ISSN No:-2456-2165

A Numerical Statistical Solution for the Time-Independent Schrödinger Equation – Part II

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Abstract:- In a previous paper we studied the extension of transition matrix chains B from the heat diffusion equation to the numerical statistical solution of the time-independent Schrödinger equation in a spatial dimension x.

In this paper, we propose the extension of transition matrix chains B to the numerical statistical solution of the time-independent Schrödinger equation in two spatial dimensions x,y.

Extending physical transition matrix chains B to the solution of the time-independent Schrödinger equation requires respecting certain limitations of the bases that we briefly explain in this article.

We present the numerical statistical solution via matrix B in two illustrative situations, namely the two-dimensional heat diffusion equation and the two-dimensional infinite potential well where the numerical results are surprisingly accurate.

I. INTRODUCTION

In a previous article, we studied the extension of transition matrix chains B to the numerical statistical solution of the time-independent Schrödinger equation in one dimension x.

In this paper, we propose the extension of transition matrix chains B to the numerical statistical solution of the time-independent Schrödinger equation in the two dimensions x,y.

The numerical results obtained via the B transition matrix chains in two distinct solutions, namely the solution of the PDE problem of thermal diffusion and quantum particles in a well of infinite potential, prove to be superior to those obtained by solving the conventional thermal PDE and the Schrödinger equation.

It is expected that, in the near future,

Thermal PDEs and Schrödinger PDEs would both become a thing of the past, while their B-matrix equivalent chains, even in their infancy, represent the future.

Note that extending physical transition matrix chains B to the solution of the time-independent Schrödinger equation requires the introduction of some basic physical and mathematical terms or concepts which we briefly state in the following ten guiding concepts,

- Square matrices are a subset of mathematical matrices and Physical square matrices that have physical meaning (such as transition matrix B) are a subset of square matrices.
- Statistical transition matrices and chains of statistical transition matrices exist and its modeling works effectively in the solution of partial differential equations.

At present, we know two of them, namely the mathematical statistical transition matrix of Markov and the transition matrix B which is the subject of this article.

However**, in Markov matrix chains we do not care about the energy density, boundary conditions, source term, average properties of the medium, etc., whereas in the case of B matrix chains we let's do this.**

- A physical transition matrix chain B for the energy density U can be defined by the recurrence relation,
  $U(x, t+ dt) = B.\ U(x,t)$

As a result, a chain transition matrix, such as the statistical transition matrix B, must be able to describe the solution trajectory through its own solution space for a given time evolution which is the energy solution E in 4- D x-t. space.

- All matrix equations "resulting from the solution of PDE via the transition matrix" are not eigen value equations.

For example, the matrix of the numerical solution of heat diffusion equation results in a system of non-homogeneous first-order linear algebraic equations while the matrix equation of the numerical solution of the Schrödinger equation is homogeneous and results in an eigenvalue problem with multiple eigenvalues.

Several eigenvalues have their corresponding eigenvectors.

BOTH time-dependent and time-independent Schrödinger equations are in-depth examples of eigenvalue equations in quantum mechanics, with their eigenvalues corresponding to the allowed energy levels of the quantum system.

Generally speaking, in statistical transition matrix B eigenvalue is the dominant eigenvalue (eigenvalue of maximal absolute value) equal to 1.

- What is Schrödinger equation and what is time-independent Schrödinger equation?

The Schrödinger equation is a second-order linear PDE in the so-called wave function $\psi(x,t)$ and is a way to probabilistically describe the time evolution of energy, momentum and position of quantum particles in space.

ISSN No:-2456-2165

His time-independent equation for ψ(x) describes the equilibrium state that occurs when the evolutionary time tends towards infinity.

It should be noted that another way to describe quantum particle dynamics is to use statistical transition matrices that completely ignore the Schrödinger equation as if it never existed in the same way as solving the heat diffusion equation without going through the thermal PDE.

- What is a numerical solution and a statistical solution!

