A Study on the Linear Algebra

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Abstract:- Linear algebra is an initial part of mathematics. It is a major branch of arithmetic related to the mathematical systems underlying the operations of addition and scalar multiplication, consisting of the principles of construction of linear equations, matrices, determinants, vector surfaces, and linear transformations. Linear algebra is a mathematical subject that deals with vectors and matrices, and more generally with vector surfaces and linear transformations. Unlike the other components of arithmetic, which are often fueled by new ideas and unsolved problems, linear algebra is very easy to understand. Its value lies in its many applications from mathematical physics to modern algebra and its use in engineering and science along with image processing and analysis.

Keywords:- *Linear Algebra; Addition; Scalar; Multiplication; Linear Equations; Matrices; Determinants; Vector Spaces;*

I. INTRODUCTION

Linear algebra is a must-read course for a wide variety of students, at least for a reason. First, such large-scale studies in various fields of mathematics, multivariate analysis, differential equations, probability in all fields of physics, biology, chemistry, economics, finance, psychology, sociology, and engineering Few subjects can claim to have a program. Second, this difficulty provides her sophomore year with an excellent opportunity to find ways to work with summary concepts. Linear algebra is one of the most recognized mathematical disciplines due to its rich theoretical underpinnings and many useful programs for technology and engineering. Solving the structure of linear equations and computing determinants are examples of important linear algebra problems that have been studied for a long time. Leibniz localized his method of definition in 1693, and Kramer in 1750 provided an approach that fixed the structure of linear equations. This is now called Cramer's Rule. This is the main basis for improving linear algebra and matrix theory. At the end of the development of virtual machines, matrix computation received a great deal of attention. John von Neumann and Alan Turing are known as pioneers of personal computer technology. They contributed greatly to improving linear PC algebra. In 1947, von Neumann and Goldstine studied the effect of rounding errors on the response of linear equations. Twelve months later, Turing [Tur48] initiated a method of factoring matrices into reduced triangular matrices using echelon matrices (the factorization is called his LU decomposition). There is a lot of interest in linear PC algebra right now. This is because today the sphere is recognize as a very important device in many areas of PC programming. This device requires long and unmanageable calculations while performed with the help of hands. For example:

PC graphics, geometric modeling, robotics, and more.

II. EASE OF USE

A. Linear algebra

The line through the origin of R3 (blue, thick) is a linear subspace and a common study of linear algebra. Linear algebra is a branch of mathematics concerned with the study of vectors, vector spaces (also called linear spaces), linear maps (also called linear transformations), and systems of linear equations. Vector spaces are a central topic in modern mathematics. Linear algebra is therefore widely used in both abstract algebra and functional analysis. Linear algebra also has concrete expressions in analytic geometry and is generalize in operator theory. Since linear models can often approximate nonlinear models, they have widespread applications in the natural and social sciences. Linear algebra is a branch of mathematics aimed at solving systems of linear equations with finite unknowns. In particular, I would like answers to the following questions:

• Solution Characterization:
Is there a solution for the given system of linear equations? How many solutions are there?

• Find solutions:
What is a solution set and what is the solution? Linear algebra is a systematic theory for solving systems of linear equations. Linear equations can be solve by applying different operations to either side of the equal sign. These operations help simplify equations, solve variables, and finally find solutions. Therefore, we can find the solution of the linear equation by following these steps:

Step 1: Simplify the formula. This includes removing brackets and other grouping characters, removing fractions, and combining synonyms.
Step 2: Isolate variables. Perform addition and subtraction to put all terms containing variables on only one side of the equation.
Step 3: Solve the equation. Multiply and divide to find the answer.

Example1: $x+2y=7$
Corresponding to the x-intercepts and y-intercepts of the graph, by setting first $x=0$ and then $y=0$
When $x=0$
we get:
$0+2y=7y=3.5$

When y=0
 we get:
x+2(0)=7x=7

So the two points are (0, 3.5) and (7, 0).

