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Solving the Problem of the Cosmological Constant and Vacuum Energy Density by Considering the Cosmic Quantum Number

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Abstract:- In this paper, it is found that the ratio between the theoretically value of the cosmological constant (and vacuum energy density) at the beginning of time, and the value determined by experiment in our present time, which takes an unjustified large quantity, is resulted from the presence of a missing link to solve the problem that is not included in its data. This missing link is the cosmic quantum number N , or maximal universal number which is obtained by Ibrahim. Therefore, the number N is taken as a correction factor for this ratio to be equal to unity, as the cosmological constant and vacuum energy density are scale independent.

Keywords:- *Quantum Gravity, Quantum Cosmology, Cosmological Constant, Vacuum Energy Density, Maximal Universal Number.*

I. INTRODUCTION

The current unsolved problems in cosmology stem from the "lack of a complete, convincing theory of quantum gravity"[1] that combines general relativity with quantum theory as a precursor to building the foundations of a unified theory. We laid down in a previous paper the foundations and principles necessary for such an expected construction [2], so it can be relied upon to solve these problems. Quantum theory is applied in understanding behavior of systems in our microscopic universe, while general relativity theory succeeds in describing physical structures in our macroscopic universe.

It is believed that there is a corresponding point at the beginning of time at which the microscopic and macroscopic systems coincide and where the foundations and principles of the unified theory can be essentially applied. Our macroscopic universe is originally governed by the mass-radius relation $Mr_c = m_c R$, which determines the condition of its equilibrium [2], where (M, R) the mass and radius of our macroscopic universe at present, (m_c, r_c) its mass and radius at the beginning of time. The macroscopic universe is still described by the same formulae that worked at the beginning of time, so its current state is a reflection of its quantum nature, then we can describe it in terms of fundamental units [2,3], and thus the laws of nature become independent on scale, but rather work similarly to the universe at beginning of time and at present.

Einstein added the cosmological constant Λ to his equations in general relativity in an attempt to obtain a static universe, but when it became clear to him - later - that our universe is expanding, he abandoned the cosmological constant and got rid of it in his theory. Elementary particle physicists and cosmologists have found that this constant is related to the minimum energy of the vacuum, whose value was very large at the beginning of time[4,5,6], but the cosmological constant Λ in present is very small and negligible. So there is an apparent contradiction in this case.This contradiction is known in scientific publications as the cosmological constant problem.

Several models have been proposed to solve this problem[4,5,6,7]. Some models, such as the vacuum decay models[8,9,10], consider that this constant diminishes in value with time, while in other models this discrepancy is attributed to the energy density of some scalar fields that prevailed in our universe at the beginning of time, when they then have some properties that give the current value observed. Physicists have looked for some symmetries that would cause this constant to zero. The constant is canceled in some models for supersymmetries, but its value does not become zero when the symmetry is broken to give the current state of our universe [7], which means that those models fail to find a decisive solution to the issue.

This problem can be easily resolved. It has become clear to us by calculation - in this paper - that the ratio between the theoretical value of the cosmic constant (or the vacuum energy density) at the beginning of time, and the value determined by experiment in present is equal to 10^{122} and not of the order 10^{120} as it is estimated in reference [13]. Since the two constants are independent on scales, then it can be assumed that there is a numerical parameter as a missing link to solve this problem that has not been seen before when calculating the two constants, it is taken as a factor to correct the ratio 10^{122} to be equal to unity. It is found that this factor is exactly what is known as the cosmic quantum number N that is obtained by Ibrahim [2].

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II. THE COSMOLOGICAL CONSTANT AND VACUUM ENERGY DENSITY

A distinctive feature of general relativity is that the energy-momentum tensor is entirely the source of the gravitational field. For other fields, only changes in energy can be measured from one state to another, and energy evaluation is then arbitrary. For example, the motion of a particle is its potential energy (x) , is like the motion of a particle of a potential energy $V(x) + V_0$, with respect to any constant. In the case of gravitation, the real value of energy is not limited to the difference between states only.

This behavior allows us to add the concept of vacuum energy[11], which is an energy density specific to empty space. One of the important features of space is that it is isotropic ,that is looks the same in all directions , that is, there is no preferred direction in it. It will then be possible to have a non-zero energy density of the vacuum ρ_v even if the energy-momentum tensor is an invariant quantity (Lorentz's invariance).

