \forall j\epsilon J.$
Where U,V, and W are pairwise disjoint subsets of (1,2,3,...m) such that $U \cup V \cup W$ =(1,2,3,...m), Q, T, and S are pairwise disjoint subsets of (1,2,3,.....n) such that $Q \cup T \cup S$ =(1,2,3,.....n)

Where,

ai = the amount of a commodity that is available at the source i
bj = the quantity of a commodity that is required at the destination j
Cij = the cost of transferring one unit of a commodity from one location to another ith source to jth destination
Xij = cost of delivering one unit of product from supply point i to destination point j based on the number of units delivered
i = set of supply points index =[1,2,3....m]
j = The demand index set = [1,2,3.....n]
m = the quantity of origins (sources)
n = the number of possible destinations

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B. Algorithm of the Modified Vogel's Approximation Method (MVAM)

This method is adapted from the Vogel's Approximation method for solving standard transportation problems. The algorithm uses the chat presented in Table 2 below.

- Calculate the difference between the minimum cost and the next minimum cost in each row and column. This is called row or column penalty.
- Chose the row or column with the highest penalty among all the penalties computed.
- Make an assignment using Table 2 as a guide.
- Remove any row or column that have been totally satisfied by the just completed assignment.
- Return to step 1 and repeat the process until an optimal solution is obtained.

The Table 2 below contains all the rules necessary for the assignment.

Supply (a_i)	Demand (b_j)	Assign Unit (,)
$=$	$=$	min (a_i, b_j)
$=$	$\leq$	$\min(a_i, b_j)$
$=$	$\geq$	a_i
$\leq$	$\leq$	0
$\leq$	$=$	$\min(a_i, b_j)$
$\leq$	$\geq$	a_i
$\geq$	$\geq$	$\max(a_i, b_j)$
$\geq$	$=$	b_j
$\geq$	$\leq$	b_j

Table 2: Summary Chart of Demand-Supply relation

III. DATA

Data was obtained from (GBC). GBC has three production sites, Awaso, Nyinahinand Kibi respectively. The shipment of bauxite was to five destinations, China, Greece, Canada, Ukraine and Brazil. The cost includes, land transportation cost, duty cost and freight cost which depends on the number of days required for each destination. The demand and supply were in metric tons (MT). The total cost incurred by the company (GBC) is Gh12,324,523 in the period of January 01- December 31, 2020.Table 3 below shows the unit costs of shipments in x 10^3 and demand and supply quantities in metric tons (MT).

↓Source/destination→	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896	4.098	3.893	3.909	5.991	$=1221$
Nyinahin	1.674	4.195	3.148	3.502	5.233	≥ 1211
Kibi	1.761	3.612	3.392	3.424	4.967	≤ 1231
Demand	$= 744$	$= 753$	≤ 807	≥ 881	≥ 478	

Table 3: Summary of data from January 01-December 31, 2020

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IV. PROBLEM FORMULATION

Let

i = supply point, i = 1, 2, 3
j = destination point, j = 1, 2, 3, 4, 5
C_{ij} =unit cost of transportation from ith source to jth destination.
Xij = is shipment from ith supply point to jth destination point

The Objective Function is:

Minimize (z) = $1.896x_{11} + 4.098x_{12} + 3.893x_{13} + 3.909x_{14} + 5.991x_{15} + 1.674x_{21} + 4.195x_{22} + 3.148x_{23} + 3.502x_{24} + 5.233x_{25} + 1.761x_{31} + 3.612x_{32} + 3.392x_{33} + 3.424x_{34} + 4.967x_{35}$

Supply Constraints are:

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 1221.$
$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \geq 1211.$
$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \leq 1231.$

Demand Constraints are:

$x_{11} + x_{21} + x_{31} = 744.$
$x_{12} + x_{22} + x_{32} = 753.$
$x_{13} + x_{23} + x_{33} \leq 807.$
$x_{14} + x_{24} + x_{34} \geq 881.$
$x_{15} + x_{25} + x_{35} \geq 478.$
$x_{ij} \geq 0$ for all i and j

Table 4 below illustrates the initial tableau of the unit costs and the supply, demand quantities. Figure 1 below is the supply and destination linkages.

