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A New Ratio Type Estimator for Double Sampling with Two Auxiliary Variables

Faweya Olanrewaju, Ajayi Esther, Akinyemi Oluwadare
Department of Statistics, Ekiti State University, Ado-Ekiti, Ekiti State, Nigeria

Abstract:- In this paper, we propose a new ratio type estimator for double sampling with two auxiliary variables. The expressions for bias and mean square error (MSE) of the proposed estimator have been obtained. Some realistic conditions have been obtained under which the proposed estimator is more efficient than the usual unbiased well-known existing estimators of double sampling using two auxiliary variables for population mean and it was found to be more efficient in many situations.

Keywords:- *Double Sampling; Ratio-Type Estimator; Bias; Mean Square Error (MSE); Auxiliary variable.*

I. INTRODUCTION

An important objective in any statistical estimation procedure is to obtain the estimators of parameters of interest with more precision. It is also well understood that incorporation of more information in the estimation procedure yields better estimator provided the information is valid and proper. Use of such auxiliary information is made through number of sampling procedures while ratio, product and regression method is used to obtain an improved estimator of population parameters such as the mean, total or ratio. In ratio technique of estimation, auxiliary information on a variable which is linearly related to the variable of interest is available and is utilized to estimate the population mean. The method of utilization of auxiliary information depends on the form in which it is available. In a situation where prior information of auxiliary variable is lacking, the appropriate technique used to get estimates of those auxiliary variables on the basis of samples is double sampling. In double sampling, the information on the auxiliary variable is collected from a large preliminary sample using simple random sampling without replacement, while the information on the variable of interest is collected from a second sample which is smaller in size than the preliminary sample size using simple random sampling without replacement.

In literatures, numbers of authors introduced many ratio, product and regression type estimators by using general linear transformation of the auxiliary variable. [15] considered the problem of estimating the population of the study variable using double sampling with a generalized chain estimator. Other authors such as [2], [3], [12], [13], [14],[11],[3],[8],[4],[9], have studied the problem of estimating population mean using two auxiliary variables under double sampling scheme.

In [5] it appears to be the first to use auxiliary information in ratio estimator when there is highly positive correlation between variable of interest and auxiliary variables. [7] were first to suggest the use of auxiliary information in selecting the population with varying probabilities. [10] gave the idea of product estimator when there is highly negative correlation.

Cochran (1940) appears to be the first to use auxiliary information in ratio estimator when there is highly positive correlation between variable of interest and auxiliary variables. Hansen and Hurwitz (1943) were first to suggest the use of auxiliary information in selecting the population with varying probabilities. Robson (1957) gave the idea of product estimator when there is highly negative correlation.

The Simple Random Sampling (SRS) estimator for a sample of size *n* drawn from a population of size *N* is defined as:

$$\bar{y}_0 = \frac{1}{n}\sum y_i = \bar{y} \qquad (1.11)$$

The mean square error is written as:

$$MSE(\bar{y}_0) = \theta S_y^2 \qquad (1.12)$$

where $\theta = \frac{n}{N}$ and $S_y^2 = \frac{1}{N-1}\sum_{i\in U}(y_i - \bar{Y})^2$

Cochran (1977) gave a classical ratio estimator based upon single auxiliary variable as given as:

$$\bar{y}_c = \hat{R}\overline{x}' \qquad (1.13)$$

where $\hat{R} = \frac{\bar{y}}{\bar{x}}$

The mean square error of $\bar{y}_C$ is given as:

$$MSE(\bar{y}_c) = \left[\theta_1 S_y^2 + (\theta_2 - \theta_1)\left(S_y^2 + \hat{R}^2 S_x^2 - 2\hat{R}S_{xy}\right)\right] \qquad (1.14)$$

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where $\theta_1 = \left(\frac{1}{n'} - \frac{1}{N}\right)$, $\theta_2 = \left(\frac{1}{n} - \frac{1}{N}\right)$

$$S_x^2 = \frac{1}{N-1}\sum_{i\in U}(x_i - \bar{X})^2 \quad and \quad S_{xy} = \frac{1}{N-1}\sum(x_i - \bar{X})(y_i - \bar{Y})$$

