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A Class of Continuous Linear Multistep Method for Solving Second Order Ordinary Differential Equations

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Abstract:- In this research work, a Class of Continuous 4-Step Method (CCSM) for solv- ing second order ordinary differential equations is provided. The step size of the approximate solution affects the coefficients of the developed approach. As a by-product of the continuous technique, a discrete second derivatives method is obtained from the continuous method. At any stage in collocation, the primary predictor needed for the implicit method assessment is of the same order as the method. The stability, consistency, and convergence aspects of the method were discussed and used to the solve linear and non-linear problems in order to demon- strate its applicability and efficiency.

Keywords:- *Interpolation, Collocation, Ordinary Differential Equation, Predictor-Corrector.*

I. INTRODUCTION

The numerical solution of the second order initial value issue in form (1) is taken into consideration.

$$y'' = f(x,y,y');\ y(a) = y_0;\ y'(a) = y_1 \quad (1)$$

Equation (1) is the result of many physical phenomena, particularly those in en- gineering, such as fluid dynamics, electrical circuits, satellite motion, and others. Researchers [9], [11], [18], and others had mentioned using this approach. Most of the time, (1) can be solved by reducing it to the equivalent system of the first ordi- nary differential equation, which may then be solved numerically according to the application. Despite the efficacy of the strategy, [2] asserts that there are certain drawbacks. These drawbacks include being uneconomical in terms of implementa- tion costs, increasing the amount of computation required and the amount of time wasted by computers, growing the size of the resulting systems of equations that must be solved, and lacking any additional details regarding a particular ordinary differential equation. If the given system of equations cannot be explicitly solved with regard to the highest order derivative, the method is worthless.A strategy for directly resolving the system of higher order ordinary differential equations has been put out by [5, [11], [12], [13], and others. According to [2] and [5], the con- tinuous linear multistep method offers more advantages. It offers a more accurate estimation of the error, simplifies the coefficient for use in subsequent analyses at various places, and assures that the solution is approximatively simple at all inte- rior points of the integration interval. A few eminent academics who have made substantial contributions to this field of study are [18], [9], [20], [1], and [5]. This study is driven by the need to develop a direct method based on collocation and interpolation to construct a continuous 4-step method for the solution of second order ordinary differential equations in order to solve the inadequacies of the ex- isting methodologies. In this work, power series was used as the basis function to create the continuous hybrid linear multistep that was developed for the solution of problem (1).

II. DERIVATION OF THE METHOD

CCSM is obtained by approximating the exact solution $y(x)$ by searching the solution $y(x,)$, which provides a discrete method as a by-product. The method has the form

$$y(x) = \sum_{j=0}^{(c+i)-1} a_j x^j \quad (2)$$

will be employed as a basis function to approximate the solution of second order initial value problems of the type (3), (2)'s second derivative is given as:

$$y''(x) = \sum_{j=0}^{(c+i)-1} j(j-1)a_j x^{j-2} \quad (3)$$

Through interpolation of (2) at $x = x_{n+1}$ and $x_{n+\frac{1}{2}}$, collocation of (3) at $x = x_{n+i}, i = 0,1,2,3,4$ to obtain $k+3$ system of equation, where k is the step number

$$y(x_{n+j}) = y_{n+j}, j = 0(4) \quad (4)$$

$$y(x_{n+j}, u) = f_{n+j}, j = 0(4)k \quad (5)$$

Equations (2) and (3) result in an equation system that is solved by the Gaussian elimination method to yield the parameter $a'_j s$. By entering the values of $a'_j s$ into equation (2), the CCSM is stated in the form after some algebraic manipulation.

$$y_{n+4} = \alpha_1 y_{n+1} + \alpha_{\frac{1}{2}} y_{n+\frac{1}{2}} + h^2(\beta_0 f_n + \beta_1 f_{n+1} + \beta_2 f_{n+2} + \beta_3 f_{n+3} + \beta_4 f_{n+4}) \quad (6)$$