The numerical solution replaces the analytical solution of the time-dependent PDE by discretizing space and time into dx and dt and replacing the differentials dy/dx by [y+dy-2y +y-dy]/2dx And d ^2 y/ dx ^2 by [y+dy-2y +y-dy]/dx^2. etc.

In other words, the numerical solution method reduces the PDE to a system of algebraic equations.

This subject called numerical differential calculus was first introduced by I. Newton through his method of differential calculus is now known as finite difference method (FDM).

It is worth mentioning that statistical transition matrices completely ignore Newton's calculus and FDM as if they never existed.

Newtonian calculus and FDM techniques exist intrinsically in statistical transition matrix chains.

- This means that, in a way, the numerical statistical solution is a subset of the numerical solution in which the differential calculus is ignored and replaced by the statistics of the transition matrix.
- In addition, we emphasize that the method of separation of variables
  W(x,y,z,t)=X(x)Y(y)Z(z) f(t)
  Is not obligatory because it is intrinsically included in the 4D inseparable unit space of the chains of matrix B.
- The Monte Carlo numerical (technical) method is closest to the Cairo technical method.

However, the numerical Monte Carlo method is a bit old and requires generating a random numerical variable thousands of times. This makes the interpretation of its numerical results long and tedious.

- **Finally, it should be mentioned that today we only know one physical transition matrix which is the transition matrix B resulting from the so-called Cairo technique.**

Note that the statistical transition matrices operate in their own space, open or closed and characterized by RO=PE/E = constant.

The space of statistical transition matrices meets the Hamilton space of the Schrödinger equation in the special case RO = 1/2.

Through this article, we examine in detail two different illustrative physical problems in the areas of the heat diffusion equation and the time-independent Schrödinger equation.

We present numerical solutions for thermal conduction in a two-dimensional thermal energy field and the steady-state distribution of quantum energy density in an infinite potential well where the numerical results are surprisingly accurate.

II. THEORY

In practice, the field of modern quantum mechanics relies entirely on the Schrödinger equation and its derivatives which constitute a subset of physics but not physics as a whole.

Bohr's original theory of the hydrogen atom introduced for the first time in history the condition of quantization of electronic energy as quantification of the circular orbits of the electron around the nucleus in orbits called allowed orbits. The so-called authorized orbits give rise to authorized atomic energy states, as opposed to prohibited energy states.

Niels Bohr's original model in 1913 was based entirely on Newton's laws of motion supplemented by Bohr's quantification hypothesis, of the principal quantum number n, namely,

$$mv.2\pi.Rn=n\,h \ldots \quad (1)$$
$$n=1,2,3.\,.\,, \text{ infinity}$$

where Rn is the nth radius of electrons circulating around the nucleus.

At the time, N. Bohr did not say a word about the electronic cloud or the superposition of quantum states.

He also said nothing about the electron cloud in a quadratic potential nor about any of the quantization numbers (n, l, m, s) other than principal quantum number n.

This is called classical quantum mechanics, where the electron is considered to be a particle whose position x, velocity v and trajectory in space are known. Although old and classic, Bohr's original hypothesis in 1913 introduced a giant step towards modern quantum mechanics and the Schrödinger equation to come in 1927.

Accordingly, the term classical quantum mechanics corresponds to the original model of quantum mechanics developed by N. Bohr's theory of the hydrogen atom in 1913 and to similar models which considered subatomic particles as a point in x-t space , before the Schrödinger equation of 1927 which was completed. by the Bohr/Copenhagen interpretation. This interpretation is known as the principle of superposition of quantum states.

A second giant step was Bohr's modern theory of the hydrogen atom in 1927, where he introduced the concept of representing the dynamics of subatomic particles in space as a probability cloud described by the Schrödinger equation which replaced Newton's laws of motion.

ISSN No:-2456-2165

The concept of a quantum point particle and a quantum particle path subject to Newton's laws of motion has been radically overturned.