Comparing the efficiency of Cochran (1977) estimator with the mean per unit estimator, the ratio estimator has larger precision as compared with mean per unit estimator only if $\rho_{yx} > {C_x}/{2C_y}$ Singh and Choudhury (2012) proposed an exponential chain ratio estimator in double sampling and defined it as:

$$\bar{y}_{SC} = \bar{y}exp\left[\frac{\bar{x}'\frac{\bar{Z}}{\bar{z}'} - \bar{x}}{\bar{x}'\frac{\bar{Z}}{\bar{z}'} + \bar{x}}\right] \qquad (1.15)$$

$$MSE(\bar{y}_{SC}) = \bar{Y}^2\left[\theta_2 C_y^2 + \frac{1}{4}((\theta_2 - \theta_1)C_x^2 + \theta_1 C_z^2) - (\theta_2 - \theta_1)C_{yx}C_x^2 - \theta_1 C_{yz}C_z^2\right] \qquad (1.16)$$

where $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, $C_z = \frac{S_z}{\bar{Z}}$, $C_{yx} = \rho_{yx}\frac{C_y}{C_x}$, $C_{yz} = \rho_{yz}\frac{C_y}{C_z}$

They showed that $\bar{y}_{SC}$ was more efficient than Chand(1975) estimator if and only if $C_{yx} < \frac{1}{4}$ and $C_{yz} < \frac{1}{4}$ Singh and Majhi (2014) developed a chain type exponential estimator for population mean and the estimator is given as:

$$\bar{y}_{SM} = \bar{y}exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right) \qquad (1.17)$$

$$MSE(\bar{y}_{SM}) = \bar{Y}^2\left[\theta_1 C_z^2 + \frac{(\theta_2-\theta_1)C_x^2}{4} + \theta_2 C_y^2 - 2\theta_1\rho_{yz}C_yC_Z - (\theta_2 - \theta_1)\rho_{yx}C_yC_x\right] \qquad (1.18)$$

where $\rho_{yz} = \frac{S_{yz}}{S_yS_z}$ and $\rho_{yx} = \frac{S_{yx}}{S_yS_x}$

Singh and Ahmed (2015) proposed a chain ratio-type exponential estimator as:

$$\bar{y}_{SA} = \bar{y}exp\left(\frac{\sqrt{\bar{x}'\frac{\bar{Z}}{\bar{z}'}} - \sqrt{\bar{x}'}}{\sqrt{\bar{x}'\frac{\bar{Z}}{\bar{z}'}} + \sqrt{\bar{x}'}}\right) \qquad (1.19)$$

Its corresponding Mean square error is given as:

$$MSE(\bar{y}_{SA}) = \bar{Y}^2\left[\theta_2 C_y^2 + \frac{1}{16}\left((\theta_2 - \theta_1)C_x^2 + \theta_1 C_z^2\right) + \frac{1}{2}\left((\theta_2 - \theta_1)\rho_{yx}C_yC_x + \theta_1\rho_{yz}C_yC_z\right)\right] \quad (1.20)$$

II. PROPOSED RATIO-TYPE ESTIMATOR FOR DOUBLE SAMPLING

The proposed unbiased estimator of the population mean, coined from the classical ratio estimator, is given as:

$$\hat{\bar{Y}}_p = \bar{y}\left[\alpha\frac{\bar{x}'}{\bar{x}} + (1-\alpha)\frac{\bar{z}'}{\bar{z}}\right] \qquad (2.11)$$

$$= \bar{y}\left[\alpha\bar{x}'\bar{x}^{-1} + (1-\alpha)\bar{z}'\bar{z}^{-1}\right]$$

Where α is a real constant

To obtain the bias and variance of the estimator $\hat{\bar{Y}}_p$, we write

$\bar{y} = \bar{Y}(1 + \varepsilon_0)$, $\bar{x} = \bar{X}(1 + \varepsilon_1)$, $\bar{x}' = \bar{X}(1 + \varepsilon_1')$, $\bar{z} = \bar{Z}(1 + \varepsilon_1)\bar{z}' = \bar{Z}(1 + \varepsilon_2')$.