The coefficients are obtained as follows using the transformation

$$t = \frac{x - x_{n+k-1}}{h}$$

$$\frac{dt}{dx} = \frac{1}{h}$$

The coefficients of y_{n+j} and f_{n+j} are obtained as:

$$\alpha_1 = 2t + 5 \quad (7)$$

$$\alpha 1 = -2t - 4 \quad (8)$$

$$\beta_0 = \frac{h^2}{23040}(32\,t^6 + 96\,t^5 - 80\,t^4 - 320\,t^3 + 113\,t - 30) \quad (9)$$

$$\beta_1 = -\frac{h^2}{2880}(16\,t^6 + 72\,t^5 - 40\,t^4 - 240\,t^3 - 1875\,t - 3750) \quad (10)$$

$$\beta_2 = \frac{h^2}{3840}(32\,t^6 + 192\,t^5 + 80\,t^4 - 960\,t^3 + 4647\,t + 4430) \quad (11)$$

$$\beta_3 = -\frac{h^2}{180}(t+2)\left(t + \frac{5}{2}\right)\left(t^4 + 3\,t^3 - 6\,t^2 - 13\,t - \frac{3}{2}\right) \quad (12)$$

$$\beta_4 = \frac{h^2}{23040}\left(32\,t^6 + 288\,t^5 + 880\,t^4 + 960\,t^3 - 339\,t + 90\right) \quad (13)$$

The first derivatives of equation (7 - 13) are as follows

$$\alpha_1' = 2 \quad (14)$$

$$\alpha_{\frac{1}{2}}' = -2 \quad (15)$$

$$\beta_0' = \frac{h^2}{23040}(192\,t^5 + 480\,t^4 - 320\,t^3 - 960\,t^2 + 113) \quad (16)$$

$$\beta_1' = -\frac{h^2}{2880}(96\,t^5 + 360\,t^4 - 160\,t^3 - 720\,t^2 - 1875) \quad (17)$$

$$\beta_2' = \frac{h^2}{3840}(192\,t^5 + 960\,t^4 + 320\,t^3 - 2880\,t^2 + 4647) \quad (18)$$

$$\beta_3' = -\frac{h^2}{180}\left(t + \frac{5}{2}\right)(t^4 + 3\,t^3 - 6\,t^2 - 13\,t - \frac{3}{2}) - \frac{h^2}{180}(t+2)(t^4 + 3\,t^3 - 6\,t^2 - 13\,t - \frac{3}{2}) - \frac{h^2}{180}(t+2)\left(t + \frac{5}{2}\right)(4\,t^3 + 9\,t^2 - 12\,t - 13) \quad (19)$$

$$\beta_4' = \frac{h^2}{23040}\left(192\,t^5 + 1440\,t^4 + 3520\,t^3 + 2880\,t^2 - 339\right) \quad (20)$$

Evaluating (3.28 − 3.34) at t = 1, yields the discrete 4-step method

$$y_{n+4} = \frac{h^2}{7680}\left(-63\, f_n + 15512\, f_{n+1} + 16842\, f_{n+2} + 7392\, f_{n+3} + 637\, f_{n+4}\right) + 7\, y_{n+1} - 6\, y_{n+\frac{1}{2}} \quad (21)$$

with first derivative

$$y'_{n+4} = \frac{1}{23040h}\left(-495\, f_n + 18392\, f_{n+1} + 19434\, f_{n+2} + 29856\, f_{n+3} + 7693\, f_{n+4}\right) + 46080\, y_{n+1} - 46080\, y_{n+\frac{1}{2}} \quad (22)$$

The predictor of the method and its derivatives are obtained as

$$y_{n+4} = \frac{h^2}{9600}\left(5495\, f_n + 41685\, f_{n+1} + 9905\, f_{n+2} + 13699\, f_{n+3} - 20384\, f_{n+\frac{1}{2}}\right) + 7\, y_{n+1} - 6\, y_{n+\frac{1}{2}} \quad (23)$$

$$y'_{n+4} = \frac{h^2}{28800h}\left(66695\, f_n + 292245\, f_{n+1} - 110335\, f_{n+2} + 91171\, f_{n+3} - 246176\, f_{n+\frac{1}{2}}\right) + 57600\, y_{n+1} - 57600\, y_{n+\frac{1}{2}} \quad (24)$$

III. ANALYSIS OF THE METHOD

Basic properties of the main method was analyzed to establish their validity. These properties namely: order, error constant, stability and consistency was obtained in this chapter. In what follows, a brief introduction of these properties are made for a better comprehension of the chapter.

