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Applications of Kushare Integral Transform in Mechanics

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Abstract:- Integral transform plays important role in various fields. Recently, Kushare, Takate and Patil introduced Kushare transform in 2021. Many researchers used this transform for solving various problems. In this paper we use Kushare transform for solving problems from mechanics. We use it to simple harmonic motion.

Keywords:- *Integral transform, Kushare transform, Simple Harmonic Motion.*

I. INTRODUCTION

Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.

In April 2022 D. P. Patil, et al [4] used Kushare transform for solving the problems on population growth and decay. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Dinkar Patil [8] used Sawi transform and its convolution theorem for solving wave equation. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9].

D .P. Patil [10] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [11] also obtained dualities between double integral transforms. Kandalkar, Gatkal and Patil [12] solved the system of differential equations using Kushare transform. D. P. Patil [13] used Aboodh and Mahgoub transforms to solve boundary value problems of the system of ordinary differential equations. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplace, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. D. P. Patil et al [16] obtained solution of Volterra Integral equations of first kind by using Emad-Sara transform. Futher Patil with Tile and Shinde [17] used Anuj transform for solving Volterra integral equations of first kind. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil [21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil used Emad-Falih transform for solving problems based on Newton's law of cooling [22].

D. P. Patil et al [23] used Soham transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [24] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY integral transform for Newton's law of cooling [25]. D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [27] introduced double kushare transform. Recently, D. P. Patil et al [28] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by Patil et al [29]. Thete, et al [30] used Emad Falih transform for handling growth and decay problems. Nikam, Patil et al [31] used, Kushare transform of error functions in evaluating improper integrals.

Wagh sisters and Patil used Kushare [32] and Soham [33] transform in chemical Sciences. Malpani, Shinde and Patil [34] used Convolution theorem for Kushare transform and applications in convolution type Volterra integral equations of first kind. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [36] developed generalized double rangaig integral transform.

Shinde, et al [37] used Kushare transform is used for solving Volterra Integro-Differential equations of first kind. Kandekar et al [38] used new general integral equation to solve Abel's integral equations. Pardeshi, Shaikh and Patil [39] used Kharrat Toma transform for solving population growth and decay problems. Patil et al [40] used Kushare transform for evaluating integrals containing Bessel's functions. Thakare and Patil [41] used general integral transform for solving mathematical models from health sciences. Rathi sisters used Soham transform for analysis of impulsive response of Mechanical and Electrical oscillators with Patil [42]. Patil [43] used KKAT transform for solving growth and decay problems. Suryawanshi et al [44] used Soham transform for solving models in health sciences and biotechnology.

We organize this paper as follows. Introduction is included in section one. Second section is devoted for preliminary concepts. Kushare transform is used to the problems in mechanics in third section.

II. PRELIMINARY

In this section we state some preliminary concepts required, i.e. definition, formulae and theorems from Kushare transform.

A. Definition of Kushare integral transform:

$$A = \left\{ f(t): \exists M, \tau_1 \tau_2 > 0, |f| < (t) M e^{\frac{|t|}{\tau_j}}, if\ t \epsilon (-1)^j * [0, \infty) \right\}$$

$$K[f(t)] = S(v) = v \int_0^\infty f(t) e^{-tv^\alpha} dt, t \geq 0, \tau_1 \leq v \leq \tau_2$$

B. Kushare transform of some functions:

In this section we state formulae of some elementary functions

SR. NO	FUNCTION	KUSHARE TRANSFORM
1	1	$\frac{1}{v^{\alpha-1}} = s(v)$
2	t^n	$\frac{n+1}{v^{\alpha(n+1)-1}}$
3	e^{at}	$\frac{v}{v^\alpha - a}$
4	$\sin(at)$	$\frac{av}{(v^{2\alpha} + a^2)}$
5	$\cos(at)$	$\frac{v^{\alpha+1}}{(v^{2\alpha} + a^2)}$

Table 1: Kushare transform of some functions

C. Kushare transform of derivative:

If $s(v)$ is Kushare transform of function $f(t)$ then

1. $K[f'(t)] = v^\alpha s(v) - vf(0)$
2. $K[f''(t)] = v^{2\alpha} s(v) - v^{\alpha+1} f(0) - vf'(0)$
3. $3. K[f^n(t)] = v^{n\alpha} s(v) - v \sum_{k=0}^{n-1} v^{\alpha(n-k-1)} f^k(0)$

III. APPLICATIONS OF KUSHARE TRANSFORM TO MECHANICS

In this section we use Kushare transform to solve problems from mechanics.

➢ ***Simple Harmonic Motion***

We consider a particle having mass m , which execute the simple harmonic motion. Let x be the displacement of the particle from the mean position at any time t. The differential equation describing the motion of the particle at time t is given by

$$x''(t) + \omega^2 x = 0, \omega^2 = k/m$$

We apply Kushare transform to find the displacement of this particle at any time t.

Assume that at $t = 0$ and $x(0) = 0$, $x'(0) = 1$

Solution: Consider the second order differential equation

$$x''(t) + \omega^2 x = 0$$

Applying Kushare integral

$$K[x''(t)] + \omega^2 K(x) = 0$$

$$v^{2\alpha} s(v) - v^{\alpha+1} x(0) - vx'(0) + \omega^2 s(v) = 0$$

$$(v^{2\alpha} + \omega^2)s(v) - 0 - v = 0$$

$$s(v) = \frac{v}{(v^{2\alpha} + \omega^2)}$$

Taking inverse on both side

$$K^{-1}[s(v)] = K^{-1}\left[\frac{v}{(v^{2\alpha+}\omega^2)}\right]$$

$$x(t) = \frac{\sin \omega t}{\omega}$$

That is $x(t) = \sqrt{\frac{m}{k}} \sin \sqrt{\frac{k}{m}} t$

It is the required displacement of the particle.