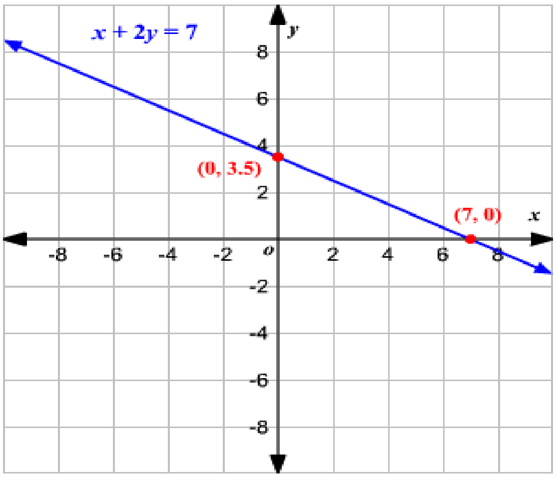

Fig.2.1 graph

B. Systems of Linear Equation

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and a single variable (to the power of 1). A linear equation can have one or more variables. Linear equations are abundant in most subfields of mathematics, especially applied mathematics. These arise quite naturally when modeling many phenomena, but by assuming that the quantity of interest deviates slightly from the 'background' state, many nonlinear equations can be converted to linear equations It is particularly useful because it can be reduced to Linear equations do not contain exponents. In this article, we consider the case of a single equation looking for the true solution. All of its content applies to complex solutions, and more generally to linear equations with coefficients and solutions in all domains.

III. SOME USEFUL THEOREMS

- Every vector space has a basis [1].
- Any two bases in the same vector space have the same cardinality. Therefore, the dimension of the vector space is well defined.
- A matrix is invertible if and only if its determinant is non-zero.
- A matrix is invertible only if the linear maps it represents are isomorphic.
- A square matrix is invertible if it has a left or right inverse.
- A matrix is positive semidefinite if and only if each of its eigenvalues is greater than or equal to zero.
- A matrix is positive definite if and only if each eigenvalue is greater than zero.
- An n × n matrix is diagonalizable iff it has n linearly independent eigenvectors (that is, we have an invertible matrix P and a diagonal matrix D with A = PDP-1).
- The spectral theorem states that we can only orthogonally diagonalizable a matrix if it is symmetric.

IV. MATRIX

A. Definition of matrix: An m × n matrix is an array of numbers with m rows and n column

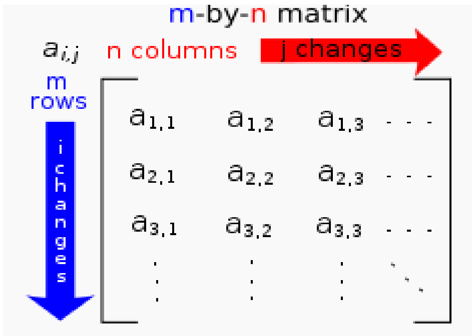

Fig.4.1 organization of matrix

In mathematics, a matrix (multiple matrices or less commonly a matrix) is a rectangular array of numbers, as shown to the right. Matrices that consist of only one column or row are vectors, while those with higher dimensions are vectors. A 3D array of numbers is a tensor. Matrices can be added, subtracted, and multiplied input by input, according to the rules corresponding to constructing linear transformations. These operations satisfy the usual identities, except that matrix multiplication is not commutative.

AB=BA ID can fail. One use of matrices is to represent linear transformations. Linear transformations are higher-dimensional analogues of linear functions of the form f(x) = cx. Where c is a constant. Matrices can also track the coefficients of a system of linear equations. For square matrices, the determinant and inverse (if any) determine the behavior of the solution of the corresponding system of linear equations, and the eigenvalues and eigenvectors provide insight into the geometry of the associated linear transformation. Matrices have many uses. Physics uses them in various fields such as matrices and matrix mechanics.

The latter also led to a detailed investigation of matrices with infinitely many rows and columns. Graph theory uses matrices to encode the distances of nodes in a graph. B. The city, connected by roads and computer graphics, uses the matrix to encode the projection of the three-dimensional space onto his two-dimensional screen. Matrix calculus generalizes classical analytical concepts such as derivatives of functions and exponentials of matrices. The latter is repeatedly needed when solving ordinary differential equations.

Serialism and dodecaphonism are 20th-century musical movements that use square matrices to determine patterns of intervals. Due to their widespread use, considerable effort has been expended to develop efficient methods of matrix computation, especially for large matrices. For this purpose, there are several matrix decomposition methods that represent matrices as products of other matrices with certain properties that simplify the computation both theoretically and practically. Sparse matrices, matrices composed mostly of zeros, such as those encountered in simulating mechanical experiments using the finite element method, often enable more specific algorithms to accomplish these tasks. Increase. The close relationship between matrices and linear transformations makes the former an important concept in linear algebra. Other types of entries are also used, such as elements and rings in the more general mathematical domain.