➢ *Order and Error Constant of the Method*

Let the linear Operator defined on the method be $[y(x); h]$,

$$[y(x);\ h] = C_0 y(x) + C_1 h y'(x) + \ldots C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) + C_{p+2} h^{p+2} y^{p+2}(x) + \ \ldots \ C_0 = C_1 = C_2 = \cdots = C_{p+3} = 0, C_{p+4} \neq 0$$

- *Definition :* The term C_{p+4} is called the error constant and it implies that the local truncation error is given as

$$T_{n+k} = C_{p+4} h^{p+4} y^{p+4}(x) + 0(h^{p+5})$$

- *Order and Error Constant for the Main Method*

The derivation of the order and error constant of the method are as follows:

$$y_{n+4} = \frac{h^2}{7680}\left(-63\, f_n + 15512\, f_{n+1} + 16842\, f_{n+2} + 7392\, f_{n+3} + 637\, f_{n+4}\right) + 7\, y_{n+1} - 6\, y_{n+\frac{1}{2}}$$

With first derivative

$$y'_{n+4} = \frac{1}{23040h}\left(-495\, f_n + 18392\, f_{n+1} + 19434\, f_{n+2} + 29856\, f_{n+3} + 7693\, f_{n+4}\right) + 46080\, y_{n+1} - 46080\, y_{n+\frac{1}{2}}$$

Expanding the method and its derivatives by Taylor series and combining coeffi- cients of like terms in h^n give

$$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = 0,\ c_7 = \frac{-139}{15360} = -0.0090495$$

Hence the method is of order 5 and the error constant is −0.0090495 while the error for the derivative is −2.453717138

$$y_{n+4} = \frac{h^2}{9600}\left(5495\, f_n + 41685\, f_{n+1} + 9905\, f_{n+2} + 13699\, f_{n+3} - 20384\, f_{n+\frac{1}{2}}\right) + 7\, y_{n+1} - 6\, y_{n+\frac{1}{2}} \quad (25)$$

$$y'_{n+4} = \frac{h^2}{28800h}\left(66695\, f_n + 292245\, f_{n+1} - 110335\, f_{n+2} + 91171\, f_{n+3} - 246176\, f_{n+\frac{1}{2}}\right) + 57600\, y_{n+1} - 57600\, y_{n+\frac{1}{2}} \quad (26)$$

Combining the coefficients of similar terms in h^n and extending the technique and its derivatives by the Taylor series results in $c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = c_6 = 0$, $c_7 = \frac{941}{19200} = 0.0490104$. Hence the first derivative of the main predictor is $\frac{941}{19200} = 0.0490104$ while the error for the derivative is $\frac{-895417}{403200} = -2.2207762$

➢ *Consistency*

The linear multi step technique is said to be consistent if it satisfies the following conditions

- the order $p \geq 1$
- $\sum_{j=0}^{k} \alpha_j = 0$
- $\rho(1) = \rho'(1) = 0$
- $\rho^{ii}(1) = 2!\sigma(1)$

- *Consistency of the Method*

$$y_{n+4} + 6\, y_{n+\frac{1}{2}} - 7\, y_{n+1} = \frac{h^2}{7680}\left(-63\, f_n + 15512\, f_{n+1} + 16842\, f_{n+2} + 7392\, f_{n+3} + 637\, f_{n+4}\right) \qquad (27)$$

✓ Condition (1) is satisfied since P = 5

$$\alpha_1 = -7;\ \alpha_{\frac{1}{2}} = 6;\ \alpha_4 = 1$$

$$\sum_{j=0}^{k} \alpha_j = \alpha_1 + \alpha_{\frac{1}{2}} + \alpha_4 = -7 + 6 + 1 = 0$$

✓ Condition (2) is satisfied since

$$\sum_{j=0}^{k} \alpha_j = 0$$

Here, $\rho(r)$ is the first characteristic polynomial while $\sigma(r)$ is the second characteristics polynomial $\rho(r) = r^4 + 6r^{\frac{1}{2}} - 7r$

$$\sigma(r) = \frac{1}{7680}\left(-63 + 15512r + 16842r^2 + 7392r^3 + 637r^4\right)$$

$$\rho(1) = r^4 + 6r^{\frac{1}{2}} - 7r = 0, \text{ when } r = 1$$
$$\rho'(r) = 4\,r^3 + \frac{3}{\sqrt{r}} - 7$$
$$\rho'(1) = 4\,r^3 + \frac{3}{\sqrt{r}} - 7 = 0, \text{ when } r = 1$$

✓ Condition (3) is satisfied since $p'(1) = 0$

$$\rho''(r) = 12\,r^2 - \frac{3}{2\,r^{\frac{3}{2}}} = \frac{21}{2}, \text{ when } r = 1$$

$$\sigma(r) = \frac{1}{7680}(-63 + 15512r + 16842r^2 + 7392r^3 + 637r^4)$$

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$$2!\sigma(r) = \frac{2!}{7680}(-63 + 15512r + 16842r^2 + 7392r^3 + 637r^4) = \frac{21}{2}$$

When $r = 1$

$$\rho''(r) = 2!\sigma(r) = \frac{21}{2}$$

✓ Condition (4) is satisfied since $\rho''(1) = 2!\sigma(1)$
Hence, the four condition of consistency are satisfied.

