Do probabilities and Statistics belong to Physics or Mathematics

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Abstract:- In general, probability and statistics are a missing part of mathematics and belong to physics rather than mathematics.

In previous papers we have shown that the solution of partial differential equations such as Laplace's and Poisson's PDE with Dirichlet boundary conditions and the time-dependent heat equation in its most general form can be solved via the physical chains of matrix B.

Moreover, we have also shown that numerical statistical integration and differentiation can be performed via the same statistical numerical method called Cairo technique.

In this paper, we extend the above work and apply the B matrix to rigorously derive the well-known normal/Gaussian statistical distribution.

In other words, we show that the law of Normal/Gaussian Distribution belongs to physics rather than mathematics.

We present numerical results for two arbitrary special cases without loss of generality.

The processing and precision of the numerical results of the new unconventional technique for the derivation and application process are striking, and its robustness is beyond doubt.

We also clarify that although the classical Gaussian mathematical law and the proposed Gaussian physical statistical law have exactly the same formula, they are completely different in their nature and concepts and therefore in the way of sampling judgment and applications.

I. INTRODUCTION

Many people think that mathematics already contains everything, but the opposite is also true. Statistical integration, statistical differentiation and statistical solution of time-dependent partial differential equations is a missing part of mathematics.

Throughout this article, we discuss the derivation and explanation of the well-known Gaussian distribution curve as another gray area in mathematics.

There are always long debates between scientists about what happens at the intersection between theoretical physics and mathematics when they meet. Here we should care more about how it works since we expect universal laws of physics and mathematical axioms to apply simultaneously.

Obviously, mathematical numerical integration and differentiation and physical statistical integration and differentiation in addition to normal/Gaussian distribution law are central objects in their intersection.

If there is a claim that mathematics is the language of physics, then the reverse is also true, physics can be the language of mathematics as in numerical differentiation and integration.

Moreover. It is important to understand that mathematics is only a tool to quantitatively describe physical phenomena, it cannot replace physical understanding.

In classical physics, mathematics, and quantum mechanics itself, time is currently understood as an external controller of all events and unwoven into 3D space to form an inseparable 4D construct block. In other words, classical time is absolute time and proper times are defined by a classical separable spacetime metric.

Perhaps this is the reason for their incompleteness in physics and mathematics.

The mathematical derivation of the classical Gaussian distribution law [1,2] is based on random sample measurements and an unexplained forced shape of the bell-shaped curve, i.e. the presupposed shape of Y(x)=const 1.Exp-(const 2.x^2).

In other words, the probability density distribution P(x) according to the superimposed semi-imperial formula:

P(x)=C1 Exp -[C2 *(x-Mue)^2].. . . . (1)

Where Mue is the median and the mean of the curve.

The numerical values of C1 and C2 could be found from the probability normalization condition, i.e. area under the curve=1.

The fundamental reason why the explanation of the effective statistical derivation of the Gaussian distribution law is absent in mathematics is that mathematics itself fails to find an adequate definition of probability and therefore appropriate statistical formulas which are essentially a probability distribution for different physical and mathematical situations.

The correct definition of transition probability must relate to time in a collective system of transition probability.

Therefore, we assume that the proper derivation of the normal/Gaussian distribution without the prior assumption of the Exp -x^2 relationship can be performed via the

ISSN No:-2456-2165

transition matrix chains as in the case of the B matrix chains in the technique from Cairo.

In other words, we should apply the statistical physical matrix B[3,4] to derive the normal/Gaussian distribution curve without the need for FDM finite difference mathematical methods or a prior assumption of the relationship Exp -x ^2.

It is extremely important to clarify that although the classical Gaussian mathematical law and the proposed unconventional Gaussian physical statistical law have exactly the same formula, they are completely different in their nature concepts and therefore in the way of sampling applications and analysis.

Briefly, the well-known classical normal/Gaussian statistical distribution law Y(x)=C1 Exp -C2 (x-Mue)^2 can be derived numerically from the statistical transition matrix of the Cairo technique.

This is the subject of this article.

II. THEORY

In fact, the mathematics itself already presents many vague approaches based on classical FDM and the prior assumption of the Exp -x^2 relationship, for the derivation and applications of the normal/Gaussian NDC distribution curve. [1,2].

Here we assume that NDC is a law forced spontaneously by nature and can be deduced by statistics from nature rather than by mathematics and the error function of integration.

