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Temperature, Velocity and Pressure Gradient Distribution of Oil Production in South West Cameroon using a Mixed Convection Heat Transfer in a Horizontal Tube

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Abstract:- A mixed convection heat transfer in an horizontal tube with uniform wall heat flux is explored in this works. Both the viscous dissipation and the buoyancy effect have been taken into account. The momentum balance equation and the energy balance equation have been derived and solved. These solution have been apply in the case of horizontal well ≪ BTMO40H ≫ of the oil fluids production at the Rio Del Rey region in South West Cameroon. The effect of the temperature, velocity distribution and pressure gradient on the debit of production have thus been derived and analyzed according to the physical configuration of the well in other to increase the rate of production.

Keywords: *Convection, Horizontal Tube, Temperature, Velocity, Pressure Gradient.*

I. INTRODUCTION

Convection heat transfer in ducts, combined force and free flows have been investigated by many authors for different geometry and boundary conditions, with reference to vertical or inclined ducts [1-4]. They attracted wide attention because, they provide excellent platform in many technological application such as the design of heat exchangers, of nuclear reactors or the cooling systems for electronic devices. In recent years analytical and experimental solution with circular and non-circular cross sections for the case of combined forced and free convection in circular duct have been studied extensively [5,6]. For instance, Choudhury et al [7] analyse the simultaneously developing laminar flow and heat transfer in an inclined pipe, with a boundary condition given by uniform wall temperature. The governing equation are solved numerically. The axial evolution of the velocity profiles as well as the Nusset number and friction factor are reported. In the same line Many others papers consider horizontal pipes, with boundary condition either of uniform wall temperature (see ref[8-10]) According to the case of vertical tube Mansour et al [11] studied analytically combined forced and free convection with uniform internal heat generation. A power series expansion of the axial velocity has been used to solve the coupled partial differential equations for fully-developed mixed convection in a vertical tube with uniform heat flux at the wall, uniform internal volumetric heat source and viscous dissipation. Explicit analytical expressions have been obtained for the temperature and velocity profiles as well as for the axial pressure gradient. They were satisfactorily validated for the case of forced convection with both heat source and dissipation, and also for cases of mixed convection with heat source and with or without dissipation. In this paper, these results have been used to analyse the physical characteristics of the laminar mixed convection of the oil flow in a horizontal well in production, in order to evaluate the influence of the temperature on the flow of oil evacuated at the end of the production. The case taken into consideration here is the horizontal well ≪ $BTMO40H$ ≫ of the oil fluids production at the Rio Del Rey region in South West Cameroon.

The second part of this paper deals with the derivation of the momentum equation and energy balance equations taking into account the horizontal tube. In the third part, the analytical solution of classical problem is revisited. The temperature and the velocity of the profiles and the pressure gradient are presented taking into account the presence of the viscous dissipation. The fourth path deals with the application of the obtained solution to analyse the evolution of all theses physical characteristics taking into account the real parameter of an existing horizontal well and their effect on the debit of oil production in that zone. The fifth part is devoted to the conclusion.

List of symbols	Greek symbols
A (Km^{-1}) axial temperature gradient	α integer
C_p ($Jkg^{-1}K^{-1}$) specific heat of fluid	β Thermal expansion coefficient
Br ($\mu v_m^2/(kDA)$) Brinkman number	ΔT Reference temperature difference
Gr($\rho_0 g\beta(AD)D^3/\mu^2$) Grashof number	θ dimensionless temperature
k ($Wm^{-1}K^{-1}$)Thermal conductivity	μ dynamic viscosity
n non negative integer	v kinetic viscosity
P (Pa) Pressure	ρ mass density
m (kgs^{-1}) Mass flow rate	ρ_0 Reference mass density
Pe ($RePr$) Peclet number	φ tangential coordinate
Pr ($\mu C_p/k$) Prandtl number	**Superscripts and subscripts**
q_w (Wm^{-2}) wall heat flux	* alternative dimensionless quantities
R_e ($\rho_0 v_m D/\mu$)Reynolds number	b bulk condition
r(m) radial coordinate	w wall
R_0(m) radius of the pipe	w_0 wall condition at the reference location
R dimensionless radius D(m) Tube diameter T (K) Temperature T_0 mean temperature v (ms^{-1})velocity component v_0 Mean value of velocity v^* dimensionless velocity Z axial coordinate z dimensionless axial coordinate	0 reference condition