From	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896	4.098	3.893	3.909	5.991	=1221
Nyinahin	1.674	4.195	3.148	3.502	5.233	$\geq$ 1211
Kibi	1.761	3.612	3.392	3.424	4.967	$\leq$1231
Demand	=744	= 753	$\leq$ 807	$\geq$ 881	$\geq$ 478	

Table 4: The time unit cost (x 10^3) and shipments

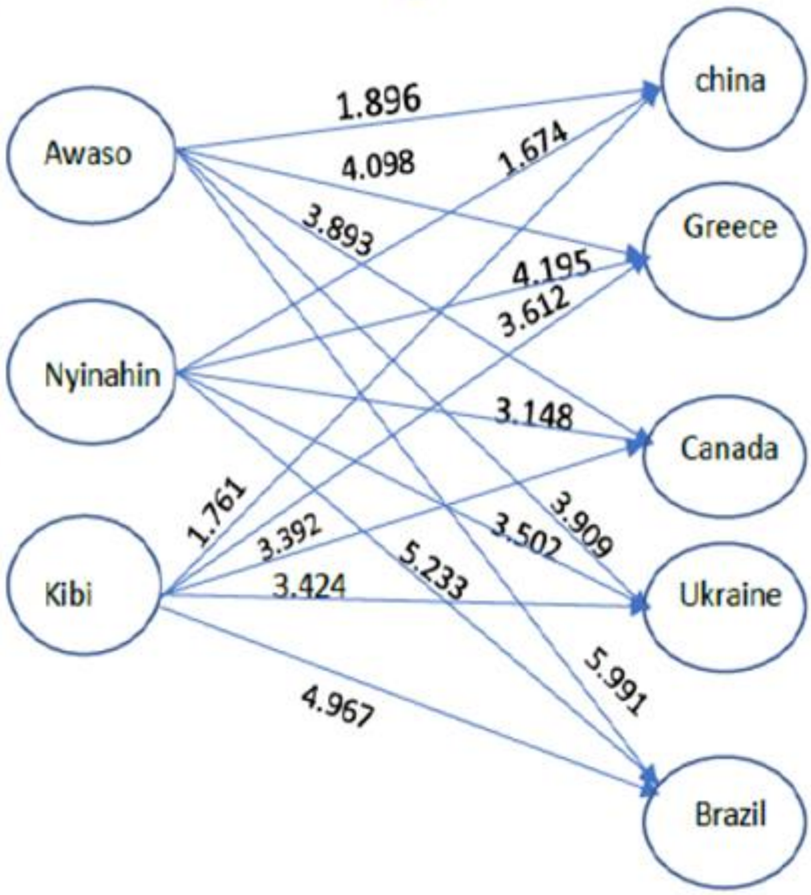

Fig. 1: Network representation of the unit costs and Shipments

V. RESULTS

- Results from initial tableau

  The Modified Vogel's Approximation Method (MVAM) was used in line with Mondalet al (2012) and Mondal and Hossain (2012). The final results of the iterations oninitial tableau is shown in Table 5 below.

From	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896 744	4.098	3.893	3.909 477	5.991	=1221
Nyinahin	1.674	4.195	3.148 807	3.502 404	5.233	$\geq$ 1211
Kibi	1.761	3.612 753	3.392	3.424	4.967 478	$\leq$ 1231
Demand	= 744	=753	$\leq$ 807	$\geq$ 881	$\geq$ 478	

Table 5: Final Result

From Table 5 , we have:

$x_{11} = 744$: Awaso - China

$x_{14} = 477$: Awaso - Ukraine

$x_{23} = 807$: Nyinahin - Canada

$x_{24} = 404$: Nyinahin - Ukraine

$x_{32} = 753$: Kibi - Greece

$x_{35} = 478$: Kibi - Brazil

The minimum total cost is

$744(1.896)+477(3.909)+807(3.148)+404(3.502)+753(3.612)+478(4.967)$

$= 12324.523$

➢ *Results for the 1st re-arrangement of the inequalities*

↓Source/destination→	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896	4.098	3.893	3.909	5.991	$\leq$ 1221
Nyinahin	1.674	4.195	3.148	3.502	5.233	$\leq$ 1211
Kibi	1.761	3.612	3.392	3.424	4.967	= 1231
Demand	= 744	$\leq$ 753	= 807	$\geq$ 881	$\leq$ 478	