➢ *Zero - Stability of the Method*

$$y_{n+4} = 7\,y_{n+1} - 6\,y_{n+\frac{1}{2}} + \frac{h^2}{7680}\left(-63\,f_n + 15512\,f_{n+1} + 16842\,f_{n+2} + 7392\,f_{n+3} + 637\,f_{n+4}\right)$$

The first characteristic polynomial according to the definition

$$\rho(r) = r^4 + 6r^{\frac{1}{2}} - 7r$$

On solving $\rho(r)$ gives

$r = 0, 1$

Which satisfies $|R_J| \geq 1, j = 1, ..., k$, the multiplicity is simple and that the roots are in the unit circle. The approach is hence zero stable.

IV. NUMERICAL EXAMPLES, RESULTS AND DISCUSSION

➢ *Introduction*

This chapter talked about how the developed method was put into practice. Fur- thermore, some numerical examples of nonlinear and linear value problems of gen- eral second order ordinary differential equations are used to test the effectiveness of the method. The absolute error of the approximate solution are computed and compared with results from existing methods. The results from the method are also discussed here.

➢ *Numerical Examples*

To test the viability of the created method, we give some numerical experiments with the four challenges below. The 4-Step Method was used to solve the following test issues.

- *Problem 1*

$$y'' = 100y;\; y(0) = 1,\; y'(0) = -10,\; h = 0.01 \text{ exact solution: } y(x) = exp(-10x)$$

- *Problem 2*

$$y'' = y':\; y(0) = 0;\; y'(0) = -1,\; h = 0.1 \text{ exact solution: } y(x) = 1 - exp(x)$$

- *Problem 3*

A stamping machine applies hammering forces on metal sheets by a die attached to the plunger which moves vertically up and down by a fly wheel makes the impact force on the metal sheet and therefore the supporting base, intermittent and cyclic. The bearing base on which the metal sheet is situated has a mass, $M = 2000kg$. The force acting on the base follows a function: $f(t) = 2000sin(10t)$, in which t=time in seconds. The base is supported by an elastic pad with an equivalent spring constant $k = 2 * 10^5 N/M$. Determine the differential equation for the instantaneous position of the base $y(t)$ if the base is initially depressed down by an amount $0.1m$.

✓ *Solution:* The mass-spring system above is modeled as differential equation:

The Bearing base mass = 2000kg

Spring constant $k = 2 * 10^5 N/M$
Force (ma) on the metal sheet = $m\frac{d^2y}{dt^2} = my''$

i.e. $ma = my'' = 2000sin(10t)$; where $a = y''$

Initial conditions on the system are

$$y(t_0) = y_0; \frac{dy}{dt}|t = 0 = y'(t_0) = y'(0);\; t_0 = 0, y'_0 = 0.1$$

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Therefore, the governing equation for the instantaneous position of the base y(t) is given by

$$My'' + ky = F(t);\ y(t_0) = y_0, y'(t_0) = y_0'$$

✓ *Theoreticalsolution:* $y(t) = \frac{1}{10}cos10t + \frac{1}{200}sin10t - \frac{t}{20}cos10t$

- *Problem 4*

$$y'' + y = 0:\ y(0) = 0;\ y'(0) = 1,\ h = 0.025 \text{ exact solution: } y = sin(x)$$

➢ *Numerical Results*

Table 1 Result of problem 1, for h=0.01

X	Exact solution	Computed Solution	Error in the new method	Error in [1]
0.01	0.904837418035960	0.904837419100000	$1.064040e-09$	$1.157e-07$
0.02	0.818730753077982	0.818730755100000	$2.022018e-09$	$3.658e-07$
0.03	0.740818220681718	0.740818222800000	$2.118282e-09$	$6.051e-07$
0.04	0.670320046035639	0.670320049600000	$3.564361e-09$	$8.502e-07$
0.05	0.606530659712633	0.606530663901465	$4.188832e-09$	$1.104e-06$
0.06	0.548811636094026	0.548811640871017	$4.776991e-09$	$1.369e-06$
0.07	0.496585303791410	0.496585308613180	$4.821770e-09$	$1.450e-06$
0.08	0.449328964117222	0.449328969922628	$5.805406e-09$	$1.597e-06$
0.09	0.406569659740599	0.406569666013090	$6.272491e-09$	$1.763e-06$
0.10	0.367879441171442	0.367879447921353	$6.749911e-09$	$1.946e-06$