$\varepsilon_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $\varepsilon_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$, $\varepsilon_2 = \frac{\bar{z}-\bar{Z}}{\bar{Z}}$ $\varepsilon_1' = \frac{\bar{x}'-\bar{X}}{\bar{X}}$, $\varepsilon_2' = \frac{\bar{z}'-\bar{Z}}{\bar{Z}}$

Such that $E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = E(\varepsilon_1') = E(\varepsilon_2') = 0$

and $E(\varepsilon_0^2) = \theta_2 C_y^2$, $E(\varepsilon_1^2) = \theta_2 C_x^2$, $E(\varepsilon_2^2) = \theta_2 C_z^2$,

$$E\left(\varepsilon_1'^2\right) = \theta_1 C_x^2 \;,\quad E\left(\varepsilon_2'^2\right) = \theta_1 C_z^2 \;,\; E(\varepsilon_0\varepsilon_1) = \theta_2\rho_{xy}C_xC_y \;,$$
$$E(\varepsilon_0\varepsilon_1') = \theta_1\rho_{xy}C_xC_y \;, E(\varepsilon_0\varepsilon_2) = \theta_2\rho_{yz}C_yC_z \;,\; E(\varepsilon_0\varepsilon_2') = \theta_1\rho_{yz}C_yC_z$$

$$E(\varepsilon_1\varepsilon_1') = \theta_1 C_x^2 \;,\; E(\varepsilon_1\varepsilon_2) = \theta_2\rho_{xz}C_xC_z \;,\quad E(\varepsilon_1\varepsilon_2') = \theta_1\rho_{xz}C_xC_z \;,$$
$$E(\varepsilon_1'\varepsilon_2') = \theta_1\rho_{xz}C_xC_z$$

where $\theta_1 = \left(\frac{1}{n'} - \frac{1}{N}\right)$, $\theta_2 = \left(\frac{1}{n} - \frac{1}{N}\right)$, , $k = \frac{n}{n'}$

$$C_y = \frac{S_y}{\bar{Y}} \;,\quad C_x = \frac{S_x}{\bar{X}} \;,\quad C_z = \frac{S_z}{\bar{Z}}, \quad C_{yx} = \rho_{yx}\frac{C_y}{C_x} \;,\quad C_{yz} = \rho_{yz}\frac{C_y}{C_z}$$

$$C_{xz} = \rho_{xz}\frac{C_x}{C_z} \;,\quad \rho_{yx} = \frac{S_{xy}}{S_xS_y}, \rho_{xz} = \frac{S_{xz}}{S_xS_z} \;,\quad \rho_{yz} = \frac{S_{yz}}{S_yS_z}$$

$$S_y^2 = \frac{1}{N-1}\sum_{i\in U}(y_i - \bar{Y})^2 \;,\quad S_x^2 = \frac{1}{N-1}\sum_{i\in U}(x_i - \bar{X})^2$$

$$S_z^2 = \frac{1}{N-1}\sum(z_i - \bar{Z})^2 \;,\quad S_{xy} = \frac{1}{N-1}\sum(x_i - \bar{X})(y_i - \bar{Y})$$

$$S_{xz} = \frac{1}{N-1}\sum(x_i - \bar{X})(z_i - \bar{Z}) \quad , S_{yz} = \frac{1}{N-1}\sum(y_i - \bar{Y})(z_i - \bar{Z})$$

The estimator $\hat{\bar{Y}}_p$ can be expressed in terms of ε's as follows

$$\hat{\bar{Y}}_p = \bar{Y}(1+\varepsilon_0)[\alpha(1+\varepsilon_1')(1+\varepsilon_1)^{-1} + (1-\alpha)(1+\varepsilon_2')(1+\varepsilon_2)^{-1}] \qquad (2.12)$$

Applying negative binomial series,

$$\hat{\bar{Y}}_p = \bar{Y}(1+\varepsilon_0)[\alpha(1+\varepsilon_1')(1-\varepsilon_1+\varepsilon_1^2-\cdots) + (1-\alpha)(1+\varepsilon_2')(1-\varepsilon_2+\varepsilon_2^2-\cdots)] \qquad (2.13)$$