Table 6: Initial tableau for Second re-arrangement of the inequalities

From	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896 744	4.098	3.893	3.909 477	5.991	$\leq$1221
Nyinahin	1.674	4.195	3.148 807	3.502 404	5.233	$\leq$ 1211
Kibi	1.761	3.612 753	3.392	3.424	4.967 478	=1231
Demand	= 744	$\leq$753	= 807	$\geq$ 881	$\leq$ 478	

Table 7: Final tableau for 2nd re-arrangement

Results for the 2nd re-arrangement of the inequalities

↓Source/destination→	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896	4.098	3.893	3.909	5.991	=1221
Nyinahin	1.674	4.195	3.148	3.502	5.233	$\leq$ 1211
Kibi	1.761	3.612	3.392	3.424	4.967	$\geq$ 1231
Demand	$\leq$744	= 753	= 807	$\geq$ 881	$\geq$ 478	

Table 8: Initial tableau 3rd re-arrangement of inequalities

From	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896 744	4.098	3.893	3.909 477	5.991	≤ 1221
Nyinahin	1.674	4.195	3.148 807	3.502 404	5.233	≤ 1211
Kibi	1.761	3.612 753	3.392	3.424	4.967 478	$= 1231$
Demand	$= 744$	≤ 753	$= 807$	≥ 881	≤ 478	

Table 9: Final tableau for the 2nd re-arrange

From Table 7 and 9 , we obtained the same results as shown below:
$x_{11} = 744$: Awaso - China
$x_{14} = 477$: Awaso - Ukraine
$x_{23} = 807$: Nyinahin - Canada
$x_{24} = 404$: Nyinahin - Ukraine
$x_{32} = 753$: Kibi - Greece
$x_{35} = 478$: Kibi - Brazil
The minimum total cost:
$744(1.896)+477(3.909)+807(3.148)+404(3.502)+753(3.612)+478(4.967)$
$= 12324.523$

➢ *Arrangement for the Standard Formulation*

↓Source/destination→	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896	4.098	3.893	3.909	5.991	$= 1221$
Nyinahin	1.674	4.195	3.148	3.502	5.233	$= 1211$
Kibi	1.761	3.612	3.392	3.424	4.967	$= 1231$
Demand	$= 744$	$= 753$	$= 807$	$= 881$	$= 478$	

Table 10: The initial tableau for the Standard formulation

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From	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896	4.098	3.893	3.909	5.991	= 1221
Nyinahin	1.674	4.195	3.148	3.502	5.233	= 1211
Kibi	1.761	3.612	3.392	3.424	4.967	= 1231
Demand	= 744	= 753	= 807	= 881	= 478	

Table 11: Initial tableau

From	China	Greece	Canada	Ukraine	Brazil	Supply
Awaso	1.896 744+△	4.098	3.893	3.909 477	5.991	1221
Nyinahin	1.674	4.195	3.148 807	3.502 404	5.233	1211
Kibi	1.761	3.612 753	3.392	3.424 △	4.967 478	1231
Demand	744	753	807	881	478	

Table 12: Initial tableau

The results above obtained using the North West Corner method and MODI. The following results which are the same as before are:

x_{11} = 744:Awaso - China

$x_{14} = 477$: Awaso - Ukraine

$x_{23} = 807$:Nyinahin - Canada

$x_{24} = 404$:Nyinahin - Ukraine

$x_{32} = 753$:Kibi - Greece

$x_{35} = 478$:Kibi - Brazil

The minimum total cost:

$$744(1.896)+477(3.909)+807(3.148)+404(3.502)+753(3.612)+478(4.967)$$
$$= 12324.523$$

In Table 13 below, we have the formulation using the original set of inequalities supplied by GBC. Variants 1 and 2 are formulations obtained by changing the arrangement of the inequalities. The last row represents the standard formulation. The results for all the formulations are same as shown below. Figure 2 is the network link age for the final solution.