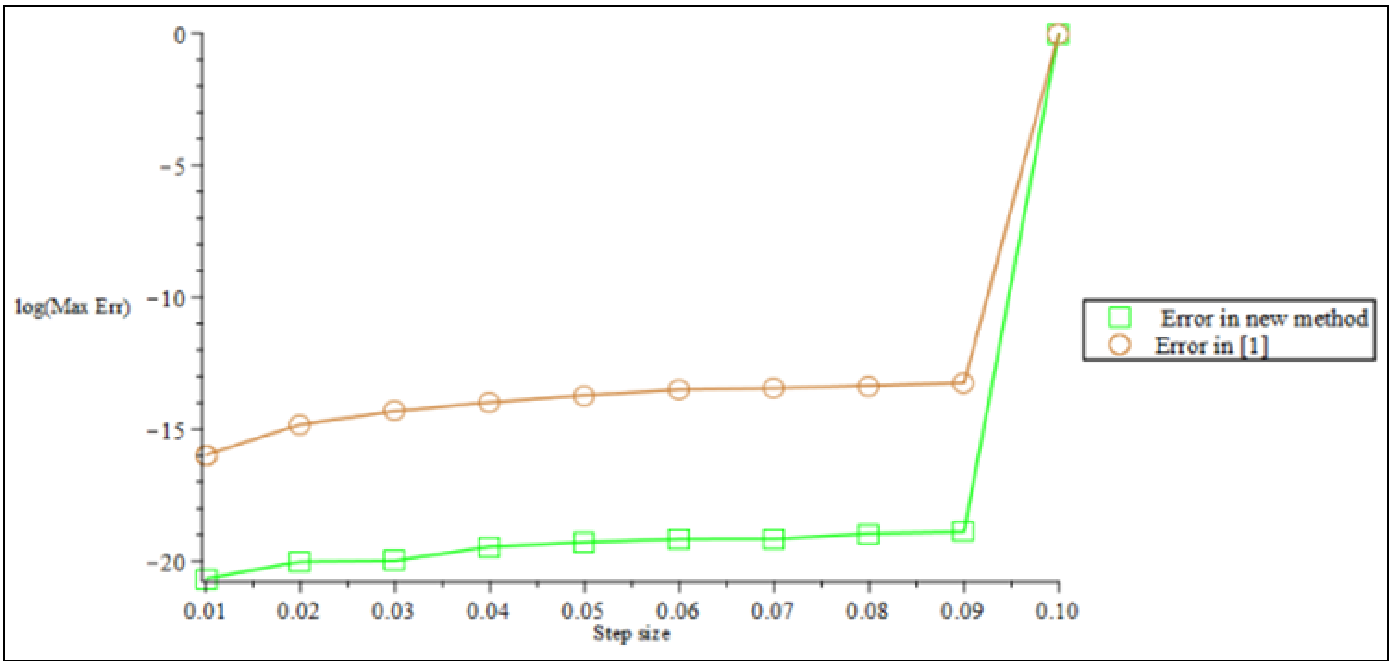

Fig 1 Efficiency Curve for Problem 1 Compared with Error in [1]

Table 2 Result of Problem 2, for h=0.1

X	Exact Solution	Computed Solution	Error in the new method	Error in [14]
0.2	-0.22140275816017	-0.221402754800000	$3.3601700e-09$	$8.17176e-07$
0.3	-0.34985880757600	-0.349858801900000	$5.6760000e-09$	$3.10356e-06$
0.4	-0.49182469764127	-0.491824690000000	$7.6412700e-09$	$6.56957e-06$
0.5	-0.64872127070013	-0.648721260202993	$1.0497137e-08$	$1.14380e-05$
0.6	-0.82211880039051	-0.822118786044656	$1.4345854e-08$	$1.79656e-05$
0.7	-1.01375270747048	-1.013752688688240	$1.8782240e-08$	$2.64474e-05$
0.8	-1.22554092849247	-1.225540905693600	$2.2798870e-08$	$3.72222e-05$
0.9	-1.45960311115695	-1.459603082898740	$2.8258210e-08$	$5.06788e-05$
1.0	-1.71828182845905	-1.718281793134250	$3.5324800e-08$	$6.72615e-05$

