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Is there a Math Free Solution for Numerical Integration

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Abstract:- It is true that statistical differentiation and statistical integration is a missing part in mathematics. Moreover, it is important to understand that mathematics is only a tool to quantitatively describe physical phenomena, it cannot replace physical understanding.

There is an assertion that mathematics is the language of physics but the converse is also true, physics can be the language of mathematics as in numerical differentiation and integration.

In this article, physical B-matrix chains are used to numerically solve single and double definite integrals as well as numerical differentiation.

We provide detailed calculations for n= 3,4, ….7 number of nodes where the efficiency and accuracy of the new technique are validated.

I. INTRODUCTION

Many people think that in mathematics there is already everything, but in fact, it is true that something important like statistical differentiation and statistical integration is a missing part of mathematics.

In this article, we apply the statistical physical matrix B to obtain a numerical integration solution without mathematical method of finite differences FDM.

Many scientists consider theoretical physics to be a subset of mathematics (Fig.1), but we should care more about what happens when they meet.

In other words, when we expect universal laws of physics and mathematical axioms to apply simultaneously.

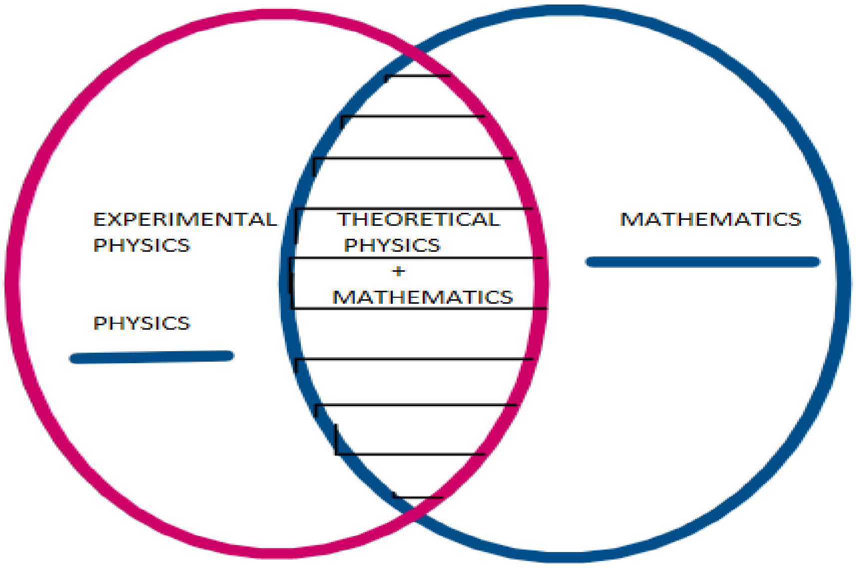

Fig. 1: Theoretical physics considered as a subset of mathematics

- What happens when they meet?

Obviously, both mathematical numerical integration and differentiation and physical statistical integration and differentiation are central to their intersection (Fig. 1).

In fact, mathematics itself already presents many modest approaches based on the classical FDM, to numerical integration such as composite trapezoidal rule, composite Simpson's rule, Newton's rule, etc. [1].

The problem is that all these methods are quadrature and based on the FDM finite difference methods invented by Newton in the 18th century. On the other hand, physics can present a more effective and much broader technique.

The question arises whether the methods of the mathematical numerical technique can be replaced by a more general and efficient statistical numerical technique.

In other words, can the physical statistical technique, especially in the field of numerical differentiation and integration, replace mathematics? This is the subject of this article.

Recall that the theory of numerical statistics technique known as Cairo technique is based on the statistical chains of the transition matrix B which is well defined and well explained [2,3,4].

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The transition probability is best defined in space and time as an element of the system transition probability matrix via sufficient and necessary physical conditions or universal physical laws, which is the case with matrix B.

The transition probability (probability of transition per unit time between two elements of n free nodes of a system, bi,j is physically defined through 4 physical conditions from universal physical laws as follows,[2,3,4].

In other words, the correct definition of transient probability requires that the time dimension be inherent in the dimensionless time evolution of the system (number of steps or jumps dt) which is satisfied by the chains of the matrix B and never satisfied elsewhere .

In classical physics and in quantum mechanics itself, time is understood as an external ("classical") concept. It is therefore assumed, as in classical physics, to exist as the controller of all motion - either as absolute time or as proper times defined by a classical spacetime metric.

