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Solution of Abel's Integral Equation by using Soham Transform

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Abstract:- Integral transform play very important role in solving ordinary, partial as well as fractional differential equations. It is also useful for solving integral equations, integro-differential equations and system of equations. In this paper we use Soham transform for solving Abel's integral equations.

Keywords:- *Integral transforms, Soham transform, Abel's integral equations.*

I. INTRODUCTION

Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.

In April 2022 D. P. Patil, et al [4] solved the problems on growth and decay by using Kushare transform. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Dinkar Patil [8] used Sawi transform and its convolution theorem for solving wave equation. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9].

D .P. Patil [10] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [11] also obtained dualities between double integral transforms. Kandalkar, Gatkal and Patil [12] solved the system of differential equations using Kushare transform. D. P. Patil [13] solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplac, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. D. P. Patil et al [16] solved Volterra Integral equations of first kind by using Emad-Sara transform. Futher Derle, Rahane and Patil[17] introduced general double rangaig integral transform. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil [21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil used Emad- Falih transform for solving problems based on Newton's law of cooling [22]. D. P. Patil et al [23] used Soham transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [24] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling [25]. D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [27] introduced double kushare transform. Recently, D. P. Patil et al [28] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by patil et al [29]. Thete et al [30] used Emad Falih transform for handling growth and decay problems. Nikam, Patil et al [31] used Kushare transform of error functions in evaluating improper integrals. Wagh sisters and Patil used Kushare [32] and Soham [33] transform in chemical Sciences. Malpani, Shinde and Patil [34] used Convolution theorem for Kushare transform and applications in convolution type Volterra integral equations of first kind. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Patil et al [36] developed generalized double rangaig integral transform. Kushare transform is used for solving Volterra Integro-Differential equations of first kind by Shinde et al [37]. Kandekar et al [38] used new general integral equation to solve Abel's integral equations. Pardeshi, Shaikh and Patil[39] used Kharrat Toma transform for solving population growth and decay problems. Patil et al [40] used Kushare transform for evaluating integrals containing Bessel's functions. Thakare and Patil [41] used general integral transform for solving mathematical models from health sciences. Rathi sisters used Soham transform for analysis of impulsive response of Mechanical and Electrical oscillators with Patil [42]. Patil [43] used KKAT transform for solving growth and decay problems.

II. PRELIMIANRY

In this section we state some preliminary concepts of Soham transform which are useful for solving Abel's integral equations. Now we state formulae of Soham transform of simple functions in following table.

Sr. No	Function	Soham transform
1	1	$\frac{1}{v^{\alpha+1}}$
2	t	$\frac{1}{v^{2\alpha+1}}$
3	t^n	$\frac{\Gamma(n+1)}{v^{\alpha n+\alpha+1}}$
4	e^{at}	$\frac{1}{v(v^\alpha+a)}$
5	sinat	$\frac{a}{v(v^{2\alpha}+a^2)}$
6	cosat	$\frac{v^\alpha}{v(v^{2\alpha}+a^2)}$
7	sinhat	$\frac{av}{v(v^{2\alpha}-a^2)}$
8	coshat	$\frac{v^\alpha}{v(v^{2\alpha}-a^2)}$

Table 1: Soham transform of simple functions

Convolution Theorem for Soham Transform, If f and g are two functions then

$$(f*g)(t) = \int_0^t (t-\tau)g(\tau)d\tau \Rightarrow \mathcal{S}(f*g)(t) = v\{F(v)G(v)\}$$

- **Inverse Soham Transform:** Inverse Soham transform of f(t) is P(v) then inverse Soham transform is defined as $\mathcal{S}^{-1}[P(v)] = f(t)$

III. SOLVING ABEL'S INTEGRAL EQUATION USING SOHAM TRANSFORM:

In this section, Soham transform is going to be used to find the exact solution of Abel's integral equation. Abel's integral equation is defined as,

$$f(x) = \int_{t=0}^{x} \frac{1}{\sqrt{(x-t)}} u(t)dt = \int_{t=0}^{x} (x-t)^{-\frac{1}{2}} u(t)dt \qquad (1)$$

Where, $\mathrm{k}(\mathrm{x},\mathrm{t}) = \frac{1}{\sqrt{x-t}} = (x-t)^{-\frac{1}{2}}$is the kernel of integral equation becomes ∞ at $t = x$, $f(x)$ function is known function, and $u(x)$ is unknown function.