In this article we describe how to apply the chains of the matrix B to describe the dependence of the total energy E on a principal quantum number n (in order not to go further to other quantum numbers l, m,s) without go through the Schrödinger equation or Bohr's quantum hypothesis Eq 1.

We assume that the proposed numerical statistical solution for the time-independent Schrödinger equation constitutes a preliminary step on a long path towards the solution of complete quantum mechanics described by the time-dependent Schrödinger equation.

We recall that, in previous articles [2,3,4] we introduced numerical statistical solutions to time-dependent partial differential equations such as the Poisson and Laplace partial differential equations, sound intensity and time of reverberation in audio rooms, digital integration and differentiation, etc.

Our numerical statistical modeling proposed for the study of time-independent SEs is based on the same chains of transition matrices B and its derived transfer matrices D, E.

The basic entries of the statistical transition matrix B(i,j) are well defined in 1D, 2D and 3D configuration space problems via four statistical conditions[2,3,4] and the resulting transfer matrices D ,E are also well defined via the following elements of the relationships:

$$E(N)=B^0 + B +B^2+ B^3 +.. . +B^N \quad (2)$$

Where,
B^0=I, the unit matrix.

If N is large enough, we arrive at the time-independent steady state solution,

$$E = 1/(I-B) \quad (3)$$

For N sufficiently large.

In all cases, transfer matrix D is defined as,

$$D=E-I \quad (4)$$

Equation 3 is the reason why we introduced the transfer matrix E to use in the first step, and then calculated the transfer matrix D from equation 4 in the second step.

This procedure is called the Cairo technique (bydistinction)[2,3,4].

We emphasize again that the Cairo technical procedure for solving the time-dependent PDE in classical physics and its proposed extension to cover QM problems is not complicated but rather lengthy and requires mastery of some prerequisites in matrix algebra and in statistical transition matrix chains [1, 2,3,4].

In the Cairo techniques approach the time dependent solution of the PDE energy density U(x,t) is given by,

$$U(x,t)=D(N) . (b +S) + IC * B^N(5)$$

Where S is the source/sink term vector and IC is the initial conditions vector.

Equation 5 is used as a time-dependent statistical PDE equivalence matrix which has been used in the solution of classical physics problems such as thermal conduction PDE and it is now proposed to find a solution for the Schrödinger equation in 1D and 2D geometric shapes.

It should be noted that equation 5 contains a term due to the initial state conditions described by IC* B^N which decreases exponentially with time because the module of matrix B is less than 1. This term tends towards zero with time and is therefore not treated in the present case of the steady state in the remainder of this article.

Note that Eq 5 is the solution of U(x,t) in 4D x-t unitary space where the real time t is completely lost and is replaced by a dimensionless integer N.

It is also worth mentioning that discretizing time t into forbidden and allowed where t = N dt and N is an integer is inherently itself a quantification of time.

Again, the integer N is the number of iterations which is number of time steps or time jumps dt.

One of the important reasons for replacing the Schrödinger equation with chains of statistical transition matrices is that you are moving from a field of SE where many questions remain unanswered to the field of modern statistical physics where almost all questions have adequate answers.

Once againwe emphasize here that the subject of the statistical equivalence of the Schrödinger equation is not complicated but is a bit long and requires prior knowledge of matrix algebra [1] and the statistical solution of the matrix B to problems of statistics and classical physics [2, 3,4, 6].

The question arises how to extend the B matrix chain solution of the Cairo technique to cover time-independent stationary situations in 2D quantum mechanics problems?

In other words, how can we process Equation 5 in order to find a statistical equivalence of the 2D Schrödinger equation?

This is the subject of current article.