In other words, the question arises whether the methods of statistical mathematical numerical technique such as the Cairo technique can successfully replace classical mathematics in a more general and efficient description.

The transition matrix B nxn which is successfully applied in the derivation of numerical integration formulas [3] is well defined by the following conditions [4,5]:

For 3D Cartesian coordinates, the inputs B i, j respector are subject to the following conditions:

- B i, j = 1/6 for i adjacent to j . . and B i, j = 0 otherwise.
  equal prior probability of all directions in space, i.e. no preferred direction.
- B i, i = RO, ie the main diagonal consists of constant entries RO .
  RO can take any value in the closed interval [0,1] .
  which corresponds to the assumption of similar residue after each time step for all free nodes.
- B i, j = B j, i, for all i, j.
  The matrix B is symmetric to conform to the symmetry of nature and detailed physical principles of reciprocity and balance.
- The sum of B i, j = 1 for all rows (or columns) away from the borders and the sum B i, j < 1 for all lines connected to the limits.

Which means that the probability of the whole space= 1

Obviously, the statistical matrix B is very different from the mathematical Laplacian matrix A and the mathematical matrix of Markov transition probability.

The physical nature of B is clear and briefly explained above through conditions i to iv which support the hypothesis of being the statistical transition matrix of nature.

However, the transition matrix B[3,4,5] is an open matrix to take into account the boundary conditions and the source term S and is suitable for solving PDEs with boundary conditions and a source term as well as to statistical numerical integration.

On the other hand, for the case of the derivation of a stationary solution of closed systems such as NDC, we introduce the closed matrix Bc.

The closed matrix that we call Bc is the same as the B matrix except that it is closed in the sense that it describes an initial condition problem of an autonomous system that evolves spontaneously over time.

In other words, condition iv is replaced by:

- The sum of B i, j = 1 for all rows (or columns) away from the borders and the sum B i, j is also equal to 1 for all the lines connected to limits.

Here, the one-dimensional matrix Bcnxn will be given by,

Bc=

line1-RO -RO/2 ½-RO/2

line 2-½-RO/2 RO ½-RO/2 0

. . . .

. . . . ½-RO/2ROlinen--½-RO/2 . . . . . Eq (2)

The time dependent solution of the system described by Equation 1 which should give the time evolution of the normal distribution curve Y(x,t) is simply given by,

Y(x,t)=Bc ^N . . . . . . (3)

Where N is the number of iterations or time step dt.

Note that time t is dimensionless and given by N.

And the time-independent stationary solution of the system is obtained after a sufficiently long time (N) and must describe the time-independent normal distribution curve is simply given by,

Y(x)= Bc ^N . . . . (4)

ISSN No:-2456-2165

For N large enough.

Note that Y(x,t) and Y(x) are vectors while Bc^N is a matrix. The gap is removed when it is known that equations (3 and 4) apply to RHS or LHS diagonals for N odd or even respectively.

That is, Y=LHS or RHS diagonal of Bc^N and not Bc^N itself.

The reason is that these two diagonals follow the probability density time-dependent scattering equation, namely,

dP/dt)partial = D . Nabla^2 P . . . . (5)

In order not to worry too much about the details of thetheory, let's move directly to illustrative applications.

III. APPLICATIONS AND NUMERICAL RESULTS

- *Case A: 11x11 Bc matrix*

Formulation of Equation 2 leads to the following expression for Bc when RO = 0 That is, when Bc is a zero main diagonal matrix,

Line1- 0 1/2 0 0 0 0 0 0 0 0 1/2
Line2 - 1/2 0 1/2 0 0 0 0 0 0 0 0
Line3 - 0 1/2 0 1/2 0 0 0 0 0 0 0
Line4 - 0 0 1/2 0 1/2 0 0 0 0 0 0
Line5 - 0 0 0 1/2 0 0 0 0 0 0 0
Line6 - 0 0 0 0 1/2 0 1/2 0 0 0 0
Line7 - 0 0 0 0 0 1/2 0 1/2 0 0 0
Line8 - 0 0 0 0 0 0 1/2 0 1/2 0 0
Line9 - 0 0 0 0 0 0 0 1/2 0 1/2 0
Line10-0 0 0 0 0 0 0 0 1/2 0 1/2
Line11-1/2 0 0 0 0 0 0 0 0 1/2 0