➢ *General Mathematical Formalism*

We consider in the framework of this work a horizontal well reduced to its column of production (modelled by a rectilinear and cylindrical vertical tube) of length L and of radius R. Figure 1 represents a longitudinal section of the column. The physical problem required the use of classical equation deducted from the momentum balance equation and the energy balance equation. The fluid here is a Newtonian fluid which steadily flows in the horizontal tube with circular cross section and radius R_0. The flow is assumed to be laminar and parallel. The z axis of the cylindrical coordinate system coincides with the tube axis and increases in the direction of flow. The flow fields is assume asymmetric and parallel to the z axis. The z axis is parallel to the gravitational acceleration g, but with opposite direction. A uniform wall heat flux is prescribed, so that both v and T depend only R and z. The continuity equation implies that the velocity fields is solenoidal,

i.e. $\frac{\partial v_z}{\partial z} = 0$. According to the Boussinesq approximation, the density is assumed to depend linearly on temperature namely

$$\rho = \rho_0 \left[1 - \beta (T - T_0)\right] \quad (1)$$

where is T_0 the wall temperature at the reference position (z = z_0) that is within the fully developed region under consideration. We suppose, even if the reference temperature T_0 depends on z coordinate, the thermo-physical properties ρ_0, β, c_p and μ involved in the momentum and energy balance equation are treated as constants.

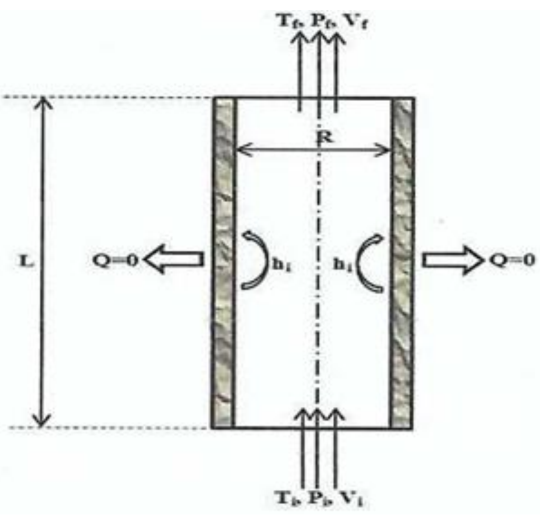

Fig 1 Schematic View of the System

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Taking into account all this consideration, the momentum balance equation is derived and given by

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)-\frac{dP}{dz}-\rho_0 g\left[T-T_0\right]=0 \tag{2}$$

thus the velocity distribution can be determined by solving eq(2). Moreover the difference between the local and the corresponding bulk fluid temperatures is a function of the radial position. Meaning that

$$\frac{\partial T}{\partial z}=\frac{dT_p}{dz}=\frac{dT_m}{dz}=A \tag{3}$$

Where A is the gradient of the axial temperature. T_b is the wall temperature and T_m is the mean value of the temperature. Taking into account the viscous damping and applying the first law thermodynamics principle to a control volume bounded by the wall fluid interface and two cross sections at z and $z+dz$ yields

$$mC_p\frac{dT_b}{dz}=q_p\pi D+\mu\int_0^{\frac{D}{2}}\left(\frac{dv_z}{dr}\right)^2 2\pi r dr \tag{4}$$

where q_p is the wall heat flux. As a consequence of the fact that the terms on the right -hand side of eq.(5) are all independent of z, the temperature in the axial direction is constant. Thus, the axial temperature verify the following assumption

$$T(r,z)-T_{po}=T(r,z)-T_p(z)-T_{po}=f(r)+A(z-z_0) \tag{5}$$

Eq (2) thus becomes

$$\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\beta\rho_0 g_z\left[T-T_p\right]=\frac{d\left[P+\beta\rho_0 z-\rho g A_z\left(0.5z-z_0\right)\right]}{dz}=\frac{\partial P^*}{\partial P} \tag{6}$$

Where P^* is the modify pressure. After some mathematical algebraic one derive the energy equation in the fully developed region given by