The time-independent Schrödinger equation, describing the square root of the probability density function ψ in all space, is expressed as follows:

- ψ(x,y,z)=-h^2/2m. Nabla^2 ψ(x,y,z) + V(x,y,z) .ψ(x,y,z) . . . (6)

Considering that the statistical equivalence approach of the Cairo techniques which is in general a time-dependent

ISSN No:-2456-2165

solution for the energy density U(x,t) as given by equation 5,

U(x,t)=D(N) . (b + S) + IC. B^N . . . . . (5)

The similarity between Equation 5 and Equation 6 is obvious and the application of Equation 5 to solving quantum mechanical problems seems natural.

In order to apply Equation 5 as a substitution for Equation 6, we propose two important natural assumptions:

- If the stationary Schrödinger equation can be interpreted as an eigenvalue equation in Hamiltonian space describing the square root of the probability density ψ then the square of ψ can also be interpreted as an eigenvalue equation in l 'space of matrix B (or any other appropriate space equation) having eigenvalues (λ ^ 2)
- the square of ψ can also be interpreted as the quantum energy density and that ψ^2 is subject to the classical physical transition matrix B described by equation 5 with secondary modification.

The modifications required for Equation 5 to describe the time-independent Schrödinger equation are briefly:

- The vector of boundary conditions b in Eq. 5 must be set equal to 0 since the function ψ extends to +/- infinity and is bounded.
- The source/sink term S in Eq. 5 is conveniently described by a function of the potential V(x), viz,
- S(x,y)= Constant C1* V(x,y) (7)

- Note that nature itself is linear and the expression S(x)=C1 V(x) in equation 7 conforms to this reality.

The choice of the constants C1 depends on the size and structure of the B matrix.

Equation 5 suggests that stationary solution of quantum energy density U(x)=E is expressed as,

E(x,y)=λ[B + C1 E(x,y)] (8)

Where λ is the dominant eigen value and is equal to 1 .

Equation 8 is the eigenvector equivalence of the B matrix of the time-independent Schrödinger equation for bounded quantum systems.

It is worth mentioning that the same transition matrix B which works efficiently for the solution of the time-dependent heat conduction equation is proposed for the solution of the time-independent Schrödinger equation but with different Ro, different BC and a different source term S.

RO is an element of the closed interval [0,1][2,3,4]) and represents the constant of motion in the statistical space of matrix B.

In the case of the thermal diffusion equation, RO is an explicit function of the thermal diffusion coefficient D.

The in-depth study of the transition matrix solutions allows all possible values of RO in the interval [0,1] for solution of the thermal conduction equation where RO is a function of the diffusivity of the material.

On the other hand we assume that the authorized values of RO for quantum mechanics problems are elements of [0,1/2]. The statistical reason is that for RO = < ½, equation 2 converges and diverges otherwise.

We also assume that the solution via matrix chains B may be, in some way, more informative than SE itself, a claim which will be explored in more detail when describing solutions to the Schrödinger equations in 3D.

It is worth mentioning that B-matrix string theory is not entirely new and has been working effectively since 2020 [2,3,4].

In order not to worry too much about the details of the theory, let's move on to the following numerical situations.

III. NUMERICAL RESULTS

A. 2D Heat diffusion equation

The heat diffusion/conduction equation has particular importance both in modern classical physics (classical physical laws supplemented by the modern definition of transition probability) and in classical and modern quantum mechanics where there are many communities of characteristics.

We start here with the 2D thermal diffusion equation problem where we find many similarities in the solution procedure. The solutions to the Schrödinger equation and that of the thermal diffusion equation are both based on the same transition matrix B, whether 1D, 2D or 3D.

In the previous part I of this article, we solved the thermal conduction/diffusion equation in 1D configuration via 1D B-matrix chains.

In this second part of the article, we solve the heat diffusion field in 2D configuration via 2D B matrix chains. Here we find and test the accuracy and precision of numerical statistics of the Cairo techniques method in two-dimensional thermal conduction/diffusion situations.