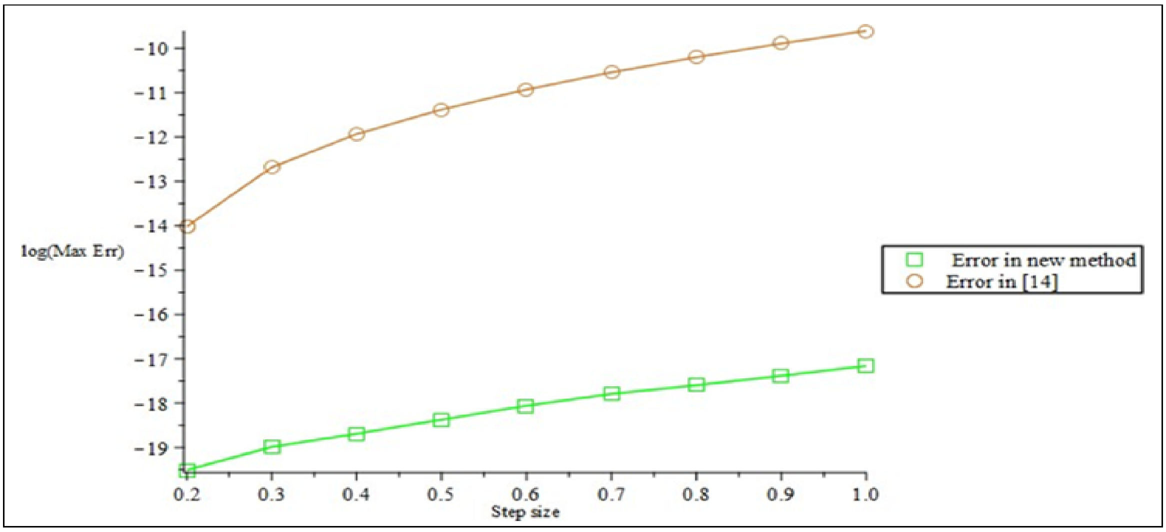

Fig 2 Efficiency Curve for Problem 2 Compared with Error in [14]

Table 3 Result of Problem 3, for h=0.0001

X	Exact Solution	Computed Solution	Error in the new method	Error in [15]
0.1	0.999999500016710	0.999999000016667	$5.49999900e-10$	$1.541367e-09$
0.2	0.999998000134000	0.999996000133333	$2.09999970e-09$	$2.976799e-09$
0.3	0.999995499778380	0.999991000449999	$4.64993300e-09$	$1.032860e-09$
0.4	0.999991999477330	0.999984000466663	$8.16984030e-09$	$2.447733e-09$
0.5	0.999987502109373	0.999975001889056	$1.27755651e-08$	$4.069112e-09$
0.6	0.999982003653984	0.999964003828369	$1.83811325e-08$	$1.337372e-08$
0.7	0.999975497241720	0.999951006433303	$2.49858426e-08$	$1.711910e-08$
0.8	0.999967995904010	0.999936009219869	$3.25609866e-08$	$1.388529e-08$
0.9	0.999959512423277	0.999919013807462	$4.12234072e-08$	$1.833084e-08$
1.0	0.999950017083162	0.999900019390904	$5.08845536e-08$	$4.286197e-08$

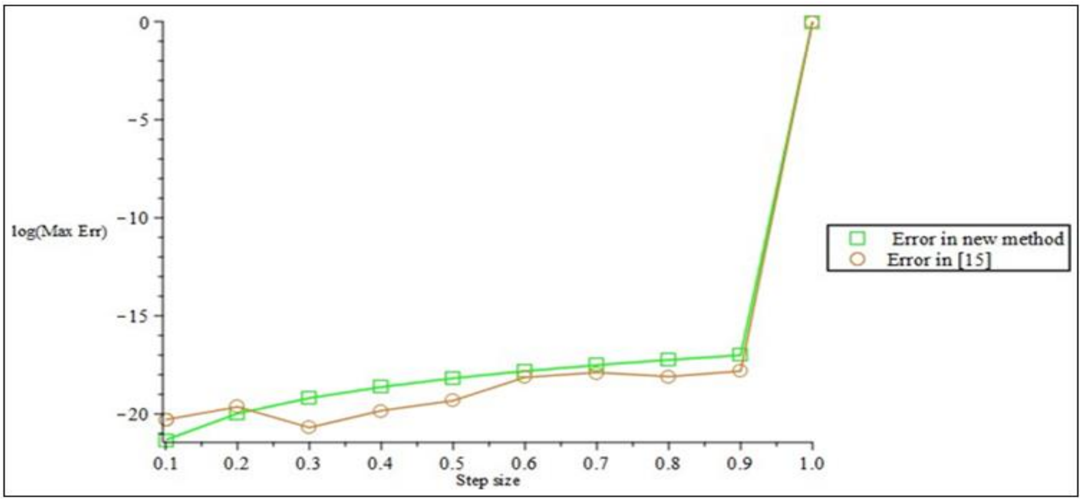

Fig 3 Efficiency Curve for Problem 3 Compared with Error in [15]