(2.13) is expanded to second-order approximation,

$$\hat{\bar{Y}}_p = \bar{Y}(1+\varepsilon_0)[\alpha(1-\varepsilon_1+\varepsilon_1^2+\varepsilon_1'-\varepsilon_1'\varepsilon_1) + (1-\alpha)(1-\varepsilon_2+\varepsilon_2^2+\varepsilon_2'-\varepsilon_2'\varepsilon_2)]$$

$$\hat{\bar{Y}}_p - \bar{Y} = \bar{Y}(1+\varepsilon_0)[(\alpha - \alpha\varepsilon_1 + \alpha\varepsilon_1^2 + \alpha\varepsilon_1' - \alpha\varepsilon_1'\varepsilon_1) + 1 - \varepsilon_2 + \varepsilon_2^2 + \varepsilon_2' - \varepsilon_2'\varepsilon_2 - \alpha + \alpha\varepsilon_2 - \alpha\varepsilon_2^2 - \alpha\varepsilon_2' + \alpha\varepsilon_2'\varepsilon_2] - \bar{Y}$$

$$\hat{\bar{Y}}_p - \bar{Y} = \bar{Y}(1+\varepsilon_0)[\alpha(\varepsilon_1^2 - \varepsilon_1 + \varepsilon_1' - \varepsilon_1'\varepsilon_1 + \varepsilon_2 - \varepsilon_2^2 - \varepsilon_2' + \varepsilon_2'\varepsilon_2) + 1 - \varepsilon_2 + \varepsilon_2^2 + \varepsilon_2' - \varepsilon_2'\varepsilon_2] - \bar{Y}$$

$$\hat{\bar{Y}}_p - \bar{Y} = \bar{Y}[\alpha(\varepsilon_1^2 - \varepsilon_1 + \varepsilon_1' - \varepsilon_1'\varepsilon_1 + \varepsilon_2 - \varepsilon_2^2 - \varepsilon_2' + \varepsilon_2'\varepsilon_2 - \varepsilon_0\varepsilon_1 + \varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2') + (1 - \varepsilon_2 + \varepsilon_2^2 + \varepsilon_2' - \varepsilon_2'\varepsilon_2 + \varepsilon_0 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2')] - \bar{Y}$$

$$\hat{\bar{Y}}_p - \bar{Y} = \bar{Y}[\alpha(\varepsilon_1^2 - \varepsilon_1 + \varepsilon_1' - \varepsilon_1'\varepsilon_1 + \varepsilon_2 - \varepsilon_2^2 - \varepsilon_2' + \varepsilon_2'\varepsilon_2 - \varepsilon_0\varepsilon_1 + \varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2') + (\varepsilon_2^2 - \varepsilon_2 + \varepsilon_2' - \varepsilon_2'\varepsilon_2 + \varepsilon_0 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2')] \qquad (2.14)$$

Taking expectation on both sides of (2.14) to get the bias of $\hat{\bar{Y}}_p$ in first-order approximation,

$$E(\hat{\bar{Y}}_p - \bar{Y}) = \bar{Y}[\alpha E(\varepsilon_1^2 - \varepsilon_1 + \varepsilon_1' - \varepsilon_1'\varepsilon_1 + \varepsilon_2 - \varepsilon_2^2 - \varepsilon_2' + \varepsilon_2'\varepsilon_2 - \varepsilon_0\varepsilon_1 + \varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_2') + E(\varepsilon_2^2 - \varepsilon_2 + \varepsilon_2' - \varepsilon_2'\varepsilon_2 + \varepsilon_0 - \varepsilon_0\varepsilon_2 + \varepsilon_0\varepsilon_2')]$$

$$E(\hat{\bar{Y}}_p - \bar{Y}) = \bar{Y}[\alpha(E(\varepsilon_1^2) - E(\varepsilon_1) + \mathrm{E}(\varepsilon_1') - \mathrm{E}(\varepsilon_1'\varepsilon_1) + E(\varepsilon_2) - \mathrm{E}(\varepsilon_2^2) - \mathrm{E}(\varepsilon_2') + \mathrm{E}(\varepsilon_2'\varepsilon_2) - \mathrm{E}(\varepsilon_0\varepsilon_1) + \mathrm{E}(\varepsilon_0\varepsilon_1') + E(\varepsilon_0\varepsilon_2) - \mathrm{E}(\varepsilon_0\varepsilon_2')) + E(\varepsilon_2^2) - E(\varepsilon_2) + E\varepsilon_2') - E(\varepsilon_2'\varepsilon_2) + \mathrm{E}(\varepsilon_0) - \mathrm{E}(\varepsilon_0\varepsilon_2) + \mathrm{E}(\varepsilon_0\varepsilon_2')]$$