By applying Soham transform on both sides of equation (1)

$$\mathcal{S}\{\mathrm{f(x)}\} = \mathcal{S}\left\{\int_{t=0}^{x} \frac{1}{\sqrt{x-t}} u(t)dt\right\}$$

$$\Rightarrow \mathcal{S}\{\mathrm{f(x)}\} = \mathcal{S}\{x^{-\frac{1}{2}} * u(x)\} \qquad (2)$$

By applying Convolution theorem of Soham transform on equation (2)

$$\mathcal{S}\{\mathrm{f(x)}\} = v.\mathcal{S}\left\{x^{-\frac{1}{2}}\right\}.\mathcal{S}\{u(x)\},$$

$$\Rightarrow \mathcal{S}\{\mathrm{f(x)}\} = v.\mathcal{S}\{u(x)\}.\frac{\Gamma\left(\frac{-1}{2}+1\right)}{v^{\alpha\left(\frac{-1}{2}\right)+\alpha+1}}$$

$$\Rightarrow \mathcal{S}\{\mathrm{f(x)}\} = v.\mathcal{S}\{u(x)\}.\frac{\Gamma\left(\frac{-1}{2}\right)}{v^{\frac{\alpha}{2}+1}} \quad \Rightarrow \mathcal{S}\{\mathrm{f(x)}\} = v.\mathcal{S}\{u(x)\}.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1}} \Rightarrow \mathcal{S}\{\mathrm{f(x)}\} = \mathcal{S}\{u(x)\}.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1-1}}$$

$$\Rightarrow \mathcal{S}\{\mathrm{f(x)}\} = \mathcal{S}\{u(x)\}.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}}} \Rightarrow \mathcal{S}\{u(x)\} = \frac{v^{\frac{\alpha}{2}}}{\sqrt{\pi}}\mathcal{S}\{\mathrm{f(x)}\} \Rightarrow \mathcal{S}\{u(x)\} = \frac{v^{\alpha}}{\pi}\left(\sqrt{\pi}.v^{\frac{-\alpha}{2}}.\mathcal{S}\{\mathrm{f(x)}\}\right) \qquad (3)$$

Let ,$h(x) = \int_0^x (x-t)^{-\frac{1}{2}} f(t)dt$ (*)

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$\mathcal{S}\{\mathrm{h(x)}\} = v.\mathcal{S}\left\{x^{-\frac{1}{2}}\right\}.\mathcal{S}\{f(x)\}$

$\therefore \mathcal{S}\{\mathrm{h(x)}\} = v\,\mathcal{S}\{f(x)\}.\frac{\Gamma\left(\frac{1}{2}\right)}{v^{\frac{\alpha}{2}+1}}$

$\Rightarrow \mathcal{S}\{\mathrm{h(x)}\} = \mathcal{S}\{f(x)\}\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}}} \Rightarrow \mathcal{S}\{\mathrm{h(x)}\} = \mathcal{S}\{f(x)\}\left(\sqrt{\pi}.v^{\frac{-\alpha}{2}}\right)$

By equation (3) $\mathcal{S}\{\mathrm{u(x)}\} = \frac{\mathrm{v}^{\alpha}}{\pi}\,\mathcal{S}\{\mathrm{h(x)}\}(4)$

Now, $\mathcal{S}[h'(x)] = v^{\alpha}P(v) - \frac{1}{v}\mathrm{f}(0) \quad \Rightarrow \mathcal{S}[h'(x)] = v^{\alpha}\,\mathcal{S}[h(x)] \Rightarrow \mathcal{S}\{\mathrm{h(x)}\} = \frac{1}{v^{\alpha}}\mathcal{S}[h(x)]$

By equation (4)

$\mathcal{S}\{u(x)\} = \frac{v^{\alpha}}{\pi}\frac{1}{v^{\alpha}}\mathcal{S}[h'(x)] \Rightarrow \mathcal{S}\{u(x)\} = \frac{1}{\pi}\,\mathcal{S}\left[h'(x)\right](5)$

Taking inverse of Soham transform equation (5);

$$u(x) = \frac{1}{\pi}h'(x)$$

$$\therefore u(x) = \frac{1}{\pi}\frac{d}{dx}h(x) \qquad (6)$$

Substituting $(*)$ in (6);

$$u(x) = \frac{1}{\pi}\left[\frac{d}{dx}\int_{t=0}^{x}\frac{1}{\sqrt{x-t}}f(t)dt\right] \qquad (7)$$

The result (7) is the required exact solution for Abel's integral equation (1).