Table 4 Result of Problem 4, for h=0.025

X	Exact Solution	Computed Solution	Error in the new method	Error in [20]
0.01	0.0249973959147123	0.024997395910000	$4.7123e-12$	$1.50000e-08$
0.02	0.0499791692706783	0.049979169270000	$6.7830e-13$	$2.46000e-07$
0.03	0.0749297072727423	0.074929707270000	$2.7423e-12$	$1.37100e-06$
0.04	0.0998334166468282	0.099833416640000	$6.8282e-12$	$2.65200e-06$
0.05	0.1246747333852280	0.124674733374188	$1.1040e-11$	$1.28450e-05$
0.06	0.1494381324735990	0.149438132462708	$1.0891e-11$	$4.27070e-05$
0.07	0.1741081375935960	0.174108137576993	$1.6603e-11$	$8.31840e-05$
0.08	0.1986693307950610	0.198669330735944	$5.9117e-11$	$2.60181e-04$

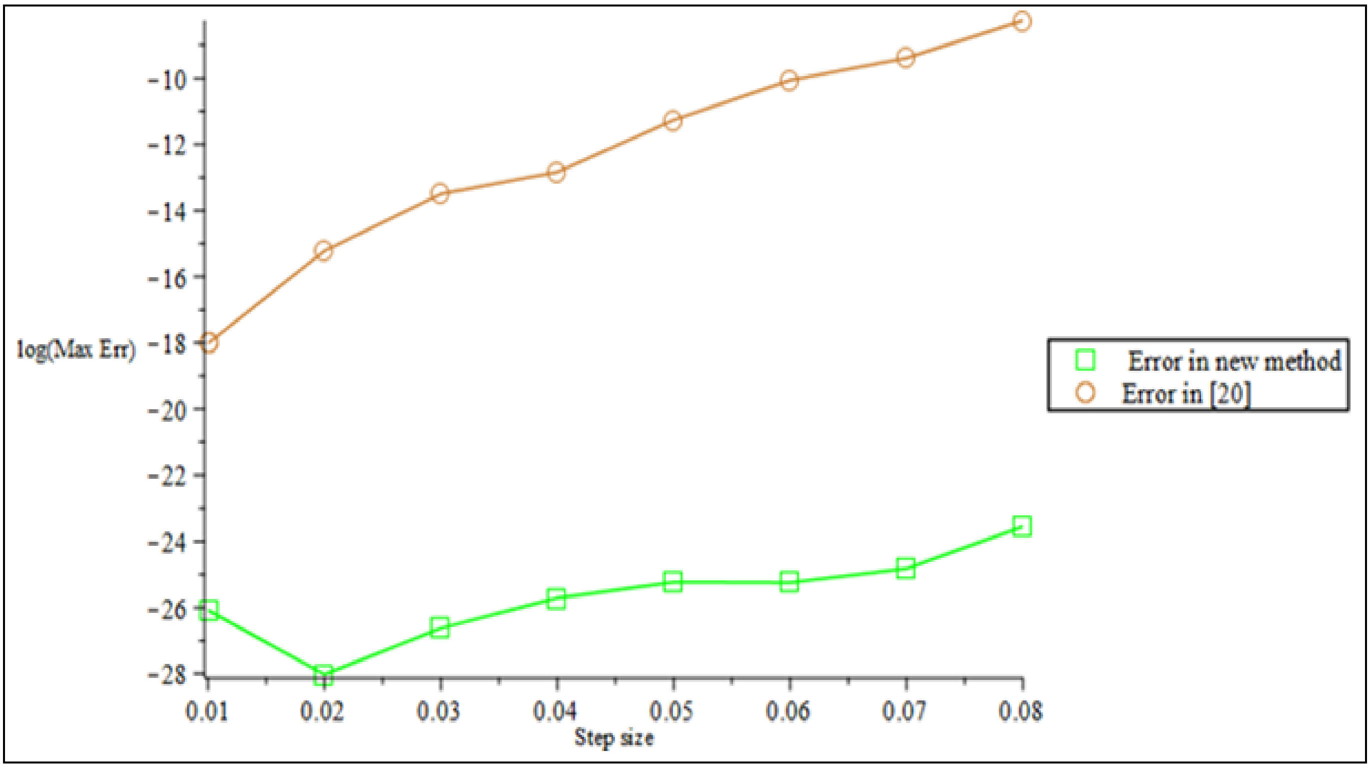

Fig 4 Efficiency Curve for Problem 4 Compared with Error in [20]

V. DISCUSSION OF THE RESULTS

In this paper, we have considered four numerical examples to test the efficiency of our method. The effect of decreasing mesh size was tested and the results showed that the smaller the mesh size, the better the accuracy. The new method gave better approximation because the proposed method requires self-starting.