This may be the reason for their incompleteness of both.

Recall that in several previous papers, the chains of the B matrix have been successfully applied to find an arithmetic solution of the partial differential equations of Laplace, Poisson and heat diffusion[2,3,4].

The statistical transition matrix B = (bi,j) itself is indeed defined by 4 statistical assumptions.

For 2D/3D Cartesian coordinates, the inputs b i, j respect or are subject to the following conditions:

i- B i, j = 1/4 for i adjacent to j .. and B i, j = 0 otherwise.

equal prior probability.

There is no preferable direction.

ii- B i, i = RO, i.e. the main diagonal is made up of constant

inputs RO

For the heat diffusion equation, RO can take any value

in the closed interval [0,1] while for Laplace and Poisson

PDE, RO = 0

That is to say that B is a null principal diagonal matrix

which corresponds to the assumption of a null residue after each time step for all the free nodes.

RO is greater than zero for hardware media. In other words, RO mainly defines and describes the thermodynamic properties of the medium expressed in thermal diffusivity D m^2/sec in the SI system.

iii-The matrix B is symmetrical to conform to the physical principle of detailed balance bi,j=bj,i.

iv- The sum of b i, j = 1 for all the rows far from the borders and the sum B i, j <1 for all the rows connected to the borders meaning that the probability of the whole space

equals 1.

Moreover, condition iv is compatible with the principle of conservation of energy.

Obviously, the statistical matrix B is very different from the Laplacian mathematical matrix A and from the Markov statistical matrix M.

the above four conditions in addition to not violating any universal physical law are sufficient to uniquely define the statistical transition matrix Bnxn which successfully replaces any mathematical description using classical FDM techniques.

It follows from the physical definition of probability above that the transition probability is most likely a rational number (condition I, iv) whereas in the mathematical definition of probability it can take any real-valued element of the interval [0,1].

Moreover, the mathematical definition of the transition probability limits its use to an external real-time domain while that of the physical transition matrix B defines it as internal in the 4D space-time domain.

If we agree that theoretical physics is a subset of mathematics, i.e. there is a subset where the two meet, the laws of physics and mathematics apply simultaneously and the four concepts of physical probability must not be violated.

By following the statements above, you can navigate the definition of physical probability in many areas such as Laplace's and Poisson's equations for electric potential, heat diffusion and, surprisingly enough, the digital integration and digital differentiation.

All of the above operations can be performed accurately and efficiently with a simple, stable, and fast algorithm. On the other hand, the current definition of mathematical probability is practically limited only for certain areas of quantum mechanics. Assuming that one day the two definitions can be positively correlated, the mystery is solved.

- ***Conclusion:***
  - It is important to understand that mathematics is only a tool to quantitatively describe physical phenomena, it cannot replace physical understanding.
  - there is an assertion that mathematics is the language of physics but the reverse is also true, physics is the language of mathematics as in numerical differentiation and integration.

In this article, matrix chains B are used to numerically solve single and double integrals as well as numerical partial differentiation.

II. THEORY

In order to solve the single definite integration

I= ∫ y dx from x=a to x=b

where y is a variable dependent on x, whether expressed in terms of the functional relation f(x) or not, then the starting point is the statistical transition matrix B and its chains.

These chains lead to stationary statistical transfer matrices Enxn and Dnxn[2,3,4]. It has been shown that for two adjacent free nodes I,j,

bi,j=1/6 for a 3D geometric configuration,

bi,j=1/4 for a 2D geometric configuration,

bi,j=1/2 for a 1D geometric configuration,

therefore, we should expect the single integration matrix B to be expressed as a matrix B with bi,j = 1/2.

The math free statistical integration itself is not complicated but you must follow the following steps precisely.

We denote the integration matrix B by BI for distinction.

Therefore, BInxn for the diagonal input elements RO=0, is expressed by:

LINE 1 0 ½ 0 0 ………………………………………....0

ROW 2 ½ 0 ½ ……………………………………………0

LINE n 0 ,0 ,0 ……………… ……………...0 ½ 0. . . . . . .(1)

The steady-state transfer matrix E follows from the relation[2,3,4],

E=B^0+B +B^2 +…………………………….B^N . . . (2)

for a sufficiently large number N.