IV. APPLICATIONS

In this section, Soham transform is going to be used to solve some Abel's integral equations.

- **Application (1):** Consider the Abel's integral equation:

$$x = \int_{t=0}^{x}\frac{1}{\sqrt{x-t}}u(t)dt \qquad (8)$$

Applying Soham transform equation (8)

$\mathcal{S}(\mathrm{x}) = \mathcal{S}\left\{\int_{t=0}^{x}\frac{1}{\sqrt{x-t}}u(t)dt\right\} \Rightarrow \frac{\Gamma(1+1)}{v^{\alpha+\alpha+1}} = v.\mathcal{S}\left\{x^{-\frac{1}{2}}\right\}.\mathcal{S}\{u(x)\} \Rightarrow \frac{\Gamma(2)}{v^{2\alpha+1}} = v.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1}}\,\mathcal{S}\{u(x)\}$

$\Rightarrow \frac{1}{v^{2\alpha+1}} = v.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1}}\,\mathcal{S}\{u(x)\} \quad \ldots\ldots\because \Gamma(2) = 1$

$\Rightarrow \quad \mathcal{S}\{u(x)\} = \frac{1}{v^{2\alpha+1}}.\frac{v^{\alpha+1}}{v.\sqrt{\pi}} \Rightarrow \mathcal{S}\{u(x)\} = \frac{v^{\frac{\alpha}{2}+1}.v^{-2\alpha-2}}{\sqrt{\pi}} \Rightarrow \mathcal{S}\{u(x)\} = \frac{v^{\frac{\alpha}{2}+1}.v^{-2\alpha-2}}{\sqrt{\pi}}$

$\Rightarrow \mathcal{S}\{u(x)\} = \frac{v^{\frac{-3\alpha}{2}-1}}{\sqrt{\pi}} \Rightarrow \mathcal{S}\{u(x)\} = \frac{1}{\sqrt{\pi}}\left[\frac{1}{v^{\frac{3\alpha}{2}+1}}\right] \Rightarrow \mathcal{S}\{u(x)\} = \frac{1}{\sqrt{\pi}}\left[\frac{1}{v^{\frac{\alpha}{2}+\alpha+1}}\right]$

Taking inverse of Soham Transform,

$$\mathrm{u(x)} = \frac{1}{\sqrt{\pi}}\,\mathcal{S}^{-1}\left[\frac{1}{v^{\frac{\alpha}{2}+\alpha+1}}\right]$$

then $\mathrm{u(x)} = \frac{2}{\pi}x^{\frac{1}{2}}$

The concluded result is the required exact solution for Abel's integral equation (8).

➢ **Application (2):**Consider the Abel's integral equation:

$$1+x+x^2=\int_{t=0}^{x}\frac{1}{\sqrt{x-t}}u(t)dt \qquad (9)$$

Applying Soham transform equation (9)

$$\mathcal{S}\{1\}+\mathcal{S}\{\mathrm{x}\}+\mathcal{S}\{x^2\}=\int_{t=0}^{x}\frac{1}{\sqrt{x-t}}u(t)dt$$

$$\therefore \frac{1}{v^{\alpha+1}}+\frac{\Gamma(2)}{v^{2\alpha+1}}+\frac{\Gamma(3)}{v^{3\alpha+1}}=\mathcal{S}\left\{x^{-\frac{1}{2}}*u(x)\right\}$$

Applying convolution theorem, $\frac{1}{v^{\alpha+1}}+\frac{1}{v^{2\alpha+1}}+\frac{2}{v^{3\alpha+1}}=.v.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1}}\ \mathcal{S}\{u(x)\}$

$$\Rightarrow \frac{1}{v^{\alpha+1}}+\frac{1}{v^{2\alpha+1}}+\frac{2}{v^{3\alpha+1}}=\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}}}\ .\mathcal{S}\{u(x)\}$$