VI. CONCLUSION AND RECOMMENDATION

➢ *Conclusion*

In this study, a class of continuous 4-step methods for second order ordinary dif- ferential equations are presented. A collocation and interpolation strategy was used to create the method. Predictor-corrector mode was used to implement the derived approach. It was found that our technique converges, zero stable, and is consistent. A few current methods have been compared to the results. The method performed better than the current methods, as shown by the comparison of errors.

➢ *Recommendation*

This paper considered a class of continuous 4-step method for solving second order ordinary differential equations. The method had been tested for usability and rec- ommended that the method be used for solving second order ordinary differential equations for efficiency and accuracy.

REFERENCES

[1]. Awari YS. (2013): Derivation and application of six-point linear multistep numerical method for solution of second order initial value problems. IOSR Journal of Mathematics;7:23-29.

[2]. Awoyemi DO. (2001): A new sixth-order algorithm for general second order ordinary differential equations. International journal of computer mathemat- ics.77:117-124.

[3]. Areo EA, Ademiluyi RA, Babatola PO (2008): Accurate Collocation Multistep method for integration of first order ODES.? J. Math. Stat. 2(1): 1-6.

[4]. Areo, E. O. and Omole, E. O.(2015) : Half - Step Symmetric Continuous Hybrid Block Method For Numerical Solution Of Fourth Order Ordinary Dif- ferential Equations.Archives of Applied Science Research. 7(10): 39-49

[5]. Awoyemi DO.(2005): An algorithm collocation methods for direct solution of special and general fourth-order initial value problems of ordinary differential equations. Journal of Nig. Math. Phy.;6:271-238.

[6]. Awoyemi DO (2003): A P-Stable Linear Multistep method for solving General third order Ordinary differential equations. Int. J. Comput. Math. 80: 8.

[7]. Awoyemi, D.O., Kayode, S.J. and Adoghe, L.O.(2015): A Six-Step Contin- uous Multi step Method For The Solution Of General Fourth Order Initial Value Problems Of Ordinary Differential Equations. Journal of Natural Sci- ences Research, 5(5): ISSN 2224 - 3186 (Paper) ISSN 2225-0921 (Online)

[8]. Dahlquist G (1978): On accuracy and unconditional stability of linear equa- tions. BIT 18: 133-136
[9]. Fatunla SO.(1988): Numerical methods for initial values problems in ordinary differential equations. New York: Academic press;
[10]. Kayode SJ.(2005): An improved numerov method for direct solution of gen- eral second order initial value problems of ordinary differential equations. In: Proceedings of the seminar organised by the national mathematical centre. Abuja. Nigeria;
[11]. Kayode SJ.(2008): An efficient zero stable numerical method for fourth order differential equation. International Journal of Mathematical Sciences.;1-10.
[12]. Kayode SJ.(2009): A zero stable method for direct solution of fourth order ordinary differential equations. American Journal of Applied Sci- ences;5(11):1461-1466.
[13]. Kayode SJ.(2010): A zero-stable optimal order method for direct solution of second order differential equations. Journal of Mathematics and Statis- tics.;6(3):367- 371
[14]. Kayode, S.J and Adeyeye O. (2013): Two step hybrid methods for general second order differential equations. African Journal of Mathematics and Com- puter Sciences Research 6(10):191-196
[15]. Kayode and Adegboro (2018): Predictor-Corrector Linear Multistep Method for Direct Solution of Initial Value Problems of Second Order Ordinary Dif- ferential Equations . Asian Journal of Physical and Chemical Sciences, 6(1)
[16]. Kayode, S.J. and Obarhua, F.O. (2015): 3 Step y-Function hybrid Methods for Direct Solution of Second Order Initial Value Problems in Ordinary Differ- ential Equations. Theoretical Mathematics and Application, Vol. 12(1): 37-48
[17]. Lambert, J. D., (1973), Computational Methods in Ordinary Differential Equations. John Wiley, New York.
[18]. Lambert, J. D. (1991): Numerical Methods for ordinary differential equations John Wiley, New York.
[19]. Olabode, B.T.(2009): Six Step Scheme for the Solution of Fourth Order Ordi- nary Differential Equations. Pacific Journal of Science and Technology 10(1): 143-148.
[20]. Owolabi KM, Ademiluyi RA (2010): A three-step discretization scheme for direct solution of second order ordinary differential equations. J.Nig.Assoc.Math.Phys.,16:265-276.