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{d}{dr}\left(T-T_p\right)\right)+\mu\frac{dv_z}{dr}=\rho_0 C_P v_z A \tag{7}$$

The conservation equations (6) and (7) constitute a system of coupled ordinary differential equations characteristics of the laminar flow fully developed with the presence of natural convection and viscous damping. The following boundary conditions will be used to solved these equations

-at $r=0$

$$v_z=0,\frac{\partial v_z}{\partial r}=0,\frac{dT}{dr}=0 \tag{8}$$

-at $r = 0.5D$

$$v_z=0,\frac{\partial v_z}{\partial r}=0,\frac{dT}{dr}=q_p \tag{9}$$

-and overall the cross-section area, the global conservation of the mass stipulate that

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$$Q_e = Q_s \Rightarrow 8\int_0^{0.5D} v_z r dr = V_m D^2 \quad (10)$$

these equations can be rewritten in a non-dimensional form as follows

$$P_e = 4\frac{q_p}{KA} + 8Br\int_0^{0.5}\left(\frac{dv_z}{dR}\right)^2 RdR \quad (11)$$

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{dv_z}{dR}\right) + \frac{Rr^*}{R_e}\theta^* = R_e\frac{dP^*}{dz} \quad (12)$$

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\theta^*}{dR}\right) + Br^*\left(\frac{dv_z}{dR}\right)^2 = P_e v_z \quad (13)$$

Where $Gr^* = \frac{\rho_0^2 g_z \beta(AD)D^3}{\mu^2} = Gr\frac{kA}{q_p}, R_e = \frac{\rho_0 V_m D}{\mu}, \Pr = \frac{\mu c_p}{k}, Br^* = \frac{\mu V_m^2}{kDA} = Br\frac{q_p}{kA}$ by solving these set of equation we will be able to determine the general profile of temperature and velocity along with the pressure gradient.

- *Solution in the Case of Mixed and Forced Convection with Viscous Damping*

- *Case of Forced Convection*

In this case the velocity profile don't depend of the temperature. Taking into account this fact, after some mathematical algebra and taking into account the boundary conditions we obtain de the well-known parabolic profile given by

$$R_e\left(-\frac{dp}{dz}\right) = 32 \quad (14)$$

and the

$$V_z = 2\left(1-4R^2\right) \quad (15)$$

using this expression one can derive the quantity

$$P_e = 4\left(\frac{q_p}{kA}\right) + 32B_r^* \quad (16)$$

To obtain the temperature profile this expression is insert into equation (13). Substitute the parabolic velocity profile and after some integration and application to the boundary conditions at R = 0 and R = 0.5 yields

$$\theta^* = \left(0.25 - R^2\right)\left[16B_r^*\left(0.25 - R^2\right) - 0.5P_e\left(0.75 - R^*\right)\right] \quad (17)$$

These set of equations can be used to have the temperature profile, the velocity and the pressure gradient. It appears that they are functions of independent number, named Peclet (P_e) and Brinkman (B_r^*)

- *Case of Mixed Convection*

In this case the differential equation are coupled. Iqbal et *al* [12] proposed a solution in the case where Q_s = 0. They combine three different method to solve these equations. Substituting θ^* into eq. (10). We leads to the following fourth order nonlinear differential equation for the axial velocity

$$v_z^{(IV)} + 2\frac{v_z^{(III)}}{R} - \frac{v_z^{(II)}}{R^2} + \frac{v_z^{(I)}}{R^3} - \frac{Gr^*}{R_e} B_r^* \left(v_z^{(I)}\right)^2 + P_e \frac{G_r^*}{R_e} v_z = 0 \qquad (18)$$

Thus the velocity profile can be expressed as a power series in terms of the radial coordinate as follows

$$v_z = \sum_{i=0}^{\infty} \alpha_i R^i \qquad (19)$$

by substituting the expression of v_z given by eq(19) into eq(18), we will obtain a serie of power of *R*. Using the asymmetric conditions one obtain the following expressions

$$\alpha_4 = \frac{-\frac{G_r^*}{R_e} P_e \alpha_0}{4^2 2^2} \qquad (20)$$

$$\alpha_6 = \frac{-\frac{G_r^*}{R_e} P_e \alpha_2 + 4B_r^* \frac{G_r^*}{R_e} P_e \alpha_2^2}{6^2 4^2} \qquad (21)$$