B. Matrix multiplication, Linear equations and Linear Transformations

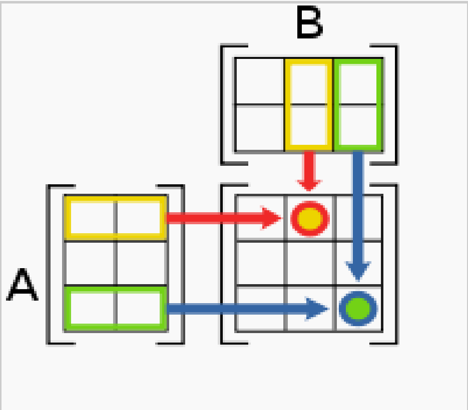

Fig.4.2 schematic depiction of the matrix

Product AB of two matrices A and B

Multiplication of two matrices is define only if the number of columns of the left matrix is the same as the number of rows of the right matrix. If **A** is an m-by-*n* matrix and **B** is an n-by-p matrix, then their matrix product **AB** is the m-by-p matrix whose entries are given by:

$$[AB]I,j = AI,1B1,j + AI,2\,B2,j + Ai,n\ Bn,j = \sum_{n=1}^{n} A\,i,r\ Br,j$$

Where $1 \le i \le m$ and $1 \le j \le p$.[5] For example (the underlined entry 1 in the product is calculated as the product 1 · 1 + 0 · 1 + 2 · 0 = 1):

Matrix multiplication follows the rules (AB)C = A(BC) (connectivity) and (A+B)C = AC+BC, and C(A+B) = CA+CB (left and right distributions). meet the rules. The size of the matrix is such that the different products are defined.[6] The product AB can be defined without BA being defined. That is, if A and B are m-by-n and n-by-k matrices, respectively, and m ≠ k. Even if both products are defined, they do not have to be equal. That is, in general AB ≠ BA.

In other words, matrix multiplication is not commutative. This contrasts sharply with numbers (rational, real, or complex) where the product does not depend on the order of the factors.

Example1:

$$\begin{bmatrix} 0 & -1 & 2 \\ 4 & 11 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (0 \times 3) + (-1 \times 1) + (2 \times 6) & (0 \times -1) + (-1 \times 2) + (2 \times 1) \\ (4 \times 3) + (11 \times 1) + (2 \times 6) & (4 \times -1) + (11 \times 2) + (2 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 1 + 12 & 0 - 2 + 2 \\ 12 + 11 + 12 & -4 + 22 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 \\ 35 & 20 \end{bmatrix}$$

C. linear equations

Linear equations are a special case of matrix multiplication and are closely related to linear equations.

(that is, an n × 1 matrix) and A is an m-by-n matrix, then the matrix equation is Ax = b. where b is part of m ×1. column vector, for systems of linear equations
A1,1x1 + A1,2x2 + ... + A1,nxn = b1
Am,1x1 + Am,2x2 + ... + Am,nxn = bm .[8]

Thus, matrices can be used to compactly describe and process multiple linear equations, or systems of linear equations.

D. Linear Transformation

Matrices and matrix multiplication reveal essential functionality in the context of linear transformations (also called linear maps). A real m-by-n matrix A leads to a linear transformation Rn → Rm that maps every vector x in Rn to the (matrix) product Ax (a vector in Rm). Conversely, any linear transformation f:

Rn → Rm from a unique m-by-n matrix A is

Explicitly the (i,j) entry of A is the ith coordinate of f(ej). where ej = (0,...,0,1,0,...,0) is the unit vector 1 at the jth position, 0 elsewhere. The matrix A represents the linear map f and A is called the transformation matrix of f.

CONCLUSION

Overall, linear algebra, in addition to its mathematical uses, has a wide range of uses and applications in most areas of engineering, medicine, and biology. As the fields of science and engineering grow, so does the use of mathematics, creating new mathematical problems and requiring new mathematical skills. In this regard, linear algebra is particularly responsive to computer science, as it plays a key role in many important computer science endeavors.

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