A fundamental hypothesis consistent with the concepts of B-matrix chains emerges:

For all nodes of a multidimensional object subject to Dirichlet boundary conditions, the boundary conditions and the source/ sink term (b-vctor source/sink term) are expressed as ,

BC=BC(x)+BC(y)+BC(z) . . . . Rule 1

S=Sx+Sy+Sz . . . . . Rule 2

Rules 1,2 are at the origin of the little-explained rule in quantum mechanics, namely:

E=Ex+Ey+Ez

ISSN No:-2456-2165

Consider the simple case of a rectangular domain with 9 equidistant free nodes, u1, u2, u3, ... u9 and 12 Dirichlet boundary conditions BC1 to BC12. These 12 BC conditions are reduced to only 9 BC when using rule 1, as shown in Figure 2.

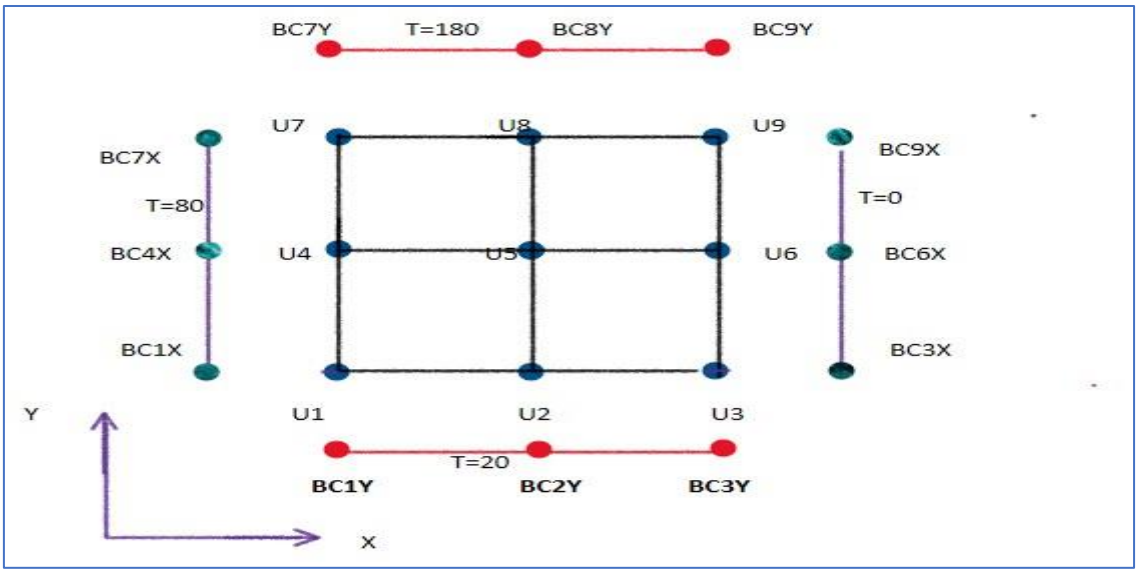

Fig. 1: A 2D rectangular domain with 9 equally spaced free nodes subject to Dirichlet BC.

Again, the 12 boundary conditions in Figure 2 can be reduced to 9 modified BCs for the 9 boundary nodes via the use of Rule 1 as follows:

BC1 = BC1X + BC1Y
BC2 = BC2X + BC2Y
. . . . . .
BC9 = BC9X + BC9Y

➢ *Step 1*

The first step consists of constructing the 9x9 two-dimensional transition matrix B in such a way as to satisfy the conditions i-iv[3,8] and to take the presupposed value of RO.

for an arbitrary RO, the 9x9 matrix B is given by,
RO 1/4-RO/4 0.0 1/4-RO / 4 0.0 0.0 0.0 0.0 0.0
1/4-RO/4 RO 1/4-RO/4 0.01/4-RO/4 0.0 0.0 0.0 0.0
0.0 1/4-RO/4 RO 0.0 1 /4-RO/ 4 0.0 0.0 0.0 0.0
1/4-RO/4 0.0 0.0 RO 1/4-RO/4 0.0 1/4-RO/4 0.0 0.0
0.0 1/4-RO /4 0.0 1/4-RO/4 RO 1/ 4 -RO /4 0.0 1/4-RO/4 0.0 0.0 0.0 1/4-RO/4 0.0 1/4-RO/4 RO 0.0 0.0 1 /4-RO/4
0.0 0.0 0.0 1/4-RO/4 0.0 0.0 RO 1/4-RO/4 0.0
0.0 0.00.0 1/4-RO/4 0.0 1/4-RO/4 0.0 RO 1/4-RO/ 4 0.0
0.0 0.00 0.00 0.00 1/4-RO/4 0.00 1/4-RO/4 RO