The Lorentz invariance of the local inertial coordinates requires that the corresponding energy-momentum tensor be proportional to the tensor $\eta_{\tilde{\mu}\tilde{\nu}}$ in the following form:

$$T^{(v)}_{\tilde{\mu}\tilde{\nu}} = \rho_v \eta_{\tilde{\mu}\tilde{\nu}} \quad (1)$$

Because $\eta_{\tilde{\mu}\tilde{\nu}}$ is only Lorentz's invariant tensor ,then relation (1) can be generalized directly from inertial to arbitrary coordinates as follows:

$$T^{(v)}_{\mu\nu} = \rho_v g_{\mu\nu} \quad (2)$$

Comparing energy-momentum tensor for a perfect fluid,we write:

$$T_{\mu\nu} = (\rho + p)V_\mu V_\nu + p g_{\mu\nu} \quad (2a)$$

We find that the vacuum appears to be analogous to a perfect fluid with an isotropic pressure whose sign is the opposite of that of the energy density.

$$p_v = -\rho_v \quad (3)$$

The energy density must be a constant in space-time, otherwise the gradient would be variable.

If we recombine the energy-momentum tensor with the matter tensor $T^{(m)}_{\mu\nu}$and the vacuum tensor $T^{(v)}_{\mu\nu} = -\rho_v g_{\mu\nu}$, Einstein's equation becomes as the following

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G\left(T^{(m)}_{\mu\nu} - \rho_v g_{\mu\nu}\right) \quad (4)$$

Einstein tried to obtain a static cosmic model, so to solve his equations for that, he needed to add a new term as follows [12,13]:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (5)$$

Comparing with (4) we find that adding the cosmological constant is exactly equivalent to adding vacuum energy density:

$$\rho_v = \frac{\Lambda}{8\pi G} \quad (6)$$

So, the concepts of the cosmological constant and vacuum energy are basically two concepts that can replace one of the other[11].

Is nonzero vacuum energy something to be expected? we have come across Hilbert's Lagrangian $(\hat{L}_H = R\ )$ function by searching for the simplest scalar quantity that can be formed from the metric tensor. then by using the identity$(\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\ g_{\mu\nu}\delta g^{\mu\nu})$, it can be clearly verified that the scalar quantity S defined by the relation:

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{8\pi G}(R - 2\Lambda) + \hat{L}_H\right] \quad (7)$$

leading to the developed equation(5), instead the Lagrange function for vacuum is simply:

$$\hat{L}_v = -\rho_v \quad (8)$$

So it is easy to provide the vacuum energy, but there is no idea of its expected value because it is entered as an arbitrary constant.

The vacuum energy is actually a constant, but there is an exception in some theories, where some space-time symmetries such as supersymmetry govern the value of the vacuum energy, here we consider a more general field theory. However, there are several distinct contributions to vacuum energy, and the sum of these is much less than the individual contributions. One such contribution comes from zero-point perturbations, which are the energies of quantum fields in their vacuum states.

Consider a simple harmonic oscillator, a particle moving in a one-dimensional space, where the potential energy is $(x) = \frac{1}{2}\omega^2 x^2$: From the classical point of view, the vacuum of this system represents the state in which the particle is at rest and at the lowest potential energy $(x = 0)$, at which point its energy vanishes. As for the quantum point of view, the principle of uncertainty prevents determining the position of the particle and its momentum with infinite accuracy, and we find that the minimum state of the system is $E_0 = \frac{1}{2}\hbar\omega$: And the system has, in the absence of

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gravitation, a completely arbitrary vacuum energy. We can add any constant to the potential energy without any change to the theory, but quantum perturbations change the zero point energy from its expected value.

Another similar case applies to field theory. If we take the Fourier transformation of a free quantum field (for simplicity, mutual interactions can be neglected), we find that it becomes an infinite number of harmonic oscillators in momentum space[14]. The frequency of any oscillator is $\omega = \sqrt{m^2 + k^2}$, where m is the mass of the field, k is the absolute value of the wave vector . If we take the vacuum energy - classically - equal to zero, then any of these forms contribute zero point energy $\frac{1}{2}\hbar\omega$. When all these contributions are added together, we get an infinite result. But if we discard all contributions to the forms of high momentum and suffice with the ultraviolet cutoff, we find that the resulting energy density is in the following form [14] :

$$\rho_v \sim \hbar k_{max}^4 \tag{9}$$

This answer can also be estimated using the dimensional analysis method. The numerical constants that does not obtain in this way depend on a more precise theory. If normal quantum field theory can be used up to the reduced Planck's scale: $\overline{m}_p = (8\pi G)^{-\frac{1}{2}} \sim 10^{18} GeV$, we expect the contribution to be of the order:

$$\rho_v \sim (10^{18} GeV)^4 \sim 10^{112} erg/cm^3 \tag{10}$$

Cosmological observations require that:

$$\left|\rho_\Lambda^{(obs)}\right| \leq (10^{-12} GeV)^4 \sim 10^{-8} erg/cm^3 \tag{11}$$

It is much smaller than the expected value extracted above.