Then,

$$\mathcal{S}\{u(x)\}=\frac{v^{\frac{\alpha}{2}}}{\sqrt{\pi}}\left[\frac{1}{v^{\alpha+1}}+\frac{1}{v^{2\alpha+1}}+\frac{2}{v^{3\alpha+1}}\right]$$

$$\mathcal{S}\{u(x)\}=\frac{1}{\sqrt{\pi}}\left[\frac{1}{v^{\frac{\alpha}{2}+1}}+\frac{1}{v^{\frac{3\alpha}{2}+1}}+\frac{2}{v^{\frac{5\alpha}{2}+1}}\right]$$

$$\mathcal{S}\{u(x)\}=\frac{1}{\sqrt{\pi}}\left[\frac{1}{v^{\frac{-\alpha}{2}+\alpha+1}}+\frac{1}{v^{\frac{\alpha}{2}+\alpha+1}}+\frac{2}{v^{\frac{3\alpha}{2}+\alpha+1}}\right]$$

Taking inverse of Soham Transform,

$$u(x)=\frac{1}{\sqrt{\pi}}\mathcal{S}^{-1}\left\{\frac{1}{v^{\frac{-\alpha}{2}+\alpha+1}}+\frac{1}{v^{\frac{\alpha}{2}+\alpha+1}}+\frac{2}{v^{\frac{3\alpha}{2}+\alpha+1}}\right\}$$

Then

$$u(x)=\frac{1}{\pi}\left[x^{\frac{-1}{2}}+x^{\frac{-1}{2}}+\frac{8}{3}x^{\frac{3}{2}}\right]$$

The concluded result is the required exact solution for Abel's " (9)"

➢ **Application (3):** Consider the Abel's integral equation:

$$3x^2=\int_{t=0}^{x}\frac{1}{\sqrt{x-t}}u(t)dt \qquad (10)$$

Applying Soham transform equation (10)

$$3\mathcal{S}=\mathcal{S}\left\{\int_{t=0}^{x}\frac{1}{\sqrt{x-t}}u(t)dt\right\}$$

$$\frac{3.2}{v^{3\alpha+1}}=\mathcal{S}\left\{x^{-\frac{1}{2}}*u(x)\right\}$$

Applying convolution theorem,

$$\frac{6}{v^{3\alpha+1}}=v.\mathcal{S}\left\{x^{-\frac{1}{2}}\right\}.\mathcal{S}\{u(x)\}$$

$$\Rightarrow \frac{6}{v^{3\alpha+1}} = v.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1}}\mathcal{S}\{u(x)\}$$

$$\Rightarrow \frac{6}{v^{3\alpha+1}} = \frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}}}\mathcal{S}\{u(x)\}$$

$$\Rightarrow \mathcal{S}\{u(x)\} = \frac{6}{v^{3\alpha+1}}.\frac{v^{\frac{\alpha}{2}}}{\sqrt{\pi}}$$

$$\mathcal{S}\{u(x)\} = \frac{1}{\sqrt{\pi}}.\frac{6}{v^{\frac{5}{2}\alpha+1}}$$

$$\mathcal{S}\{u(x)\} = \frac{1}{\sqrt{\pi}}\left[\frac{6}{v^{\frac{3}{2}\alpha+\alpha+1}}\right]$$

Taking inverse of Soham Transform,

$$u(x) = \frac{6}{\sqrt{\pi}}\mathcal{S}^{-1}\left\{\frac{1}{v^{\frac{3}{2}\alpha+\alpha+1}}\right\}$$

Then

$$u(x) = \frac{8}{\pi}x^{\frac{3}{2}}$$

The concluded result is the required exact solution for Abel's " (10)"

- **Application (4):** Consider the Abel's integral equation:

$$\frac{4}{3}x^{\frac{3}{2}} = \int_{t=0}^{x}\frac{1}{\sqrt{x-t}}u(t)dt \qquad (11)$$

Applying Soham transform equation (11)

$$\frac{4}{3}\mathcal{S}\left\{x^{\frac{3}{2}}\right\} = \mathcal{S}\left\{\int_{t=0}^{x}\frac{1}{\sqrt{x-t}}u(t)dt\right\}$$

Then $$\frac{4}{3}\frac{\Gamma\left(\frac{3}{2}+1\right)}{v^{\frac{3}{2}\alpha+\alpha+1}} == \mathcal{S}\left\{x^{-\frac{1}{2}} * u(x)\right\}$$

$$\Rightarrow \frac{4}{3}\frac{\Gamma\left(\frac{5}{2}\right)}{v^{\frac{5}{2}\alpha+1}} = \mathcal{S}\left\{x^{-\frac{1}{2}} * u(x)\right\} \quad \Rightarrow \frac{4}{3}.\frac{3}{4}\frac{\sqrt{\pi}}{v^{\frac{5\alpha}{2}+1}} = \mathcal{S}\left\{x^{-\frac{1}{2}} * u(x)\right\} \Rightarrow \frac{\sqrt{\pi}}{v^{\frac{5\alpha}{2}+1}} = \mathcal{S}\left\{x^{-\frac{1}{2}} * u(x)\right\}$$