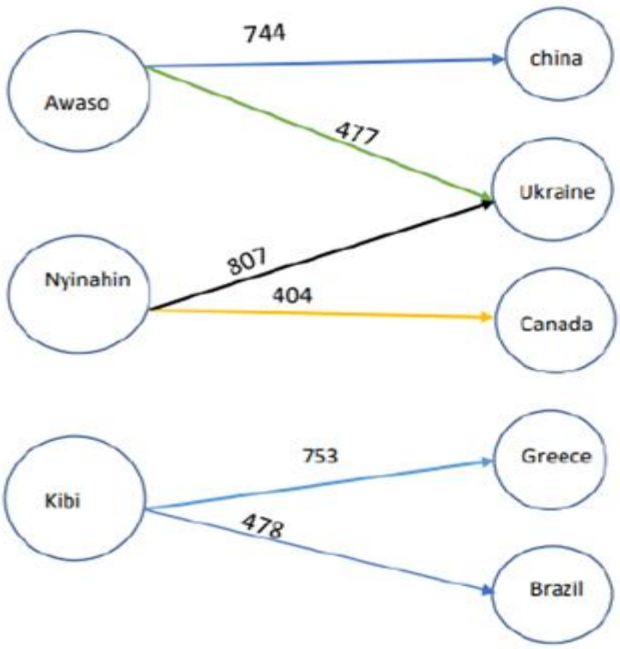

Fig. 2: Network diagram showing the solution results and shipments

Variant	Method	Total Cost	Decision Variable
Case study 1	MVAM	12324.523	$x_{11} = 744$
Mixed Variant 1	MVAM	12324.523	$x_{14} = 477$
Mixed Variant 2	MVAM	12324.523	$x_{23} = 807$
Standard Model	NWCM/MODI	12324.523	$x_{24} = 404$
			$x_{32} = 753$
			$x_{35} = 478$

Table 13: Summary of the results from the other solutions

VI. DISCUSSION

From the results in Table 13 above, the solution for all the variants including the standard model are the same for the optimal solution and the total cost. The variation was extended to the case study described in the Mondal et al (2012).In Table 14 below, the first supply column and demand row are the inequalities provided by Mondal et al (2012). The inequalities in the last supply column and demand row are those generated by the authors of this paper.

↓Source/destination→	A	B	C	D	Supply	
1	12	4	9	5	$= 55$	≥ 55
2	8	1	6	6	≥ 40	≤ 40
3	1	2	4	7	≤ 30	≥ 30
Demand	$= 40$	$= 20$	≤ 45	≤ 20		
	≥ 40	$= 20$	≥ 45	≤ 20		

Table 14: Mondal et al (2012)

Both order of inequalities arrangement in Table 14 above gave the same total cost of 605 and the optimal solution:

$X_{13} = 35$
$X_{14} = 20$
$X_{21} = 10$
$X_{22} = 20$
$X_{23} = 10$
$X_{31} = 30$

It is observed that when the model inequalities were changed and the resulting model solved by MVAM, the optimal shipments and the total cost were always the same.

VII. CONCLUSION

The optimal shipments obtained for the original mixed transportation problem using the MVAM are as follows;

From Awaso to China is Gh744000, Awaso to Ukraine is Gh477000, from Nyinahinto Canada is Gh807000, Nyinahin to Ukraine is Gh404000, from Kibi to Greece isGh753000 and Kibi to Brazil also gave Gh478000.

The total optimal cost computed is Gh12,324,523 which is 27.04% savings of the current total transportation cost of Gh 16,892,753. The saving made is Gh4,568,230. The variation of the order of the inequalities gave the same results as shown above. The standard model solution also gave the same results as above.

Therefore, the result so far shows that the Modified Vogel Approximation Method(MVAM) for mixed transportation does not differentiate between order of arrangement of the inequalities that produce different variant of formulation and the therefore the process is not also unique for the different variants.

The same scenario was observed with the case study provided in Mondal et al (2012).However, the advantage of the MVAM is that, it is simple to use, it is also faster due to less number of iteration. It does not involve the problem of degeneracy as compared to the standard method.

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