We call it matrix **M1**
Note that all diagonal elements of M1 (Mi,j) are equal to RO.

➢ *Step 2*

Calculate the transfer matrix E from equation 3.

E=
67/5611/28 1/811/28 ¼ 3/28 1/8 3/28 .3/56
11/2837/2811/28¼ 1/2 1/4 3/28 5/28 3/28
1/8 11/2867/563/28 ¼ 11/28 3/56 3/28 1/8
11/28 1/4 3/28 37/281/2 5/28 11/281/4 3/28
¼ ½ ¼ 1/2 3/2 1/2 1/4 1/2 ¼
3/28 1/4 11/28 5/28 ½ 37/28 3/28¼ 11/28
1/8 3/28 3/56 11/281/4 3/28 67/5611/281/8
3/28 5/28 3/28 1/4 1/2 1/4 11/2837/2811/28

3/56 3/28 1/8 3/28 1/4 11/28 1/8 11/2867/56

➢ *Step 3*

The last step 3 is to use equation 5 to obtain the temperature distribution vector, namely:

U(x, y)=D(N) . (b +S) + IC * BN (5)

With source term S=0 and initial conditions IC =0

Mathews [1,pp 524] classically solved the same thermal system shown in Figure 1 via 9 linear algebraic equations resulting from the FDM technique.

He calculated the steady-state temperature distribution using the Gaussian elimination method in a more efficient scheme by extending the tridiagonal algorithm to the more sophisticated penta-diagonalalgorithm for its arbitrarily chosen vector boundary conditions,

b = [100,20,20,80,0,0,260, 180 180]^T. . . . . (9)

Mathews arrived at the temperature solution vector:

T=[55,7143, 43,2143, 27,1429, 79,6429,70,0000,45,3571,112,357,111,786,84,2857] ^T... . (10)

Now, in the proposed statistical solution, the Dirichlet boundary conditions are modified via rule 1. It follows that the vector b corresponding to Figure 1 and the arbitrary Mathews boundary conditions is simply rewritten as follows:

b= [100/4, 20/4, 20/4, 20/ 4, 80/4, 0., 0,260/4, 180/4, 180/4]^ T

Using equation 5 and the previous vector b, we arrive at,

T=[390/7,605/14, 190/7,1115/14, 70,635/14,790/7,1565/14,
590/7]^T . . . . ..(11)

Note that:

- Comparing the results of Mathews Eq 10 for the 2D thermal equation applied in Figure 1 with the numerical results obtained via the matrix transition statistical chains B Eq 11, we find a striking accuracy.
- **The proposed numerical statistical method bypasses the Heat PDE and FDM techniques.**

B. Quantum particle in a 2D infinite potential well

➢ *Step 1*

Construct the 2D statistical matrix B corresponding to Figure 2 which represents a quantum particle in a 2D infinite potential well.

