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Mathematical Model of Male Circumcision in HIV/AIDS Preventions

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Abstract:- In this research, we have been able to establish the mathematical model of male circumcision in HIV/AIDS preventions. This could be a traditional ritual, religious rituals or medical procedure circumcision. The model is formulated under several assumptions.

The steady states were established, and shown that these steady states are stable if $(\sigma + \mu) > -B$ and $\sigma > -(\mu + v_c)$.

From the analysis of the model, we observed that if the sum of the rate of death due to natural incidence (μ) and that due to infection is reduced (v_c) while the rate of circumcision is increased, this will bring the reproduction number of infectious process to less than one and the epidemic will die out of the system.

Keywords:- *Circumcision, Male, HIV/AIDS, Dynamical System Theory, Equilibrium*

I. INTRODUCTION

Essentially, male circumcision is the surgical removal of all or parts of the foreskin of the penis. It can be carried out as part of a religious ritual, medical procedure or traditional /cultural ritual performed as recruitment into male adulthood.

Right from the 1980s over thirty (30) studies have been monitored in heterosexual men. These studies point to the fact that male circumcision can offer effective protection against the acquisition of HIV [1]. The main aim of mathematical modelling in HIV transmission is to predict population level outcome from individual level inputs (Susan C., et al (2008)) Some of the possible outcomes that can be studied with a model are; the number of time infection occur, the prevalence of infection or the doubling time of the epidemic. The most prominent outcome however is the probability of occurrence of the infection. That is, is there any transmission rate sufficient enough to sustain a chain of the infection? In classic epidemic theory, this outcome is seen as the reproduction numberR_0. of infectious process which is a simple summary statistics In a susceptible population the R_0. the expected secondary infections generated by the initial infected individuals. If $R_0 > 1$ an epidemic is expected to grow, if $R_0 < 1$ the infection is likely going to be eradicated [(Susan C., et al (2008))]

For complex epidemic like HIV/AIDS there is no established medical cure, it is discovered that HIV/AIDS may be eradicated provided that the net transmission rate of the infected individual is sufficiently reduced [(Susan C., etal (2008))].

In three different random but controlled clinical pilot carried out recently in some of the African countries suggested that when adult male with negative HIV status are circumcised, the risk of acquiring HIV infection through penile–vaginal sex is reduced drastically [Auvert , et al (2005), Bailey, et al. (2007), Gray,et al. (2007)]. In this different trial, up to two years of monitoring showed those adult males who were randomly circumcised got less incidence rate of infection, when compared with adult male who were uncircumcised. About a range of 51% to 60% reduction estimate was observed in the risk of HIV infection and the risk reduction ranged from 55% to 76% per-protocol estimates was also observed.

Circumcision of adult male has recently been shown to reduce the susceptibility of infection with HIV by approximately 60% [Joint United Nations Programme on HIV/AIDS (2007), Sullivan PS, et al. (2007)].

A research carried out in targeted sub-Saharan African countries, shows that outstanding numbers of adolescent males have had access to voluntary medical male circumcisions (VMMC) services of the number needed to reach adult male circumcision prevalence. Yet strategies must be employed to reach optimal circumcision prevalence needed to develop and maintain a reduced tempo in HIV incidence so that AIDS is no longer seen as a public health menace in the nearest possible future. Prominent among these strategies is mathematical modelling, because of optimal cost and ease of implementation. Some of the mathematical models used already are Decision Makers' Program Planning Tool Version 2.0 (DMPPT 2.0), the Actuarial Society of South Africa (ASSA2008) model, and the age structured mathematical (ASM) model. These models help countries to look at the possible effects on program impact and cost-effectiveness of considering specific sub-populations for voluntary medical male circumcision programmes, for instance, by client age, HIV-non negative status, risk group, and geographical location. The modelling also considers long-term maintenance strategies, such as adolescent and/or early infant male circumcision, to preserve voluntary medical male circumcision coverage gains achieved during rapid scale-up. [**Catherine Hankins, et al** (2016)],

Agwu, et al (2018) considered the mathematical model of a prey-predator fishery in a three patch aquatic environment when the predator and prey populations is harvested selectively. Attempt was made to use of numerical simulation to examine the stability behaviour and co-existence steady state solution in an interaction between prey and predator populations because of the variation of the harvesting effort qualitatively, when other model parameters are constant. The fraction of harvest and un-harvest resource biomass for prey and predator populations has been determined using this innovative simulation technique. The values of the maximum sustainable yield (MSY) and the corresponding populations' level were obtained explicitly. Presenting the solution of the model graphically, Some sort of control was pointed out so that the resource biomass is not exploited.