Or equivalent,

E=(I-B)^-1 . . . . (3)

It follows that the transfer matrix D is given by,

D=E-I . . . . (4)

And finally, the BI integration matrix should be given by,

BI=(n-1).D/ sum. ∑∑bi,j over all i,j . . . . . . (5)

Let us denote the term n-1/∑∑bi,j over all I,j by the statistical integration factor STI.

STI=n-1/∑∑bi,j .. . . . (6)

The finite unique integration or area under the curve follows as the

Sum of elements of vector V,

V=STI.DI.(1 , 1 . .1)T . . . . . (7)

Where V is V(x) the vertical data to integrate.

In other words, the statistical integration formula will be given by,

V1+V2+. . . +Vn. . (8)

This statistical technique can be directly extended to the case of the limited double integral of Z= ∬fn(x,y)dy dx (volume under the area) by replacing only the value of bi,j by ¼ instead of ½ for the concerned geometric bag.

Similarly, this statistical technique can be directly extended to the case of a bounded triple integral, U= ∭ f(x, y, z) dx dy dz , by simply replacing the value of bi,j by 1/6 at the instead of ½ for the required geometric volume considered.

In order not to concern ourselves too much with the details of the theory, let us move on to some illustrative applications and their numerical results.

III. APPLICATIONS AND NUMERICAL RESULTS

A. Applications to numerical statistical integration

T. Simpson was a mathematician best known for his work on interpolation and numerical mathematical methods of integration.

Below we compare our statistical results with those of Simpson's mathematical rule, the trapezoidal mathematical rule and the analytical solution for n= 3,4,5,6 and 7 free nodes [1,5].

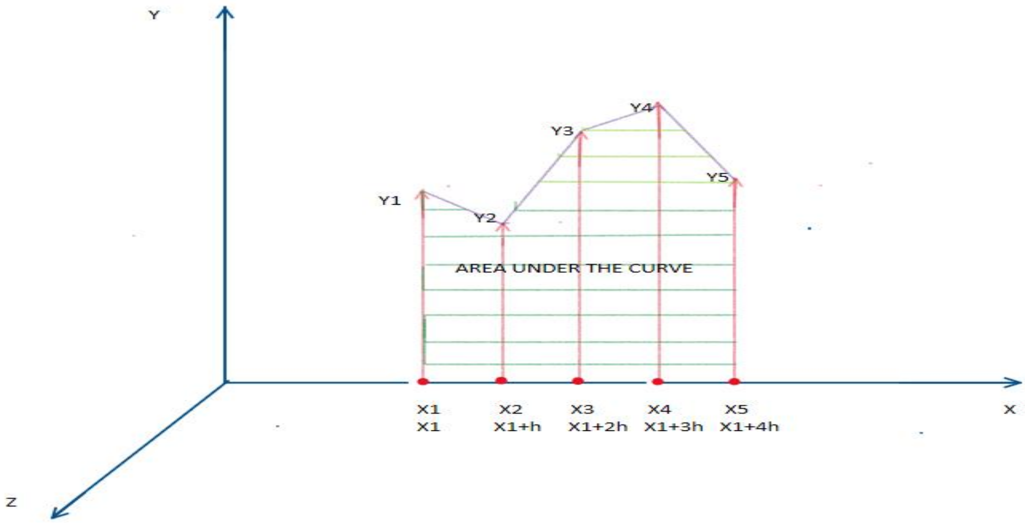

Fig. 2: Simple definite numerical integration for 5 equidistant free nodes with equal steps h. The integration ∫ y dx is equal to the area under the curve.

- **Case (a)**
  3 free nodes.
  Here we get the DI3x3 integration steady state transfer matrix via Eq(4) expressed in rational fractional probability as,

  DI3x3=

½	**1**	**1/2**
1	**1**	**1**
½	**1**	**½**

∑∑ bi,j for all the 9 Statistical elements equals 7 and hence the statistical integration factor STI =2 /7 ….[Eq.6]

It follows that,

2/7. DI3x3 . $(1,1,1)^T = (4/7,6/7,4/7)^T$

which is the statistical equivalence of Simpson rule.

Therefore, the numerical statistical integration I is given by.

I=4/7 Y1 + 6/7 Y2 + 4/7 Y3

On the other hand, Simpson numerical integration rule

I=h/3 (Y1 + 4Y2 + Y3-Eps)

As compared to numerical statistical integration rule,

I=h/7 (4Y1 + 6Y2 +4Y3)

The two rules are similar in appearance but they do not have the same nature.

Eps is the truncation error assumed sufficiently small.