and the destination matrix of Bc(11x11)^N for N=10 is given by Bc ^10 is expressed as follows:
line1 0.246093750 9.76562500E-04 0.205078125 9.76562500E-03 0.117187500 4.39453125E-02 4.39453125E-02 0.117187500 9.76562500E-03 0.205078125 9.76562500E-04

line2 9.76562500E-04 0.246093750 9.76562500E-04 0.205078125 9.76562500E-03 0.117187500 4.39453125E-02 4.39453125E-02 0.117187500 9.76562500E-03 0.205078125

line3 0.205078125 9.76562500E-04 0.246093750 9.76562500E-04 0.205078125 9.76562500E-03 0.117187500 4.39453125E-02 4.39453125E-02 0.117187500 9.76562500E-0

line4 9.76562500E-03 0.205078125 9.76562500E-04 0.246093750 9.76562500E-04 0.205078125 9.76562500E-03 0.117187500 4.39453125E-02 4.39453125E-02 0.117187500

line5 0.117187500 9.76562500E-03 0.205078125 9.76562500E-04 0.246093750 9.76562500E-04 0.205078125 9.76562500E-03 0.117187500 4.39453125E-02 4.39453125E-0

line6 4.39453125E-02 0.117187500 9.76562500E-03 0.205078125 9.76562500E-04 0.246093750 9.76562500E-04 0.205078125 9.76562500E-03 0.117187500 4.39453125E-02

lime7 4.39453125E-02 4.39453125E-02 0.117187500 9.76562500E-03 0.205078125 9.76562500E-04 0.246093750 9.76562500E-04 0.205078125 9.76562500E-03 0.117187500

line8 0.117187500 4.39453125E-024.39453125E-02 0.117187500 9.76562500E-03 0.205078125 9.76562500E-04 0.246093750 9.76562500E-04 0.205078125 9.76562500E-03

line9 9.76562500E-03 0.117187500 4.39453125E-02 4.39453125E-02 0.117187500 9.76562500E-03 0.205078125 9.76562500E-04 0.246093750 9.76562500E-04 0.205078125

line10 0.205078125 9.76562500E-03 0.117187500 4.39453125E-02 4.39453125E-02 0.117187500 9.76562500E-03 0.205078125 9.76562500E-04 0.246093750 9.76562500E-0

line11 9.76562500E-04 0.205078125 9.76562500E-03 0.117187500 4.39453125E-02 4.39453125E-02 0.117187500 9.76562500E-03 0.205078125 9.76562500E-04 0.246093750

Notice that the LHS diagonal 11 elements are,

ISSN No:-2456-2165

9.76562500E-04 9.76562500E-034.39453125E-02 0.117187500 0.2050781250.246093750 0.2050781250.117187500 4.39453125E-02 9.76562500E-03 9.76562500E-04 . . . . (Distribution 1)

We call the 11 elements of the LHS diagonal that of the Distribution emerging from the Bc 11x11 matrix (Distribution 1).

- Distribution 1 is the required unconventional numerical Gaussian distribution law that corresponds to the classical Gaussian distribution curve,

  Y(x)=Exp-0.5{(Mue-x)/(σ) }^2/{(σ) . Sqrt(2 Pie)}. . . . . (6)

  for mean/median Mue=6 andnormalization factorA=1./Sqrt(2Pie) of 0.2461

- the total sum of the 11 elements of the left diagonal (Distribution 1) which represents the area under the normal/Gaussian distribution curve is equal to 1.0000000, which is surprisingly accurate.
- The value of the normalization constant A in the proposed unconventional Gaussian law for Bc 11x11 is 0.2461 which corresponds precisely to the value of 1 /SQRT(2 Pie) in the classic Gaussian law.

To further test the correctness of Distribution 1, compare its numerical values with the classical Gaussian distribution law [6] in the simplified form:

Y(x)=C1 Exp-C2*(x-6)^2 . . . . . (7)

Table I presents Equation 7 which is the simple form of the classical Gaussian distribution versus distribution 1 which is the proposed unconventional Gaussian distribution. In equation 7 of the classical Gaussian law, we substitute the median/mean Mue of 6. We substitute C1=0.2461 and C2=0.185 as an exponential fit which corresponds to 1/(σ)^2 explained in section IV, (Disccussion) .