Udofia and Inyama (2012) considered the mathematical model of the impact of vaccination on the transmission dynamics of fowl pox in poultry, which is a system differential equation. The model was analysed using methods from dynamical system theory together with Routh Harwitz theorem. The local stability of the infection free equilibrium was established if the effective reproductive ratio in the presence of vaccination is less than one and unstable if it is greater than one. The condition for control was used to determine the critical proportion that must be vaccinated to attain immunity for fowl pox. The researchers demonstrated that fowl pox can be eradicated from the poultry through vaccination provided the critical proportion is attained .

Udofia, and Sampson (2014a) presented the Mathematical model in HIV/AIDS preventions that examine the impact of complacency. The model developed under useful assumptions resulted in a system of differential equations. Analysis of the model showed that 1 < (μ+ λ), is the condition of the stability infection free state, which is the rate of progression to AIDS plus the rate of natural death is greater than one. Furthermore the asymptotic stability of endemic equilibrium state was established. When this happens, the infection will be eradicated from the community/. For this to be realised there must be a limit on the rate of progression to AIDS; through sustained campaign against HIV/AIDS.

Udofia, and Inyama, (2013). In their work on mathematical structural strategy(delayed first intercoure) in HIVAIDS prevention, established that the steady states $(X^0, Y^0, Z^0\ S^0, I^0, A^0) = (0.\ 0,\ 0,0,0,0)$ *and* $\left(\frac{b_1}{a}.\ 1, \frac{m_2}{d_3}, \frac{b_2}{c}, \frac{\beta b_2}{cd}, \frac{\gamma\beta b_2}{cd\lambda})Y^0\right)$ of the system are stable if the rate of at which the under aged get mature into sexually active population is greater is greater than the rate at which they die. Also if the natural death rate of both the infectious individual and susceptible ones is greater than the rate of infection transmission in the affected community.

A discrete delay mathematical model for the epidemiology of fowl pox infection transmission was studied by Udofia, and Sampson (2014b) The model being a system discrete delay differential equations has delay parameter $\tau \geq 0$. They realised that the number of infected birds at lag from the epidemic inception date is constant. The stability of the system in the absence of lag at the disease free equilibrium and endemic equilibrium points was discovered. The system became unstable with increasing delay if the rate at which birds die with the rate at which infection transmission is taking place is less the rate at which the birds are brought into the system.

Udofia and Inyama (2012) looked at the mathematical model of the transmission dynamics of fowl pox infection in poultry. It describe the interaction between the susceptible and the infected birds which results in a system of ordinary differential equation. When control, which is the effort in applying chemoprophylaxis control u_1 and treatment control u_2 in birds with fowl pox is incorporated, the system became a system of ordinary differential equations with control. The optimal control problem here involves that in which the number of birds with latent and active fowl pox infections and the cost of treatment controls $u_1(t)$ and $u_2(t)$ are minimized subject to the model differential. This involved the number of birds with active and exposed fowl pox respectively as well as the cost of applying chemoprophylaxis control u_1 and treatment u_2 in birds with fowl pox. Optimality conditions and optimal effort necessary to reduce the transmission rate of fowl pox in the poultry were determined appropriately using Pontryagin's Maximum Principle.