- *Let a body A of mass 1 gram moves on –axis. It is attracted towards the origin 0 with a force equals to 4x. Also, assume that initially it is at rest when $x = 5$ then; determine its position by considering no any other forces acting on it.*

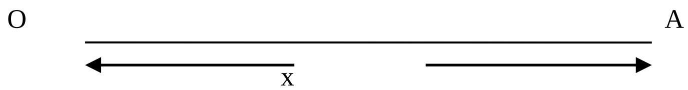

Figure 1: Motion of the body along x axis

- **Solution:** See above figure number 1,

For x > 0 the net force towards left is given by -4x, while

For x < 0 the net force towards right is given by -4x.

Thus, for both cases, the net force is equal to -4x.

By Newton's second law of motion: Mass * accelation = net force

Therefore we can obtain second order differential equation,

$$\frac{d^2x}{dt^2} = -4x$$

$$\frac{d^2x}{dt^2} + 4x = 0$$

The initial conditions are

$$x(0) = 5 \ and \ x'(0) = 0$$

$$x''(t) + 4x = 0$$

Applying Kushare integral transform

$$K[x''(t)] + 4K(x) = 0$$

$$v^{2\alpha}s(v) - v^{\alpha+1}x(0) - vx'(0) + 4s(v) = 0$$

$$(v^{2\alpha} + 4)s(v) - 5v^{\alpha+1} - 0 = 0$$

$$(v^{2\alpha} + 4)s(v) = 5v^{\alpha+1}$$

$$s(v) = \frac{5v^{\alpha+1}}{(v^{2\alpha} + 4)}$$

Taking inverse on both side

$$K^{-1}[s(v)] = K^{-1}\left[\frac{5v^{\alpha+1}}{(v^{2\alpha+} + 4)}\right]$$

$$x(t) = 5\cos 2t$$

- *The relationship between resistive force of air and the velocity of a freely falling body is given by* $dy/dt = g - \alpha v$, *where* $v(t)$ *is the velocity at any time t. Initially the body is at rest.*
- **Solution:**

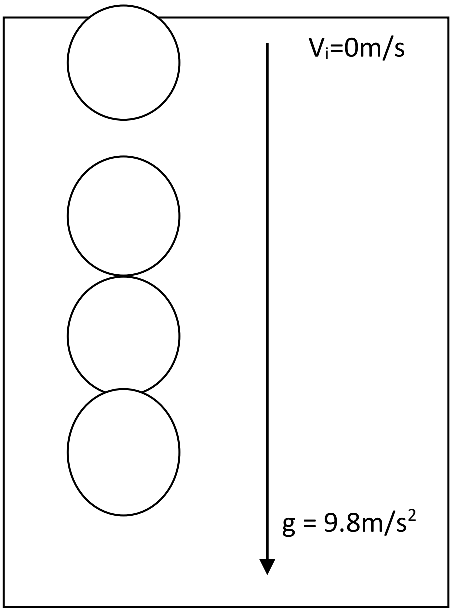

Fig. 2: Freely falling body

- **Solution**: See above figure number 2:
  The equation for the motion of the body moving under constant gravitational acceleration (as shown in figure) is

$$\frac{dv}{dt} = g - \gamma v$$

i.e $x^{'} = g - \gamma x$

Applying kushare integral transform on both side

$$K[x^{'}(t)] = gK(1) - \gamma K(x)$$

$$v^{\alpha}s(v) - vf(0) = g\left(\frac{1}{v^{\alpha-1}}\right) - \gamma s(v)$$

$v^{\alpha}s(v) - 0 = g\left(\frac{1}{v^{\alpha-1}}\right) - \gamma s(v)$

$$v^{\alpha}s(v) + \gamma s(v) = g\left(\frac{1}{v^{\alpha-1}}\right)$$

$$(v^{\alpha} + \gamma)s(v) = \frac{g}{v^{\alpha-1}}$$

$$s(v) = \left(\frac{g}{v^{\alpha-1}}\right)\left(\frac{1}{v^{\alpha} + \gamma}\right)$$

$$s(v) = \left(\frac{g}{v^{\alpha-1}}\right)\left(\frac{1}{\gamma} - \frac{v^{\alpha}/\gamma}{v^{\alpha} + \gamma}\right)$$

$$s(v) = \left(\frac{g}{v^{\alpha-1}}\right)\left(\frac{1}{\gamma} - \frac{v^{\alpha}}{\gamma(v^{\alpha}+\gamma)}\right)$$

$$s(v) = \left(\frac{g}{\gamma}\right)\left(\frac{1}{v^{\alpha-1}} - \frac{v^{\alpha}}{(v^{\alpha-1})(v^{\alpha}+\gamma)}\right)$$

$$s(v) = \left(\frac{g}{\gamma}\right)\left(\frac{1}{v^{\alpha-1}} - \frac{v}{(v^{\alpha}+\gamma)}\right)$$

Applying inverse on both side

$$K^{-1}[s(v)] = \left(\frac{g}{\gamma}\right)K^{-1}\left(\frac{1}{v^{\alpha-1}} - \frac{v}{(v^{\alpha}+\gamma)}\right)$$

$$K^{-1}[s(v)] = \frac{g}{\gamma}\left[K^{-1}\left(\frac{1}{v^{\alpha-1}}\right) - K^{-1}\left(\frac{1}{v^{\alpha}+\gamma}\right)\right]$$

$v(t) = \frac{g}{\gamma}(1 - e^{-\gamma t})$

It is required solution.

IV. CONCLUSION

We have successfully used Kushare transform for solving problems occurring in mechanics and obtain solution exactly or very closer to the solutions obtained by another transform.

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