X	Y(x) Classical gaussian	Y(x) Distribution1 unconventional Gaussian
x = 1	2.41267937E-03	9.76562500E-04
x = 2	1.27526345E-02	9.76562500E-03
x =3	2.41267937E-03	4.39453125E-02
x =4	0.117417730	0.117187500
x =5	0.204534754	0.205078125
x = 6	0.246099994	0.246093750
x =7	0.204534754	0.20507813
x =8	0.1174177	0.117187500
x =9	4.65598218E-02	4.39453125E-02
x =10	1.27526345E-02	9.76562500E-03
x =11	2.41267937E-03	9.76562500E-04 .

Table 1: Classical Gaussian distribution compared to the proposed results of the Bc matrix for n = 11.

Note that x-6 is the distance to the median/mean.

Table I shows that the proposed unconventional Gaussian distribution (distribution 1) is in good agreement with the classical Gaussian distribution.

Moreover, the value of the maximum of the bell curve (0.246099994) is in excellent condition with the Gaussian normalization factor 1/Sqrt(2 . Pie).

- *Case B :15x15 Bc matrix*

Again, equation 1 is used to define the elements of the matrix Bc for RO = 0 in the same way as in case A, except that the number of columns and rows is 15 instead of 11.

ISSN No:-2456-2165

And the destination matrix Bc^10 is given by, Matrix Bc(15X15)^N when N = 10 is expressed as,

1 0.24609375 0.000000 0.2050781 0.00000 0.1171875 9.765625E-4 4.394531E-2 9.765625E-3 9.765625E-3 4.394531E-2 9.765625E-4 0.11718750 0.00000 0.2050783 0.00000

2 0.0000 0.2460938 0 .0000 0.20507813 0.00000 0.117187500 9.765625E-4 4.3945313E-2 9.765625E-3 9.765625E-3 4.3945313E-2 9.765625E-4 0.1171875 0.00000 0.20507813

3 0.20507813 0.000000 0.2460938 0.00000 0.20507813 0.00000 0.1171875 9.765625E-4 4.3945313E-2 9.765625E-3 9.765625E-3 4.3945313E-2 9.765625E-4 0.1171875 0.0000

4 0.00000 0.20507813 0.00000 0.246093750 0.00000 0.20507813 0.00000 0.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.39453125E-2 9.7656250E-4 0.11718750

5 0.11718750 0.000000 0.20507813 0.00000 0.2460938 0.00000 0.20507813 0.00000 0.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.3945313E-2 9.7656250E-4

6 9.7656250E-4 0.11718750 0.00000 0.20507813 0.00000 0.246093750 0.000000 0.205078125 0.00000 0.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.3945313E-02

7 4.3945313E-2 9.7656250E-4 .11718750 .00000 .20507813 0.00000 .246093750 0.00000 .20507813 0.00000 0.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3

8 9.7656250E-3 4.3945313E-2 9.7656250E-4 .11718750 0.00000 0.20507813 0.00000 .246093750 0.00000 0.20507813 0.00000 0.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3

9 9.7656250E-3 9.7656250E-3 4.3945313E-2 9.7656250E-4 0.117187500.00000 0.20507813 0.0000 0.246093750 0.00000 .20507813 0.00000 0.11718750 9.7656250E-4 4.39453125E-2

10 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.3945313E-2 9.7656250E-4 .11718750 0.00000 0.20507813 0.00000 .246093750 0.00000 0.20507813 0.00000 0.11718750 9.7656250E-4

11 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.3945313E-2 9.7656250E-4 0.117187500.00000 0.20507813 0.00000 0.246093750 0.00000 0.20507813 0.00000 0.11718750

120.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.3945313E-2 9.7656250E-4 0.11718750 0.000000 0.20507813 0.00000 0.246093750 0.00000 0.20507813 0.00000

13 0.00000 0.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.39453125E-2 9.7656250E-4 0.11718750 0.00000 0.20507813 0.0000 0.246093750 0.00000 0.20507813

14 0.20507813 0.0000 0.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.3945313E-2 9.7656250E-4 0.11718750 0.00000 0.20507813 0.00000 .246093750 0.00000

15 0.0000 0.20507813 0.00000 0.11718750 9.7656250E-4 4.3945313E-2 9.7656250E-3 9.7656250E-3 4.3945313E-2 9.7656250E-4 0.117187500.00000 0.20507813 0.00000 0.24609375

We follow a procedure similar to that of case A.