For $m \succeq 4$, the coefficients are given by this general expression

$$\alpha_{m+4} = \frac{-P_e \frac{G_r^*}{R_e} \alpha_m + B_r^* \frac{G_r^*}{R_e} \sum_{k=1}^{m+2} k\alpha_k \left(m+4-k\right) \alpha\left(m+2-k\right)}{\left(m+4\right)^2 \left(m+2\right)^2} \qquad (22)$$

Thus, all the coefficients are functions of the values of α_0 and α_2. These two unknown values, can be determined by applying the no-slip condition (12) and the integral mass balance eq(13). These yields

$$\alpha_0 + \sum_{i=1}^{\infty} \frac{\alpha_{2i}}{\alpha^{(2i)}} = 0 \qquad (23)$$

And

$$\sum_{i=1}^{\infty} \frac{\alpha_{2i}}{\left(2i+2\right) 2^{2i+2}} = \frac{1}{8} \qquad (24)$$

which can be solved numerically to determine α_0 and α_2. Hence, the distribution of temperature can be determine using this expression

$$\theta^* = \frac{R_e}{G_r^*} \left[R_e \frac{dD}{dz} - \sum_{i=1}^{\infty} \left(2i\right)^2 \alpha_{2i} R^{2i-2} \right] \qquad (25)$$

since $\theta^* = 0$ at $R = 0.5$ the pressure gradient can be deducted and given by

$$R_e \frac{dP}{dz} = \sum_{i=1}^{\infty} \frac{\left(2i\right)^2 \alpha_{2i}}{2^{2i-2}} \qquad (26)$$

the ratio $\frac{q_w}{KA}$ can be evaluated from eq(12).

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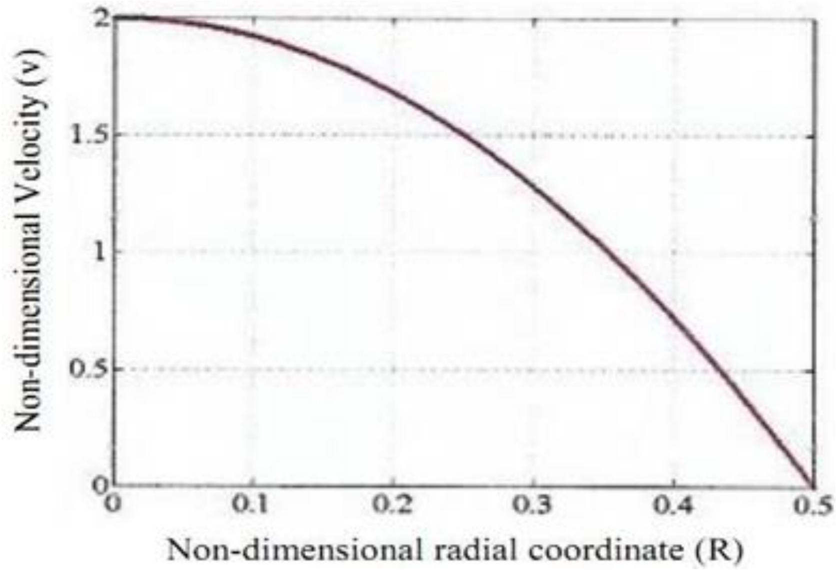

Fig 2 Profile of the Velocity

➢ *Application to the Case of Horizontal Well "BTMO40H"of Rio Del Rey Region in South West Cameroon*

In order to predict the influence of the discharged oil flow on the temperature, velocity and pressure distributions in a horizontal well in production, we will apply to the previous general expression of the horizontal well ≪ *BTMO*40*H* ≫ situated at the Rio Del Rey in the southwest region of Cameroon.