$$E\left(\hat{\bar{Y}}_p - \bar{Y}\right) = \bar{Y}\big[\alpha\big(\theta_2C_x^2 - \theta_1C_x^2 - \theta_2\rho_{xy}C_xC_y + \theta_1\rho_{xy}C_xC_y - \theta_2C_z^2 + \theta_1C_z^2 + \theta_2\rho_{yz}C_yC_z - \theta_1\rho_{yz}C_yC_z\big) + \big(\theta_2C_z^2 - \theta_1C_z^2 - \theta_2\rho_{yz}C_yC_z + \theta_1\rho_{yz}C_yC_z\big)\big] \qquad (2.15)$$

$$E\left(\hat{\bar{Y}}_p - \bar{Y}\right) = \bar{Y}\big[\alpha\big((\theta_2-\theta_1)C_x^2 + (\theta_2-\theta_1)\rho_{yz}C_yC_z - (\theta_2-\theta_1)\rho_{xy}C_xC_y - (\theta_2-\theta_1)C_z^2\big) + \big((\theta_2-\theta_1)C_z^2 - (\theta_2-\theta_1)\rho_{yz}C_yC_z\big)\big]$$

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$$\text{Bias}\left(\hat{\hat{Y}}_p\right) = E\left(\hat{\hat{Y}}_p - \bar{Y}\right) = \bar{Y}\left[\alpha\left((\theta_2-\theta_1)(C_x^2+\rho_{yz}C_yC_z-\rho_{xy}C_xC_y\ -C_z^2)\ \right)+\left((\theta_2-\theta_1)(C_z^2-\rho_{yz}C_yC_z)\right)\right]$$
(2.16)

So, estimator $\hat{\hat{Y}}_p$ is unbiased if the value of the constant is

$$\alpha = \frac{(\rho_{yz}C_yC_z - C_z^2)}{(C_x^2+\rho_{yz}C_yC_z-\rho_{xy}C_xC_y\ -C_z^2)} \qquad (2.17)$$

Re-writing (2.14) to the first degree of approximation,

$$\hat{\hat{Y}}_p - \bar{Y} = \bar{Y}[\alpha(\varepsilon_1' - \varepsilon_1 + \varepsilon_2 - \varepsilon_2') + (\varepsilon_2' - \varepsilon_2 + \varepsilon_0)] \qquad (2.18)$$

Squaring both sides & neglecting terms of ε's involving power greater than two,

$$\left(\hat{\hat{Y}}_p - \bar{Y}\right)^2 = \bar{Y}^2[\alpha(\varepsilon_1' - \varepsilon_1 + \varepsilon_2 - \varepsilon_2') + (\varepsilon_2' - \varepsilon_2 + \varepsilon_0)]^2 \quad (2.19)$$

$$\left(\hat{\hat{Y}}_p - \bar{Y}\right)^2 = \bar{Y}^2[\alpha^2(\varepsilon_1' - \varepsilon_1 + \varepsilon_2 - \varepsilon_2')^2 + (\varepsilon_2' - \varepsilon_2 + \varepsilon_0)^2 + 2\alpha(\varepsilon_1' - \varepsilon_1 + \varepsilon_2 - \varepsilon_2')(\varepsilon_2' - \varepsilon_2 + \varepsilon_0)]$$

$$\left(\hat{\hat{Y}}_p - \bar{Y}\right)^2 = \bar{Y}^2[\alpha^2(\varepsilon_1^2 - \varepsilon_1'\varepsilon_1 + \varepsilon_1\varepsilon_2' - \varepsilon_1\varepsilon_2 + \varepsilon_1'\varepsilon_2 - \varepsilon_1\varepsilon_2 + \varepsilon_2'\varepsilon_1 + \varepsilon_2^2) + (\varepsilon_0^2 + \varepsilon_0\varepsilon_2' - \varepsilon_0\varepsilon_2 + \varepsilon_2'\varepsilon_0 - \varepsilon_0\varepsilon_2 - \varepsilon_2\varepsilon_2' + \varepsilon_2^2) + 2\alpha(\varepsilon_1'\varepsilon_0 - \varepsilon_0\varepsilon_1 - \varepsilon_1\varepsilon_2' + \varepsilon_1\varepsilon_2 - \varepsilon_0\varepsilon_2' + \varepsilon_0\varepsilon_2 + \varepsilon_2\varepsilon_2' - \varepsilon_2^2)] \qquad (2.20)$$