Applying convolution theorem,

$$\frac{\sqrt{\pi}}{v^{\frac{5\alpha}{2}+1}} == v.\mathcal{S}\left\{x^{-\frac{1}{2}}\right\}.\mathcal{S}\{u(x)\}$$

$$\therefore \frac{\sqrt{\pi}}{v^{\frac{5\alpha}{2}+1}} == v.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1}}.\mathcal{S}\{u(x)\}$$

$$\Rightarrow \mathcal{S}\{u(x)\} = \frac{v^{\frac{\alpha}{2}}}{v^{\frac{5\alpha}{2}+1}} \quad \Rightarrow \mathcal{S}\{u(x)\} = \frac{1}{v^{2\alpha+1}} \quad \Rightarrow \mathcal{S}\{u(x)\} = \frac{1}{v^{\alpha+\alpha+1}}$$

Taking inverse of Soham Transform,

$$u(x) = \mathcal{S}^{-1}\left\{\frac{1}{v^{\alpha+\alpha+1}}\right\}$$

Then $u(x) = x$

The concluded result is the required exact solution for Abel's integral equation(11).

➢ **Application (5):** Consider the Abel's integral equation:

$$2\sqrt{x} + \frac{8}{3}x^{\frac{3}{2}} = \int_{t=0}^{x} \frac{1}{\sqrt{x-t}} u(t)dt \tag{12}$$

Applying Soham transform equation (11)

$$2\mathcal{S}\left\{x^{\frac{1}{2}}\right\} + \frac{8}{3}\mathcal{S}\left\{x^{\frac{3}{2}}\right\} = \mathcal{S}\left\{\int_{t=0}^{x} \frac{1}{\sqrt{x-t}} u(t)dt\right\}$$

$$\Rightarrow\ 2\frac{\Gamma\left(\frac{1}{2}+1\right)}{v^{\frac{\alpha}{2}+\alpha+1}} + \frac{8}{3}\frac{\Gamma\left(\frac{3}{2}+1\right)}{v^{\frac{3\alpha}{2}+\alpha+1}} = \mathcal{S}\left\{\int_{t=0}^{x} \frac{1}{\sqrt{x-t}} u(t)dt\right\}$$

$$\Rightarrow 2\frac{\Gamma\left(\frac{3}{2}\right)}{v^{\frac{\alpha}{2}+\alpha+1}} + \frac{8}{3}\frac{\Gamma\left(\frac{5}{2}\right)}{v^{\frac{3\alpha}{2}+\alpha+1}} = \mathcal{S}\left\{\int_{t=0}^{x} \frac{1}{\sqrt{x-t}} u(t)dt\right\}$$

Applying convolution theorem,

$$2\frac{\sqrt{\pi}}{2(v^{\frac{\alpha}{2}+\alpha+1})} + \frac{8}{3}.\frac{3}{4}\frac{\sqrt{\pi}}{v^{\frac{3\alpha}{2}+\alpha+1}} = v.\mathcal{S}\left\{x^{-\frac{1}{2}}\right\}.\mathcal{S}\{u(x)\}$$

$$\Rightarrow \frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+\alpha+1}} + 2\frac{\sqrt{\pi}}{v^{\frac{3\alpha}{2}+\alpha+1}} = v.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1}}.\mathcal{S}\{u(x)\} \quad \Rightarrow \sqrt{\pi}\left(\frac{1}{v^{\frac{3\alpha}{2}+1}} + \frac{2}{v^{\frac{5\alpha}{2}+1}}\right) = \frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}}}.\mathcal{S}\{u(x)\}$$

$$\Rightarrow \mathcal{S}\{u(x)\} = v^{\frac{\alpha}{2}}\left(\frac{1}{v^{\frac{3\alpha}{2}+1}} + \frac{2}{v^{\frac{5\alpha}{2}+1}}\right) \Rightarrow \mathcal{S}\{u(x)\} = \frac{1}{v^{\alpha+1}} + \frac{2}{v^{2\alpha+1}} \ \Rightarrow \mathcal{S}\{u(x)\} = \frac{1}{v^{\alpha+1}} + \frac{2}{v^{\alpha+\alpha+1}}$$

Taking inverse of Soham Transform,

$$u(x) = \mathcal{S}^{-1}\left\{\frac{1}{v^{\alpha+1}} + \frac{1}{v^{\alpha+\alpha+1}}\right\}$$

Then, $u(x) = 1 + 2x$

The concluded result is the required exact solution for Abel's integral equation (12).