For optimal ingredient allocation and mixing to be achieved, (Udofia and Etukudo 2019) proposed a mathematical model of efficient allocation of these ingredients. The problem of this allocation in the production of biscuit is modelled as a linear programming problem and analyzed using an invariant property based algorithm approach. An Optdesolver, a computer program was used to obtain the optimal solution to this problem. The algorithm obtained the optimal allocation of the available resources for the maximum profit of the modelled system

Udofia and Idungafa (2018) studied a system of differential equation that was formulated to describe the mathematical model of bacteria-nutrient harvesting in a cultivated habitat, with the assumption that the bacteria harvesting rate is constant. The researchers were able to establish that the product of the number of cells produced per unit of nutrient uptake with the maximum nutrient uptake per cell is constant given as $\boldsymbol{ln2} + \boldsymbol{h}$. On the assumption of the varied rate of harvesting of these bacteria, a system of equation was also formed. Seeking to establish the stability of the system, from analysis it was discovered that, the first steady state was unstable while the second was globally asymptotically stable if the carrying capacity of the habitat has a lower limit , showing the ratio of the harvesting coefficient of the bacteria, cost per unit effort per unit price of the bacteria.

Udofia et al,(2016) developed a deterministic model to demonstrate that media coverage on giving prompt report about the number of infections of fowl pox can have great impact in reducing and subsequently eradicating fowl pox in poultry. The equilibrium stability of the model at both disease free equilibrium and endemic equilibrium were determined. The reproductive ratio R_0 - called the threshold parameter was analytically gotten and used to discuss he local stability of the disease free equilibrium.

ONUOHA, et al,2014 described the transmission dynamics of swine flu among swine and humans with the vaccination of newborns in differential equations. The model assumed a vaccine with a life-long immunity. Analysing the model at Disease-free Equilibrium (DFE) shows that it will be stable if there is a limit on the rate of transmission from swine to swine (βs) and the rate of transmission from human to human (βH). Also analysing the model at Endemic Equilibrium (EE) shows that the disease will persist if there is a lower limit on the rate of transmission from swine to swine (βS) and on the rate of newborn babies vaccinated (VH).

Udofia and Inyama (2011) looked at two models that examines the dynamics of transmission of fowl pox infection persistence within birds according to the medium of transmission. Analysing the system at equilibrium state the first model establishes the stability of the system at infection free equilibrium if $\alpha N < (d_1 + \mu + \gamma), \beta < \gamma$. If $\beta - \gamma < \frac{\alpha(d_1+\mu+\gamma)}{k}$.then the asymptotic stability of the endemic equilibrium state is established. That is fowl pox can be eradicated from the poultry if the rate at which the susceptible birds become infectious γ is less than the rate at which the susceptible birds β are brought into the poultry .It was also discovered that $R_0 < 1$ if $S_0 > S_c$ where $S_c = \frac{(d_1+\mu+\gamma)}{\alpha}$ $R_0 = \frac{\gamma S_0(d_1+\mu+\gamma)}{\alpha}$. The study of the second system establishes stability the birds recovery rate and mortality rate of mosquito are above average Also if the death rate of mosquito is greater than the growth rate of mosquito

In this work, we seek to formulate and analyse the mathematical model of male circumcision in HIV/AIDS preventions resulting in differential equations which investigate the effect and dynamics of male circumcision on HIV/AIDS preventions.

II. MODEL FORMULATION

A. *Assumptions and Parameters*

B. *Model Assumptions*

- We assume that there is a proportionate recruitment rate of individuals into the heterosexual population.
- There is proportionate rate of circumcision of both the susceptible and infected individuals.

C. *Model Parameters*

$S_c(t)$ = Number of susceptible individuals that are circumcised at time t , $t > 0$

$S_{nc}(t)$ = Number of susceptible individuals that are not circumcised at time t , $t > 0$

$S(t) = S_c(t) + S_{nc}(t) =$ Susceptible population at time t , $t > 0$

$I_c(t)$ = Number of infected individuals that are circumcised at time t , $t > 0$

$I_{nc}(t)$ = Number of infected individuals that are not circumcised at time t , $t > 0$