Again, Table II presents Equation 7 which is the simplified form of a classical Gaussian distribution versus distribution 2 (LHS diagonal of Bc^10 matrix) which is the proposed unconventional Gaussian distribution. In equation 7 of the classical Gaussian law, we substitute the median/mean Mue of 8.

We substitute C1=0.2461 and C2=0.185 as an exponential fit that corresponds to 1/sgma^2 .

Table II

Equation 7	Proposed distribution 2
x = 12.84598027E-05	0.0000
x = 23.15290148E-04	0.0000
x =32.41267937E-03	9.7656250E-4
x =41.27526345E-02	9.7656250E-3

ISSN No:-2456-2165

x =5 4.65598218E-02	4.3945313E-2
x = 60.117417730	0.11718750
x =70.204534754	0 20507813
x =80.246099994	0.246093750
x =9 0.204534754	0.20507813
x =100.117417730	0.11718750
x =114.65598218E-02	4.3945313E-2
x =121.27526345E-02	9.7656250E-3
x =132.41267937E-03	9.765625E-4
x =143.15290148E-04	0.00000
x =152.84598027E-05	0.00000

Table 2: Classical Gaussian distribution compared to the proposed results of the Bc matrix for n = 15.

Note again that,

- Distribution 2 is the required unconventional numerical Gaussian distribution law that corresponds to the classical Gaussian distribution curve.

  Y(x)= Exp-0.5{(Mue-x)/(σ)}^2/ {(σ).Sqrt (2 Pie)} . . . . (6)

  OR,

  Y(x)=C1 Exp-C2*(x-8) ^2 . . . . . (7)

  for mean/median Mue= 8 and normalization factor A=1./Sqrt(2Pie) of 0.2461

- the total sum of the 15 elements of the left diagonal (Distribution 2) which represents the area under the normal/Gaussian distribution curve is equal to 1.0000000, which is surprisingly accurate.
- The value of the normalization constant A in the unconventional Gaussian law proposed for any B 15x15 is still 0.2461 which corresponds precisely to the value of 1 /SQRT(2 Pie) in the classic Gaussian law.

IV. DISCUSSION

It has been shown in previous papers that the B-matrix chain technique is able to process and derive adequate mathematical formulas for different physical and mathematical situations [3,4,5,6], which validates the hypothesis that ***matrix channel B is nature's matrix chain or how nature's energy fields live and function in 4D x-t space.***

In science and medicine, what makes a result reliable enough to be taken seriously is its statistical significance, but also judgments about the norms that make sense in a given situation.

Note that the standard deviation is a measure of how unusual a data set is if an assumption is true. Physicists express standard deviations in units called sigma, σ. The higher the sigma number, the more inconsistent the data with the hypothesis.

In short The unit of measurement of the spread of the statistical curve or of its statistical significance is the standard deviation, expressed by the lowercase Greek letter sigma (σ)[2].

It therefore remains here how to find the corresponding sigma standard deviation and the normalization constant A from the distribution formula 1 or 2 which we explain briefly here in order not to weigh down the article.

In other words, what is the relationship or how to calculate A and sigma from C1 and C2.

It is simple and we briefly give the relevant derived equations by comparing equations 6 and 7.

We conclude the first sigma of the relationship,
sigma=2. C2
then find A via the relation,
A=1/ sigma.Sqrt(2 Pie)
that is to say .
A=0.3989424/sigma

V. CONCLUSION

In general, probability and statistics are a missing part of mathematics and belong to physics rather than mathematics.

In previous papers we have shown that the solution of partial differential equations such as Laplace's and Poisson's PDE with Dirichlet boundary conditions and the time-dependent heat equation in its most general form can be solved via the physical chains of matrix B.

Moreover, we have also shown that numerical statistical integration and differentiation can be performed via the same statistical numerical method called Cairo technique.

Throughout this paper, we have explained the derivation and applications of the proposed unconventional normal distribution and compared our numerical results with those of the well-known classical Gaussian distribution curve through two arbitrarily chosen special cases without loss of generality.

The agreement between the two formulas is striking and confirms the validation of the new technique.

In other words, the B matrix chain technique is able to process and derive adequate mathematical formulas for different physical and mathematical situations, which validates the hypothesis that

Matrix B channels are nature's matrix channels or how nature's energy fields live and function in 4D x-t space.

NB. All calculations in this article were produced using the author's double-precision algorithm to ensure maximum accuracy, as follows by ref. 7 for example.

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