The data given by that well are grouped in the table below:

Table 1 Characteristics Parameters of the Horizontal Eell ≪*BTMO*40*H*≫

Tank	Permeability	1350*MD*
	Bottom pressure	132.5*bar*
	Bottom temperature	73°*C*
Well	Depth	3350*m*
	Diameter tubing	3"1/2
	Type of completion	Open hole
	Drain	500*m*
	Duration of production	16*years*
Fluids	Density	759kg/m^3
	Viscosity	0.810C_p
	Bubble pressure	133.59*bar*
	Specific heat	19W/m^2

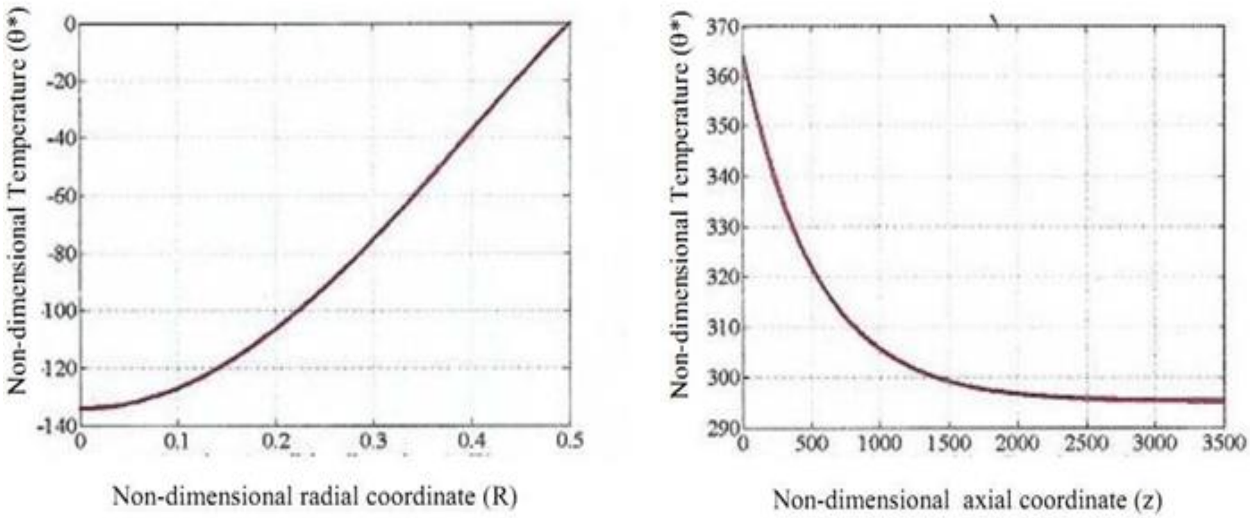

Fig 3 Profile of the Temperature

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Taking into account all these parameters, we plotted in figure 2 the evolution of the velocity of fluids as function of the non dimensionless radial coordinate parameter *R*. It appears that as the radial coordinate increased the velocity decrease and vanish at R = 0.5. It should be noted that this value of R corresponds to the position of the walls of the horizontal well. For this purpose we can conclude that the oil will move more quickly in the center of the well than its edges. It is therefore clear that oil production will be faster and more optimal by putting the extraction devices at the center than at the walls of the well. The evolution

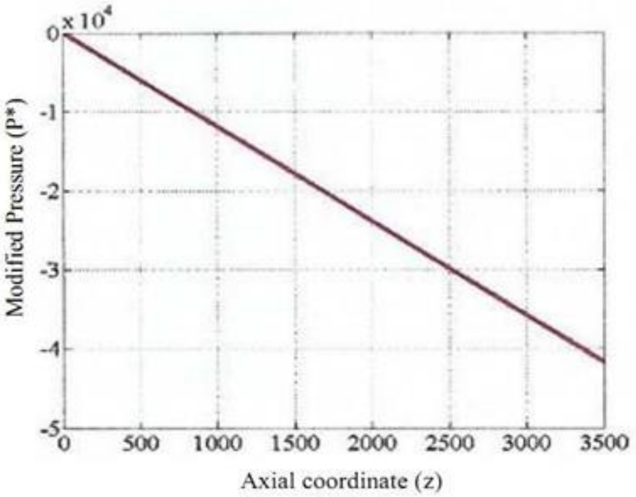

Fig 4 Profile of the Modified Pressure Gradient

of temperature as function of radial and axial coordinates is display in figure 3. Taking into account the case of radial coordinate (figure 3a) we notice that there is a positive gradient of temperature. In other words, as the radial component increases, the temperature decreases. Physically this implies that on a flat section of the horizontal well, the oil will be warmer to the walls of the well than to its center. In the case where axial coordinate varies (see figure 3b), we notice a drop of about 3^0C every $100m$ in height and this cooling will necessarily lead to the precipitation and the deposit of organic compounds (paraffin) on the walls of the well and will thus impact the debit of the evacuated oil flow.