The ratio between (10)and (11)is the origin of the contradiction between the theoretical and experimental values of the cosmological constant[14].

$$\frac{\rho_v}{\left|\rho_\Lambda^{(obs)}\right|} \sim 10^{120} \tag{12}$$

This expresses a very large apparent difference between theoretical predictions and experimental observations.

III. SOLUTION OF THE PROBLEM

Relation(12) basically expresses the ratio between the value of the energy density ρ_c^v at Planck's time , and its value ρ^v in our present time, so we write:

$$\rho_c^v = \frac{m_c c^2}{r_c^3} \tag{13}$$

$$\rho^v = \frac{M c^2}{R^3} \tag{13a}$$

Where (m_c, r_c) the mass and radius of our universe at the beginning of time , (M, R) its mass and radius in our present time.

We write the ratio (12) using (13)and(13a) as follows:

$$\frac{\rho_c^v}{\rho^v} = \frac{m_c R^3}{M r_c^3} \tag{14}$$

But:

$$\frac{m_c R}{M r_c} = 1 \tag{15}$$

It is the mass-radius relation, i.e. the equilibrium condition that controls our macroscopic universe [2,3].

So relation (14) becomes:

$$\frac{\rho_c^v}{\rho^v} = \frac{R^2}{r_c^2} = \frac{\Lambda_c}{\Lambda} \tag{16}$$

Where Λ_c the cosmological constant at the beginning of time for our universe, Λ the cosmological constant in our present time.

Relation (16) can be written as:

$$\frac{\rho_c^v}{\rho^v} = \frac{\Lambda_c}{\Lambda} = N^2 = 10^{122} \tag{17}$$

Whereas $N = 10^{61}$ is Ibrahim's number [2,3], which expresses the "maximal universal number", or the "the cosmic quantum number". This number forms the basis of an important cosmic principle "the principle of cosmic equilibrium" or the cosmic equilibrium condition, whereby any of maximal universal quantities $(X, Y, \dots)$, and themselves of minimal universal quantities $(x, y, \dots)$, are linked by a simple relation as follows: $\frac{X}{x} = \frac{Y}{y} = \cdots = N$ Thus, a compact bridge is established between quantum theory and cosmology, It is then confirmed that there is a

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close link between the macroscopic universe and the microscopic universe.

There is previous work[15,16,17] to find such a "larger" number, by dividing the radius of a large celestial body by the radius of a small quantum particle, and those numbers are different from N $(10^{42}, 10^{60}, 3 \times 10^{60})$

The ratio(17) is a hundred times greater than the ratio(12), and more specific and accurate than it, since according to relation (11) it is possible to take:

$$\rho^{\upsilon} < 10^{-8}\ erg\ /cm^3 \qquad (18)$$

Also, relation(10) is as follows:

$$\rho_c^{\upsilon} > 10^{112} erg/cm^3 \qquad (19)$$

This is if we consider the fundamental unified energy[2] $\sim 10^{19} GeV$ instead of Planck's energy $\sim 10^{18} GeV$ from which the expected value in the relation has been estimated.

The energy density of the vacuum at Planck's time can also be expressed in the following form:

$$\rho_c^{\upsilon} = \Lambda_c (\frac{q_c}{r_c})^2 \qquad (20)$$

Its value at present is as follows:

$$\rho^{\upsilon} = \Lambda (\frac{Q}{R})^2 \qquad (21)$$

Where: q_c , the critical charge[2,3]. As for the quantity $Q = Nq_c$, it expresses the total amount of critical charges in the universe - just as the quantity expresses the total amount of critical masses in the universe $M = Nm_c$ - and we can call it the name: "cosmic charge" .The specific charge of the universe is a constant, and is related to Newton's constant of gravitation G, as follows:

$$\frac{Q}{M} = \frac{q_c}{m_c} = \sqrt{G} \qquad (21a)$$

We take in relation: $Q = Nq_c$,the absolute value of q_c because it is a result of square root of a quantity.

From (20)and(21) we find

$$\frac{\rho_c^{\upsilon}}{\rho^{\upsilon}} = \frac{\Lambda_c}{\Lambda} = N^2 \qquad (22)$$

Thus, the two relations(17)and(22) explain the origin of the apparent contradiction between the theoretical value of cosmological constant (or vacuum energy density) at Planck's time, and the value determined by experiment at present , as the ratio between them is equal to 10^{122} and is basically related to the cosmic quantum number N.