Consider the numerical case I=∫ y dx from x=1 to x=3 where y=X^2.

That's to say,

x=1 2 3

y=1 4 9

Simpson rule,

I=h/3(1+16+9) =**8.667 square units.**

Trapezoidal rule,

I=(Y1/2+Y2+Y3/2)=**9.0 square units.**

Analytic integration expression,

I=X^3/3=(27-1)/3=**8.667 square units.**

Statistical integration method,

I=h/7(4Y1+6Y2+4Y3)=h (54) /7 =**7.71 square units.**

- **Case (b)**
  4 free nodes

DI 4x4 Integration matrix via Eq (4) is given by,

0.6	1.2	.8	0.4
1.2	1.4	1.6	0.8
0.8	1.6	1.4	1.2
0.4	0.8	1.2	0.6

∑∑ bi,j for all the 16 Statistical elements equals and hence the integration factor STI =3 /16 ….[Eq.6]

and,

3/16. DI 4x4. (1,1 ,1,1)T ,gives,

I=0.563 Y1 + 0.938 Y2 + 0.938 Y3 +0.563Y4) which is the statistical equivalence of Simpson's ruleConsider the case I=∫ y dx from x=4 to x=7 where y=X^2. That is,

X= 4 5 6 7

y=16 25 36 49

We get,

Simpson rule,

I=h/3(Y1+4Y2+Y3+Y4)=201/3=**67 square units.**

Trapezoidal rule,

I=h*(Y1/2+Y2+Y3+Y4/2)=126/2=**63 square units.**

Analytic integration expression,

I=X^3/3=(343-64)/3=**93 square units.**

Statistical integration method,

I=h.(0.563 Y1 + 0.938 Y2 + 0.938 Y3 +0.563Y4)=**76 square units.**

- **Case (c)**

5 free nodes.

DI 5x5 Integration matrix via Eq(4) is given by,

2/3	4/3	1	2/3	1/3
4/3	5/3	2	4/3	2/3
1	2	2	2	1
2/3	4/3	2	5/3	4/3
1/3	2/3	1	4/3	2/3

∑∑ bi,j for all the 16 Statistical elements equals Eq.6,

=4+7+8+7+4=30

Then SIF factor= (5-1)/30 =4/30

The statistical integration formula for five nodes is given by,

I=4h/30 (4 Y1+7 Y2+8 Y3 + 7 Y4+ 4 Y5)

I= h* (16/30 Y1+28/30 Y2+32/30 Y3 + 28/30 Y4+ 16/30 Y5)

which is the statistical equivalence of Simpson rule for 5 nodes.

Consider the case I=∫ y dx from x=2 to x=6 where y=X^2. That is,

X = 2 3 4 5 6

Y= 4 9 16 25 36

Composite or extended Simpson's rule obtained by recruisive application of the 3-node formula [5,6],

I=h/3 (Y1+4Y2+2Y3 +4Y4+Y5+Eps)

I=h/3 (4+36+32 +100+36) =**69.333 square units.**

Trapezoidal rule,

I =h*(Y1/2+Y2+Y3+Y4 +Y5/2)

2+9+16+25+36/2=**70 square units.**

Analytic integration expression,

I=X^3/3=(216-8)/3=**69.33 square units.**

Statistical integration formula,

4 h/30(4 Y1+7 Y2+8 Y3 + 7 Y4+ 4 Y5)

4h/30 (16+63+128+175+144) =**70.133 square units.**

Note that as n increases, the precision increases.

- **Case (d)**

6 free nodes

∑∑ bi,j for all the 16 Statistical elements equals Eq.6,

=5+9+11+11+9+5=50

Then SIF factor=(6-1)/50 =1/10

The statistical integration formula for five nodes is given by,

I=h/10 (5 Y1+9 Y2+11 Y3 + 11 Y4+ 9 Y5+5Y6)

which is the statistical equivalence of Simpson rule for 6 nodes.

Consider the case I=∫ y dx from x=2 to x=7 where y=X^2. That is,

X = 2 3 4 5 6 7

Y= 4 9 16 25 36 49

Trapezoidal rule,

I =h*(Y1/2+Y2+Y3+Y4 +Y5+Y6/2)=**112.5 square units**

Analytic integration expression,

I=X^3/3=(343-16)/3=**109 square units.**

Statistical integration formula,

h/10 (5 Y1+9 Y2+11 Y3 + 11 Y4+9Y5 + 5Y6)

(20+81+176+275+324+245) =**112.1 square units.**

Again, note that as n increases, the precision increases.