➢ **Application (6):** Consider the Abel's integral equation:

$$\frac{3}{8}\pi x^2 = \int_{t=0}^{x} \frac{1}{\sqrt{x-t}} u(t)dt \tag{13}$$

Applying Soham transform equation (13)

$$\frac{3}{8}\pi\mathcal{S}\{x^2\} = \mathcal{S}\left\{\int_{t=0}^{x} \frac{1}{\sqrt{x-t}} u(t)dt\right\}$$

Applying convolution theorem, $\frac{3}{8}\frac{2\pi}{v^{3\alpha+1}} = v.\mathcal{S}\left\{x^{-\frac{1}{2}}\right\}.\mathcal{S}\{u(x)\}$

$$\Rightarrow \frac{3}{4}\frac{\pi}{v^{3\alpha+1}} = v.\frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}+1}}.\mathcal{S}\{u(x)\} \Rightarrow \frac{3}{4}\frac{\pi}{v^{3\alpha+1}} = \frac{\sqrt{\pi}}{v^{\frac{\alpha}{2}}}.\mathcal{S}\{u(x)\} \Rightarrow \mathcal{S}\{u(x)\} = \frac{3}{4}\frac{\pi}{v^{3\alpha+1}}\frac{v^{\frac{\alpha}{2}}}{\sqrt{\pi}}$$

$$\Rightarrow \mathcal{S}\{u(x)\} = \frac{3}{4}\frac{\sqrt{\pi}}{v^{\frac{5\alpha}{2}+1}}$$

Taking inverse of Soham Transform, $u(x) = \frac{3\sqrt{\pi}}{4}\mathcal{S}^{-1}\left\{\frac{1}{v^{\frac{5\alpha}{2}+1}}\right\}$

Then, $u(x) = x^{\frac{3}{2}}$

The concluded result is the required exact solution for Abel's integral equation (13).

V. CONCLUSION

In this paper, Soham transform has been used to find the exact solution for the Abel's integral equation. It has been concluded from solving some Abel's problems using Soham transform that Soham transform is successful tool which gives the same solution given by using other integral transform easily.

REFERENCES

[1.] S. R. Kushare, D. P. Patil and A. M. Takate, The new integral transform, "Kushare transform", International Journal of Advances in Engineering and Management , Vol.3, Issue 9, Sept.2021, PP. 1589-1592

[2.] D. P. Patil and S. S. Khakale, The new integral transforms "Soham transform, International Journal of Advances in Engineering and Management, Vol.3, issue 10, Oct. 2021.

[3.] R. S. Sanap and D. P. Patil, Kushare integral transform for Newton's law of Cooling, International Journal of Advances in Engineering and Management, vol.4, Issue1, January 2022, PP. 166-170

[4.] D. P. Patil, P. S. Nikam, S. D. Shirsath and A. T. Aher, kushare transform for solving the problems on growth and decay; journal of Emerging Technologies and Innovative Research, Vol. 9, Issue-4, April 2022, PP h317 – h-323.

[5.] D. P. Patil, Sawi transform in Bessel functions, Aayushi International Interdisciplinary Research Journal, Special Issue No. 86, PP. 171-175.

[6.] D. P. Patil, Application of Sawi transform of error function for evaluating Improper integrals, Vol. 11, Issue 20 June 2021, PP 41-45 .

[7.] D. P. Patil , Applications of integral transforms (Laplace and Shehu) in Chemical Sciences , Aayushi International Interdiscipilinary Research Journal , Special Issue 88 PP.437-477 .

[8.] D. P. Patil, Sawi transform and Convolution theorem for initial boundary value problems (Wave equation), Journal of Research and Development, Vol.11, Issue 14 June 2021, PP. 133-136 .

[9.] D. P. Patil, Application of Mahgoub transform in parabolic boundary value problems, International Journal of Current Advanced Research, Vol-9, Issue 4(C), April.2020 , PP. 21949-21951.