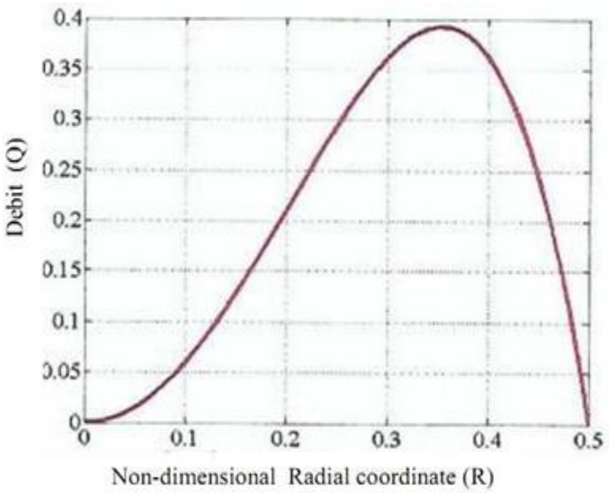

Fig 5 Spatiale Profile of the Debit of Petrol

According to the spatial profile of the debit of oil, in figure 4 we show that the debit Q increased when the radial position is situated between 0 and 0.35 and then decrease until R = 0.5. Thus the maximum production is obtained for a quantity of fluids situated in the circle of radius R = 0.35. Taking a look on the evolution of the debit as a function of the velocity, we realise that the debit increase when the velocity is situated between 0 and 1.0 and decrease for a velocity situated between 1 and 2.0.

We can conclude that oil production will be very important when the fluid will move with a dimensionless speed in the vicinity of 1. The effect of the temperature on the debit is also analyse (see figure 5). The flow rate increases for low temperatures and decreases for slightly elevated temperatures. Flow is important for a non dimensionless temperature around −50. Thus the oil production will be accelerated when the fluid is at this temperature

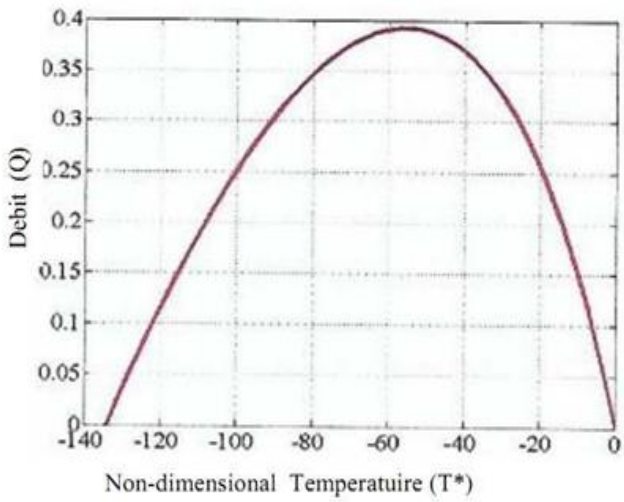

Fig 6 Profile of the Debit as a Function of the Temperature

❖ *Declaration of Competing Interests:*

Authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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II. CONCLUSION

The purpose of this paper was to analyze the combined and free convection of a Newtonian flows in a horizontal duct with circular cross section. The flows has been considered laminar and parallel, and the effects of viscous dissipation in the fluid have been taken into account. A boundary condition of prescribed and uniform wall heat flux has been assumed. The momentum balance equation of and the energy balance equation have been written in a dimensionless form and solved. The solutions obtained allowed us to determine the general expressions of the velocity and temperature profiles of the flow, as well as the pressure gradient. We have therefore applied to these general expressions the characteristic parameters of the horizontal well ≪ *BTMO*40*H* ≫ in order to analyse their influence on the production. In view of the different results obtained, we can conclude that in the vicinity of the walls of the well, the average temperature of the oil is substantially equal to the temperature of the wall. The fluid velocity is almost zero. It then develops, in the near walls of the hydrodynamic and thermal boundary layers due to the paraffin deposits along the pipes and causing the reduction of the Tubing diameter. This phenomenon leads to loss of charges leading to decreases in the flow of oil evacuated.

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