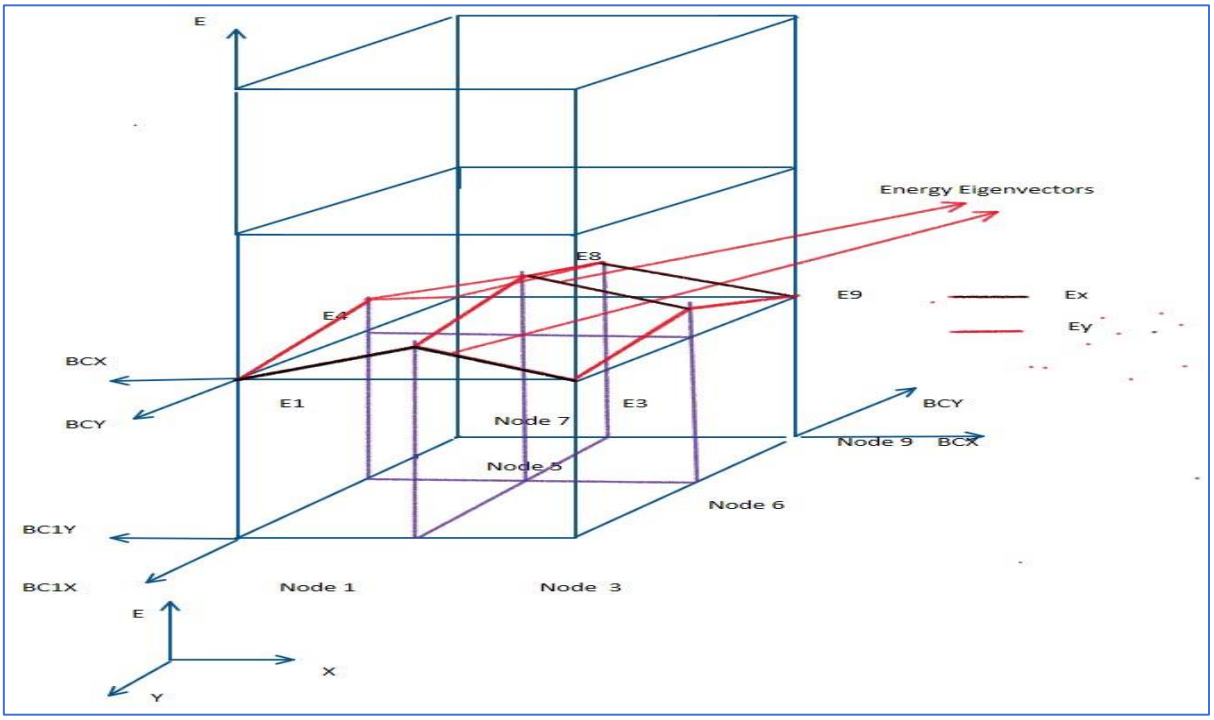

Fig. 2: A quantum particle in a 2D infinite potential well

The basis for generating an eigen or proper matrix is the 2D matrix B with 9 equidistant free nodes as shown in Figure 2, nodes 1-9.

Note that RO=0 because PE is zero.

It is expressed by the same matrix **M1** explained previously in the example of 2D thermal conduction.

➢ ***Step 2***

Compose the proper or eigen matrix M2 as given by,

M2=M1+S(x,y)

Where S(x,y) is a diagonal matrix and S=C1*V(x,y)

The resulting eigenmatrix M2 will be given by,

M2=

1/14	**1/4**	**0**	**1/4**	**0**	**0**	**0**	**0**	**0**
1/4	**4/14**	**1/4**	**0**	**1/4**	**0**	**0**	**0**	**0**
0	**1/4**	**1/14**	**0**	**0**	**1/4**	**0**	**0**	**0**
1/4	**0**	**0**	**4/14**	**1/4**	**0**	**1/4**	**0**	**0**
0	**1/4**	**0**	**1/4**	**9/14**	**1/4**	**0**	**1/4**	**0**
0	**0**	**1/4**	**0**	**1/4**	**4/14**	**0**	**0**	**1/4**
0	**0**	**0**	**1/4**	**0**	**0**	**1/14**	**1/4**	**0**
0	**0**	**0**	**0**	**1/4**	**0**	**1/4**	**4/14**	**1/4**
0	**0**	**0**	**0**	**0**	**1/4**	**0**	**1/4**	**1/14**

Where C1 is substituted for by the factor 1/14.