$I(t) = I_c(t) + I_{nc}(t) =$ infected population at time t , $t > 0$

$N = S(t) + I(t) = S_c(t) + S_{nc}(t) + I_c(t) + I_{nc}(t) =$ total population under the

b = Recruitment rate into the population

μ = Natural death rate of the population

$V_c =$ Death rate of circumcised infected individuals

$V_{nc} =$ Death rate of uncircumcised infected individuals

$\sigma = \mathit{The\ rate\ at\ which\ susceptible\ indiduals\ are\ being\ circumcised}$

$\rho = \mathit{The\ rate\ at\ which\ infected\ individuals\ are\ being\ circumcised.}$

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$\beta\ =\ The\ probability\ of\ transmission\ by\ individuals\ in\ class\ \mathrm{I}_{nc}$

$\alpha\ =\ The\ probability\ of\ transmission\ by\ individuals\ in\ class\ \mathrm{I}_{c}$

$c\ =\ Average\ number\ of\ contact\ or\ partners\ per\ unit\ time$

$c\beta\ and\ c\alpha\ are\ net\ transmission\ of\ individuals\ in\ class\ \mathrm{I}_{nc}\ and\ \mathrm{I}_{c}$ respectively

D. The Model Equations

The combination of the above assumptions and parameters result in the following model equation for male circumcision in HIV/AIDS preventions.

$$\frac{dS_c(t)}{dt} = \sigma S_{nc}(t) - B(t)S_c(t) - \mu S_c(t)$$

$$\frac{dS_{nc}(t)}{dt} = bN - B(t)S_{nc}(t) - \sigma S_{nc}(t) - \mu S_{nc}(t)$$

$$\frac{dI_c(t)}{dt} = B(t)S_c(t) - (\mu + v_c)I_c + \sigma I_{nc}(t)$$

$$\frac{dI_{nc}(t)}{dt} = B(t)S_{nc}(t) - (\mu + v_{nc})I_{nc} - \sigma I_{nc}(t)$$

Where

$$B(t) = \frac{c\beta I_{nc}(t) + c\alpha I_c(t)}{N} = incidence\ rate\ of\ infection$$

III. MODEL ANALYSIS

The model is analysed using dynamical system theory.

At equilibrium

$$\frac{dS_c(t)}{dt} = \frac{dS_{nc}(t)}{dt} = \frac{dI_c(t)}{dt} = \frac{dI_{nc}(t)}{dt} = 0$$

This implies that

$$\sigma S_{nc}^0(t) - B(t)S_c^0(t) - \mu S_c^0(t) = 0 \quad \dots\dots\dots\dots\dots\dots (4.1)$$

$$bN - B(t)S_{nc}^0(t) - (\sigma + \mu)S_{nc}^0(t) = 0 \quad \dots\dots\dots\dots\dots\dots (4.2)$$

$$B(t)S_c^0(t) - (\mu + v_c)I_c^0 + \sigma \mathrm{I}_{nc}^0(t) = 0 \quad \dots\dots\dots.. \quad (4,3)$$

$$B(t)S_{nc}^0(t) - (\mu + v_{nc})I_{nc}^0 - \sigma I_{nc}^0(t) = 0 \quad \dots\dots\dots\dots. (4.4)$$

Let $\sigma + \mu = K_1$ $\mu + v_c = K_2$, $\mu + v_{nc} = K_3$. then (4.1), (4.2), (4.3), (4.4) becomes

$$\sigma S_{nc}^0(t) - B(t)S_c^0(t) - \mu S_c^0(t) = 0 \quad \dots\dots\dots\dots\dots\dots \ (4.5)$$

$$bN - B(t)S_{nc}^0(t) - K_1 S_{nc}^0(t) = 0 \quad \dots\dots\dots\dots\dots\dots.. \quad (4.6)$$

$$B(t)S_c^0(t) - K_2 I_c^0 + \sigma \mathrm{I}_{nc}^0(t) = 0 \quad \dots\dots\dots.. \quad (4,7)$$

$$B(t)S_{nc}^0(t) - K_3 I_{nc}^0 - \sigma I_{nc}^0(t) = 0 \quad \dots\dots\dots\dots. \quad (4.8)$$

Where $S_c^0(t), S_{nc}^0(t), I_c^0(t), I_{nc}^0(t)$ are steady state for $S_{nc}, S_c, I_{nc}, I_c,$ respectively.