[10.] D. P. Patil, Solution of Wave equation by double Laplace and double Sumudu transform, Vidyabharti International Interdisciplinary Research Journal, Special Issue IVCIMS 2021 , Aug 2021 , PP.135-138.

[11.] D. P. Patil, Dualities between double integral transforms, International Advanced Journal in Science , Engineering and Technology , Vol.7 , Issue 6 , June 2020 , PP.74-82.

[12.] Dinkar P. Patil, Shweta L. Kandalkar and Nikita D. Gatkal, Applications of Kushare transform in the system of differential equations, International Advanced Research in Science, Engineering and Technology, Vol. 9, Issue 7, July 2022, pp. 192-195.

[13.] D. P. Patil, Aboodh and Mahgoub transform in boundary Value problems of System of ordinary differential equations, International Journal of Advanced Research in Science, communication and Technology, Vol.6, Issue 1, June 2021, pp. 67-75.

[14.] D. P. Patil, Double Mahgoub transform for the solution of parabolic boundary value problems, Journal of Engineering Mathematics and Stat., Vol.4, Issue (2020).

[15.] D. P. Patil, Comparative Sttudy of Laplace ,Sumudu , Aboodh , Elazki and Mahgoub transform and application in boundary value problems , International Journal of Reasearch and Analytical Reviews , Vol.5 , Issue -4 (2018) PP.22-26.

[16.] D .P. Patil , Y .S. Suryawanshi , M .D. Nehete , Application of Soham transform for solving volterra Integral Equation of first kind , International Advanced Research Journal in Science , Engineering and Technology , Vol.9, Issue 4 (2022) .

[17.] D. P. Patil, M. S. Derle and N. K. Rahane, On generalized Double rangaig integral transform and applications, Stochastic Modeling and Applications, VOl. 26, No.3, January to June special issue 2022 part-8, pp. 533- 545. ISSN: 0972-3641.

[18.] D. P. Patil, Shweta Rathi and Shrutika Rathi, The new integral transform Soham transform for system of differential equations, International Journal of Advances in Engineering and Management, Vol. 4, Issue 5 , May 2022, PP. 1675- 1678.

[19.] D. P. Patil, Shweta Vispute and Gauri Jadhav, Applications of Emad-Sara transform for general solution of telegraph equation, International Advanced Research Journal in Science , Engineering and Technology, Vol. 9, Issue 6, June2022, pp. 127-132.

[20.] D. P. Patil, K. S. Kandakar and T. V. Zankar, Application of general integral transform of error function for evaluating improper integrals, International Journal of Advances in Engineering and Management, Vol. 4, Issue 6, June 2022.

[21.] Dinkar Patil, Prerana Thakare and Prajakta Patil, A double general integral transform for the solution of parabolic boundary value problems, International Advanced Research in Science, Engineering and Technology, Vol. 9, Issue 6, June 2022, pp. 82-90.

[22.] D. P. Patil, S. A. Patil and K. J. Patil, Newton's law of cooling by Emad- Falih transform, International Journal of Advances in Engineering and Management, Vol. 4, Issue 6, June 2022, pp. 1515-1519.

[23.] D. P. Patil, D. S. Shirsath and V. S. Gangurde, Application of Soham transform in Newton's law of cooling, International Journal of Research in Engineering and Science, Vol. 10, Issue 6, (2022) pp. 1299- 1303.

[24.] Dinkar Patil, Areen Fatema Shaikh, Neha More and Jaweria Shaikh, The HY integral transform for handling growth and Decay problems, Journal of Emerging Technology and Innovative Research, Vol. 9, Issue 6, June 2022, pp. f334-f 343.

[25.] Dinkar Patil, J. P. Gangurde, S. N. Wagh, T. P. Bachhav, Applications of the HY transform for Newton's law of cooling, International Journal of