Taking expectation on both sides of (2.20), the variance of the estimator to first degree of approximation gives:

$$\begin{aligned} MSE\left(\hat{\hat{Y}}_p\right) = \bar{Y}^2\big[&\alpha^2(\theta_2C_x^2 - \theta_1C_x^2 + \theta_1\rho_{xz}C_xC_z - \theta_2\rho_{xz}C_xC_z + \theta_1\rho_{xz}C_xC_z - \theta_2\rho_{xz}C_xC_z + \theta_2C_z^2 - \theta_1C_z^2) \\ &+ \left(\theta_2C_y^2 + \theta_1\rho_{yz}C_yC_z - \theta_2\rho_{yz}C_yC_z + \theta_1\rho_{yz}C_yC_z - \theta_2\rho_{yz}C_yC_z - \theta_1C_z^2 + \theta_2C_z^2\right) \\ &+ 2\alpha\left(\theta_1\rho_{xy}C_xC_y - \theta_2\rho_{xy}C_xC_y - \theta_1\rho_{xz}C_xC_z + \theta_2\rho_{xz}C_xC_z - \theta_1\rho_{yz}C_yC_z + \theta_2\rho_{yz}C_yC_z + \theta_1C_z^2 - \theta_2C_z^2\right)\big] \end{aligned}$$

$$\begin{aligned} MSE\left(\hat{\hat{Y}}_p\right) = \bar{Y}^2\Big[&\alpha^2\left(C_x^2(\theta_2-\theta_1) - 2\rho_{xz}C_xC_z(\theta_2-\theta_1) + C_z^2(\theta_2-\theta_1)\right) + \left(\theta_2C_y^2 - (\theta_2-\theta_1)2\rho_{yz}C_yC_z + (\theta_2-\theta_1)C_z^2\right) \\ &+ 2\alpha\left((\theta_2-\theta_1)\rho_{xz}C_xC_z - (\theta_2-\theta_1)\rho_{xy}C_xC_y + (\theta_2-\theta_1)\rho_{yz}C_yC_z\ (\theta_2-\theta_1)C_z^2\right)\Big] \end{aligned}$$

$$MSE\left(\hat{\hat{Y}}_p\right) = \bar{Y}^2\left[\alpha^2(C_x^2 - 2\rho_{xz}C_xC_z + C_z^2)(\theta_2-\theta_1) + \left((C_z^2 - 2\rho_{yz}C_yC_z)(\theta_2-\theta_1) + \theta_2C_y^2\right) + 2\alpha(\theta_2-\theta_1)\left(\rho_{xz}C_xC_z - \rho_{xy}C_xC_y + \rho_{yz}C_yC_z - C_z^2\right)\right] \qquad (2.21)$$

Minimizing 2.21 w.r.t ,

$$\alpha^{opt} = \frac{C_z^2 - \rho_{xz}C_xC_z + \rho_{xy}C_xC_y - \rho_{yz}C_yC_z}{C_x^2 - 2\rho_{xz}C_xC_z + C_z^2} \qquad (2.22)$$

The Mean square error of the estimate is:

$$MSE\left(\hat{\hat{Y}}_p\right) = \bar{Y}^2\left[\theta_2C_y^2 + (\theta_2-\theta_1)\left(C_z^2 - 2\rho_{yz}C_yC_z - \frac{(C_z^2 - \rho_{xz}C_xC_z + \rho_{xy}C_xC_y - \rho_{yz}C_yC_z)^2}{C_x^2 - 2\rho_{xz}C_xC_z + C_z^2}\right)\right] \quad (2.23)$$

III. MATHEMATICAL COMPARISON OF PROPOSED ESTIMATOR OVER WELL-KNOWN EXISTING ESTIMATORS

In this section, we obtain the efficiency conditions for the proposed new ratio-type estimator by comparing its MSE with the MSE of the well-known existing estimators of double sampling. The mathematical efficiency of the proposed ratio type estimator is given as:

A. Comparison with Simple Random Sampling Estimator

$$MSE(\bar{y}_0) - MSE(\bar{y}_p)$$

$$= \bar{Y}^2\left[(\theta_1-\theta_2)\left(C_z^2 - 2\rho_{yz}C_yC_z - \frac{(C_z^2 - \rho_{xz}C_xC_z + \rho_{xy}C_xC_y - \rho_{yz}C_yC_z)^2}{C_x^2 - 2\rho_{xz}C_xC_z + C_z^2}\right)\right] \geq 0 \quad (3.1)$$