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From (4.6) $S^0_{nc} = \frac{bN}{B+K_1}$, From (4.5) $S^0_c = \frac{\sigma bN}{(B+K_1)(B+N)}$,

From (4.8) $I^0_{nc} = \frac{BbN}{(B+K_1)(K_3+\sigma)}$ From (4.7) $I^0_c = \frac{B\sigma bN[(K_3+\sigma)+(\beta+\mu)]}{(B+K_1)(B+N)(\beta+\mu)K_2}$

Hence, the steady value of the system is

$$(S^0_c(t), S^0_{nc}(t), I^0_c(t), I^0_{nc}(t)) =$$

$$\left(\frac{\sigma bN}{(B+K_1)(B+N)}, \frac{bN}{B+K_1}, \frac{B\sigma bN[(K_3+\sigma)+(\beta+\mu)]}{(B+K_1)(B+N)(\beta+\mu)K_2}, \quad \frac{BbN}{(B+K_1)(K_3+\sigma)}\right)$$

We obtain the Jacobian matrix of the system as

$$J = \begin{pmatrix} -(B+K_1) & 0 & 0 & 0 \\ 0 & -(B+\mu) & 0 & 0 \\ 0 & B & -K_2 & \sigma \\ B & 0 & 0 & -(K_2+\sigma) \end{pmatrix}$$

$$|J-\lambda I| = \begin{vmatrix} -(B+K_1)-\lambda_1 & 0 & 0 & 0 \\ 0 & -(B+\mu)-\lambda_2 & 0 & 0 \\ 0 & B & -K_2-\lambda_3 & \sigma \\ B & 0 & 0 & -(K_2+\sigma)-\lambda_4 \end{vmatrix}$$

$$|J-\lambda I| = -(B+K_1)-\lambda_1 \begin{vmatrix} -(B+\mu)-\lambda_2 & 0 & 0 \\ B & -K_2-\lambda_3 & \sigma \\ 0 & 0 & -(K_2+\sigma)-\lambda_4 \end{vmatrix}$$

This implies that

$\lambda_1 = -(B+K_1), \lambda_2 = -(B+\mu), \lambda_3 = -K_2, \lambda_4 = -(K_2+\sigma)$

We recall that $\sigma+\mu = K_1$ $\mu + v_c = K_2$

$$\lambda_1 < 0 \Rightarrow -(B+\sigma+\mu) < 0$$

$$\Rightarrow (\sigma+\mu) > -B$$

That is, the sum of the natural death rate and the rate of circumcision must be greater than the negative of incidence rate.

$$\lambda_4 < 0 \Rightarrow -(K_2+\sigma) < 0, \quad -(\mu+v_c+\sigma) < 0$$

That is, $-(\mu+v_c) < \sigma, \Rightarrow \sigma > -(\mu+v_c)$

This implies that the rate of circumcision is greater than the sum of the rate of natural death and death due to infection.

Since all the real part of the eigenvalues of the system are negative, then the system is stable.

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IV. CONCLUSION

In this research, we have been able to establish the mathematical model of male circumcision in HIV/AIDS preventions. This could be a traditional ritual, religious rituals or medical procedure circumcision. The model is formulated under several assumptions.

The steady state were establish to be

$$(S_c^0(t), S_{nc}^0(t), I_c^0(t), I_{nc}^0(t)) = \left(\frac{\sigma bN}{(B+K_1)(B+N)}, \frac{bN}{B+K_1}, \frac{B\sigma bN[(K_3+\sigma)+(\beta+\mu)]}{(B+K_1)(B+N)(\beta+\mu)K_2}, \frac{BbN}{(B+K_1)(K_3+\sigma)}\right)$$

It was shown that these steady states are stable if $(\sigma+\mu) > -B$ and $\sigma > -(\mu+v_c)$

From the analysis of the model, we observed that if the sum of the rate of death due to natural incidence (μ) and that due to infection is reduced (v_c) while the rate of circumcision is increased, this will bring the reproduction number of infectious process to less than one and the epidemic will die out of the system.

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