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On the Other Forms of Near Perfect Number

Danielle Enriquez[1] and Eman C. Ahmad[2]
Department of Mathematics and Statistics College of Science and Mathematics Western Mindanao State University
Normal Road, Baliwasan, Zamboanga City, Philippines

Abstract:- In this paper, we introduce a new form of near perfect number where these numbers are the product of two numbers that are relatively prime and we investigate these results concerning the near perfect numbers with fixed redundant divisor.

Keywords:- *Perfect numbers, Near Perfect Numbers, Redundant divisors*

I. INTRODUCTION

Numbers have different classifcation and characterization. One of these numbers is the perfect number which is a positive number n which is equal to the sum of all its positive divisors excluding n itself[1].

Equivalently, $\sigma(n) = 2n$. In 1965, Sierpinski[4] defined that near perfect numbers are very special class of pseudo perfect numbers. In 2012, Pollack and Shevelev[1] introduced and defined the near perfect number n as the sum of all its positive divisors(excluding n), except for one of them. The missing divisor d is called the *redundant divisor* [1]. Equivalently, n is a near perfect number with redundant divisor d if $\sigma(n) = 2n + d$, where d is a proper divisor of n[1].

II. MAIN RESULTS

This section consists of new results concerning other forms of Near Perfect Number.

A. Lemma 2.1

If p and $2^{3p} - 3$ are primes then $(2^{3p-1}, 2^{3p} - 3) = 1$.

Proof: Assume to the contrary that $(2^{3p-1}, 2^{3p} - 3) = d > 1$. Then, $d|2^{3p-1}$ and $d|2^{3p} - 3$. Now, $d|2^{3p-1} \Rightarrow d \in \{2^5, 2^8, ..., 2^{3p-1}\} = A$ and $d|2^{3p} - 3 \Rightarrow d \in \{2^{3p} - 3\} = B$. Thus, $d \in A$ and $d \in B \Rightarrow d \in A \cap B$ which contradicts the fact that $A \cap B = \emptyset$. Hence, $(2^{3p-1}, 2^{3p} - 3) = 1$.

B. Theorem 2.2

If p and $2^{3p} - 3$ are primes then $n = 2^{3p-1}(2^{3p} - 3)$ is a near perfect number with redundant divisor 2.

Proof: Let $n = (2^{3p-1}, 2^{3p} - 3)$. Then,

$$
\begin{aligned}
d &= \sigma(n) - 2n \\
&= \sigma(2^{3p-1}(2^{3p} - 3)) - 2(2^{3p-1}(2^{3p} - 3)) \\
&= \sigma(2^{3p-1})\sigma(2^{3p} - 3) - 2(2^{6p-1} - 3(2^{3p-1})) \\
&= (2^{3p} - 1)(2^{3p} - 2) - 2^{6p} + 3(2^{3p}) \\
&= 2^{6p} - 3(2^{3p}) + 2 - 2^{6p} + 3(2^{3p}) \\
&= 2.
\end{aligned}
$$

C. Lemma 2.3

If $2^t - 9$ is a prime then $(2^{t-1}, 2^t - 9) = 1$.

Proof: Assume to the contrary that $(2^{t-1}, 2^t - 9) = d > 1$. Then, $d|2^{t-1}$ and $d|2^t - 9$. Now, $d|2^{t-1} \Rightarrow d \in \{2^3, 2^4, ..., 2^{t-1}\} = A$ and $d|2^t - 9 \Rightarrow d \in \{2^t - 9\} = B$. Thus, $d \in A$ and $d \in B \Rightarrow d \in A \cap B$ which contradicts the fact that $A \cap B = \emptyset$. Hence, $(2^{t-1}, 2^t - 9) = 1$.

D. Theorem 2.4

If $2^t - 9$ is a prime then $n = 2^{t-1}(2^t - 9)$ is a near perfect number with redundant divisor 8.

Proof: Let $n = 2^{t-1}(2^t - 9)$. Then,

$$
\begin{aligned}
d &= \sigma(n) - 2n \\
&= \sigma(2^{t-1}(2^t - 9) - 2(2^{t-1}(2^t - 9)) \\
&= \sigma(2^{t-1})\sigma(2^t - 9) - 2(2^{2t-1} - 9(2^{t-1})) \\
&= (2^t - 1)(2^t - 8) - 2(2^{2t-1} - 9(2^{t-1})) \\
&= 2^{2t} - 9(2^t) + 8 - 2^{2t} + 9(2^t) \\
&= 8.
\end{aligned}
$$

III. CONCLUSION

This paper is an extension of Xiao-Zhi Ren and Yong-Gao Chen's work [3] on the types of near perfect numbers. In this result, type 1 construction is considered where k = 1, t = 3p and k = 3, t with fixed redundant divisor.

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