This association with the number N reveals the secret of the issue, and indicates the "missing link" to solve it. Therefore, for the constancy of the value Λ (or ρ^{υ}) over all stages of our macroscopic universe life, we can then redefine them so that they independent on scales.But first, we will define an important quantity that has a close relation with the cosmological constant and vacuum energy density, which is the length density $\lambda_{(n)}$ of the number n of particles with critical masses m_c in our macroscopic universe , so we write:

$$\lambda_{(n)} = \frac{n_c}{r_c} = \frac{N}{R} \qquad (23)$$

$N = 10^{61}$, Where $n_c = 1$

Then we define the cosmological constant by squaring $\lambda_{(n)}$, that is:

$$\Lambda = \frac{n_c^2}{r_c^2} = \frac{N^2}{R^2} \qquad (24)$$

This relation expresses the constancy of Λ from the beginning of time until our present time, and the ratio between its value in Planck's time and its current value is equal to unity.

$$\frac{n_c^2}{N^2}\frac{R^2}{r_c^2} = 1 \qquad (25)$$

This relation and (23) are equivalent to the condition(15) by which the universe is in equilibrium.

Also, the energy density of the vacuum is as follows:

$$\rho^{\upsilon} = \frac{n_c^2 m_c}{r_c^3} c^2 = \frac{N^2 M}{R^3} c^2 \qquad (26)$$

That is, the ratio between the two values at the beginning of time and in our present time becomes equal to unity.

Thus, it becomes clear that relation (11) gave a very small value to the energy density of the vacuum because the cosmic quantum number (Ibrahim's number N) was not taken into account, as it was not known before. So we write relation (11) in its correct form as follows:

$$\rho^{\upsilon} \sim N^2 (10^{-8} erg/cm^3) \qquad (27)$$

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Or we write

$$\rho^{v} \sim 10^{114} erg/cm^3 \quad (27a)$$

Thus, the ratio between ρ_c^v and ρ^v in this case is of the order of unity, according to the considerations that were determined by the two relations (18)and(19) .

Relation (26)can be written as:

$$\rho^{v} = \lambda_{(n)}^2 \frac{m_c}{r_c} c^2 = \lambda_{(n)}^2 \frac{M}{R} c^2 \quad (28)$$

$$\rho^{v} = \lambda_{(n)}^2 \lambda_{(m)} c^2 \quad (28a)$$

Where $\lambda_{(m)} = \frac{m_c}{r_c} = \frac{M}{R}$ is the length density of our macroscopic universe .

So then

$$\rho^{v} = \Lambda\, \lambda_{(m)} c^2 \quad (28b)$$

Or we write

$$\rho^{v} = \Lambda f_c = constant \quad (28c)$$

Relation (28c) links the cosmological constant Λ with the vacuum energy density ρ^v, and does not depend on scales. It is equivalent to relations (20) and(21), so indicates that "the concepts of the cosmological constant and vacuum energy are basically two concepts that can replace each other" [13].The quantity $f_c = \lambda_{(m)} c^2$ represents the fundamental unified force.

- *Note:- The Unit System is CGS for all Relations in this Paper*

IV. RESULTS DISCUSSION

It has been shown - in this paper - that the ratio of theoretical value of the cosmological constant (or the vacuum energy density) at the beginning of time, to the value determined by experiment is equal to 10^{122} which is related to the cosmic quantum number N , not of the order 10^{120} as it is estimated in reference[13]. The number N is taken as a correction factor for this ratio to be equal to unity, as the cosmic constant Λ and the vacuum energy density ρ^v are independent on scales. Thus, it becomes clear that the current state of our universe is a reflection of its quantum nature , then we can described it in terms of fundamental units (m_c, r_c, t_c) , then laws of nature of our macroscopic universe are independent on scales, but work similarly at the beginning of time and in our present time. In this paper, a new important physical quantity related to cosmology and elementary particles is defined, namely: cosmic charge $= Nq_c$, which expresses the total amount of critical charges q_c in the universe, as the quantity $M = Nm_c$ expresses the total amount of critical masses m_c in the universe.

V. CONCLUSION

Relation(28c) combined the cosmic constant Λ with vacuum energy density ρ^v, and it does not depend on scales, relation(11) gave a very small value to vacuum energy density because it did not take into account Ibrahim's number N that was used as a key to the solution in this research, and thus the origin of the apparent conflict between the theoretical value at Planck's time and the value determined by experiment was shown in this paper.

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