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- **Case (e)**

7 free nodes

In brief, we arrive at,

The statistical integration formula for 7 nodes is given by,

I= Y=6 h/77(6.Y1 +11. Y2 + 14. Y3+15. Y4 +14. Y5 + 11.Y6 + 6.Y7)

which is the statistical equivalence of Simpson rule for 7 nodes.

Consider the case I=∫ y dx from x=2 to x=8 where y=X^2. That is,

X =	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Y=	**4**	**9**	**16**	**25**	**36**	**49**	**64**

Trapezoidal rule,

I =h*(Y1/2+Y2+Y3+Y4 +Y5+Y6+Y7/2)

=h*(2+9+16+25+36+49+32=**169 square units.**

Analytic integration expression,

I=X^3/3=(512-8)/3=**168 square units.**

Finally, the statistical integration formula for 7 nodes is given by**,**

I=6 h/77(6*4 +11* 9 + 14* 16+15* 25 +14* 36 + 11*49 + 6*64)

I=167.455 square units.

Meaning is that the statical integration is quite accurate.

Now we go to more complex relation y=x^3 over x=1 to x=7 which results in,

X=	1	2	3	4	5	6	7
Y=	1	8	27	64	125	216	343

Trapezoidal rule,

I =h*(Y1/2+Y2+Y3+Y4 +Y5+Y6+Y7/2)=**612 square uits**

Analytic integration expression,

I=X^4/4=(2401-1)/4=**600 square units.**

Simpson rule: ------ ------ ------ -----

Finally, the statistical integration formula for 7 nodes is given by,

I=6 h/77(6*1 +11* 8 + 14* 27+15* 64 +14* 125 + 11*216 + 6*343)=**598.13 square units**

- **Conclusion**
  - The statistical integration technique is fairly accurate and works for any number of nodes n odd or even.
  - The only limitation is the memory capacity of the computer memory and processor to meet the excessive number of double precision calculations.
  - When the y-x functional relation is more complicated than quadrature or there is no known function at all then classical numerical integration methods does not work and the only alternative is the statistical integration technique.

B. Applications to numerical statistical differentiation

In mathematical numerical analysis[1,6] centered finite divided difference

dy/dx=[Y(x+h)-Y(x-h)]/2h

Can be replaced by a DI statistical numerical analysis matrix and its resulting weights for n=3 or n=5.

Dy/dx=[Y(x+h)-Y(x-h)].4/7/2h…….formula for 3 nodes

Or,

Dy/dx=[Y(x+h)-Y(X+h)].28/30/2h……. formula 5 nodes

Moreover, when we describe a physical phenomenon such as the heat diffusion equation, a differential equation (ordinary or partial) arises. Differential equations are extremely important because many kinds of things in the world around us can be described by differential equations.

since, the exact analytical solution of PDE is not always possible.

The numerical mathematical solution of the simplest solution of the partial differential equation is the separation of the variables and then the use of the finite difference method FDM to transform the PDE into difference equations which is a set of algebraic equations easy-to-solve linear.

Again, all these complicated processes can be replaced by statistical numerical integration without mathematics [2,3,4,8,9].

This was the case in several previous articles where we renounced the method of separation of variables by finite differences, the FDM method is a method of linear algebraic equations simple to solve. The method used to transform the PDE into difference equations resulting in a definite statistical integration matrix effectively replaces both.

IV. CONCLUSION

It is true that statistical differentiation and statistical integration is a missing part in mathematics. there is an assertion that mathematics is the language of physics but the reverse is also true, physics is the language of mathematics as in numerical differentiation and integration.

We present the procedure and numerical results 5 different applications for statistical integration where the number of nodes vary from 3,4,…7.

We compare the numerical results of the new statistical technique with the classical Simpson and trapezoidal methods where it proved.

- Advantageous.

The statistical integration technique is fairly accurate and works for any number of nodes odd or even.

The only limitation is the memory capacity of the computer memory and processor to meet the excessive number of double precision calculations.

Finally, the ultimate idea of this article is that we are really able to produce statistical integration tables for any number of nodes and upload them to mathematical libraries to be ready to use in a procedure of mathfree integration.

NB. All calculations in this article were produced through the author's double precision algorithm to ensure maximum accuracy, as followed by Ref. 10 for example.

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