➢ *Step 3*

The energy eigenvector E(x,y) is equal to the principal diagonal of the matrix A which gives the following eigenvector equation,

2/14	1/4	0	1/4	0	0	0	0	0
1/4	4/14	1/4	0	1/4	0	0	0	0
0	1/4	2/14	0	0	1/4	0	0	0
1/4	0	0	4/14	1/4	0	1/4	0	0
0	1/4	0	1/4	9/14	1/4	0	1/4	0
0	0	1/4	0	1/4	4/14	0	0	1/4
0	0	0	1/4	0	0	2/14	1/4	0
0	0	0	0	1/4	0	1/4	4/14	1/4
0	0	0	0	0	1/4	0	1/4	2/14

*

[2/144/142/144/149/144/142/144/142/14]T
is equal to,

[8/49123/392
8/49123/392137/196123/3928/49123/392 8/49] T

Showing that the energy eigenvector is=
[2/144/142/144/149/144/142/144/142/14] T

with a dominant eigenvalue almost equal to 1.

The reason why we multiply the nodes 1,3,6 and 9 by the factor 2 is that these nodes are located at the four intersections of the two axes x and y where the rule E=Ex+Ey applies.

The x-oriented eigenvectors and the y-oriented eigenvectors are shown in Figure 2 in black and red lines.

A quantum mechanical case similar to Figure 2 but slightly more complicated is that of 25 equidistant free nodes of rectangular shape.

Here, the eigenmatrix of 25x25 entries and the 25 eigenvectors are presented in the following matrix form,

2/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.25	4/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.25	9/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.25	4/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.25	2/40	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.25	0.0	0.0	0.0	0.0	4/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.25	0.0	0.0	0.0	0.25	9/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.25	0.0	0.0	0.0	0.25	16/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.25	9/40	0.25	0.0	0.0	0.0	0.25	0.0
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.25	4/40	0.0	0.0	0.0	0.0	0.25
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0	9/40	0.25	0.0	0.0	0.0
	0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.25	16/40	0.25	0.0	0.0
	0.0	0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.25	25/40	0.25	0.0
	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.25	16/40	0.25
	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.25	9/40
	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0
	4/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0
	0.25	9/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25	0.0	0.0
	0.0	0.25	16/40	0.25	0.0	0.0	0.0	0.25	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25	0.0
	0.0	0.0	0.25	9/40	0.25	0.0	0.0	0.0	0.25	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.25
	0.0	0.0	0.0	0.25	4/40	0.0	0.0	0.0	0.0	0.25				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.25	0.0	0.0	0.0	0.0	2/40	0.25	0.0	0.0	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.25	0.0	0.0	0.0	0.25	4/40	0.25	0.0	0.0				

ISSN No:-2456-2165

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.25	0.0	0.0	0.0	0.25	9/40	0.25	0.0				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.25	4/40	0.25				
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.25	0.0	0.0	0.0	0.25	2/40				

*

[2/404/409/40 4/40 2/404/40 9/40 9/4016/40 9/40 4/40 9/4016/4025/4016/40 . . . . … .]T

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[21/40027/200321/1600 27/200 21/400 27/200481/160097/200481/160027/200321/1600 97/200253/320 97/200 . . etc]T

It is clear that in this case the proportionality constant C1 =1/40.

IV. CONCLUSION

Extending the physical transition matrix chains B to the solution of the time-independent Schrödinger equation is not complicated but it is a bit long and requires respecting certain limitations of the bases that we briefly explain in this article.

The present study shows that the statistical chains of the B matrix can be applied to the solution of the 2D heat equation and the time-independent 2D Schrödinger equation.

We present the numerical solution via the statistical transition matrix B in two illustrative situations, namely the 2D heat diffusion equation and the two-dimensional infinite potential well where the numerical results are of excellent accuracy.

NB:In the previous calculations, the author used his own double precision algorithm as explained in ref. 8.

No ready-made algorithms such as Python or MATLAB are needed.

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