Research and Analytical Reviews, Vol. 9, Issue 2, June 2022, pp. 740-745.
[26.] D. P. Patil, Sonal Borse and Darshana Kapadi, Applications of Emad-Falih transform for general solution of telegraph equation, International Journal of Advanced Research in Science, Engineering and Technology, Vol. 9, Issue 6, June 2022, pp. 19450-19454.
[27.] Dinkar P. Patil, Divya S. Patil and Kanchan S. Malunjkar, New integral transform, " Double Kushare transform" , IRE Journals, Vol.6, Issue 1, July 2022, pp. 45-52.
[28.] Dinkar P. Patil, Priti R. Pardeshi, Rizwana A. R. Shaikh and Harshali M. Deshmukh, Applications of Emad Sara transform in handling population growth and decay problems, International Journal of Creative Research Thoughts, Vol. 10, Issue 7, July 2022, pp. a137-a141.
[29.] D. P. Patil, B. S. Patel and P. S. Khelukar, Applications of Alenzi transform for handling exponential growth and decay problems, International Journal of Research in Engineering and Science, Vol. 10, Issue 7, July 2022, pp. 158-162.
[30.] D. P. Patil, A. N. Wani and P. D. Thete, Solutions of Growth Decay Problems by "Emad-Falih Transform", International Journal of Innovative Science and Research Technology, Vol. 7, Issue 7, July 2022, pp. 196-201.
[31.] Dinkar P. Patil, Vibhavari J. Nikam, Pranjal S. Wagh and Ashwini A. Jaware, Kushare transform of error functions in evaluating improper integrals, International Journal of Emerging Trends and Technology in Computer Science, Vol. 11, Issue 4, July-Aug 2022, pp. 33-38.
[32.] Dinkar P. Patil, Priyanka S. Wagh, Pratiksha Wagh, Applications of Kushare Transform in Chemical Sciences, International Journal of Science, Engineering and Technology, 2022, Vol 10, Issue 3.
[33.] Dinkar P. Patil, Prinka S. Wagh, Pratiksha Wagh, Applications of Soham Transform in Chemical Sciences, International Journal of Science, Engineering and Technology, 2022, Vol 10, Issue 3, pp. 1-5.
[34.] Dinkar P. Patil, Saloni K. Malpani, Prachi N. Shinde, Convolution theorem for Kushare transform and applications in convolution type Volterra integral equations of first kind, International Journal of Scientific Development and Research, Vol. 7 , Issue 7, July 2022, pp. 262-267.
[35.] Dinkar Patil and Nikhil Raundal, Applications of double general integral transform for solving boundary value problems in partial differential equations, International Advanced Research Journal in Science, Engineering and Technology, Vol. 9, Issue 6, June 2022, pp. 735-739.
[36.] D. P. Patil, M. S. Derle and N. K. Rahane, On generalized Double rangaig integral transform and applications, Stochastic Modeling and Applications, Vol. 26, No.3, January to June special issue 2022 part-8, pp. 533- 545.
[37.] D. P. Patil, P. S. Nikam and P. D. Shinde; Kushare transform in solving Faltung type Volterra Integro-Differential equation of first kind , International Advanced Research Journal in Science, Engineering and Technology, vol. 8, Issue 10, Oct. 2022,
[38.] D. P. Patil, K. S. Kandekar and T. V. Zankar; Application of new general integral transform for solving Abel's integral equations, International Journal of All Research Education and Scientific method, vol. 10, Issue 11, Nov.2022, pp. 1477-1487.
[39.] Dinkar P. Patil, Priti R. Pardeshi and Rizwana A. R. Shaikh, Applications of Kharrat Toma Transform in Handling Population Growth and Decay Problems, Journal of Emerging Technologies and Innavative Research, Vol. 9, Issue 11, November 2022, pp. f179-f187.
[40.] Dinkar P. Patil, Pranjal S. Wagh, Ashwini A. Jaware and Vibhavari J. Nikam; Evaluation of integrals containing Bessel's functions using Kushare transform, International Journal of Emerging Trends and Technology in Computer Science, Vol. 11, Issue 6, November- December 2022, pp. 23-28.
[41.] Dinkar P. Patil, Prerana D. Thakare and Prajakta R. Patil, General Integral Transform for the Solution of Models in Health Sciences, International Journal of Innovative Science and Research Technology, Vol. 7 , Issue 12, December 2022, pp. 1177- 1183.
[42.] Dinkar P. Patil, Shrutika D. Rathi and Shweta D. Rathi; Soham Transform for Analysis of Impulsive Response of Mechanical and Electrical Oscillators, International Journal of All Research Education and Scientific Method, Vol. 11, Issue 1, January 2023, pp. 13-20.
[43.] D. P. Patil, K. J. Patil and S. A. Patil; Applications of Karry-Kalim-Adnan Transformation(KKAT) in Growth and Decay Problems, International Journal of Innovative Research in Technology, Vol. 9, Issue 7, December 2022, pp. 437- 442.