B. Comparison with Cochran(1977) Estimator

$$MSE(\bar{y}_C) - MSE(\bar{y}_p)$$

$$= (\theta_2 - \theta_1)\left[\hat{R}^2 S_x^2 - 2\hat{R}S_{xy} - \bar{Y}^2\left(C_z^2 - 2\rho_{yz}C_yC_z - \frac{(C_z^2 - \rho_{xz}C_xC_z + \rho_{xy}C_xC_y - \rho_{yz}C_yC_z)^2}{C_x^2 - 2\rho_{xz}C_xC_z + C_z^2}\right)\right] \geq 0 \ (3.2)$$

C. Comparison with Singh and Majhi (2014) Estimator

$$MSE(\bar{y}_{SM}) - MSE(\bar{y}_p)$$

$$\bar{Y}^2\left[\theta_1(2C_z^2 - 4\rho_{yz}C_yC_z) + \frac{1}{4}(\theta_2 - \theta_1)C_x^2 + \theta_2(2\rho_{yz}C_yC_z - C_z^2) + (\theta_2 - \theta_1)\left(\frac{(C_z^2 - \rho_{xz}C_xC_z + \rho_{xy}C_xC_y - \rho_{yz}C_yC_z)^2}{C_x^2 - 2\rho_{xz}C_xC_z + C_z^2} - \rho_{xy}C_xC_y\right)\right] \geq 0 \quad (3.3)$$

D. *Comparison with Singh and Ahmed (2015) Estimator*

$$MSE(\bar{y}_{SA}) - MSE(\bar{y}_p)$$

$$= \bar{Y}^2\left[\left(\frac{(\theta_2 - \theta_1)C_x^2 + \theta_1 C_z^2}{16}\right) + \left(\frac{(\theta_2 - \theta_1)\rho_{xy}C_xC_y + \theta_1\rho_{yz}C_yC_z}{2}\right) - (\theta_2 - \theta_1)\left(C_z^2 - 2\rho_{yz}C_yC_z - \frac{(C_z^2 - \rho_{xz}C_xC_z + \rho_{xy}C_xC_y - \rho_{yz}C_yC_z)^2}{C_x^2 - 2\rho_{xz}C_xC_z + C_z^2}\right)\right] \geq 0 \quad (3.4)$$

IV. EMPIRICAL COMPARISON OF ESTIMATORS

This research work used secondary data obtained from an anthropometric and physiological test conducted on some youths in Ekiti State, South West, Nigeria. The double sampling used rate pressure product as the study variable (y), systolic measure as the first auxiliary variable (x) and diastolic measure as the second auxiliary variable (z). Table 1 below shows the standard error and the coefficient of variation for each of the estimators

Estimator	$\hat{\bar{y}}_p$	$\bar{y}_0$	$\bar{y}_C$	$\bar{y}_{SM}$	$\bar{y}_{SA}$
MSE	0.3916	0.8496	0.4007	0.4299	1.2354
CV	30.43%	43.49%	30.79%	31.41%	52.34%
Conclusion	Highest Precision	High Precison	Higher Precision	High Precison	High Precision

Table 1: Summary of the MSE and CV of each estimator

*MSE-Mean Square Error, CV-Coefficient of Variation

Table 1 above reveals that the proposed estimator has the least variance and the highest precision.

Table 2 below shows the relative efficiency of the proposed estimator to other estimators

	Relative efficiency
$\hat{\bar{y}}_p$	100%
$\bar{y}_0$	217%
$\bar{y}_C$	102%
$\bar{y}_{SM}$	110%
$\bar{y}_{SA}$	315%

Table 2: Summary of the relative efficiency of the proposed estimator to each estimator

It is revealed in Table 2 above that the proposed estimator is 217% more efficient that Simple Random Sampling estimator. It is 102% efficient than Cochran(1977) estimator. The proposed estimator is 110% more efficient that Singh &Majhi (2014) estimator. It is 315% more efficient than Singh &Ahmed(2015) estimator.

V. CONCLUSION

In this work, a ratio type estimator for double sampling with two auxiliary variables has been proposed. Mathematical and Empirical Comparisons were made with well-known estimators of double sampling. One can readily see that the new proposed estimator is more precise than all other estimators mentioned in Section 2, so the new estimator provides more accurate estimate about population mean.

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